

DERIVATION OF RELATIONS AND ANALYSIS OF TUBE BENDING PROCESSES USING DISCONTINUOUS FIELDS OF PLASTIC STRAINS

Part I. Derivation of geometric-analytic relationships

Z. ŚLODERBACH

Opole University of Technology
Faculty of Applications of Chemistry and Mechanics
45-036 Opole, Luboszycka 7, POLAND
E-mail: z.sloderbach@po.opole.pl

The generalized strain scheme in bending metal tubes at bending machines with the use of a mandrel presented in Śloderbach (1999; 2002; 2013_{1,2}; 2014) satisfies initial and boundary kinematic conditions of bending, conditions of continuity and inseparability of strains. This paper introduces three formal simplifications gradually imposed into forms of principal components of the generalized strain model giving suitable simplifications of the 1st, 2nd and 3rd types. Such mathematical simplifications cause that the obtained strain fields do not satisfy the condition of consistency of displacements and strain continuity. The simplified methods determine safer values of the wall thickness than those from the generalized continuous strain scheme. The condition of plastic incompressibility was used for the derivation of an expression for distribution of wall thickness of the bent elbow in the layers subjected to tension and compression for three examples of discontinuous kinematic strain fields.

Key words: bending tubes, three simplifications, strains, wall thickness, discontinuous strain fields.

1. Introduction

This paper continues the author's previous work (Śloderbach, 1999; 2000; 2002; 2013_{1,2}; 2014). As it was mentioned in the previous papers, tube bending at bending machines with a rotational template and using a mandrel always causes reduction of the wall thickness in the elongated layers, as well as wall thickening in the compressed layers, it also causes ovalization and corrugations which deform the tube cross-section. Those unfavourable deformations dependent on the bending technology applied should be included into the tolerance limits determined by the valid standards or requirements defined by technical inspection. Thus, at the stage of elaboration of a given technology of tube bending it is necessary to predict the future deformations. It will make it possible to chose a suitable initial material and obtain a product (an elbow of a pipeline) of the required quality and operation life.

Tube bending simulation was considered in the papers (Śloderbach, 1999; 2000; 2002; 2013; 2014), presenting a model of geometry of deformations used for the calculation of component deformations formed during bending thin- and thick-walled tubes for the bending angle $\alpha_b \in \langle 0^\circ; 180^\circ \rangle$. The basic equation applied for the calculation of the actual thickness of the bent tube in the bending zone becomes an algebraic equation of the 3rd degree when $\lambda_i = 1$. Solving such an equation seems to be difficult (Śloderbach, 1999; 2002; 2014). Thus, it seems to be necessary to obtain approximate expressions for calculations of suitable deformations in the bent tube which could be applied under industrial conditions, repair works or modernizations, using an electronic calculator.

After a suitable analysis of the calculation results obtained from the relationships derived in Śloderbach (1999; 2002; 2013₁; 2014), this paper proposes three formal simplifications imposed on measures of suitable logarithmic constituent strains. In general, these simplifications are not only a formal mathematical operation making calculation easier, but they have a physical sense as well. If a suitable mandrel is applied (especially a segment one with an adjustable diameter) (Franz, 1961; Korzowski, 1971),

we can neglect oblateness of the elbow cross section because longitudinal strains $\varphi_2 \approx 0$, and according to the European standard (EN 13445-4, 2009) a permissible ovalization of the cross section is up to 6 %. If a bending radius R is relatively big, and the bending angle α_b is small, we can assume that reduction of the actual external radius of the bent tube r_i is relatively small as compared with the external radius r_{ext} (Śloderbach, 1999; 2002; 2013; 2014). However, those simplified analytical schemes of strain fields do not satisfy the conditions of continuity of displacements and deformations, see (Olszak *et al.*, 1985; Szczepiński, 1969; Życzkowski, 1981). In particular cases, when ($\lambda_i = 1$), for the assumed simplifications of the 1st and 2nd order we obtained suitable algebraic equations of the 2nd degree for a searched actual thickness of the tube wall in the bending zone instead of previously mentioned equation of the 3rd degree. For the assumed simplification of the 3rd order we obtained an algebraic equation of the 1st degree. Simplification of the 1st order is very precise, and differences in the calculated wall thickness in the elongated layers are about $\sim 1\%$, and for simplifications of the 2nd and 3rd orders the differences reach even some % as compared with the results obtained from the equation of the 3rd degree derived and presented in Śloderbach (1999; 2002; 2013; 2014) for a general strain scheme. The calculation results obtained in this paper for the elongated layers ($\lambda_1 = 1$) and compressed layers ($\lambda_2 = 1$ and $\lambda_2 = 0.5$) are shown in graphs and table. From the point of view of resistance, exploitation durability, work safety, an analysis of the vertex points of the elongated layers seems to be the most reasonable because in these points we can usually observe the first cracks and other degradation processes caused by the greatest reduction of the wall thickness and negative influence of big strains on the material structure.

2. Geometric and analytic description of

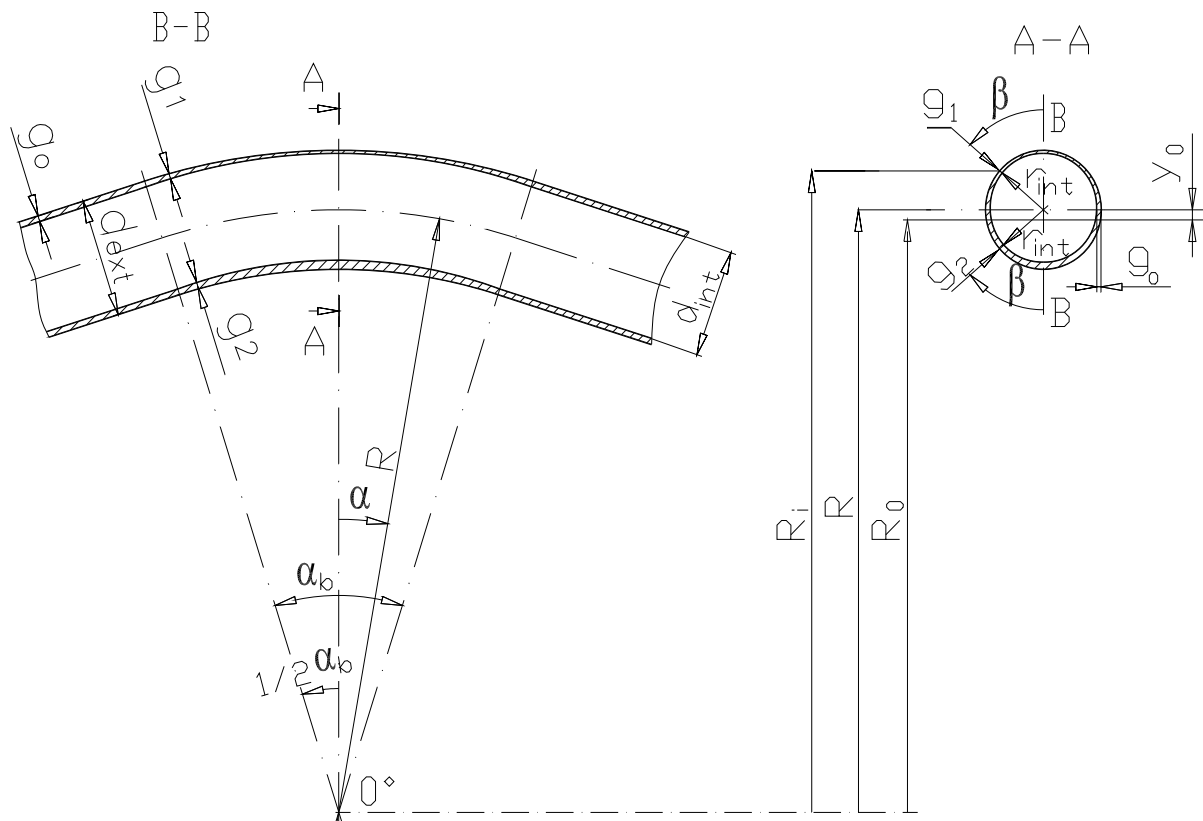


Fig.1. Geometric and dimensional quantities occurring during tube bending.

2.1. Approximate methods of strain calculations

a) Simplification of the 1st type

The simplification of the 1st type concerns the „big actual radius” $R_i = 2(R - y_0) \pm (d_i \cos \beta_i \pm 2y_0) \left(\cos(k\alpha) - \cos\left(k \frac{\alpha_b}{2}\right) \right)$ in the bending zone, see Śloderbach (1999; 2002; 2013₁; 2014), where the diameter d_i is formally replaced by the diameter d_{ext} , or the radius ($r_i = r_{int} + g_i$) is formally replaced by the radius ($r_{ext} = r_{int} + g_0$). The suitable quantities for component strains and the expressions obtained under simplification of the 1st type were noted by the symbol ($'$). Thus

$$R'_i = (R - y_0) \pm [(r_{int} + g_0) \cos \beta_i \pm y_0] \left[\cos(k\alpha) - \cos\left(k \frac{\alpha_b}{2}\right) \right], \quad (2.1)$$

and

$$\varphi'_1 = \lambda_i \ln \frac{2(R - y_0) \pm (d_{ext} \cos \beta_i \pm 2y_0) \left(\cos(k\alpha) - \cos\left(k \frac{\alpha_b}{2}\right) \right)}{2(R - y_0)}, \quad (2.2)$$

$$\varphi'_2 = \ln \frac{d'_i}{d_{ext}}, \quad \varphi'_3 = \ln \frac{g'_i}{g_0}$$

where $d'_i = d_{int} + 2g'_i = d_{ext} - 2(g_0 - g'_i)$.

The strain intensity and the condition of plastic incompressibility are

$$\varphi'_{(i)} = \left[\frac{2}{3} (\varphi'^2_1 + \varphi'^2_2 + \varphi'^2_3) \right]^{0.5}, \quad \varphi'_1 + \varphi'_2 + \varphi'_3 = 0. \quad (2.3)$$

Substituting expressions (2.2) to the condition of incompressibility (2.3)₂ and after transformations we obtain the following equation for the actual wall thickness g'_i in the bending zone

$$\left[2(R - y_0) \pm (d_{ext} \cos \beta_i \pm 2y_0) \left(\cos(k\alpha) - \cos\left(k \frac{\alpha_b}{2}\right) \right) \right]^{\lambda_i} d'_i g'_i + (2(R - y_0))^{\lambda_i} d_{ext} g_0 = 0. \quad (2.4)$$

Considering only the vertex points ($\alpha = \beta_i = \theta^0$) in Eq.(2.4), i.e., the points of maximum strains in the elongated layers for ($\lambda_i = 1$), after a solution related to g'_i , we obtain the following expression satisfying the actual bending conditions

$$g'_i = -\frac{(d_{\text{ext}} - 2g_0)}{4} + \sqrt{\left(\frac{d_{\text{ext}} - 2g_0}{4}\right)^2 + \frac{(R - y_0)d_{\text{ext}}g_0}{2(R - y_0) + (d_{\text{ext}} + 2y_0)\left(1 - \cos\left(k\frac{\alpha_b}{2}\right)\right)}} \quad (2.5)$$

where g'_i - local actual thickness of the bent elbow in the bending zone of the elongated layers calculated in logarithmic measures of strain.

The other root being a solution of Eq.(2.4) takes negative values, so it does not satisfy real conditions of bending. It is quite easy to make calculations according to expression (2.5). The expression (2.5) is simplified when the bending angle reaches ($k\alpha_b = 180^\circ$), i.e., the extreme values are reached at that point (formation of the maximum strains and initiation of the „plateau” zone). Then

$$g'_{\text{min}} = -\frac{(d_{\text{ext}} - 2g_0)}{4} + \sqrt{\left(\frac{d_{\text{ext}} - 2g_0}{4}\right)^2 + \frac{(R - y_{0\text{max}})d_{\text{ext}}g_0}{2(R - y_{0\text{max}}) + (d_{\text{ext}} + 2y_{0\text{max}})}} \quad (2.6)$$

Equations (2.4), (2.5) or (2.6) are applied for calculations of suitable wall thickness, and the calculated value is substituted to the relationship (2.2), and suitable deformation components φ'_1 , φ'_2 and φ'_3 are calculated, Eq.(2.3)₁ is applied for the calculation of an equivalent strain $\varphi'_{(i)}$.

b) Simplification of the 2nd type

In this case, let us denote suitable quantities as ($''$). In simplification of the 2nd type it is assumed that a value of the circumferential strain is negligibly low in relation to two other strain components. From the available experimental data (Franz, 1961; Korzowski, 1971; Śloderbach, 2002) it results that those strains can be almost five times less than the other strain components. Physically, it means the application of a certain „quasi-plane” strain state. Quantities of the strain components and the expressions obtained under simplification of the 2nd type are as follow

$$R''_i = (R - y_0) \pm [(r_{\text{int}} + g''_i)\cos\beta_i \pm y_0] \left[\cos(k\alpha) - \cos\left(k\frac{\alpha_b}{2}\right) \right], \quad (2.7)$$

and

$$\varphi''_i = \lambda_i \ln \frac{2(R - y_0) \pm (d''_i \cos\beta_i \pm 2y_0) \left(\cos(k\alpha) - \cos\left(k\frac{\alpha_b}{2}\right) \right)}{2(R - y_0)}, \quad \varphi''_2 \approx 0, \quad \varphi''_3 = \ln \frac{g''_i}{g_0}, \quad (2.8)$$

where $d''_i = d_{\text{int}} + 2g''_i = d_{\text{ext}} - 2(g_0 - g''_i)$.

Strain intensity and the condition of plastic incompressibility are

$$\varphi''_{(i)} = \left[\frac{2}{3} (\varphi''_1^2 + \varphi''_3^2) \right]^{0.5}, \quad \varphi''_1 + \varphi''_3 = 0. \quad (2.9)$$

Substituting the expressions (2.8) to the condition of incompressibility (2.9)₂ and after transformations we obtain the following equation for the actual wall thickness g_i'' in the bending zone

$$\left[2(R - y_0) \pm (d_i'' \cos \beta_i \pm 2y_0) \left(\cos(k\alpha) - \cos\left(k \frac{\alpha_b}{2}\right) \right) \right]^{\lambda_i} g_i'' - (2(R - y_0))^{\lambda_i} g_0 = 0. \quad (2.10)$$

Let us consider only the vertex points ($\alpha = \beta_i = \theta^0$) in Eq.(2.10), i.e., the points of maximum strains in the elongated layers ($\lambda_i = 1$). Solving the problem related to g_i'' , we obtain the following expression satisfying the real bending conditions

$$g_i'' = \frac{-a'' + \sqrt{(a'')^2 + 16(R - y_0)g_0 \left(1 - \cos\left(k \frac{\alpha_b}{2}\right) \right)}}{4 \left(1 - \cos\left(k \frac{\alpha_b}{2}\right) \right)} \quad (2.11)$$

where
$$a'' = 2(R - y_0) + (d_{\text{ext}} - 2g_0) \left(1 - \cos\left(k \frac{\alpha_b}{2}\right) \right).$$

The other root being a solution of Eq.(2.10) takes negative values, so it does not satisfy real bending conditions. Also the expression (2.11) can be easily used for calculations with the use of an electronic calculator. The expression (2.11) is subjected to further simplification in the moment when the bending angle reaches the value ($k\alpha_b = 180^\circ$), i.e., when the extreme values are reached in that point (formation of the maximum strains and initiation of the „plateau” zone). Then

$$g_{i\text{min}}'' = \frac{-\left(2(R - y_{0\text{max}}) + d_{\text{ext}} - 2g_0\right)}{4} + \frac{\sqrt{\left(2(R - y_{0\text{max}}) + d_{\text{ext}} - 2g_0\right)^2 + 16(R - y_{0\text{max}})g_0}}{4}. \quad (2.12)$$

As previously, Eqs (2.10), (2.11) or (2.12) are applied for calculations of proper wall thickness, and next the calculated value is substituted to the relationship (2.8). Then, we calculate suitable components of the deformation φ''_1 and φ''_3 and Eq.(2.9)₁ is applied for the determination of the equivalent strain $\varphi''_{(i)}$.

c) Simplification of the 3rd type

In the case of this simplification, suitable quantities are denoted as (' '). This simplification is assumed to be a composition of simplifications of the 1st and 2nd types. Thus, we can state that the „big actual radius” R_i in the bending zone, see Eqs (2.1) or (2.7), is such that the radius r_{int} is formally replaced by the radius r_{ext} , and a value of the circumferential strain is negligibly low in relation to two other strain components. Physically, it also means the application of a certain „quasi plane” state of strain. Thus, the strain components and expressions obtained in the case of simplification of the 3rd type are

$$R_i''' = (R - y_0) \pm \left[(r_{\text{int}} + g_0) \cos \beta_i \pm y_0 \right] \left[\cos(k\alpha) - \cos\left(k \frac{\alpha_b}{2}\right) \right], \quad (2.13)$$

and

$$\varphi_i''' = \lambda_i \ln \frac{2(R - y_0) \pm (d_{\text{ext}} \cos \beta_i \pm 2y_0) \left(\cos(k\alpha) - \cos\left(k \frac{\alpha_b}{2}\right) \right)}{2(R - y_0)}, \quad (2.14)$$

$$\varphi_2''' \approx 0, \quad \varphi_3''' = \ln \frac{g_{ib}'''}{g_0},$$

where $d_i''' = d_{\text{int}} + 2g_i''' = d_{\text{ext}} - 2(g_0 - g_i''')$.

Strain intensity and the condition of plastic incompressibility are the following

$$\varphi_{(i)}''' = \left[\frac{2}{3} (\varphi_1'''^2 + \varphi_3'''^2) \right]^{0.5}, \quad \varphi_1''' + \varphi_3''' = 0. \quad (2.15)$$

Substituting (2.14) to the condition of incompressibility (2.15)₂ and after transformations we obtain the following equation for the actual wall thickness g_i''' in the bending zone

$$\left[2(R - y_0) \pm (d_{\text{ext}} \cos \beta_i \pm 2y_0) \left(\cos(k\alpha) - \cos\left(k \frac{\alpha_b}{2}\right) \right) \right]^{\lambda_i} g_i''' - (2(R - y_0))^{\lambda_i} g_0 = 0 \quad (2.16)$$

Let us consider only the vertex points of the bent elbow ($\alpha = \beta_i = 0^\circ$) in Eq.(2.16) (points of the maximum strains in the elongated layers ($\lambda_i = 1$)). Solving the problem related to g_i''' , we obtain the following expression satisfying real bending conditions

$$g_i''' = \frac{2(R - y_0) g_0}{2(R - y_0) + (d_{\text{ext}} + 2y_0) \left(1 - \cos\left(k \frac{\alpha_b}{2}\right) \right)}. \quad (2.17)$$

It is easy to make the calculations according to the expression (2.17) with the use of an electronic calculator. This expression is subjected to further simplification when the bending angle reaches a value ($k\alpha_b = 180^\circ$), i.e., when in that point the extreme values (formation of the maximum strains and initiation of the „plateau” zone) are reached. Then

$$g_{i\text{min}}''' = \frac{2(R - y_{0\text{max}}) g_0}{2R + d_{\text{ext}}}, \quad \text{when } (y_{0\text{max}} = 0) \quad \text{to} \quad g_{i\text{min}}''' = \frac{2R g_0}{2R + d_{\text{ext}}}. \quad (2.18)_{1,2}$$

The expression (2.18)₂ was previously cited in publications of the Polish Office of Technical Inspection (UDT) and applied in industrial practice (UDT CONDITIONS, 2003).

The following remark seems to be important here. The expressions (2.14) are similar to a certain plane strain state. It is not a „pure” plane strain state because in the expression for strain φ_i''' there should be ($r_i''' = r_{\text{int}} + g_i'''$), and there is ($r_i''' = r_{\text{ext}}$). Thus, it is a certain „quasi-plane” strain state.

Substituting to Eqs (2.4)₁ and (2.14)₁, the vertex in the bending zone, when ($\alpha = 0^0$, $\cos(k\alpha) = 1$) and for ($\lambda_l = 1$ and $y_0 = 0$), and also including the condition of reaching the maximum strains in the bending zone $\cos\left(k\frac{\alpha_b}{2}\right) = 0$, then these equations become similar to the expression for calculations of strain values during the tube bending process $\left(\varepsilon' = \varepsilon'' = \frac{d_{\text{ext}}}{2R_m}\right)$, according to the Polish and European Standards (EN 13445-4, 2009; Zdankiewicz, 1998). Here R_m – a mean bending radius, $R_m \in \langle y_{0\text{max}}; R \rangle$ (Śloderbach, 2014). Eqs (2.4), (2.10) and (2.16) render it possible to obtain results for the reverse problem, i.e. we can determine, as in (Śloderbach, 2000; 2002; 2013₁), allowable (critical) values of the bending angle α_{ball} , corresponding to the allowable (critical) wall thickness $g_{l\text{all}}$ in the vertex point of maximum strains ($\alpha = \beta = 0^0$). For the elongated layers ($\lambda_l = 1$), it is the minimum allowable wall thickness $g_{l\text{all}}$.

- **For simplification of the 1st type**

$$\cos\left(k\frac{\alpha'_{\text{ball}}}{2}\right) = 1 - (R - y_0) \frac{2[d_{\text{ext}}g_0 - g'_{l\text{all}}(d_{\text{ext}} + 2y_0)]}{g'_{l\text{all}}(d_{\text{ext}} + 2y_0)[d_{\text{ext}} - 2(g_0 - g'_{l\text{all}})]}. \quad (2.19)$$

- **For simplification of the 2nd type**

$$\cos\left(k\frac{\alpha''_{\text{ball}}}{2}\right) = 1 - (R - y_0) \frac{2(g_0 - g''_{l\text{all}})}{[d_{\text{ext}} - 2(g_0 - g''_{l\text{all}}) + 2y_0]}. \quad (2.20)$$

- **For simplification of the 3rd type**

$$\cos\left(k\frac{\alpha'''_{\text{ball}}}{2}\right) = 1 - (R - y_0) \frac{2(g_0 - g'''_{l\text{all}})}{g'''_{l\text{all}}(d_{\text{ext}} + 2y_0)}. \quad (2.21)$$

From Eqs (2.19)-(2.21) it appears that when a suitable allowable wall thickness is equal to g_0 , then angle cosines $\cos\left(k\frac{\alpha_b}{2}\right) = 1$ and $y_0 = 0$, and it means no bending. Thus, the angles $\alpha'_{\text{ball}} = \alpha''_{\text{ball}} = \alpha'''_{\text{ball}} = 0$, respectively, because $k > 0$.

From the above Eqs (2.19)-(2.21) it appears that when a suitable allowable wall thickness is equal to g_0 , then suitable angle cosines $\cos\left(k\frac{\alpha_b}{2}\right) = 1$, and it means no bending. Thus, suitable angles $\alpha_b = 0$, because $k > 0$.

3. Initial and boundary conditions for simplified methods

Expressions derived in the point 2 for simplified methods satisfy the following initial and boundary conditions during the pipe bending process:

- a) when $\alpha = \left(\frac{\alpha_b}{2}\right) = 0$ - the beginning of the bending process (or lack of bending),

- b) when $\alpha = \left(\frac{\alpha_b}{2}\right) \neq 0$ - the beginning and the end of the bending zone,
- c) when $(\beta_i = 90^\circ \pm \beta_0)$ - the layers of zero displacements on the R_0 radius in the bending zone, where $(i = 1, 2)$. Then $\varphi'_i = \varphi'_{(i)} = \varphi''_i = \varphi''_{(i)} = \varphi'''_i = \varphi'''_{(i)} = 0$ and $g'_i = g_0$, $g''_i = g_0$, $g'''_i = g_0$. Hence $\varphi'_2 = \varphi''_2 = \varphi'''_2 = 0$ and $\varphi'_3 = \varphi''_3 = \varphi'''_3 = 0$,
- d) when $(k\alpha_b = 180^\circ)$ - beginning of the state of maximum deformations (start of the plateau zone) and $(\alpha = \beta_i = 0^\circ)$ - the apex point of the bending zone in the elongated and compressed layers, then appropriate thicknesses (g'_i, g''_i, g'''_i) reach their extreme values (minimal and maximum) for $(i = 1, 2)$,
- e) when $d_{\text{ext}} = d_{\text{int}}$ (internal area), then $\varphi_2 = \varphi'_2 = \varphi''_2 = \varphi'''_2 = 0$.

4. Summary and conclusions

1. The paper presents the derived approximate (simplified) formulas for calculations of suitable strain components and strain intensity, and expressions for calculations of changes of wall thickness during technological processes of metal tube bending at bending machines. The derived simplifications have a certain physical sense and they are formal and mathematical operations making calculations easier. Three types of formal simplifications were proposed, namely simplifications of the 1st, 2nd and 3rd type, respectively.. In the case of simplification of the first type, the basic equation for calculations of the actual wall thickness of the bend elbow is an algebraic equation of the 2nd degree, and for simplifications of the 2nd and 3rd types – an equation of the 1st degree. These simplifications allow making calculations with the use of a calculator. The derived simplified relationships define low – i.e., safer - values for determination of the allowable bending angle α_{ball} , or the allowable equivalent strain $\varphi_{(i)\text{all}}$.
2. The introduced simplifications of the 1st, 2nd and 3rd types are not only formal mathematical operations making expressions and calculations simpler. They have a physical sense, too. In the case of simplification of the 1st type, as the wall thickness becomes thinner, the neutral layer of plastic bending proportionally displaces downward (in the direction to the centre of rotation). In the considered case this displacement is not big, and we can write that $(r_{\text{ext}}/R \approx \text{const})$ and $R_0 = R_0(\alpha_b) \neq \text{const}$. From the tests and analytic calculations it also appears that the radius determining the position of the neutral layer is $[R_0 \approx R - (g_0 - g_i)]$. In the case of simplification of the 2nd type we can state that circumferential strains are sometimes smaller than longitudinal strains (along the axes and thickness). Thus, they can be neglected ($\varphi_2 \approx 0$). Simplification of the 3rd type is a composition of simplifications of the 1st and 2nd type.
3. The derived simplifications of the 1st, 2nd and 3rd types for components of the kinematically permissible (continuous) field of strains (Śloderbach, 1999; 2002; 2013; 2014) cause that the obtained simplified schemes (fields) of strains do not satisfy the conditions of mutual agreement and continuity of strains between the main components of displacements and strains (there is a non-continuous change of their magnitude).

Nomenclature

- d_i – local „actual diameter” of the bent elbow, $d_i = 2r_i$ (subscript $i=1$ for elongated layers, and $i=2$ for compressed layers)
- d_{ext} and d_{int} – external and internal diameters of the bent tube, respectively

- d_{ext} – it also means the initial (for calculations) external diameter of the considered layer of the wall of the bent tube, such that $d_{\text{ext}} > d_{\text{int}}$ and also $d_{\text{ext}} = 2r_{\text{ext}}$ and $r_{\text{ext}} = r_{\text{int}} + g_0$
- g_0 – initial thickness of the bent tube
- g_i – local actual thickness of the bent elbow in the bending and bent zone where ($i = 1$ or $i = 2$)
- g_{ir} – local actual thickness of the bent elbow in the bending and bent zone calculated according to real (logarithmic) measures of strain
- $g_{1\text{min}}$ – minimum wall thickness in the elongated layer
- $g_{2\text{max}}$ – maximum wall thickness in the compressed layer
- R – bending radius (initial position of the neutral layer)
- R_i – local big active actual radius of the bent elbow connected with the longitudinal strain
- R_0 – radius of the actual and local position of the neutral layer in the tube bending process
- \tilde{r} – relative bending radius, $\tilde{r} = R/d_{\text{ext}}$
- r_i and r^* – local small actual and locally small “active” actual radius in the bending zone
- r_{ext} and r_{int} – external and internal radius of the bent tube
- y_0 – local and instantaneous displacement of the neutral axis, see Fig. 1
- α – actual angle of the bending zone determined in the main bending plane and in the parallel planes, $\alpha \in \left\langle 0^\circ, \frac{\alpha_b}{2} \right\rangle$. When $\alpha_b = 0$ (no bending), then $\alpha = 0$
- α_b – bending angle measured in the bending zone, $\alpha_b \in \left\langle 0^\circ, 180^\circ \right\rangle$
- $\alpha_{b\text{all}}$ and $\alpha_{b\text{cr}}$ – allowable and critical values of the bending angle respectively
- α_0 – bend angle (angle of rotation of the template at the bending machine), in practice $\alpha_0 \in \left\langle 0^\circ, 180^\circ \right\rangle$
- In the bending zone the bend and bending angles are equal, so ($\alpha_0 = \alpha_b$). When the plateau zone arised, then $\alpha_0 = \alpha_b + \alpha_{pl}$, where α_{pl} – the plateau zone angle (Śloderbach, 2002; 2014).
- β – actual angle defined in the plane perpendicular to the bending plane, $\beta \in \left\langle 0^\circ, 90^\circ \right\rangle$
- β_i – angle of circulation of elongated and compressed layers ($\beta_i = 90^\circ \pm \beta_0$) where ($i = 1, 2$)
- φ_1 – logarithmic longitudinal strain (along the axis) in the bending plane
- φ_2 – logarithmic circumferential strain in the plane perpendicular to the main bending plane
- φ_3 – logarithmic radial strain (along thickness)
- $\varphi_{(i)} \equiv$ intensity of the logarithmic strain (logarithmic equivalent strain), where $\varphi_{(i)} \equiv \varphi_i$
- λ_i – correction coefficients of strain distribution (especially important in the description and analysis of the compressed layers) of the bending zone, dependent on technological parameters of bending, $i = 1$ for t elongated layers, $i = 2$ for compressed layers, and $\varphi_1 = \varphi_1^p$, $\varphi_2 = \varphi_2^p$, $\varphi_3 = \varphi_3^p$, $\varphi_i = \varphi_{(i)}^p$, where the superscript p means plastic strains

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DERIVATION OF RELATIONS AND ANALYSIS OF TUBE BENDING PROCESSES USING DISCONTINUOUS FIELDS OF PLASTIC STRAINS

Part II. Discussion and analysis of the obtained results

Z. ŚLODERBACH

Opole University of Technology
Faculty of Applications of Chemistry and Mechanics
45-036 Opole, Luboszycka 7, POLAND
E-mail: z.sloderbach@po.opole.pl

This paper presents the calculation results obtained on the basis of three formal simplifications (derived in Part I) gradually led into the main components of the generalized model of strain where suitable simplifications of the 1st, 2nd and 3rd type have been obtained. The paper also presents the results of considerations on the wall thickness distribution in the vertex point of the elongated layers of the bent elbow, and values of main components of the strain state and intensity of the strain (equivalent strain) depending on the bending angle for the generalized strain scheme and three simplified methods. The results are shown in the form of graphs and table.

Key words: three simplifications, strains, wall thickness, allowable strains and ovalization.

1. Introduction

This part of the paper presents the results of numerical calculations of the wall thickness distribution in the vertex point of the elongated layers of the elbow subjected to bending, and values of main strain components of the strain state and intensity of strain (equivalent strain) depending on the bending angle for the generalized scheme of strain (Śloderbach, 1999; 2002; 2013; 2014) and for three simplified methods. Variations of the wall thickness in the main bending plane and the perpendicular plane passing through the vertex of the elbow previously bent to the angle $k\alpha_b = 180^\circ$ were presented, too. The obtained results were shown as tables and graphs which can be useful as nomograms.

From the calculation results obtained it appears that simplification of the 1st order is very precise and the differences in calculations of the wall thickness in the elongated layers are of the order of $\sim 1\%$. In the case of simplifications of the 2nd and 3rd orders the differences reach some % as compared to the results obtained from the equation of the 3rd order derived and presented in Śloderbach (1999; 2002; 2013; 2014) for the generalized scheme of strains. The calculation results obtained in this paper for the elongated layers ($\lambda_1 = 1$) and the compressed layers ($\lambda_2 = 1$ and $\lambda_2 = 0.5$) are shown in graphs and tables. From the point of view of resistance, exploitation durability or work safety, an analysis of the vertex points of the elongated layers seems to be the most desirable because the maximum plastic strains and the first cracks and other degradation processes usually occur in such points (Dobosiewicz, 1988; Dzikowski, 1998; 2001; EL-Sebaie and Mellor, 1972; Gabryszewski, 1991; 2001; Marcinak, 1971; Moore and Wallace, 1964/65; Seyna and Ginalski, 1987; 1989; Śloderbach, 1999; 2000; 2002; 2013_{1,2}; 2014). They are caused by the maximum reduction of the wall thickness and negative influence of big strains on the material structure.

2. Discussion and analysis of the results

According to the experimental data (Franz, 1961), in the calculations it was assumed that $y_0 \cong 0$, so $\beta_0 \cong 0$. Values indicated without upper commas (g_i , φ_i and $\varphi_{(i)}$) are taken from (Śloderbach, 1999, 2002, 2013_{1,2}, 2014), where $\varphi_{(i)} \equiv \varphi_i$, (see Part I).

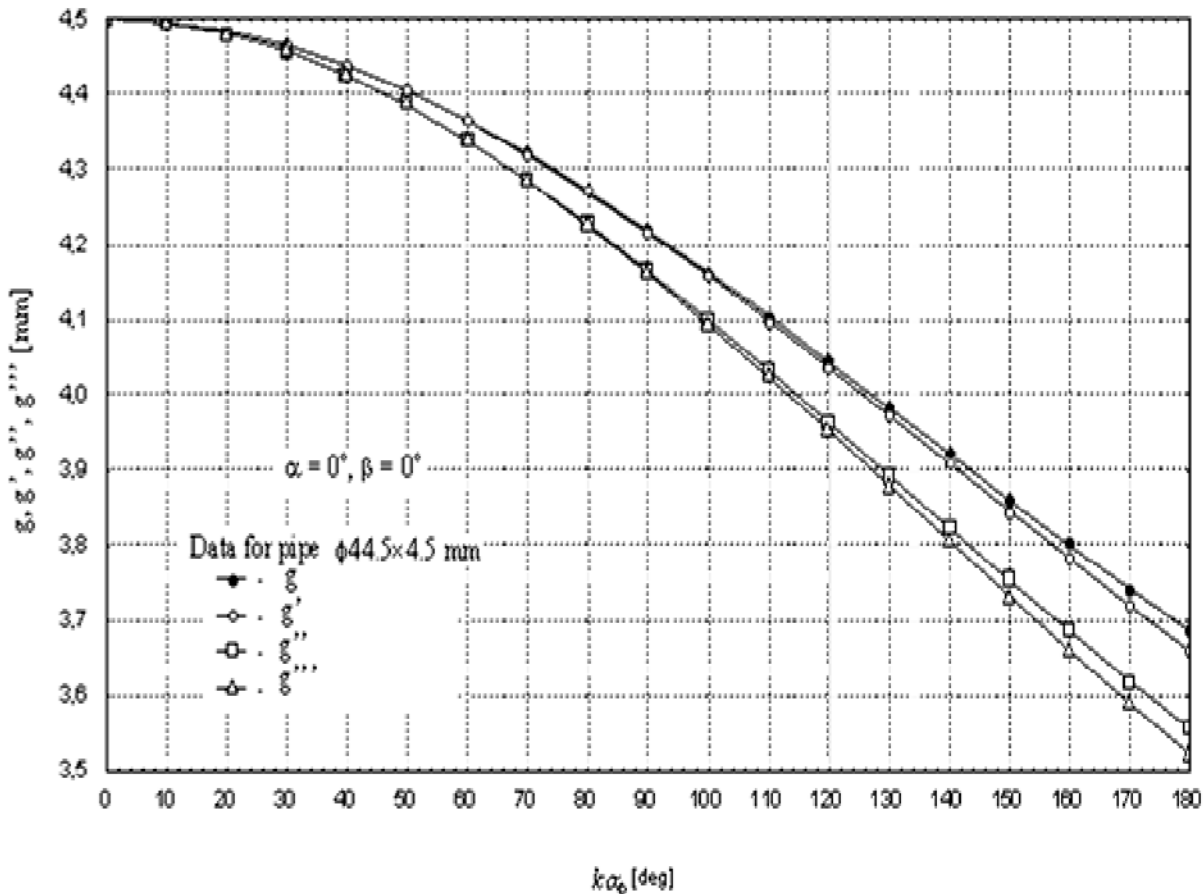


Fig.1. Variation of the wall thickness value at the apex of the elbow versus the bending angle according to four computing methods: the author's own results.

Figure 1 shows the calculation results of variation of the wall thickness (g_l , g_l' , g_l'' , g_l''') depending on the bending angle $k\alpha_b$. The thickness is calculated in the vertex point of the elbow ($\alpha = \beta = 0^\circ$) of the elongated layers for ($\lambda_l = 1$), for the tube $\phi 44.5 \times 4.5$ mm bent at the bending angle $R = 80$ mm such that ($R \cong 1.73 \times d_{ext}$), made of the steel St 35.8 according to the standard DIN 17175 [6]. The graph for thickness g_l was obtained from the papers (Śloderbach, 1999, 2002), and for thickness (g_l' , g_l'' , g_l''') from Eqs (2.5), (2.11) and (2.17) from Part I, respectively.

Figure 2 shows the results of calculations of components of logarithmic measures of strains and strain intensity (equivalent strains) (φ_i , φ_i' , φ_i''') depending on the bending angle ($k\alpha_b$) calculated from the expressions for the generalized model (Śloderbach, 1999, 2002, 2013_{1,2}, 2014) and the derived simplified expressions of the 1st and 3rd order presented in Part I. Calculations were made for the vertex point of the elbow in the bending zone and the central point of the elbow at the same time ($\alpha = \beta = 0^\circ$) in the elongated layers ($\lambda_l = 1$). Only two extreme simplifications (the 1st and 3rd order) are considered because including additional graphs for simplification of the 2nd order could make the graphs not very clear and readable. When the experimental permissible values of the equivalent strain are placed on the Y-axis, it is possible to determine an approximate value of the permissible (critical) bending angle α_{bcr} (after the previous determination of the coefficient k). When values of the angle α_{bcr} are exceeded, values of the permissible strains A_5 or A_{all} in a test of uniaxial tension for tube steels (Franz, 1961; Korzowski, 1971) are exceeded, too. Here, A_{all} means an allowable (permissible) value of uniform strains.

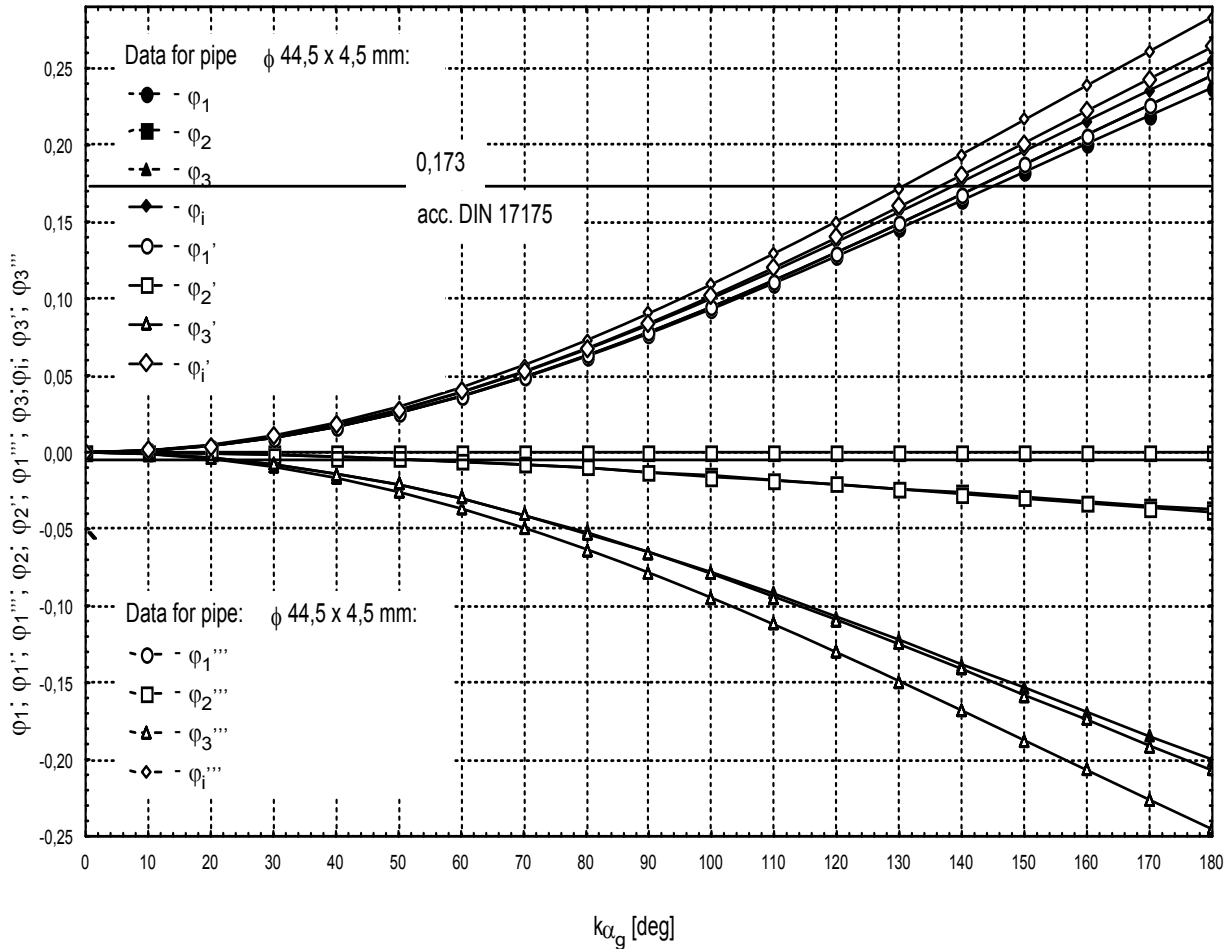


Fig.2. Strain components and strain intensity versus the bending angle according to three computing methods, where ($\alpha_g \equiv \alpha_b$).

Figure 2 shows the value of φ_i equal to 0.173 defined for steel St 35.8 (Franz, 1961) in a test of simple tension according to the standard DIN 17175. On the X-axis, the values of the bending angle α_g for suitable values of (φ_i , φ_i'' , and φ_i''') determined for three methods of calculations respectively (the precise method and two simplified methods of the 2nd and 3rd type) oscillate around the following angle values ($\alpha_{bcr} \approx 140^\circ$ for $k = 1$), ($\alpha_{bcr} \approx 57^\circ$ for $k = 2.5$) and ($\alpha_{bcr} \approx 47^\circ$ for $k = 3$).

From the graphs in Figs 1 and 2 it also follows that application of simplified measures of the 1st, 2nd and 3rd order, respectively causes determination of a greater reduction of a material thickness in the bending zone and greater values of the strain components and the equivalent strain as compared to the results obtained from the solution of the general model. It means that these methods give lower (safer) values, i.e., safer limitations imposed into the permissible bending angle α_{ball} or α_{bcr} . When the values of the angles α_{ball} or α_{bcr} are exceeded, effects connected with localization of plastic deformations or any other form of stability loss or cracking points can occur (Dobosiewicz, 1988; Dzikowski, 1998; 2001; EL-Sebaie and Mellor, 1972; Gabryszewski, 1991; 2001; Marcinak, 1971; Moore and Wallace, 1964/65; Seyna and Ginalski, 1987; 1989; Śloderbach, 1999; 2000; 2002; 2013_{1,2}; 2014; Tang, 2000; Życzkowski, 1981;1997).

Thus, the simplified measures formally introduced in Part I can also be applied in the cases, when a given bending process causes greater reductions in the wall thickness in the elongated layers, suitably greater components of strains and the equivalent strain. Because of simplified forms of the expressions, the

calculations can be made with the use of a calculator, for example under operating conditions, during repair in situ or at the object.

In future, the determined simplified forms for the principal strain components should make it possible to find whether a given tube can be subjected to bending according to the method of determination and analysis of approximate curves of boundary strains (Dzidowski and Cisek, 2001; Gabryszewski and Gronostajski, 1991; Gabryszewski, 2001; Marciniak, 1971; Moore and Wallace, 1964/65), as in the case of drawability of sheets. As compared to the expressions for the generalized strain scheme, the methods are based on simple calculations and they seem to be safer estimations.

Figure 3 shows variations of the wall thickness (g_I, g_I', g_I'' and g_I''') of the elongated layers ($\lambda_I = l$) of the elbow in the main bending plane ($\beta = 0^\circ$) and in the perpendicular plane ($\alpha = 0^\circ$), for the bent and bending angles ($\alpha_0 = k\alpha_b = 180^\circ$). Let us note that the graphs in those planes suitably coincide, and it directly results from Eq.(2.1), and Eqs. (2.5), (2.11) and (2.17) of Part I. In the case of other bending angles ($k\alpha_b \neq 180^\circ$), there is no such coincidence and the graphs have different courses.

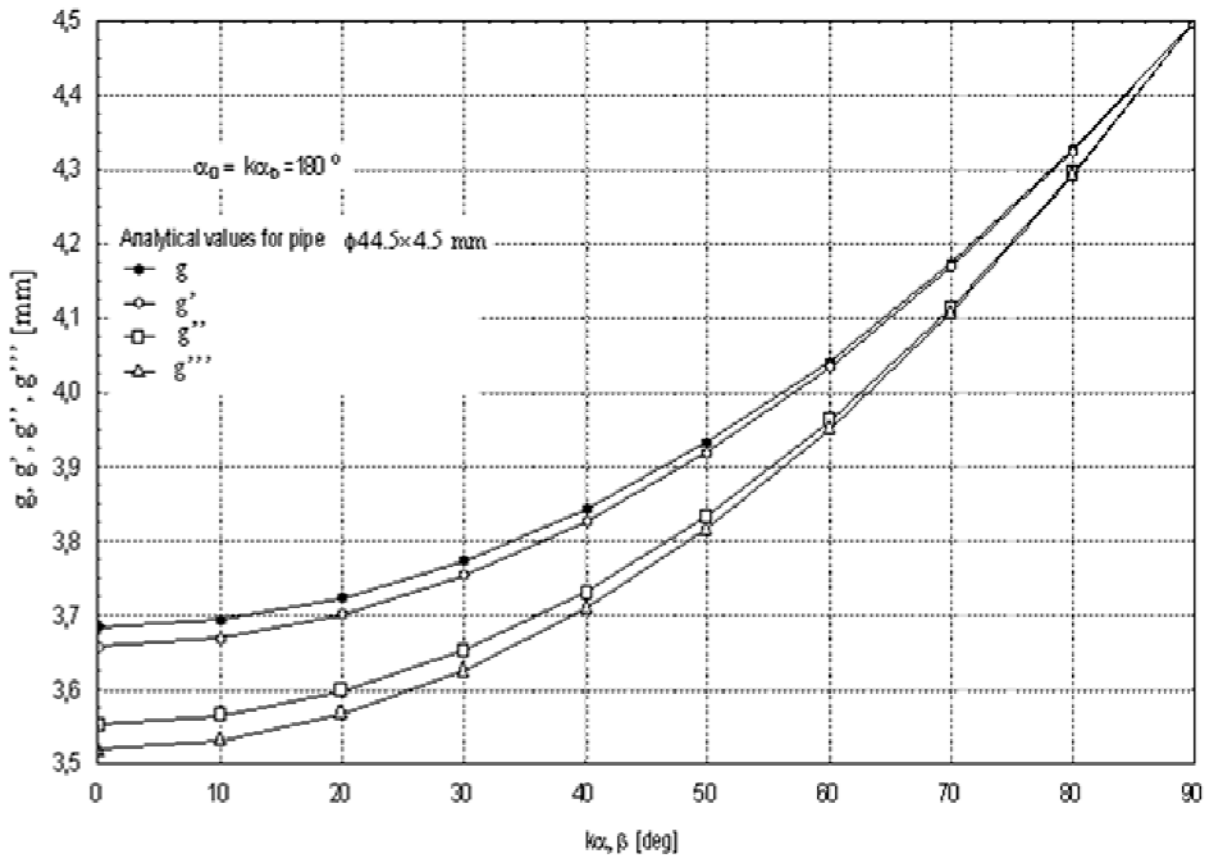


Fig.3. Variation of the wall thickness on the principal plane of bending and the plane perpendicular to it as given by four computing methods.

From the calculations it appears that approximate and simplified methods determine lower values of wall thickness than those obtained from the general model. Thus, the following inequalities take place ($g_I > g_I' > g_I'' > g_I'''$) for the same of bending parameters. It means that the simplified methods determine a more unfavourable distribution of thickness, higher values of strain intensity, so they give lower values for determination of the permissible bending angle. Thus, in a given bending process such estimations are safer from the point of view of strength of a material, operating life or safe service.

On the basis of the graphs from Figs 1-3, we can formulate the following inequalities for the elongated layers

$$\begin{cases} \text{when } \varphi_I < \varphi'_I < \varphi''_I < \varphi'''_I, \\ \text{then } \varphi_{(i)} < \varphi'_{(i)} < \varphi''_{(i)} < \varphi'''_{(i)}, \\ \text{and } g_I > g'_I > g''_I > g'''_I. \end{cases} \quad (2.1)$$

In the compressed layers of the tube subjected to bending the above inequalities (also their gradual arrangement) are not valid.

From the formal analysis of the expressions presented in Śloderbach (1999; 2002; 2013₁; 2014) and Part I we can draw another conclusion, useful in our further considerations. Values of strain components, the equivalent strains and wall thickness for the general strain scheme and three approximate methods calculated depending on the external diameter and internal diameter ($d_{\text{int}} = d_{\text{ext}} - 2g_0$) of a tube are the same. It applies both to the elongated and compressed layers. Thus

$$\begin{aligned} \varphi_j(d_{\text{ext}}) &= \varphi_j(d_{\text{int}}), & \varphi'_j(d_{\text{ext}}) &= \varphi'_j(d_{\text{int}}), \\ \varphi''_j(d_{\text{ext}}) &= \varphi''_j(d_{\text{int}}), & \varphi'''_j(d_{\text{ext}}) &= \varphi'''_j(d_{\text{int}}), \text{ where } j = 1, 2 \text{ and } 3, \\ \varphi_{(i)}(d_{\text{ext}}) &= \varphi_{(i)}(d_{\text{int}}), & \varphi'_{(i)}(d_{\text{ext}}) &= \varphi'_{(i)}(d_{\text{int}}), \\ \varphi''_{(i)}(d_{\text{ext}}) &= \varphi''_{(i)}(d_{\text{int}}), & \varphi'''_{(i)}(d_{\text{ext}}) &= \varphi'''_{(i)}(d_{\text{int}}), \\ g_i(d_{\text{ext}}) &= g_i(d_{\text{int}}), & g'_i(d_{\text{ext}}) &= g'_i(d_{\text{int}}), \\ g''_i(d_{\text{ext}}) &= g''_i(d_{\text{int}}) \text{ and } g'''_i(d_{\text{ext}}) &= g'''_i(d_{\text{int}}), \text{ where } i = 1, 2. \end{aligned} \quad (2.2)$$

However, this statement is not valid for calculations of the initial wall thickness of tubes which have to be subjected to bending (Śloderbach, 2002). The actual thickness in Eqs (2.2) are calculated according to logarithmic measures of strain (see Part I).

Also the derived simplified equations and relationships satisfy the initial and boundary conditions of tube bending specified in item 3, Part. I. Let us note that the derived approximate expressions determine lower values, i.e., lower wall thickness, so they are safer from the point of view of critical strains, the permissible bending angle or the bending radius. Thus, the estimations seem to be more safe from the point of view of technology and design, strength of material, working safety, operation life and so on. It means that for a given bending angle ($k\alpha_b$) the obtained reductions in thickness of the elbow wall in the bending zone and suitable longitudinal components of deformation, as well as the strain intensity will be greater than the relative quantities obtained from the general solution. It means that for a given permissible wall thickness in the bending zone, the permissible values of the bending angle, components of longitudinal strains and the equivalent strain will be relatively lower than those from the general model. The safest and the easiest values from the point of view of calculating are obtained on the basis of a model resulting from simplification of the 3rd order. The above conclusions can be easily verified on the basis of the graphs and calculations. The considered simplifications have no physical sense (except for a certain sense resulting from simplification of the 2nd order), and they are only a formal mathematical operation making calculations easier. The derived

simplified schemes of strain do not satisfy conditions of conformity of displacement and continuity of strains.

Figure 4 gives a comparison of results obtained from the equations derived in Śloderbach (1999; 2002) with those obtained from the tests presented in (Franz, 1961). The bending angle was $\alpha_0 = 180^\circ$ as measured on the principal plane of bending, i.e., for $\beta = 0^\circ$. As it can be seen from the plot, the $\varphi_1, \varphi_2, \varphi_3$ values for the apex point of the bending zone ($\alpha = \beta = 0^\circ$) and within the elongated fibre area are in very good agreement with the experimental data. They do not coincide so well for the whole angle range ξ - but the discrepancy can be easily explained: ideal conditions of bending assumed in the model differ from the actual ones which involve friction, a projecting mandrel and the limited value of $k = 1$. It can be noted that for $k = 3$ the calculated and experimental values agree fairly well, especially for the left-hand half of the plots in Fig.4, both for the bending zone and the plateau zone. For the right-hand side of Fig.4, the bending zone may be adequately represented by the measure presented in (Śloderbach, 1999). The above statements were confirmed by the author when his calculated data were compared with experimental ones quoted in Franz, (1961), Korzowski (1971). The other data in Franz (1961) obtained in tests using either a not-projecting mandrel or no mandrel at all conform well with the author's theory if one assumes $k = 2.5$.

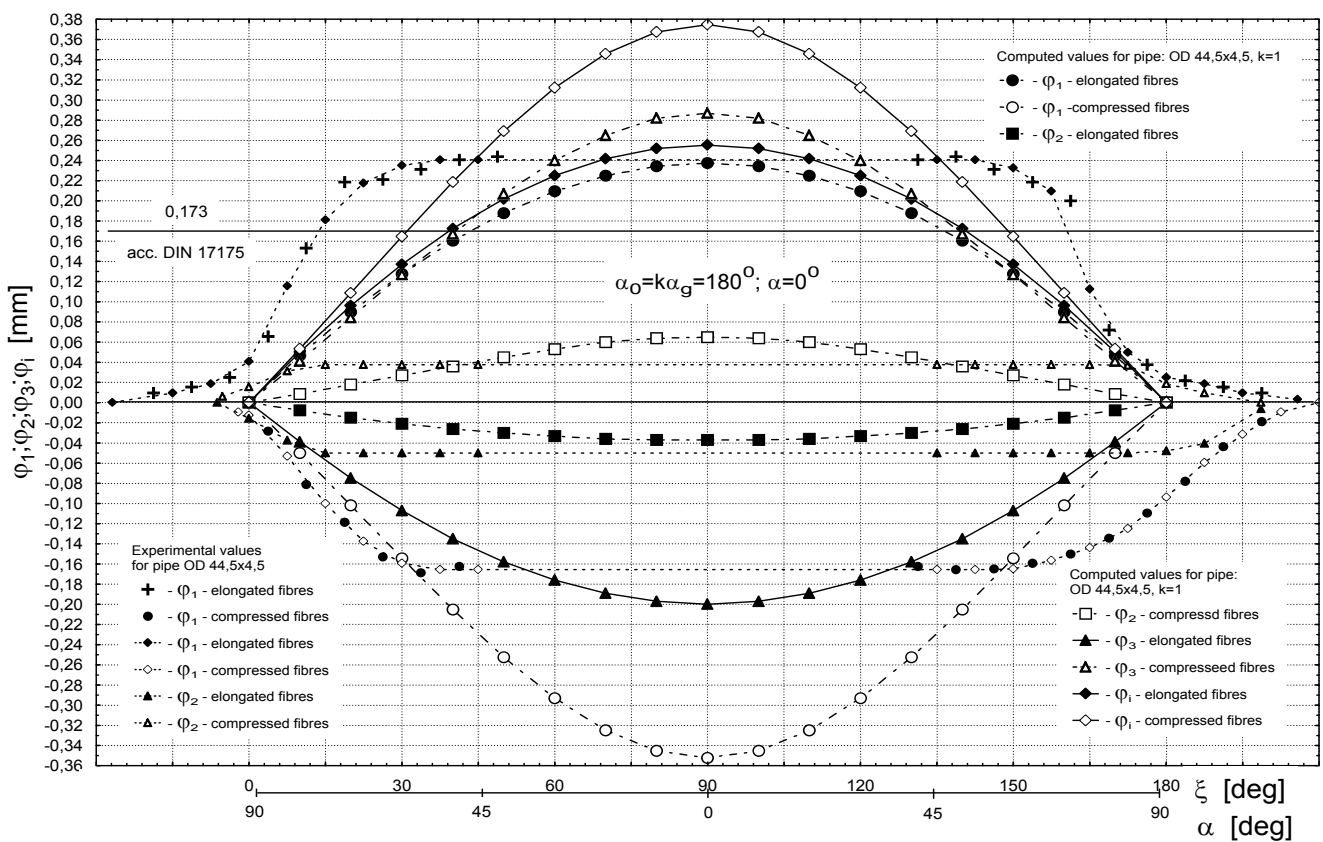


Fig.4. Plots of strains at the bending plan.

The bending zone-plateau zone ratio deserves a short comment. If $k = 1$, then the whole zone defined by the range $[0^\circ; 180^\circ]$ is the bending zone without the plateau portion. When k tends to infinity, then the bending zone extent tends to zero and the whole bend area is occupied by the plateau zone. The running wall thickness g_i is then constant and at the bend ends a sharp change in thickness occurs, so that $g_i \neq g_0$ ($g_1 < g_0$ for elongated fibres and $g_2 > g_0$ for compressed ones). The strains $\varphi_{(i)}$, φ_0 and thickness g_i depend solely upon angle β rather than on angles α and α_b . The obvious conclusion is that for each value of k belonging to the set $k \in [0^\circ; \infty)$ we have $\alpha_0 \geq \alpha_b$.

It follows from the bending operation model presented herein that the maximum angular range of bending portion is equal to 180° while the bend angle range is, at least in theory, unlimited. The difference $(\alpha_o - \alpha_b)$ between the bend angle and bending angle gives the plateau zone range. It must be emphasized that for compressed fibres the model for $(\lambda_2 = 1)$ fails to predict true values of strains. The error may exceed 100%, so this model can be well applicable for $(\lambda_2 = 0.5)$.

It can be generally stated that the coefficient k becomes larger, firstly, with a decreasing bending radius R , secondly, if a more projecting and stiffer mandrel is used and, thirdly, if thick-walled tubes fabricated of less ductile material are involved. A real value of k is also to some degree affected by a particular measurement method used and its accuracy.

A comparison of the author's own experimental data with the obtained calculation results is presented in Tab.1.1. Values of g_1 and g_2 are taken from Śloderbach (1999; 2002).

Table 1.1. Data coming from the tests carried out by the author and the calculation results for $\lambda_1=1$ and $\lambda_2=0.5$.

Pipe ($d_{ext} \times g_o$)[mm]	R [mm] ($\tilde{r} \times d_{ext}$)[mm]	g_{1exp} [mm]	g_{2exp} [mm]	O_{exp} [%]	g_1 [mm]	g_2 [mm]	g'_1 [mm]	g'_2 [mm]	g''_1 [mm]	g''_2 [mm]	g'''_1 [mm]	g'''_2 [mm]	O_m [%]
31.8×3.6 steel K18	50 mm 1.57× d_{ext}	2.85	4.30	5.20	2.89	4.23	2.86	4.20	2.77	4.41	2.73	4.36	2.64
31.8×4.5 steel K18	50 mm 1.57× d_{ext}	3.55	5.45	5.05	3.65	5.26	3.62	5.22	3.47	5.53	3.41	5.45	5.70
44.5×3.2 steel K18	80 mm 1.8× d_{ext}	2.55	3.75	5.85	2.59	3.70	2.58	3.69	2.52	3.79	2.50	3.77	2.88
44.5×4.5 steel K18	80 mm 1.8× d_{ext}	3.60	5.30	5.00	3.68	5.17	3.66	5.15	3.55	5.34	3.52	5.30	4.10
57×4.5 steel K10	142.5 mm 2.5× d_{ext}	3.85	5.05	5.15	3.85	4.96	3.84	4.95	3.77	5.04	3.75	5.03	2.29
57×6.3 steel K10	142.5 mm 2.5× d_{ext}	5.45	7.05	4.80	5.44	6.91	5.42	6.90	5.28	7.07	5.25	7.04	3.24
76.1×6.3 steel K18	190 mm 2.5× d_{ext}	5.35	7.10	5.00	5.40	6.94	5.38	6.93	5.27	7.06	5.25	7.05	2.40
76.1×7.1 steel K18	190 mm 2.5× d_{ext}	6.10	7.85	4.90	6.10	7.82	6.88	7.80	5.95	7.96	5.92	7.94	2.72
139.7×4.5 steel R35	280 mm 2.0× d_{ext}	3.65	5.65	5.55	3.66	5.15	3.65	5.15	3.61	5.20	3.60	5.20	1.14
139.7×8 steel R35	280 mm 2.0× d_{ext}	3.80	5.35	4.85	6.56	9.11	6.54	9.09	6.43	9.26	6.40	9.23	2.05
159×4.5 steel R35	400 mm 2.5× d_{ext}	3.80	5.35	5.25	3.79	5.01	3.79	5.00	3.76	5.03	3.75	5.03	0.91
159×10 steel R35	400 mm 2.5× d_{ext}	8.50	1.05	4.60	8.53	11.05	8.50	11.03	8.37	11.19	8.34	11.17	1.81

where O_m - mean coefficient of ovalization of the cross section inside the elbow caused by variation in wall thickness, O_{all} - allowable coefficient of section ovalization ($O_{all} = 6\%$ according to EN 13445-4; 2009), the index (exp) -stands for experimental data.

3. Final statements and conclusions

1. Simplifications of the 1st, 2nd and 3rd type determine greater strains, decrease and increase of the wall thickness as well as greater initial wall thickness than those resulting from a generalized model of strain.
2. From the experimental data presented in Tab.1.1 and calculation results it appears that:

- The smaller value of the relative bending radius \tilde{r} , ($\tilde{r} = R/d_{\text{ext}}$) (Śloderbach, 2002), the „sharper” bending, the greater variation of the elbow wall thickness and ovalization of the cross section of the elbow are,
- The ovalization coefficient for the elbow cross section determined from the tests and defined by the method of arithmetic averaging from calculations O_{all} , does not exceed the allowable value ($O_{\text{all}} = 6\%$) assumed in, see e.g. European Standard EN 13445-4; 2009; UDT Conditions, 2003; Zdankiewicz, 1998,
- The experimental data were obtained from tests for bending an elbow to the right angle ($\alpha_0 = 90^\circ$) and the straight angle ($\alpha_0 = 180^\circ$),
- The calculations were made for the moment when the maximum strains were reached in the bending zone, i.e. initiation of the „plateau” zone in the central points of the elbow,
- The calculation results were obtained for technological-material correction coefficients of strain distribution in the bending zone λ_i ; for elongated layers ($\lambda_1 = 1$) and compressed layers ($\lambda_2 = 0.5$),
- Relatively big differences between ovalization coefficients coming from the tests O_{exp} and calculations, mathematically averaged O_m result from the fact that the experimental data include the effects resulting from the cross section flattening and non-uniform variation of the elbow wall thickness, and the calculation results for ovalization include only the effects of non-uniform variation of wall thickness.

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