

## **THEORETICAL AND EXPERIMENTAL ANALYSIS OF HEAT TRANSFER IN THE LAYERS OF ROAD PAVEMENT**

Piotr ALIAWDIN<sup>1</sup>, Jakub MARCINOWSKI<sup>2</sup>, Piotr WILK<sup>3</sup>

<sup>1</sup> University of Zielona Góra, prof. Z. Szafrana St. 2,  
65-516 Zielona Góra, Poland

<sup>2</sup> University of Zielona Góra, prof. Z. Szafrana St. 2,  
65-516 Zielona Góra, Poland

<sup>3</sup> Directorate of voivodeship roads In Zielona Góra, Niepodległości St. 32,  
65-042 Zielona Góra, Poland

A heat transfer problem for the road pavement system “layered plate – embankment – soil base” is formulated. There are suggested mathematical models in this work; they are realized with finite element analysis and design software COSMOS/M by solving layer plate on the ground foundation. Transient thermal state for this road structure is defined in numerical analysis by discretization of the system “layered plate – embankment – soil base” with triangular finite elements described by different material properties. The numerical results are compared with the experiment data for the real road pavement of Lubuski province road No 297 on the route Zagan-Kozuchow in Poland.

**Keywords:** road pavement system, embankment, heat transfer problem, mathematical models, MES, tests, experimental and numerical results

### **1. INTRODUCTION**

The problem of analysis and regulation of the water and heat regime for the road structure, including pavement and subgrade, is of great practical importance [1]. This theme, for instance, was discussed not long ago on the pages in senior journals of Russian road industry [2, 3]. The scientists also are active analyzing the problems of heat influences on the upper surface of the soils and pavements [4, 5].

In this work a problem of heat transfer is examined for the road pavement (pavement and base) as layered isotropic plate on the soil embankment for the soil base. The conductivity and convection are investigated, i.e. the change of the temperature field in the air, pavement, base of the embankment and in soil. The influences of radiation, thermoelasticity as well as change of the moisture of materials were not considered. Mathematical models for such problem were formulated; they were realized by finite element analysis and design software COSMOS/M. The numerical results were compared with the experiment data for the real road pavement in Poland.

## 2. MATHEMATICAL MODEL OF PROBLEM

A two dimensional problem of heat transfer is examined for the road pavement as layered isotropic plate on the soil embankment and the soil base (Fig. 1). The system of axial coordinate  $Oxy$  (comp. Fig. 2) is used. The governing differential equation for heat transfer for the peace-homogeneous body in the presence of heat sources is as follows [6, 7]:

$$\frac{\partial T}{\partial t} = a(\mathbf{x}) \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \frac{1}{\rho c_p} Q(\mathbf{x}, t), \quad t > 0, \quad \mathbf{x} \geq 0, \quad (1)$$

where  $T(\cdot) = T(\mathbf{x}, t)$  is a temperature, K;  $\mathbf{x} = (x, y)$  is a vector of body point,  $\mathbf{x} \in \Omega$ ;  $\Omega$  is a region of two dimensional space for variables  $\mathbf{x}$ ,  $\Omega \subseteq \mathbf{R}^2$ ;  $t$  is a time, s;  $a(\mathbf{x})$  is a heat capacity as function of coordinates  $\mathbf{x}$ , constant for the every layer of body,  $\text{m}^2/\text{c}$ ;  $a = \frac{\lambda}{\rho c_p}$ ,  $\lambda$  is a thermal conductivity,  $\text{W}/(\text{m}\cdot\text{K})$ ;  $\rho$  is a density of material,  $\text{kg}/\text{m}^3$ ;  $c_p$  is a specific heat for constant pressure,  $\text{kJ}/(\text{kg}\cdot\text{K})$ ;  $Q(\mathbf{x}, t)$  is a volumetric heat generation rate (e.g. from geothermal energy [8, 9]),  $\text{kJ}/(\text{s}\cdot\text{m}^3)$ .

Further, the temperature of the body satisfies initial as well as boundary conditions. The first condition

$$T(\mathbf{x}, 0) = \Phi_0(\mathbf{x}), \quad \mathbf{x} \in \Omega, \quad (2)$$



As a result we have two dimensional mixed boundary-value transient problem of the heat transfer (1)-(6) with boundary conditions I, II and IV type. Stationary problem will be easier, due to  $\frac{\partial T}{\partial t} = 0$  in the Eqn (1) and  $T(\cdot) = T(\mathbf{x})$ , without initial condition (2); it includes Eqn (1) and conditions (3)-(6).

### 3. FINITE ELEMENT METHOD FOR THE HEAT TRANSFER PROBLEM

Numerical solution of the given boundary problem will be performed by finite element method. To this aim the equivalent variational formulation will be used. Let us consider at the beginning the stationary problem when conditions (4)–(6) will not be taken into account. In this case for nonhomogenous (layered) body these conditions are fulfilled automatically. Let us divide the space  $\Omega$  into finite number of separate subspaces or just finite elements  $\Omega_e$ . The temperature field for every element will be approximated by linear combination of piecewise continuous functions, namely

$$\bar{T}(x, y) = \sum_{\Omega_1}^{i=1, P} C_i^{(1)} N_i^{(1)}(x, y) + \sum_{\Omega_2}^{i=1, P} C_i^{(2)} N_i^{(2)}(x, y) + \dots + \sum_{\Omega_M}^{i=1, P} C_i^{(M)} N_i^{(M)}(x, y) \quad (7)$$

where  $P$  is a number of independent  $C_i^e$  parameters in every  $e$ -th element,  $N_i^e(x, y)$  is the given shape function of the  $e$ -th element,  $M$  is a global number of elements.

In accordance to Euler theorem, the equation (1) and condition (3) are equivalent to the problem of minimization of the corresponding functional. The condition of extremum existence of this functional has the following form [10-12]:

$$\int_{\Omega} \left[ \frac{\partial N_i}{\partial x} \lambda \frac{\partial \bar{T}}{\partial x} + \frac{\partial N_i}{\partial y} \lambda \frac{\partial \bar{T}}{\partial y} \right] dx dy - \int_{\Omega} N_i Q dx dy = 0. \quad (8)$$

or taking into account the relation (7), after summation of components for every  $e$ -th element

$$\sum_{e=1}^M \int_{\Omega_e} \left[ \frac{\partial N_i^e}{\partial x} \lambda \frac{\partial \bar{T}^e}{\partial x} + \frac{\partial N_i^e}{\partial y} \lambda \frac{\partial \bar{T}^e}{\partial y} \right] dx dy - \sum_{e=1}^M \int_{\Omega_e} N_i^e Q dx dy = 0, \quad (9)$$

where

$$\bar{T}^e(x, y) = \sum_{\Omega_e}^{i=1, P} C_i^e N_i^e(x, y), \quad (10)$$

$\Omega_e$  is area of  $e$ -th element,  $e \in [1:M]$ .

For the separate finite element the integration under summation symbol of the relation (9) can be written in the following way

$$K_{ij}^e T_j^e - f_i^e \quad (i, j) \in [1:P]; \quad (11)$$

where  $K_{ij}^e$  is the conductivity matrix,

$$K_{ij}^e = \int_{\Omega_e} \left[ \frac{\partial N_i^e}{\partial x} \lambda \frac{\partial N_j^e}{\partial x} + \frac{\partial N_i^e}{\partial y} \lambda \frac{\partial N_j^e}{\partial y} \right] dx dy \quad (i, j) \in [1:P]; \quad (12)$$

$f_i^e$  is a load vector

$$f_i^e = \int_{\Omega_e} N_i^e Q dx dy \quad i \in [1:P]; \quad (13)$$

$T_j^e := C_i^e$  are unknown values of temperature at  $j$ -th nodes of  $e$ -th finite element,  $e \in [1:M]$ .

As a result of the aggregation of all finite elements, or after summation taken in approximation equations (9), these relations can be written in the form of the following matrix equation

$$[K_{ij}] \{T_j\} = \{f_i\}, \quad (i, j) \in [1:P], \quad (14)$$

where

$$K_{ij} = \sum_{e=1}^M K_{ij}^e, \quad (15)$$

$$f_i = \sum_{e=1}^M f_i^e, \quad (16)$$

$P$  is the global number of unknown values of temperature  $T_j$  at the  $j$ -th node,  $j \in [1:P]$ .

In this way the global conductivity matrix  $K_{ij}$  and load vector  $f_i$  are obtained as a result of aggregation of local objects for separate elements.

At the second step the transient problem of heat transfer (1)–(3) with conditions  $\frac{\partial T}{\partial t} \neq 0$  and  $T(\cdot) = T(\mathbf{x}, t)$  will be considered. In this case the problem will be the same as the steady problem provided the left hand side of (1) is included into the second term of the right hand side of this equation. It can be treated then just as the value proportional to the volumetric heat generation rate  $Q(x, t)$ . After some rearrangements analogous to these done above, the following matrix differential equation is obtained

$$[K_{ij}][T_j] + [C_{ij}]\frac{\partial}{\partial t}\{T_j\} = \{f_i\}, \quad (i, j) \in [1:P], \quad (17)$$

where

$$C_{ij} = \sum_{e=1}^M C_{ij}^e, \quad (18)$$

$$C_{ij}^e = \int_{\Omega_e} N_i^e \mu N_j^e dx dy, \quad i \in [1:P]; \quad (19)$$

and  $\mu := \rho c_p$ .

The numerical solution of the boundary problem (1)–(6) for the transient heat transfer was realized by means of the COSMOS/M System. Various boundary conditions were taken into account and particularly arbitrary form of initial conditions and actual law of the temperature versus time variation of the air or of the external surface of the road cover.

It is worthy to mention that also the finite difference method is admissible to this problem ([13], [14]).

#### 4. DESCRIPTION OF ROAD STRUCTURE

Elements of given embankment-type road structure are shown in the Fig.1. From the catalogue of standard structures of flexible and semi rigid pavements, at the Lubuski province, road No 297 in Poland on the route Zagan-Kozuchow were built up two experimental road sectors of 3 m width and 4 m length everyone, with following cross-sections:

##### Section 1.

1. SMA mixture-wearing course 0-12,8 grading – 5 cm thick.
2. Asphalt concrete binder course 0-20 grading – 6 cm thick.
3. Asphalt concrete base course 0-20 grading – 7 cm thick.

4. Crusher-run base course – 20 cm thick.
5. Natural base, sand with middle particles.

### Section 2.

1. SMA mixture-wearing course 0-12,8 grading – 5 cm thick.
2. Asphalt concrete binder course 0-20 grading – 7 cm thick.
3. Asphalt concrete base course 0-20 grading – 7 cm thick.
4. Cement stabilized granular aggregate base course 20 cm thick.
5. Natural base, sand with middle particles.

Characteristics of the materials are given in Table 1. Note, that in the literature the data for the soil materials are visibly different; here are taken their average values.

Road structures were built up as embankment-type, formed from non-swelling soils; group G1 of soil bearing capacity – middle dimension of sandy particles.

Table 1. Characteristics of the layer materials

Materials	Thermal conductivity $\lambda$ , W/(m·K)	Specific heat $c_p$ , kJ/(kg·K)	Density $\rho$ , kg/m <sup>3</sup>
SMA/asphalt concrete	1.4	0.47	2530
Crushed stone, stabilized mechanically	0.4	0.35	1940
Soil, stabilized by cement	0.99	0.25	1880
Sand with middle particles	0.58	0.23	1600

The temperature sensors were set into the pavement at the same time. They were placed on the axis of moving vehicle (between the wheels) for the both sectors on the following depth from the pavement surface:

- ◆ 1 cm under surface, in the wearing course ((1-0), (2-0), Fig. 1);
- ◆ 5 cm under the wearing course ((1-1), (2-1));
- ◆ 18 cm under the bituminous base course ((1-2), (2-2));
- ◆ 38/39 cm under the subbase ((1-3), (2-3));
- ◆ 80 cm, in the soil foundation ((1-4), (2-4)).

## 5. NUMERICAL AND EXPERIMENTAL RESULTS

The numerical solution of the temperature distribution was obtained by means of the finite element system COSMOS/M [15] for the adopted parameters of the road construction. The finite element TRIANG was used to discretize the do-

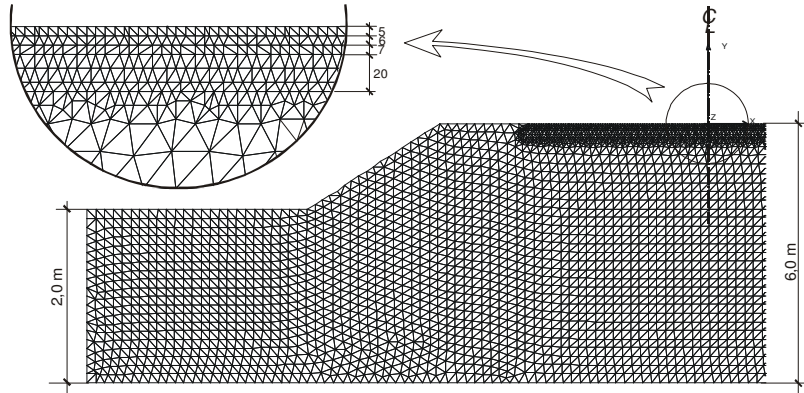


Fig. 2. The FEM mesh used in calculations

main of the road construction. The adopted mesh was shown in Fig. 2. The total number of elements was 9189, and the number of nodes, 4820, was the number of unknown nodal temperatures.

The 17 hours time interval with initial temperature conditions, which followed from the measurement data, was considered, and the time step was constant,  $\Delta t = 0,2$  h. It means that the whole analysis was performed at 85 equal time steps.

The numerical analysis was performed on the basis of temperature regis-

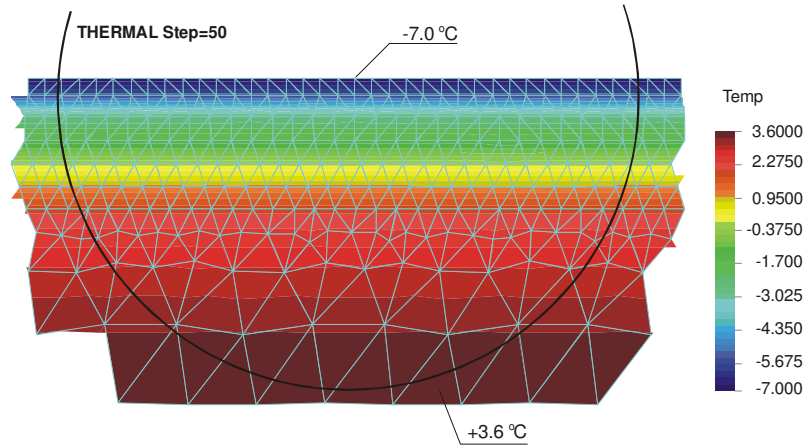


Fig. 3. Temperature distribution at tenth hour of analysis

trations which were done between 8 pm (20:00) on 01.02.2003 and 1 pm (13:00) on 02.02.2003 on the section 1 of the road No 297 in Poland. The temperature



distribution registered at 8 pm on 01.02.2003 was treated as the initial condition  $\Phi_0(\mathbf{x})$  for the numerical analysis. The temperature  $T_{mes}$  registered on the external layer surface ( $y = 0$  cm) of the road during the analysed 17 hours period was the main data  $\Phi(\cdot)$  to the transient heat transfer problem. We assumed additionally that on the depth  $y = -80$  cm the temperature is constant all the time and is equal to  $\Phi(\cdot)=3,6$  °C. As far as the material parameters are concerned, the data shown in Table 1 were adopted.

Table 2. Results of the experimental measurements and numerical analysis of the temperature distribution in the road pavement

$t$	$T_p$	$T_{mes}$	$T_{mes}$	$T_{cal}$	$\Delta T$
	°C				
$h$	y, cm				
	15	0	-5		
20:00	-6.7	-5,6	-3,7	-3,80	0,10
22:00	-7,4	-6,5	-4,3	-4,67	0,37
24:00	-7,3	-6,4	-4,7	-5,13	0,43
2:00	-7,8	-7,1	-5,1	-5,51	0,41
4:00	-7,1	-6,5	-5,3	-5,64	0,34
6:00	-8,1	-7,0	-5,7	-5,76	0,06
8:00	-7,8	-6,7	-5,8	-5,84	0,04
10:00	-4,9	-4,5	-4,4	-5,09	0,69
12:00	-2,2	-1,1	-0,9	-3,42	2,52
13:00	-1,1	0,2	0,7	-2,4	3,10

Table 2 (continuation)

$t$	$T_{mes}$	$T_{cal}$	$\Delta T$	$T_{mes}$	$T_{cal}$	$\Delta T$
	°C					
$h$	y, cm					
	-18			-38		
20:00	0,1	0,12	-0,02	2,4	2,47	-0,07
22:00	-0,3	-0,61	0,31	2,4	2,36	0,04
24:00	-0,7	-0,93	0,23	2,4	2,27	0,13
2:00	-1,1	-1,19	0,09	2,4	2,16	0,24
4:00	-1,4	-1,45	0,05	2,3	2,05	0,25
6:00	-1,7	-1,67	-0,03	2,3	1,92	0,38
8:00	-2	-1,88	-0,12	2,3	1,81	0,49
10:00	-2,2	-2,04	-0,16	2,2	1,68	0,52
12:00	-1,7	-2,07	0,37	2,2	1,56	0,64
13:00	-1,2	-2	0,8	2,2	1,51	0,69

Results of the performed analysis and experimental test data are shown in Table 2. The second column contains the temperature of air  $T_p$ . The differences

$\Delta T$  between registered temperatures  $T_{mes}$  and temperatures obtained as a result of numerical analysis  $T_{cal}$  is quite good in spite of the fact that material parameters were taken just as average for similar materials. Actual material parameters for the every road layer will be probably different.

The temperature distribution at 10-th hour of analysis (6 am on 02.02.2003) was shown in Fig. 3.

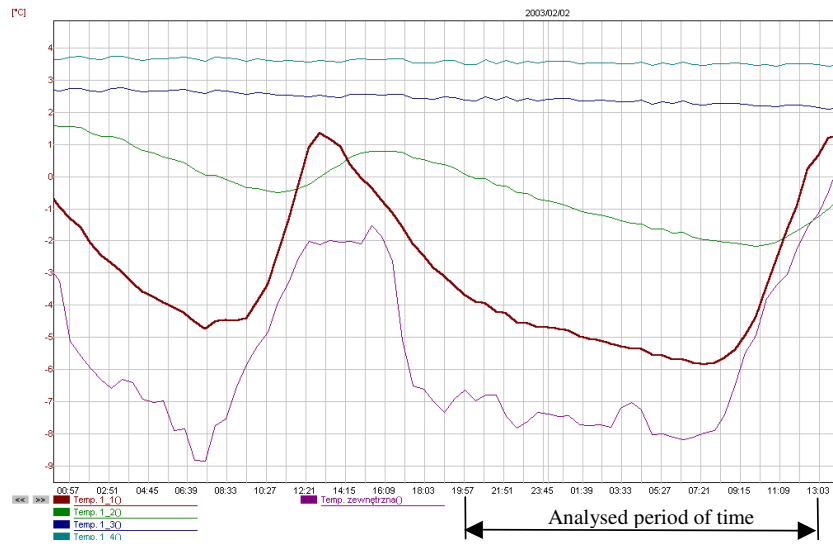


Fig. 4. The temperature distribution registered on 01÷02.02.2003 for road pavement construction with crusher-run base course

## 6. CONCLUSIONS

Analysis of the 2D- and 3D-problems for the heat transient regime of layered road structure, in the form of embankment or cavity, in general case is advisable to realize by finite element method, using suitable computer software.

It is necessary to continue the numerical and experimental investigations of these problems in order to obtain more specific information about thermal-physic parameters of road and soil materials, as well as development of numerical analysis.

---

**LITERATURE**

1. *Handbook for drafting a method of regulating moisture-temperature regime of the upper layer of ground pavement* (to SNiP 2.05.02-85). – Strojizdat, Moscow, 1989. (In Russian).
2. Ruwinski W.I.: *How to estimate the frost resistance of road pavement – it remains a riddle*, Science and technology in road construction, 2 (2002) 6-7. (In Russian).
3. Nikolski Yu.E.: *Comments on published articles*, Science and technology in road construction, 4 (2002) 34. (In Russian).
4. Peck L., Jordan R., Koenig G.: *SNTHERM-RT Predictions of Pavement Temperature*, U.S. Army Engineer Research and Development Center Cold Regions / Research and Engineering Laboratory (CRREL) August 2002.
5. Mieczkowski P.: *Calculating the temperature of upper layer of asphalt pavement – the physical model*, Drogownictwo, 8 (2001) 230-235. (In Polish).
6. Luikov A.W.: *Theory of heat and mass transfer*. Izdatelstwo Academy Nauk BelSSR, Minsk, 1961. (In Russian).
7. Luikov A.W., Michajłow Ju.A.: *Theoretical fundamentals of construction thermal physics*. Gosenergoizdat, Moscow, 1963. (In Russian).
8. *On using geothermal energy in civil engineering*, Engineering and civil engineering, 4 (2003) 238. („Beton + Zement”, 4 (2002)). (In Polish).
9. H. Zeng, N. Diao, Z. Fang.: *Heat transfer analysis of boreholes in vertical ground heat exchangers*, International Journal of Heat and Mass Transfer, **46**, 21 (2003) 4467-4481.
10. Zienkiewicz O.C., Taylor R. L. The finite element method. Ed. 5. Vol. 1: The basis, 689 p.; Vol. 2: Solid mechanics, , 459 p.; Vol. 3: Fluid dynamics, 334 p. Oxford: Butterworth-Heinemann, 2000.
11. Hou-Cheng Huang, Asif S. Usmani.: *Finite Element Analysis for Heat Transfer: Theory and Software*. Springer-Verlag, London, 1994.
12. T. Linsz.: *Layer-adapted meshes for convection-diffusion problems [review article]*, Computer Methods in Applied Mechanics and Engineering, **192**, 9-10 (2003) 1061-1105.
13. Samarski A. A., Vabischevich P.N.: *Computational Heat Transfer*, vol. 1 - *Mathematical Modelling*, vol. 2 - *The Finite Difference Methodology*, Wiley, 1995.
14. Samarski A. A., Vabischevich P.N.: *Numerical methods for solving of convection-diffusion problems*. Moscow, Editorial URSS, 1999. (In Russian).
15. COSMOS/M. Designer II. A Complete Design Analysis System. Version 1.2. Structural Research and Analysis Corporation. Los Angeles, California. Copyright. 1996.

## ANALIZA TEORETYCZNA I BADANIA DOŚWIADCZALNE WYMIANY CIEPŁA W WARSTWACH JEZDNI DROGOWEJ

### Streszczenie

W pracy został przedstawiony problem wymiany ciepła w nawierzchni drogowej jako w układzie „płyta wielowarstwowa - nasyp - podłoże gruntowe”. Podano matematyczne modele zagadnienia, które zostały rozwiązane przy pomocy metody elementów skończonych za pośrednictwem oprogramowania COSMOS/M. Stan termiczny danej konstrukcji drogowej dla nieustalonego przewodzenia ciepła został zdefiniowany w rozwiązaniach numerycznych poprzez dyskretyzację układu „płyta wielowarstwowa - nasyp - podłoże gruntowe” z trójkątnymi elementami skończonymi opisanymi przez różne właściwości termiczne materiałów. Obliczenia numeryczne zostały porównane z danymi zmierzonymi w przekroju drogi wojewódzkiej nr 297 w na odcinku Żagań-Kożuchów województwa lubuskiego w Polsce.