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ON ONE ALGORITHM FOR SOLVING THE PROBLEM OF SOURCE FUNCTION RECONSTRUCTION

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In the paper, the problem of source function reconstruction in a differential equation of the parabolic type is investigated. Using the semigroup representation of trajectories of dynamical systems, we build a finite-step iterative procedure for solving this problem. The algorithm originates from the theory of closed-loop control (the method of extremal shift). At every step of the algorithm, the sum of a quality criterion and a linear penalty term is minimized. This procedure is robust to perturbations in problems data.

Keywords: reconstruction, source function, feedback control.

1. Introduction and problem formulation

In the theory of differential equations, the following problem is well-known: it is required to determine parameters of a differential equation provided a given function plays the role of its solution. Unknown parameters (generally speaking, not constant and depending on time) can be controls, dynamical disturbances, coefficients, some system characteristics and so on. As a rule, such problems are ill-posed. First, a number of parameter values may correspond to a given solution. This property rather often results in a new problem of choosing parameters. For example, in the problem of control reconstruction, the interest is usually in finding a control with an extremal (maximal or minimal) energy resource. Second, the mapping "solution $\longrightarrow$ parameter" is discontinuous in the general case. Therefore, this mapping cannot be used to approximate a desired parameter in the case when a disturbed solution is given instead of an exact one. If some function, which is not a solution, is given instead of a disturbed solution of an equation, additional difficulties arise. Under such conditions, the "approximative" feature should be provided by some regularizing procedures. Besides, there exists the problem of sufficiently convenient and constructive description and determination of the mapping "solution $\longrightarrow$ parameter". Issues of determining some parameters through equation solutions are often called reconstruction (identification) problems. Recently, methods of solving reconstruction problems have been intensively developed. A considerable number of works is devoted to solving problems of reconstructing right-hand parts (e.g., a source function) of parabolic equations through results of sensor observations. A typical problem from this field is as follows.

Let us imagine a water reservoir occupying an area Ω and n contamination sources located in subareas $\Omega_1, \ldots, \Omega_n$ of Ω. It is assumed that an input concentration rate of some contaminant at every point ξ in the source area Ω_j is modeled as $u_j(t)\omega_j(\xi)$, where t is current time. The positive function $u_j(t)$ is a measure of the time-varying intensity of the source located in Ω_j; $u_j(t)$ represents the current rate of the contamination inflow in Ω_j. Let $x(t, \xi)$ be the current concentration of the contaminant at a point ξ in Ω. Some information on the distribution of $x(t, \xi)$ in Ω is registered by m sensors. The sensors register weighted average concentrations $z_1(t), \ldots, z_m(t)$ in fixed subareas $\Theta_1, \ldots, \Theta_m$ of Ω:

$$z_k(t) = \int_{\Theta_k} p_k(\xi) x(t, \xi) \, \mathrm{d}\xi, \quad k = 1, \ldots, m,$$

the positive weight coefficients $p_k(\xi)$ are supposed to be given. The problem of reconstructing the right-hand part (the intensity reconstruction problem) is as follows. Observing the weighted average concentrations $z_1(t), \ldots, z_m(t)$ of the contaminant in the areas $\Theta_1, \ldots, \Theta_m$, reconstruct the intensities of the contamination sources, $u_1(t), \ldots, u_n(t)$, in the source areas

$\Omega_1, \dots, \Omega_m$. This problem has been studied by many authors (see, e.g., (Omatu and Seinfeld, 1989; Korbicz and Zgurowski, 1991; Uciński, 1999)).

In the present work, a new algorithm for solving such a problem for an abstract parabolic equation is suggested. A particular case of such an equation is a diffusion equation describing the process of contaminant propagation in the atmosphere or liquid media. This algorithm rests upon constructions of the theory of stable dynamical inversion based on the combination of methods of the theory of ill-posed problems and that of positional control. The essence of the technique described in (Blizorukova and Maksimov, 1998; Digas *et al.*, 2003; Kryazhimskii and Osipov, 1987; Kryazhimskii *et al.*, 1997) is that a reconstruction algorithm is represented as a control algorithm for some auxiliary dynamical system. It should be noted that this technique exploits the idea of stabilizing appropriate functionals of the Lyapunov type by means of extremal shift (Krasovskii and Subbotin, 1988). Thus, the technique combines the stabilization principle with that of extremal shift in some scheme of control with a model. In the beginning, some functional treated as a Lyapunov type one is introduced. Then, a control law for an auxiliary system is chosen. This law uses the idea of extremal shift providing a "weak growth" of the functional in time.

Let us pass to the statement of the problem under investigation. In a real Hilbert space $(H, |\cdot|_H)$, the following parabolic equation is considered:

$$\dot{x}(t) + Ax(t) = Bu(t) + f(t), \qquad (1)$$
$$t \in T = [0, \vartheta], \quad x(0) = x_0 \in H.$$

Here, $A : V \to V^*$ is a linear continuous symmetric operator satisfying (for some $c > 0$ and $\lambda \in \mathbb{R}$) the coercivity condition

$$\langle Ay, y\rangle_V + \lambda |y|_H^2 \ge c\|y\|^2, \quad \forall y \in V.$$

$(V, \|\cdot\|)$ is a separable and reflexive Banach space, which is densely and continuously embedded in the space H identified with its conjugate space ($H = H^*$); $\langle \cdot, \cdot \rangle_V$ is the duality between V and V^*; $x(t)$ is the phase state of the system (1) at the moment t; $u(t) \in U$ is a disturbance generating the motion $x(\cdot)$; the space of disturbances $(U, |\cdot|_U)$ is a Hilbert space with a scalar product $(\cdot, \cdot)_U$; $f(\cdot) \in L_2(T; H)$ is a given input action; $B : U \to H$ is a linear continuous operator.

A solution of the system (1) (which is understood in the weak sense) is defined to be a unique continuous function $x(\cdot) = x(\cdot; x_0, u(\cdot))$ of the form

$$x(t) = S(t)x_0 + \int_0^t S(t-\tau)(Bu(\tau) + f(\tau))\, d\tau, \quad t \in T.$$

Here $S(t) : H \to H$ ($t > 0$) is the semigroup of continuous linear operators generated by the operator A. As is known (Bensoussan *et al.*, 1992), for all $u(\cdot) \in L_2(T; U)$, $f(\cdot) \in L_2(T; H)$, $x_0 \in H$, there exists a unique solution of the system (1). Below it is assumed that the set of admissible system inputs $u(\cdot)$ is of the form

$$U_* = \{u(\cdot) \in L_2(T; U) : \ u(t) \in P \text{ for a. a. } t \in T\},$$

$P \subset U$ is a convex bounded and closed set.

The problem under consideration consists in the following. Let Eqn. (1), an action $f(\cdot)$, an initial state x_0, and the set of admissible inputs U_* be known. In addition, at moments $t \in T$, values

$$z(t) = Gx(t) \qquad (2)$$

are inaccurately measured. Here G is a given linear continuous operator acting from the space of states H into a Hilbert space of measurements H_1. Results of the measurements $\xi^h(t)$ are, generally speaking, inaccurate:

$$|\xi^h(t) - z(t)|_{H_1} \le h, \quad t \in T. \qquad (3)$$

Here h is the measurement accuracy.

Let U_z be the set of all admissible inputs $u(\cdot)$ compatible with some output $z(\cdot)$, i.e.,

$$U_z = \{u(\cdot) \in U_* : \ Gx(t; 0, x_0, u(\cdot)) = z(t), \quad \forall t \in T\},$$

$$J(u(\cdot)) = \int_0^\vartheta \omega(t, u(t))\, dt \qquad (4)$$

is a given performance index. Here $\omega(\cdot, \cdot) : T \times U \to \mathbb{R}_+ = [0, +\infty)$ is a functional, which is convex with respect to the second argument. It is assumed that the functional $J(u(\cdot))$ is defined on the set U_* and is lower semicontinuous. It is necessary to construct a stable algorithm for calculating an extremal value

$$J_z^0 = \min\{J(u(\cdot)) : u(\cdot) \in U_z\} \qquad (5)$$

and an extremal input action $u_0(\cdot) \in U_0(z)$, where

$$U_0(z) = \arg\min\{J(u(\cdot)) : u(\cdot) \in U_z\}.$$

Since the functional J is convex and the set U_* is bounded and closed, the set $U_0(z)$ is a non-empty convex and closed set. Therefore, the problem in question has a solution. However, the precise calculation of J_z^0 and $u_0(\cdot)$ is impossible, in particular, due to inaccuracies in measuring the values $z(t)$, $t \in T$ (see (3)). In this case, it is necessary to design a stable algorithm for approximate determination of the value J_z^0 and the extremal input action $u^h(\cdot) = u(\cdot; \xi^h(\cdot))$. The stability of the algorithm is understood in the following sense:

$$J(u^h(\cdot)) \to J_z^0, \quad u^h(\cdot) \to U_0(z)$$
$$\text{weakly in } L_2(T; U) \text{ as } h \to +0.$$

The last relation means that any convergent (in $L_2(T;U)$) sequence $\{u^{h_l}(\cdot)\}$, $h_l \to +0$ as $l \to +\infty$, converges to some element from the set $U_0(z)$. Below we present an algorithm for solving the problem considered in the case when the operators A and B, as well as the functional J and the initial state x_0, are inaccurately known.

Note that the problem of source function reconstruction formulated above can be interpreted as an optimal control problem for the parabolic equation (1) subject to state constraints. Then the performance index takes the form (4), whereas the state constraints are given by (2). A solving algorithm suggested in the paper is oriented to the case of inaccurate data on the problem parameters, i.e., on the system structure, the quality criterion, and the state constraints.

2. Auxiliary results

Before passing on to solving the problem in question, let us give auxiliary statements. Introduce the operator $F : L_2(T;U) \to L_2(T;H_1)$ and the element $b(\cdot) \in L_2(T;H_1)$:

$$(Fu(\cdot))(\eta) = \int_0^\eta GS(\eta - t)Bu(t)\,\mathrm{d}t, \quad \eta \in T, \ u(\cdot) \in U_*,$$

$$b(\eta) = z(\eta) - GS(\eta)x_0, \qquad \eta \in T.$$

Here the function z is defined by (2). Then the problem of calculating the extremal value J_z^0 and extremal input action $u_0(\cdot) \in U_0(z)$ is equivalent to the following extremal problem.

It is required to find

$$u^0 = \arg\min\{J(u) : u \in U_*, Fu = b\}$$

and

$$J^0 = \min\{J(u) : u \in U_*, Fu = b\},$$

where J is the quality criterion (4). Note that the set $\{u \in U_* : Fu = b\}$ is non-empty. Therefore, there exists a solution to the last problem, and $J^0 = J_z^0$ (see (5)). If the function $\omega(\cdot,\cdot)$ is strictly convex with respect to the second argument, then the set $U_0(z)$ is a singleton and, in addition, $u^0 = u_0 = U_0(z)$.

Let non-negative numbers ν^F, ν^b, and ν^J, linear continuous operators $F^\nu : L_2(T;U) \to L_2(T;H_1)$, elements $b^\nu \in L_2(T;H_1)$, and convex functionals $J^\nu(\cdot)$ defined on U_* be given in such a way that

$$|F^\nu u - Fu|_{L_2(T;H_1)} \le \nu^F, \quad \forall\, u \in U_*, \tag{6}$$

$$|b^\nu - b|_{L_2(T;H_1)} \le \nu^b, \tag{7}$$

$$|J^\nu(u) - J(u)| \le \nu^J, \quad \forall\, u \in U_*. \tag{8}$$

For simplicity, in what follows it is assumed that $\nu^F, \nu^b, \nu^J \in [0,1)$.

Our goal is to design an algorithm for approximate determination of the value J^0 and the element u^0. Namely, it is required to construct an algorithm which, using values F^ν, b^ν, and J^ν known instead of F, b, and J, forms elements $\{u_j\} \in U_*$, $j = 1, \ldots$, with the properties

$$|Fu_j - b|_{L_2(T;H_1)} \to 0, \tag{9}$$

$$J(u_j) \to J^0 \quad \text{as} \quad j \to +\infty, \tag{10}$$

if $\nu^F = \nu_j^F \to 0$, $\nu^b = \nu_j^b \to 0$, and $\nu^J = \nu_j^J \to 0$ as $j \to +\infty$.

Let us pass on to the description of this algorithm.

Set

$$\begin{aligned} K_0 &= \sup\{J(u) : u \in U_*\}, \\ U_\gamma &= \{u \in U_* : |Fu - b|_{L_2(T;H_1)} \le \gamma\}, \\ J^0(\gamma) &= \inf\{J(u) : u \in U_\gamma\}. \end{aligned}$$

From the results of (Vasiliev, 1981, p. 182), we obtain what follows.

Theorem 1. *Let $u_j \in U_*$, $\alpha_j > 0$, $\gamma_j > 0$, $|J^{\nu_j}(u) - J(u)| \le \nu_j^J$ $\forall u \in U_*$,*

$$\begin{gathered} |Fu_j - b|^2_{L_2(T;H_1)} + \alpha_j J(u_j) - \alpha_j J^0 \le \gamma_j, \\ \nu_j^J \to 0, \quad \alpha_j \to +0, \quad \gamma_j \to +0, \\ \gamma_j/\alpha_j \to 0 \quad as \quad j \to +\infty. \end{gathered} \tag{11}$$

Then we have

(a) $J^{\nu_j}(u_j) \to J^0$ *as* $j \to +\infty$,

(b)

$$\begin{aligned} &J^0((\gamma_j + 2K_0\alpha_j)^{1/2}) - \nu_j^J \\ &\qquad \le J^{\nu_j}(u_j) \le J^0 + \gamma_j/\alpha_j + \nu_j^J. \end{aligned}$$

Any element $w_* \in W$ is called an *ε-solution* ($\varepsilon > 0$) of the extremal problem $\varphi(w) \to \inf$, $w \in W \neq \varnothing$, if $\varphi(w_*) \le \inf\{\varphi(w) : w \in W\} + \varepsilon$. We denote by the symbol $Y_j^\nu(\delta,\alpha,\varepsilon)$ the set of elements y_i, $i = 0, 1, \ldots, j$, from U such that

$$y_0 = 0, \tag{12}$$

$$y_{i+1} = y_i + u_i\delta, \quad i = 0, 1, \ldots, j-1, \tag{13}$$

where u_i is an ε-solution of the problem

$$2\langle F^\nu y_i - i\delta b^\nu, F^\nu u\rangle + \alpha J^\nu(u) \to \inf, \quad u \in U_*. \tag{14}$$

Hereinafter, the symbol $\langle\cdot,\cdot\rangle$ denotes the scalar product in $L_2(T;H_1)$.

Let

$$K_1 = |b|_{L_2(T;H_1)}, \quad |Fu|_{L_2(T;H_1)} \le K_2 \ \forall u \in U_*.$$

Based on the approach from (Kryazhimskii and Osipov, 1987; Kryazhimskii *et al.*, 1997), the following lemma is proved.

Lemma 1. *The inequality*

$$|F(y_j/(\delta j)) - b|^2_{L_2(T;H_1)} + \alpha\{J(y_j/(\delta j)) - J^0\}/(\delta j) \le \delta_j^*$$

is valid for any $j \ge 1$, *where*

$$\begin{aligned}\delta_j^* &= k_1\nu^F + k_2\nu^b + (k_3(\nu^F)^2 + 2\alpha\nu^J)/(j\delta) \\ &\quad + k_4/j + \varepsilon/(\delta j), \\ k_1 &= 4(K_1 + 2K_2), \\ k_2 &= K_2 + 1, \\ k_3 &= 4, \\ k_4 &= (K_1 + K_2)^2.\end{aligned}$$

Proof. Let us estimate the change of the value

$$\begin{aligned}\Lambda_{i+1} &= |F(y_i + \delta u_i) - t_{i+1}b|^2_{L_2(T;H_1)} \\ &\quad + \alpha\int_0^{t_{i+1}} J(\dot y(\tau))\,\mathrm{d}\tau - \alpha J(u^0)t_{i+1} \\ &= \Lambda_i + \mu_i + \delta^2|Fu_i - b|^2_{L_2(T;H_1)}, \quad i \ge 0,\end{aligned}$$

where

$$\begin{aligned}&\dot y(t) = u_i \quad \text{for} \quad t \in [t_i, t_{i+1}), \\ &i = 0, 1, \dots, \quad t_i = i\delta, \quad y(0) = 0, \\ &\mu_i = 2\langle Fy_i - t_ib, Fu_i - b\rangle\delta + \alpha\delta\{J(u_i) - J(u^0)\}.\end{aligned}$$

Since u^0 is a solution of the problem under consideration, the equality

$$Fu^0 = b$$

is fulfilled. Therefore,

$$\begin{aligned}\mu_i &= 2\langle Fy_i - t_ib, Fu_i - b\rangle\delta - 2\langle Fy_i - t_ib, Fu^0 - b\rangle\delta \\ &\quad + \alpha\delta\{J(u_i) - J(u^0)\} \\ &= 2\langle Fy_i - t_ib, Fu_i\rangle\delta + \alpha\delta J(u_i) \\ &\quad - 2\langle Fy_i - t_ib, Fu^0\rangle\delta - \alpha\delta J(u^0).\end{aligned}$$

Moreover,

$$\begin{aligned}\lambda_i^* &\equiv 2\langle Fy_i - t_ib, Fu_i\rangle\delta + \alpha J(u_i)\delta \\ &\quad - 2\langle F^\nu y_i - t_ib^\nu, F^\nu u_i\rangle\delta + \alpha J^\nu(u_i)\delta = \sum_{j=1}^{3}\lambda^{(j)}, \\ \lambda_i^{(1)} &= 2\langle Fy_i - t_ib, (F - F^\nu)u_i\rangle\delta, \\ \lambda_i^{(2)} &= 2\langle (F - F^\nu)y_i - t_i(b - b^\nu), F^\nu u_i\rangle\delta, \\ \lambda_i^{(3)} &= \alpha\{J(u_i) - J^\nu(u_i)\}\delta.\end{aligned}$$

In addition, by (6) we obtain

$$|Fy_i|_{L_2(T;H_1)} = |\sum_{j=0}^{i-1} Fu_j\delta|_{L_2(T;H_1)} \le t_{i-1}K_2,$$

$$|(F - F^\nu)y_i|_{L_2(T;H_1)} = \Big|\sum_{j=0}^{i-1}(F - F^\nu)u_j\delta\Big|_{L_2(T;H_1)} \le \nu^F t_i,$$

$$\begin{aligned}|F^\nu u|_{L_2(T;H_1)} &\le |(F^\nu - F)u|_{L_2(T;H_1)} + |Fu|_{L_2(T;H_1)} \\ &\le K_2 + \nu^F, \quad u \in U_*.\end{aligned}$$

Note that, due to the convexity and completeness of the set P, the rule for choosing the elements u_i implies the inclusion $y_i/(\delta i) \in U_*$. Thus, we have (see (6)–(8))

$$\begin{aligned}\lambda_i^{(1)} &\le 2(K_2 + K_1)\nu^F\delta t_i, \\ \lambda_i^{(2)} &= 2(\nu^F + \nu^b)(K_2 + \nu^F)\delta t_i, \\ \lambda_i^{(3)} &\le \alpha\delta\nu^F.\end{aligned}$$

Therefore,

$$\lambda_i^* \le \delta\nu^F K_{1i} + \delta\nu^b K_{2i} + \alpha\nu^J\delta,$$

where

$$\begin{aligned}K_{1i} &= 2((2K_2 + K_1)t_i + \nu^F), \\ K_{2i} &= 2t_i(K_2 + \nu^F)\delta.\end{aligned}$$

A similar estimate holds if we replace u_i by u^0. Hence,

$$\begin{aligned}\mu_i &\le 2\langle F^\nu y_i - t_ib^\nu, F^\nu u_i\rangle\delta - 2\langle F^\nu y_i - t_ib^\nu, F^\nu u^0\rangle\delta \\ &\quad + \alpha\delta\{J^\nu(u_i) - J^\nu(u^0)\} \\ &\quad + 2\{K_{1i}\nu^F + K_{2i}\nu^b + \alpha\nu^J\}\delta.\end{aligned}$$

From (14) we deduce that

$$\mu_i \le 2\{K_{1i}\nu^F + K_{2i}\nu^b + \alpha\nu^J\}\delta + \varepsilon\delta.$$

It is easily seen that

$$|Fu_i - b|^2_{L_2(T;H_1)}\delta^2 \le \{(K_2 + K_1)\delta\}^2.$$

In addition, due to the convexity of J, we obtain

$$J(y(t)/t) = J\Big(\frac{1}{t}\int_0^t \dot y(\tau)\,\mathrm{d}\tau\Big) \le \frac{1}{t}\int_0^t J(\dot y(\tau))\,\mathrm{d}\tau. \tag{15}$$

Accordingly,

$$\begin{aligned}\Lambda_{i+1} &\le \Lambda_i + 2\{K_{1i}\nu^F + K_{2i}\nu^b + \alpha\nu^J\}\delta \\ &\quad + \{(K_2 + K_1)\delta\}^2 + \varepsilon\delta \\ &\le \Lambda_i + 2\{2((2K_2 + K_1)(i\delta + \nu^F)\nu^F\delta \\ &\quad + 2i\delta(K_2 + \nu^F)\nu^b\delta + \alpha\nu^J\delta\} \\ &\quad + \{(K_2 + K_1)\delta\}^2 + \varepsilon\delta, \qquad i \ge 0.\end{aligned}$$

Using (12), we obtain $\Lambda_0 = 0$. Then

$$\begin{aligned}\Lambda_{i+1} &\le 2\{((2K_1 + 4K_2)i\delta + 2\nu^F)\nu^F\delta \\ &\quad + 2i\delta(K_2 + \nu^F)\nu^b\delta + \alpha\nu^J\delta\}i \\ &\quad + \{(K_1 + K_2)\delta\}^2 i + \varepsilon\delta i, \qquad i \ge 0.\end{aligned}$$

Dividing the right-hand and left-hand sides by t_{i+1}^2 and using (15), we get

$$\begin{aligned}
&|F(y(t_{i+1})/t_{i+1}) - b|^2_{L_2(T;H_1)}\\
&\quad + \alpha/t_{i+1}\{J(y(t_{i+1})/t_{i+1}) - J(u^0)\}\\
&\le 2\Big\{\frac{2K_1+4K_2}{\delta} + \frac{2\nu^F}{(i+1)\delta^2}\Big\}\nu^F\delta + \varepsilon/(\delta(i+1))\\
&\quad + 4(K_2+\nu^F)\nu^b + 2\alpha\nu^J/((i+1)\delta)\\
&\quad + (K_1+K_2)^2/(i+1)\\
&= 4\{(K_1+2K_2)\nu^F + (K_2+\nu^F)\nu^b\}\\
&\quad + 2\{2(\nu^F)^2 + \alpha\nu^J\}/((i+1)\delta)\\
&\quad + (K_1+K_2)^2/(i+1) + \varepsilon/(\delta(i+1)).
\end{aligned}$$

■

The next theorem follows from Theorem 1 and Lemma 1.

Theorem 2. *Let*

(a) $\delta_j \to +0$, $\varepsilon_j \to +0$ *as* $j \to +\infty$*;*

(b) sequences $\{\alpha_j\}$ *and* $\{\gamma_j\}$ *satisfy the conditions (11);*

(c) inequalities

$$\begin{aligned}
k_1\nu_j^F + k_2\nu_j^b + (k_3(\nu_j^F)^2 + 2\alpha_j\nu_j^J)/(j\delta_j)&\\
+ k_4/j + \varepsilon_j/(\delta_j j) \le \gamma_j&
\end{aligned}$$

be true;

(d) $\{y_i\}_{i=0}^j = Y_j^\nu(\delta_j, \alpha_j, \varepsilon_j)$.

Then the sequence of elements $\{u_j\}_{j=1}^{+\infty}$,

$$u_j = y_j/(\delta_j j) \in U_*,$$

satisfying the conditions (9) and (10), solves the problem of approximate calculation of u^0 *and* J^0.

As is seen from Lemma 1 and Theorem 2, choosing appropriate relations between the values $\delta, i, \nu^F, \nu^b, \nu^J$, and α, we provide a "slow" growth of the Lyapunov functional

$$\begin{aligned}
\Lambda(t) &= |F(y(t)/t) - b|^2_{L_2(\cdot;H_1)}\\
&\quad + \alpha\int_0^t J(\dot{y}(\tau))\,\mathrm{d}\tau/t - \alpha J(u^0)/t,\ \ t>0.
\end{aligned}$$

The rule (14) for choosing controls u_i is, in essence, a modification of the extremal shift principle from the theory of differential games (Krasovskii and Subbotin, 1988).

Let

$$\nu_1(\alpha,\delta,j,\varepsilon,\nu^F,\nu^b,\nu^J) = \delta_j^* + 2K_0\alpha/(\delta j),$$
$$\nu_2(\alpha,\delta,j,\varepsilon,\nu^F,\nu^b,\nu^J) = \delta_j^*\delta j/\alpha.$$

The next inequalities follow from Lemma 1:

$$|F(y_j/(\delta j)) - b|^2_{L_2(T;H_1)} \le \nu_1(\alpha,\delta,j,\varepsilon,\nu^F,\nu^b,\nu^J), \tag{16}$$

$$J(y_j/(\delta j)) - J^0 \le \nu_2(\alpha,\delta,j,\varepsilon,\nu^F,\nu^b,\nu^J). \tag{17}$$

The following lemma holds.

Lemma 2. *Let* $\nu_j^F \to 0$, $\nu_j^b \to 0$, $\nu_j^J \to 0$, $\alpha_j \to +0$, $\alpha_j/(\delta_j j) \to +0$, $(\delta_j+\varepsilon_j)/\alpha_j \to +0$, $\varepsilon_j/(\delta_j j) \to +0$, $\nu_j^F j\delta_j/\alpha_j \to 0$, $\nu_j^b j\delta_j/\alpha_j \to 0$ *as* $j \to +\infty$.
Then

$$|F(y_j/(\delta_j j)) - b|^2_{L_2(T;H_1)} \to 0,$$
$$J(y_j/(\delta_j j)) \to J^0 \quad \text{as} \quad j \to +\infty.$$

Condition 1.

$$j = j(h) = [1/h], \quad \nu_j^F = a_1 h, \quad \nu_j^b = a_2 h,$$
$$\nu_j^J = a_3 h, \quad \varepsilon_j = a_4\delta_j, \quad \alpha_j = \delta_j^{1/2},$$
$$\delta_j = h^{1-\varkappa}, \quad \varkappa = \text{const} \in (0,1).$$

Here the symbol $[1/h]$ denotes the integer part of $1/h$; a_j, $j = 1,\dots,4$, are some constants. In this case, $j = j(h) \to +\infty$ as $h \to +0$ and the elements $y_j/(\delta_j j)$ depend on h, i.e.,

$$u^h = y_{j(h)}/(h^{1-\varkappa}[1/h]) \in U_*. \tag{18}$$

The next statement can be formulated.

Corollary 1. *Let Condition 1 be fulfilled and* $h \in (0, 1-\varepsilon_*)$, $\varepsilon_* = \text{const} \in (0,1)$. *Then the inequalities*

$$|F(u^h) - b|^2_{L_2(T;H_1)} \le Ch, \tag{19}$$

$$J(u^h) - J^0 \le C_0 h^{1/2-\varkappa/2} \tag{20}$$

hold. Here $C = C(\varepsilon_*)$, $C_0 = C_0(\varepsilon_*)$, *and the element* u^h *is defined by (18).*

The validity of Corollary 1 follows from the inequalities

$$\nu_1(\alpha,\delta,j,\varepsilon,\nu^F,\nu^b,\nu^J) \le Ch,$$
$$\nu_2(\alpha,\delta,j,\varepsilon,\nu^F,\nu^b,\nu^J) \le C_0 h^{1/2-\varkappa/2}$$

and the inequalities (16) and (17).

Let a sequence of positive numbers $\{h_l\}_{l=0}^{+\infty}$, $h_l \to +0$ as $l \to +\infty$ be fixed. Let Condition 1 be fulfilled. The symbol y_l is used for the sequence constructed according to (12)–(14) for $j = j_l = [1/h_l]$, $h = h_l$, $\delta = \delta_l = h_l^{1-\varkappa}$, and $\varkappa = \text{const} \in [0,1)$. Then the following theorem is valid.

Theorem 3. *Let* $u_l = y_{j_l}/(j_l h_l^{1-\varkappa})$, $l = 1,2,\dots$. *Then any weakly convergent subsequence of the sequence* $\{u_l\}_{l=1}^{+\infty}$ *weakly converges in* $L_2(T;U)$ *to the set* $U_0(z)$. *If* $J(u) = |u|^2_{L_2(T;U)}$, *then this convergence is strong.*

3. Solving the algorithm

Let us come back to the problem of approximate calculating of the value J_z^0 and the extremal input action $u^h(\cdot)$. Let the operators A and B, as well as the initial state x_0, be inaccurately known. Namely, we have some operators $A^\nu : V \to V$ and $B^\nu : U \to H$, and also an element $x_0^\nu \in H$ such that

$$|B^\nu - B|_{L(U;H)} \le \nu^B, \tag{21}$$
$$|x_0^\nu - x_0|_H \le \nu^{x_0}. \tag{22}$$

The operator A^ν generates a semigroup of linear continuous operators $S^\nu(t), t \ge 0$, such that

$$\sup_{t\in T} |S^\nu(t) - S(t)|_{L(H;H)} \le \nu^A.$$

We also assume that, instead of the functional $\omega(t, u)$, we get functionals $\omega^\nu(t, u)$ which are convex with respect to u, such that the functionals

$$J^\nu(u(\cdot)) = \int_0^\vartheta \omega^\nu(t, u(t))\,\mathrm{d}t, \tag{23}$$

are defined on the set U_*, are lower semicontinuous and satisfy the inequalities

$$|J^\nu(u(\cdot)) - J(u(\cdot))| \le \nu^I, \quad \forall u(\cdot) \in U_*.$$

Here ν^A, ν^B, ν^{x_0}, and $\nu^I \in [0, 1)$ are given numbers.

Let the sequence $\{(y_i(\cdot), \psi_i(\cdot)\}_{i=0}^j$ of elements from $L_2(T; U) \times L_2(T; H)$ be defined by the rule:

$$y_{i+1}(\cdot) = y_i(\cdot) + \delta w_i(\cdot) \in L_2(T; U), \quad y_0(\cdot) = 0, \tag{24}$$

$$\psi_{i+1}(\cdot) = \psi_i(\cdot) + \delta\zeta_i(\cdot) \in L_2(T; H), \tag{25}$$
$$\psi_0(\cdot) = 0, \quad i = 0, 1, \dots, j-1,$$

where $w_i(\cdot)$ is a β_0-solution to the problem

$$\int_0^\vartheta \big\{2(B^{\nu*}\psi_i(t), w(t))_U + \alpha\omega^\nu(t, w(t))\big\}\,\mathrm{d}t \to \inf,$$
$$w(\cdot) \in U_*; \tag{26}$$

$$\gamma_i(\cdot) \in C(T, H), \; |\gamma_i(\cdot) - \bar\gamma_i(\cdot)|_{C(T,H)} \le \beta_2, \tag{27}$$

$\bar\gamma_i(\cdot)$ is a solution on T to the Cauchy problem

$$\dot\gamma(t) = A^\nu\gamma(t) + B^\nu w_i(t), \quad \gamma(0) = 0; \tag{28}$$

$$|\zeta_i(\cdot) - \bar\zeta_i(\cdot)|_{C(T,H)} \le \beta_1, \tag{29}$$

$\bar\zeta_i(\cdot)$ is a solution on T to the Cauchy problem

$$\dot\zeta(t) = -A^{\nu*}\zeta(t) - G^*\kappa_i(t), \quad \zeta(\vartheta) = 0; \tag{30}$$
$$\kappa_i(t) = G\gamma_i(t) - b^\nu(t), \; t \in T. \tag{31}$$

Here the symbol G^* stands for the operator adjoint to G,

$$b^\nu(t) = \xi^h(t) - GS^\nu(t)x_0^\nu.$$

Note that, in this case, we have

$$(F^\nu u(\cdot))(\eta) = \int_0^\eta GS^\nu(\eta - t)B^\nu u(t)\,\mathrm{d}t, \quad \eta \in T, \; u(\cdot) \in U_*.$$

Let

$$\psi^*_{i+1}(\cdot) = \psi^*_i(\cdot) + \delta\zeta^*_i(\cdot) \in L_2(T; H),$$
$$i = 0, 1, \dots, j-1, \tag{32}$$

where $\zeta^*_i(\cdot)$ is a solution on T of the problem

$$\dot\varrho(t) = -A^{\nu*}\varrho(t) - G^*\kappa^*_i(t), \qquad \varrho(\vartheta) = 0, \tag{33}$$
$$\kappa^*_i(t) = G\bar\gamma_i(t) - b^\nu(t), \qquad t \in T. \tag{34}$$

Here $\bar\gamma_i(\cdot)$ is a solution of the problem (28) on T.

Lemma 3. *For any $w(\cdot) \in U_*$, the following equality*

$$\int_0^\vartheta \big\{2(B^{\nu*}\psi^*_i(t), w(t))_U + \alpha\omega^\nu(t, w(t))\big\}\,\mathrm{d}t = 2\Psi_i(w(\cdot)) + \alpha J^\nu(w(\cdot)),$$
$$i = 0, \dots, j-1, \tag{35}$$

holds, where

$$\Psi_i(w(\cdot)) = \langle F^\nu y_i(\cdot) - i\delta b^\nu, F^\nu w(\cdot)\rangle. \tag{36}$$

Proof. Taking into account the structure of the functional $J^\nu(\cdot)$ (23), we conclude that (35) is equivalent to the equality

$$\int_0^\vartheta (B^{\nu*}\psi^*_i(t), w(t))_U\,\mathrm{d}t = \Psi_i(w(\cdot)),$$
$$i = 0, \dots, j-1. \tag{37}$$

Introduce the notation

$$\nu_i(\eta) = (F^\nu y_i(\cdot))(\eta) - i\delta b^\nu(\eta) = \int_0^\eta GS^\nu(\eta - t)B^\nu y_i(t)\,\mathrm{d}t - i\delta b^\nu(\eta), \quad \eta \in T. \tag{38}$$

Then

$$\Psi_i(w(\cdot)) = \int_0^\vartheta (\nu_i(\eta)\,\mathrm{d}\eta,$$

$$\int_{t_0}^{\eta} GS^{\nu}(\eta - t)B^{\nu}w(t)\,\mathrm{d}t)_{H_1}\,\mathrm{d}\eta$$
$$= \int_0^{\vartheta} (B^{\nu *}\chi_i(t), w(t))_U\,\mathrm{d}t,$$

where the symbol $(\cdot,\cdot)_{H_1}$ stands for the scalar product in H_1,

$$\chi_i(t) = \int_t^{\vartheta} S^{\nu *}(\eta - t)G^{*}\nu_i(\eta)\,\mathrm{d}\eta. \tag{39}$$

To prove (35), it is sufficient to show that

$$\psi_i^{*}(\cdot) = \chi_i(\cdot) \tag{40}$$

for $i = 0,\dots,j-1$.

Let us prove (40) by induction. For $i = 0$, we have $y_0(\cdot) = 0$. By virtue of (38) and (39), we obtain $\chi_0(\cdot) = 0 = \psi_0^{*}(\cdot)$. Assume that the equalities (40) hold for some $i < j-1$. Show that the equality

$$\psi_{i+1}^{*}(\cdot) = \chi_{i+1}(\cdot) \tag{41}$$

is also true. From (38) and (24), it follows that

$$\nu_{i+1}(\eta) = \nu_i(\eta) + \delta\{G\phi_i(\eta) - b^{\nu}(\eta)\}, \tag{42}$$

where

$$\phi_i(\eta) = \int_0^{\eta} S^{\nu}(\eta - t)B^{\nu}w_i(t)\,\mathrm{d}t.$$

Note that $\phi_i(\cdot)$ is a solution of the Cauchy problem (28), i. e., $\phi_i(\cdot) = \bar{\gamma}_i(\cdot)$. Then (see (42) and (33))

$$\nu_{i+1}(\cdot) = \nu_i(\cdot) + \delta\kappa_i^{*}(\cdot). \tag{43}$$

Further, from (39) and (43), it follows that

$$\chi_{i+1}(\cdot) = \chi_i(\cdot) + \delta\rho_i(\cdot). \tag{44}$$

where

$$\rho_i(t) = \int_t^{\vartheta} S^{\nu *}(\eta - t)G^{*}\kappa_i^{*}(\eta)\,\mathrm{d}\eta.$$

Consequently, the function $\rho_i(\cdot)$ is a solution of the Cauchy problem (33), i.e., $\rho_i(\cdot) = \bar{\zeta}_i(\cdot)$. Hence, due to (44), (34), and (40), we get (41). ■

Introduce the constants

$$C_1 = \sup\{|u|_U : u \in U_*\},$$
$$C_2 = (|B|_{L(U;H)} + 1)C_1\vartheta^{1/2},$$
$$C_3 = |G|^2_{L(H;H_1)}\vartheta K_3,$$
$$K_3 = \sup_{t\in T}\sup_{\nu\in(0,1]} |S^{\nu}(t)|_{L(H;H)}.$$

Lemma 4. *Let the elements $\{y_i(\cdot),\psi_i(\cdot)\}$ be defined according to (24)–(31). Then $\{y_i(\cdot)\}_{i=0}^{j} = Y_j^{\nu}(\delta,\alpha,\varepsilon)$, where*

$$\varepsilon = jC_2\delta(\beta_1 + C_3\beta_2) + \beta_0.$$

Proof. By definition, $w_i(\cdot)$ is a β_0-solution of the problem (26). Taking into account (35), we show that, for all $\beta_0,\beta_1,\beta_2 \ge 0$, $w_i(\cdot)$ is an ε-solution of the problem

$$2\Psi_i(w(\cdot)) + \alpha J^{\nu}(w(\cdot)) \to \inf, \quad w(\cdot) \in U_*. \tag{45}$$

It is sufficient to prove that $w_i(\cdot)$ is an ε_i-solution of the problem (45), where $\varepsilon_i = \mu_i + \beta_0$,

$$\mu_i = iC_2\delta(\beta_1 + C_3\beta_2) \tag{46}$$

(clearly, $\varepsilon_i \le \varepsilon$ for $i \le j$). For this purpose, it is sufficient to prove that the values of the functionals to be minimized in the problems (26) and (45) (for an arbitrary $w(\cdot) \in U_*$) differ by no more than μ_i or (see (36) and (37)):

$$\varepsilon_i(w(\cdot)) = \Big|\int_0^{\vartheta} 2(B^{\nu *}\psi_i^{*}(t), w(t))_U\,\mathrm{d}t - \int_0^{\vartheta} 2(B^{\nu *}\psi_i(t), w(t))_U\,\mathrm{d}t\Big| \le \mu_i. \tag{47}$$

Using the Cauchy-Bunyakovsky inequality, we get

$$\varepsilon_i(w(\cdot)) \le \Big(\int_0^{\vartheta} |B^{\nu}|^2_{L(U;H)}|w(t)|^2_U\,\mathrm{d}t\Big)^{1/2} \times \Big(\int_0^{\vartheta} |\psi_i^{*}(t) - \psi_i(t)|^2_H\,\mathrm{d}t\Big)^{1/2}$$
$$\le (|B|_{L(U;H)} + \nu^B)C_1\varepsilon_i^{(1)}\vartheta^{1/2} = C_2\varepsilon_i^{(1)},$$

where

$$\varepsilon_i^{(1)} = |\psi_i^{*}(\cdot) - \psi_i(\cdot)|_{C(T,H)}.$$

Therefore, for (47) it is sufficient to prove that

$$\varepsilon_i^{(1)} \le i\delta(\beta_1 + C_3\beta_2). \tag{48}$$

We prove the inequalities (48) by induction. Since $\psi_i^{*}(\cdot) = \psi_i(\cdot) = 0$, the relation (48) holds for $i = 0$. Assume that the relation holds for some i and prove that

$$\varepsilon_{i+1}^{(1)} \le (i+1)\delta(\beta_1 + C_3\beta_2). \tag{49}$$

Due to (31) and (34), we have

$$|\kappa_i^{*}(t) - \kappa_i(t)|_H \le |G|_{L(H;H_1)}\beta_2, \quad t \in T. \tag{50}$$

Then, by virtue of (50), the solution $\bar{\zeta}_i(\cdot)$ of the Cauchy problem (30) solves the Cauchy problem

$$\dot{\zeta}(t) = -A^{\nu *}\zeta(t) - G^{*}\kappa_i^{*}(t) + \lambda_i(t), \;\; \zeta(\vartheta) = 0,$$

where

$$\lambda_i(t) = G^*(\kappa_i(t) - \kappa_i^*(t)), \quad |\lambda_i(t)|_H \le |G|^2_{L(H;H_1)}\beta_2.$$

Therefore, the function $\zeta_i(\cdot) = \bar\zeta_i(\cdot) - \zeta_i^*(\cdot)$ is a solution on T of the Cauchy problem

$$\dot\zeta(t) = -A^{\nu *}\zeta(t) + \lambda_i(t), \quad \zeta(\vartheta) = 0.$$

Thus,

$$\begin{aligned}|\bar\zeta_i(\cdot) - \zeta_i^*(\cdot)|_{C(T,H)} &\\ \le \vartheta\beta_2|G|^2_{L(H;H_1)} \sup_{t\in T}\sup_{\nu\in[0,1]} |S^\nu(t)|_{L(H;H)} &\\ = C_3\beta_2.\end{aligned}$$

Hence, using (29), we derive

$$|\zeta_i(\cdot) - \zeta_i^*(\cdot)|_{C(T;H)} \le \beta_1 + C_3\beta_2. \tag{51}$$

By virtue of (25) and (32), we conclude from (51) that

$$\begin{aligned}\varepsilon^{(1)}_{i+1} \le \varepsilon^{(1)}_i + \delta|\zeta_i(\cdot) - \zeta_i^*(\cdot)|_{C(T;H)}&\\ \le \varepsilon^{(1)}_i + \delta(\beta_1 + C_3\beta_2).\end{aligned}$$

This implies (48). ■

Note that the inequalities

$$\begin{aligned}&|Fu - F^\nu u|_{L_2(T;H_1)}\\ &= \Big(\int_0^\vartheta \Big|\int_0^\eta G(S(\eta-t)B - S^\nu(\eta-t)B^\nu)u(t)\,dt\Big|^2_{H_1} d\eta\Big)^{1/2}\\ &\le \vartheta|G|_{L(H;H_1)}\{\nu^A|B|_{L(U;H)} + K_3\nu^B\}C_1\\ &\le \nu^F = C_4(\nu^A + \nu^B),\end{aligned} \tag{52}$$

$$\begin{aligned}&|b^\nu - b|_{L_2(T;H_1)}\\ &= \Big(\int_0^\vartheta \Big|G(S^\nu(\eta)x_0^\nu - S(\eta)x_0)\Big|^2_{L(H;H_1)} d\eta\Big)^{1/2}\\ &\quad + \Big(\int_0^\vartheta |\xi^h(t) - z(t)|^2_{H_1}\,dt\Big)^{1/2}\\ &\le \vartheta^{1/2}(|G|_{L(H;H_1)}(K_3\nu^{x_0} + |x_0|_H\nu^A) + h)\\ &\le \nu^b = C_5(\nu^{x_0} + \nu^A + h)\end{aligned} \tag{53}$$

are fulfilled. Here

$$C_4 = \vartheta|G|_{L(H;H_1)}C_1\max\{K_3, |B|_{L(U;H)}\},$$
$$C_5 = \vartheta^{1/2}(|G|_{L(H;H_1)}\max\{K_3, |x_0|_H\} + 1).$$

The next statement follows from Theorems 1 and 2 together with Lemma 4.

Theorem 4. *Let $\nu_j^A \to 0$, $\nu_j^B \to 0$, $\nu_j^{x_0} \to 0$, $h_j \to +0$, $\beta_{0,j} \to +0$, $\beta_{1,j} \to +0$, $\beta_{2,j} \to +0$ and Conditions (a)–(c) of Theorem 2, where $\nu_j^F = C_4(\nu_j^A + \nu_j^B)$, $\nu_j^b = C_5(\nu_j^{x_0} + \nu_j^A + h_j)$, $\varepsilon_j = jC_2\delta_j(\beta_{1,j} + C_3\beta_{2,j}) + \beta_{0,j}$, be fulfilled. Let the sequences $\{y_i(\cdot), \psi_i(\cdot)\}_{i=0}^j$ be defined according to (24)–(31), where $\delta = \delta_j$, $\alpha = \alpha_j$, $h = h_j$, $\beta_0 = \beta_{0,j}$, $\beta_1 = \beta_{1,j}$, $\beta_2 = \beta_{2,j}$, $\nu = \nu_j$, $|J^{\nu_j}(u(\cdot)) - J(u(\cdot))| \le \nu_j^I \equiv \nu_j^J\ \forall u(\cdot) \in U_*$. Then the sequence of elements $\{u_j\}_{j=1}^{+\infty}$,*

$$u_j = y_j(\cdot)/(\delta_j j),$$

satisfies the conditions (9) and (10), i.e., it solves the problem of approximate determination of u^0 and J^0. In addition, the inequalities (b) of Theorem 1 are valid and any weakly convergent subsequence of the sequence $\{u_j\}_{j=1}^{+\infty}$ weakly converges in $L_2(T;U)$ to the set $U_0(z)$.

Consequently from Theorem 4 and the inequalities (52), (53), the next statement follows.

Corollary 2. *Let $j = j(h) = [1/h]$, $\nu_j^A = k^{(1)}h$, $\nu_j^B = k^{(2)}h$, $\nu_j^{x_0} = k^{(3)}h$, $\delta_j = h^{1-\varkappa}$, ($\varkappa = \text{const} \in (0,1)$), $\alpha_j = h^{1/2-\varkappa/2}$, $\beta_{1,j} = k^{(4)}h$, $\beta_{2,j} = k^{(5)}h$, $\beta_{0,j} = k^{(6)}h$, $\nu_j^I = k^{(7)}h$, and $h \in (0, 1-\varepsilon_*)$, $\varepsilon_* = \text{const} \in (0,1)$. Then the inequalities (19) and (20), in which C and C_0 are some constants depending on ε_*,*

$$u^h = y_j(\cdot)/(h^{1-\varkappa}[1/h]),$$

are fulfilled. The element $y_j(\cdot)$ is defined according to (24)–(31) for $j = j(h)$.

4. Conclusions

In the present work, the problem of source function reconstruction was under investigation. A new algorithm for solving such a problem for an abstract differential equation was suggested. This algorithm relies upon constructions of the theory of stable dynamical inversion based on the combination of methods of the theory of ill-posed problems and that of feedback control. The inversion theory exploits the idea of stabilizing appropriate functionals of the Lyapunov type by means of the extremal shift.

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