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EXISTENCE AND EXPONENTIAL STABILITY OF A PERIODIC SOLUTION FOR FUZZY CELLULAR NEURAL NETWORKS WITH TIME–VARYING DELAYS

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Fuzzy cellular neural networks with time-varying delays are considered. Some sufficient conditions for the existence and exponential stability of periodic solutions are obtained by using the continuation theorem based on the coincidence degree and the differential inequality technique. The sufficient conditions are easy to use in pattern recognition and automatic control. Finally, an example is given to show the feasibility and effectiveness of our methods.

Keywords: fuzzy cellular neural networks, global exponential stability, periodic solution, coincidence degree.

1. Introduction

Consider the following fuzzy cellular neural networks with time-varying delays:

$$\begin{aligned} x_i'(t) = &-c_i x_i(t) + \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) \\ &+ \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) + I_i(t) \\ &+ \bigwedge_{j=1}^{n} T_{ij}(t) u_j(t) + \bigvee_{j=1}^{n} \beta_{ij}(t) f_j(t-\tau_{ij}(t)) \\ &+ \bigvee_{j=1}^{n} H_{ij}(t) u_j(t), \quad i = 1, 2, \dots, n, \end{aligned} \tag{1}$$

where n corresponds to the number of units in a neural network, $x_i(t)$ corresponds to the state vector of the i-th unit at time t, c_i represents the rate with which the i-th unit will reset its potential to the resting state in isolation when disconnected from the network and external input, $a_{ij}(t)$ denotes the strength of the j-th unit on the i-th unit at time t, $\bigwedge$ and $\bigvee$ denote fuzzy AND and fuzzy OR operations, respectively, $f_j(\cdot)\,(j = 1, 2, \dots, n)$ are signal transmission functions, $\alpha_{ij}(t)$ and $\beta_{ij}(t)$ are respectively the elements of fuzzy feedback MIN and fuzzy feedback MAX at time t, $T_{ij}(t)$ and $H_{ij}(t)$ are respectively the elements of fuzzy feed-forward MIN and fuzzy feed-forward MAX at time t, $u_j(t)$ denotes the external inputs at time t, and $I_i(t)$ denotes the bias of the i-th unit at time t.

It is well known that the Fuzzy Cellular Neural Network (FCNN) first introduced by Yang and his co-workers (Yang and Yang, 1996; Yang *et al.*, 1996) is another type of cellular neural network model, which combines fuzzy operations (fuzzy AND and fuzzy OR) with cellular neural networks. As dynamical systems with a special structure, FCNNs have many interesting properties that deserve theoretical studies. In recent years, autonomous FCNNs have been extensively studied and successfully applied to image processing and to solve nonlinear algebraic equations. Such applications rely on the qualitative properties of stability (Huang, 2006; Liu and Tang, 2004; Yuan *et al.*, 2006; Zhang and Xiang, 2008; Zhang and Luo, 2009; Liu *et al.*, 2009; Niu *et al.*, 2008). During hardware implementation, time delays occur due to finite switching speeds of the amplifiers and communication time. Time delays may lead to oscillations and, furthermore, to network instability. Therefore, the study of the stability of FCNNs with delay is required in practice. However, non-autonomous phenomena often occurs in many realistic systems, particularly, when we consider long-time dynamical behavior of a system. The system parameters and time delays will usually change in time. Thus the research on non-autonomous FCNNs is very important, just like that on autonomous FCNNs.

So for, many important results on CNNs have been obtained regarding the existence of equilibria, global asymptotic stability, global exponential stability (Kosto, 1987; 1988; Gopalsmy and He, 1994; Cao and Wang, 2002; Cao, 2003; Cao and Dong, 2003; Chen *et al.*, 2004; Liu *et al.*, 2003; Liao and Yu, 1998; Zhao, 2006; 2002; Arik and Tavsanoglu, 2005; Tian *et al.*, 2010; Wang *et al.*, 2007; Raja *et al.*, 2011). Especially the investigation of CNNs with periodic coefficients and delays has attracted more and more attention of researchers (Liu and Tang, 2006; Liu and Huang, 2006). To the best of our knowledge, few authors consider the stability of fuzzy cellular neural networks with periodic coefficients and time-varying delays. Motivated by the above discussion, in this paper, by using the continuation theorem of coincidence degree theory and the differential inequality technique, we will give some sufficient conditions for the existence and exponential stability of periodic solutions to the system (1).

Throughout this paper, we always assume that $a_{ij}(t), \alpha_{ij}(t), \beta_{ij}(t), \tau_{ij}(t), T_{ij}(t), H_{ij}(t), u_j(t)$ and $I_i(t)$ are continuous ω-periodic functions, where $i, j = 1, 2, \ldots, n$, $\tau = \max_{1\le i,j\le n}\{\max_{t\in[0,\omega]} \tau_{ij}(t)\}$.

For convenience, we introduce the following notation. Let $r(t)$ be a ω-periodic solution defined on $\mathbb{R}$,

$$r^+ = \max_{0\le t\le\omega} |r(t)|, \quad \overline{r} = \frac{1}{\omega}\int_0^\omega r(t)\,\mathrm{d}t,$$

$$\|r\|_2 = \left(\int_0^\omega |r(t)|^2\,\mathrm{d}t\right)^{1/2}.$$

We will use $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ to denote a column vector (symbol 'T' denotes the transpose of a vector). For a matrix $D = (d_{ij})_{n\times n}$, D^T denotes the transpose of D, and E_n denotes the identity matrix of size n. For a matrix or a vector $D \ge 0$ means that all entries of D are greater than or equal to zero. $D > 0$ can be defined similarly. For a matrix or a vector, $D \ge E$ (respectively, $D > E$) means that $D-E \ge 0$ (respectively, $D-E > 0$).

The initial conditions associated with the system (1) are of the form

$$x_i(s) = \varphi_i(s), \quad s \in [-\tau, 0], \quad i = 1, 2, \ldots, n, \tag{2}$$

where

$$\varphi = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t))^T \in C([-\tau, 0], \mathbb{R}^n).$$

Throughout this paper, we make the following assumptions:

(A1) $f_j(\cdot)$ is Lipschitz continuous on $\mathbb{R}$ with Lipschitz constants p_j ($j = 1, 2, \ldots, n$), and $f_j(0) = 0$. That is, for all $x, y \in \mathbb{R}$,

$$|f_j(x) - f_j(y)| \le p_j|x - y|.$$

(A2) There exist non-negative constants p_j and q_j such that $|f_j(x)| \le p_j|x| + q_j$, for $j = 1, 2, \ldots, n, x \in \mathbb{R}$.

Definition 1. The periodic solution $z^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ of the system (1) with the initial value $\varphi^* = (\varphi_1^*, \varphi_2^*, \ldots, \varphi_n^*)^T \in C([-\tau, 0], \mathbb{R}^n)$ is said to be *globally exponentially stable* if there exist constants $\lambda > 0$ and $M \ge 1$ such that

$$|x_i(t) - x_i^*(t)| \le M\|\varphi - \varphi^*\|e^{-\lambda t}, \qquad \forall t > 0, \quad i = 1, 2, \ldots, n,$$

for every solution $z(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ of the system (1) with the initial value $\varphi \in C([-\tau, 0], \mathbb{R}^n)$.

Definition 2. A real matrix $A = (a_{ij})_{n\times n}$ is said to be an *M-matrix* if $a_{ij} \le 0, i, j = 1, 2, \ldots, n, i \ne j$, and $A^{-1} \ge 0$.

Lemma 1. (Liao and Yu, 1998) *Let $A = (a_{ij})$ be an $n\times n$ matrix with non-positive off-diagonal elements. Then the following statements are equivalent:*

(i) A is an M-matrix.

(ii) The real parts of all eigenvalues of A are positive.

(iii) There exists a vector $\eta > 0$ such that $A\eta > 0$.

(iv) There exists a vector $\xi > 0$ such that $\xi^T A > 0$.

Lemma 2. (Yang and Yang, 1996) *Suppose x and y are two states of the system (1). Then we have*

$$\Big|\bigwedge_{j=1}^n \alpha_{ij}(t)f_j(x) - \bigwedge_{j=1}^n \alpha_{ij}(t)f_j(y)\Big| \le \sum_{j=1}^n |\alpha_{ij}(t)||f_j(x) - f_j(y)|,$$

and

$$\Big|\bigvee_{j=1}^n \beta_{ij}(t)f_j(x) - \bigvee_{j=1}^n \beta_{ij}(t)f_j(y)\Big| \le \sum_{j=1}^n |\beta_{ij}(t)||f_j(x) - f_j(y)|.$$

The rest of this paper is organized as follows. In Section 2, we will prove the existence of a periodic solution by using the continuation theorem of coincidence degree theory. In Section 3, we establish the result that periodic solutions are globally exponentially stable by using the Lyapunov function method. In Section 4, an example will be given to illustrate the feasibility and effectiveness of our methods. General conclusions are drawn in Section 5.

2. Existence of a periodic solution

In this section, based on Mawhin's continuation theorem, we shall study the existence of at least one periodic solution of (1). To do so, we need some prerequisites.

For ease of exposition, throughout this paper will adopt the following notation:

$$|x_i|_\infty = \max_{t\in[0,\omega]} |x_i(t)|,$$
$$u(t) = (x_1(t), x_2(t), \dots, x_n(t))^T,$$
$$|x_i|_2 = \left(\int_0^\omega |x_i(t)|^2 \,\mathrm{d}t\right)^{1/2}, \quad i = 1, 2, \dots, n.$$

We denote by X the set of all continuously ω-periodic solutions $u(t)$ defined on $\mathbb{R}$, and write

$$\|u\|_X = \max\{|x_1|_\infty, |x_2|_\infty, \dots, |x_n|_\infty\}.$$

Consider the following abstract equation in the Banach space X:

$$Lx = \lambda Nx, \tag{3}$$

where $L : \mathrm{Dom} L \cap X \to X$ is a Fredholm mapping of index zero and $\lambda \in [0, 1]$ is a parameter. There exist two linear and continuous projectors P and Q,

$$P : X \cap \mathrm{Dom}\, L \to \mathrm{Ker}\, L, \quad Q : X \to X/\mathrm{Im}\, L,$$

such that $\mathrm{Im}\, P = \mathrm{Ker}\, L$, $\mathrm{Ker}\, Q = \mathrm{Im}\, L$. Since $\dim \mathrm{Im}\, Q = \dim \mathrm{Ker}\, L$, there exists an algebraical and topological isomorphism $J : \mathrm{Im}\, Q \to \mathrm{Ker}\, L$.

Lemma 3. (Gaines and Mawhin, 1990) *Let X be a Banach space and let L be a Fredholm mapping of index zero. Assume that $N : \overline{\Omega} \to X$ is L compact on $\overline{\Omega}$ with Ω open and bounded in X. Furthermore, suppose that*

(a) for each $\lambda \in (0, 1), x \in \partial\Omega \cap \mathrm{Dom}\, L, Lx \neq \lambda Nx$,

(b) for each $x \in \partial\Omega \cap \mathrm{Ker}\, L, QNx \neq 0$,

(c) $\deg\{QNx, \Omega \cap \mathrm{Ker}\, L, 0\} \neq 0$.

Then the equation $Lx = Nx$ has at least one solution in $\overline{\Omega}$, where $\overline{\Omega}$ is the closure to Ω, $\partial\Omega$ is the boundary of Ω.

Theorem 1. *Assume that $(A2)$ holds, and the following condition is satisfied:*

$(A3)$ $E_n - D$ is an M-matrix, where $D = (d_{ij})_{n\times n}$, $d_{ij} = \frac{1}{c_i}(a^+_{ij} + \alpha^+_{ij} + \beta^+_{ij})p_j$, $\quad i, j = 1, 2, \dots, n$. Then the system (1) has at least one ω-periodic solution.

Proof. In order to use the continuation theorem of coincidence degree theory to establish the existence of a periodic solution, let

$$\begin{aligned}
&(Nu)_i(t)\\
&\quad = -c_i x_i(t) + \sum_{j=1}^n a_{ij}(t) f_j(x_j(t - \tau_{ij}(t)))\\
&\qquad + \bigwedge_{j=1}^n \alpha_{ij}(t) f_j(x_j(t - \tau_{ij}(t))) + I_i(t)\\
&\qquad + \bigwedge_{j=1}^n T_{ij}(t) u_j(t) + \bigvee_{j=1}^n \beta_{ij}(t) f_j(t - \tau_{ij}(t))\\
&\qquad + \bigvee_{j=1}^n H_{ij}(t) u_j(t), \quad i = 1, 2, \dots, n.
\end{aligned} \tag{4}$$

$$(Lu)(t) = u'(t) = (x'_1(t), x'_2(t), \dots, x'_n(t))^T, \tag{5}$$
$$\mathrm{Dom}\, L = \{u(t) : u(t) \in X, u'(t) \in X\}, \tag{6}$$

$$\begin{aligned}
Pu = Qu &= \frac{1}{\omega}\int_0^\omega u(t)\,\mathrm{d}t\\
&= \left(\frac{1}{\omega}\int_0^\omega x_1(t)\,\mathrm{d}t, \dots, \frac{1}{\omega}\int_0^\omega x_n(t)\,\mathrm{d}t\right)^T
\end{aligned}$$

for $u(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in X \cap \mathrm{Dom}\, L$. It is easy to prove that L is a Fredholm mapping of index zero, $P : X \cap \mathrm{Dom}\, L \to \mathrm{Ker}\, L$ and $Q : X \to X/\mathrm{Im}\, L$ are two projectors, and N is L compact on $\overline{\Omega}$ for any given open bounded set.

In view of (4)–(6), the operator equation $Lx = \lambda Nx, \lambda \in (0, 1)$, is equivalent to the following one:

$$\begin{aligned}
x'_i(t) = \lambda \Bigg[&-c_i x_i(t) + \sum_{j=1}^n a_{ij}(t) f_j(x_j(t - \tau_{ij}(t)))\\
&+ \bigwedge_{j=1}^n \alpha_{ij}(t) f_j(x_j(t - \tau_{ij}(t))) + I_i(t)\\
&+ \bigwedge_{j=1}^n T_{ij}(t) u_j(t) + \bigvee_{j=1}^n \beta_{ij}(t) f_j(t - \tau_{ij}(t))\\
&+ \bigvee_{j=1}^n H_{ij}(t) u_j(t) \Bigg], \quad i = 1, 2, \dots, n.
\end{aligned} \tag{7}$$

Suppose that $u(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in X$ is a solution of the system (7) for a certain $\lambda \in (0, 1)$. Then $x_i(t)$ is continuously differentiable $(i = 1, 2, \dots, n)$. Therefore, there exists $t_i \in [0, \omega]$ such that $|x_i(t_i)| = \max_{t\in[0,\omega]} |x_i(t)|$. Hence $x'_i(t_i) = 0$ $(i =$

$1,2,\ldots,n$). This implies that, for $i=1,2,\ldots,n$,

$$
\begin{aligned}
c_i x_i(t_i) = &\sum_{j=1}^{n} a_{ij}(t_i) f_j(x_j(t_i-\tau_{ij}(t_i))) \\
&+ \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(x_j(t_i-\tau_{ij}(t_i))) + I_i(t_i) \\
&+ \bigwedge_{j=1}^{n} T_{ij}(t_i) u_j(t_i) + \bigvee_{j=1}^{n} H_{ij}(t_i) u_j(t_i) \\
&+ \bigvee_{j=1}^{n} \beta_{ij}(t_i) f_j(t_i-\tau_{ij}(t_i)).
\end{aligned}
\tag{8}
$$

Thus

$$
\begin{aligned}
|x_i(t_i)| = &\Bigg| \frac{1}{c_i}\Big[\sum_{j=1}^{n} a_{ij}(t_i) f_j(x_j(t_i-\tau_{ij}(t_i))) \\
&+ \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(x_j(t_i-\tau_{ij}(t_i))) + I_i(t_i) \\
&+ \bigwedge_{j=1}^{n} T_{ij}(t_i) u_j(t_i) + \bigvee_{j=1}^{n} H_{ij}(t_i) u_j(t_i) \\
&+ \bigvee_{j=1}^{n} \beta_{ij}(t_i) f_j(t_i-\tau_{ij}(t_i))\Big]\Bigg| \\
\le &\sum_{j=1}^{n} \frac{1}{c_i} |a_{ij}(t_i)||f_j(x_j(t_i-\tau_{ij}(t_i)))| \\
&+ \bigwedge_{j=1}^{n} \frac{1}{c_i} |\alpha_{ij}(t)||f_j(x_j(t_i-\tau_{ij}(t_i)))| \\
&+ \bigvee_{j=1}^{n} \frac{1}{c_i} |\beta_{ij}(t_i)||f_j(t_i-\tau_{ij}(t_i))| \\
&+ \bigvee_{j=1}^{n} \frac{1}{c_i} |H_{ij}(t_i)||u_j(t_i)| + \frac{1}{c_i}|I_i(t_i)| \\
&+ \bigwedge_{j=1}^{n} \frac{1}{c_i} |T_{ij}(t_i)||u_j(t_i)| \\
\le &\sum_{j=1}^{n} \frac{1}{c_i} (a_{ij}^{+} + \alpha_{ij}^{+} + \beta_{ij}^{+}) p_j |x_j(t_i-\tau_{ij}(t_i))| \\
&+ \sum_{j=1}^{n} \frac{1}{c_i} (a_{ij}^{+} + \alpha_{ij}^{+} + \beta_{ij}^{+}) q_j \\
&+ \bigwedge_{j=1}^{n} \frac{1}{c_i} T_{ij}^{+} u_j^{+} + \bigvee_{j=1}^{n} \frac{1}{c_i} H_{ij}^{+} u_j^{+} + \frac{1}{c_i} I_i^{+} \\
\le &\sum_{j=1}^{n} \frac{1}{c_i} (a_{ij}^{+} + \alpha_{ij}^{+} + \beta_{ij}^{+}) p_j |x_j(t_j)| \\
&+ \sum_{j=1}^{n} \frac{1}{c_i} (a_{ij}^{+} + \alpha_{ij}^{+} + \beta_{ij}^{+}) q_j + \bigwedge_{j=1}^{n} \frac{1}{c_i} T_{ij}^{+} u_j^{+} + \frac{1}{c_i} I_i^{+} \\
\le &\sum_{j=1}^{n} d_{ij} |x_j(t_j)| + G_i,
\end{aligned}
\tag{9}
$$

where

$$
\begin{aligned}
G_i = &\sum_{j=1}^{n} \frac{1}{c_i} (a_{ij}^{+} + \alpha_{ij}^{+} + \beta_{ij}^{+}) q_j \\
&+ \bigwedge_{j=1}^{n} \frac{1}{c_i} T_{ij}^{+} u_j^{+} + \frac{1}{c_i} I_i^{+}, \quad i=1,2,\ldots,n.
\end{aligned}
$$

From (9), it follows that

$$
\begin{aligned}
&(E_n - D)(|x_1(t_1)|, |x_2(t_2)|, \ldots, |x_n(t_n)|)^T \\
&\quad \le (G_1, G_2, \ldots, G_n)^T := G.
\end{aligned}
\tag{10}
$$

Since $E_n - D$ is an M-matrix, from (A3) and Lemma 1 it follows that there exists a vector $\eta = (\eta_1, \eta_2, \ldots, \eta_n) > (0,0,\ldots,0)$ such that

$$
\overline{\eta} = (\overline{\eta}_1, \overline{\eta}_2, \ldots, \overline{\eta}_n) = \eta(E_n - D) > (0,0,\ldots,0),
\tag{11}
$$

which, together with (10), implies that

$$
\begin{aligned}
&\min\{\overline{\eta}_1, \overline{\eta}_2, \ldots, \overline{\eta}_n\}(|x_1(t_1)| + \cdots + |x_n(t_n)|) \\
&\quad \le \overline{\eta}_1|x_1(t_1)| + \overline{\eta}_2|x_2(t_2)| + \cdots + \overline{\eta}_n|x_1(t_n)| \\
&\quad = \eta(E_n - D)(|x_1(t_1)|, |x_2(t_2)|, \ldots, |x_n(t_n)|)^T \\
&\quad \le \eta(G_1, G_2, \ldots, G_n)^T \\
&\quad = \eta_1 G_1 + \eta_2 G_2 + \cdots + \eta_n G_n.
\end{aligned}
\tag{12}
$$

Therefore,

$$
\begin{aligned}
|x_i|_\infty &= \max_{t\in[0,\omega]} |x_i(t)| = |x_i(t_i)| \\
&\le \frac{1}{\min\limits_{1\le i\le n}\{\overline{\eta}_i\}} (\eta_1 G_1 + \eta_2 G_2 + \cdots + \eta_n G_n) \\
&:= \delta.
\end{aligned}
\tag{13}
$$

From (A3) and Lemma 1, we have that there exists a vector $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^T > (0,0,\ldots,0)^T$ such that $(E_n - D)\xi > 0$. Therefore, we can choose a positive number $a > 1$ such that $\overline{\xi} = (\overline{\xi}_1, \overline{\xi}_2, \ldots, \overline{\xi}_n)^T = (a\xi_1, a\xi_2, \ldots, a\xi_n)^T = a\xi$ and

$$
\overline{\xi}_i = a\xi_i > \delta, \quad (E_n - D)\overline{\xi} > G.
\tag{14}
$$

Take

$$
\Omega = \{u : u(t) \in X, |u(t)| < \overline{\xi}, \forall t \in \mathbb{R}\},
\tag{15}
$$

which satisfies the condition (a) of Lemma 3. If $u(t) = (x_1(t), x_2(t), \ldots, x_n(t)) \in \partial\Omega \cap \operatorname{Ker} L$, then $u(t)$ is a constant vector on $\mathbb{R}^n$. Hence there exist some $i \in \{1,2,\ldots,n\}$ such that $|x_i| = \overline{\xi}_i$. It follows that

$$(QNu)_i = -c_i x_i + \sum_{j=1}^{n} f_j(x_j)\frac{1}{\omega}\int_0^{\omega} a_{ij}(t)\,\mathrm{d}t + \bigwedge_{j=1}^{n} f_j(x_j)\frac{1}{\omega}\int_0^{\omega}\alpha_{ij}(t)\,\mathrm{d}t + \bigwedge_{j=1}^{n}\frac{1}{\omega}\int_0^{\omega} H_{ij}(t)u_j(t)\,\mathrm{d}t + \frac{1}{\omega}\int_0^{\omega} I_i(t)\,\mathrm{d}t + \bigvee_{j=1}^{n} f_j(x_j)\frac{1}{\omega}\int_0^{\omega}\beta_{ij}(t)\,\mathrm{d}t + \bigvee_{j=1}^{n}\frac{1}{\omega}\int_0^{\omega} T_{ij}(t)u_j(t)\,\mathrm{d}t. \tag{16}$$

We claim that

$$|(QNu)_i| > 0, \quad i = 1,2,\ldots,n. \tag{17}$$

To get a contradiction, assume that $|(QNu)_i| = 0$, namely,

$$0 = -c_i x_i + \sum_{j=1}^{n} f_j(x_j)\frac{1}{\omega}\int_0^{\omega} a_{ij}(t)\,\mathrm{d}t + \bigwedge_{j=1}^{n} f_j(x_j)\frac{1}{\omega}\int_0^{\omega}\alpha_{ij}(t)\,\mathrm{d}t + \bigwedge_{j=1}^{n}\frac{1}{\omega}\int_0^{\omega} H_{ij}(t)u_j(t)\,\mathrm{d}t + \bigvee_{j=1}^{n} f_j(x_j)\frac{1}{\omega}\int_0^{\omega}\beta_{ij}(t)\,\mathrm{d}t + \bigvee_{j=1}^{n}\frac{1}{\omega}\int_0^{\omega} T_{ij}(t)u_j(t)\,\mathrm{d}t + \frac{1}{\omega}\int_0^{\omega} I_i(t)\,\mathrm{d}t.$$

Then there exists some $t^* \in [0,\omega]$ such that

$$0 = -c_i x_i + \sum_{j=1}^{n} a_{ij}(t^*)f_j(x_j) + \bigwedge_{j=1}^{n}\alpha_{ij}(t^*)f_j(x_j) + \bigvee_{j=1}^{n}\beta_{ij}(t^*)f_j(x_j) + \bigwedge_{j=1}^{n} T_{ij}(t^*)u_j(t^*) + \bigvee_{j=1}^{n} H_{ij}(t^*)u_j(t^*) + I_i(t^*).$$

Therefore,

$$\begin{aligned}\overline{\xi}_i &= |x_i| \\ &\le \sum_{j=1}^{n}\frac{1}{c_i}|a_{ij}(t^*)||f_j(x_j)| + \bigwedge_{j=1}^{n}\frac{1}{c_i}|\alpha_{ij}(t^*)||f_j(x_j)| \\ &\quad + \bigvee_{j=1}^{n}\frac{1}{c_i}|\beta_{ij}(t^*)||f_j(x_j)| + \bigwedge_{j=1}^{n}\frac{1}{c_i}|T_{ij}(t^*)||u_j(t^*)| \\ &\quad + \bigvee_{j=1}^{n}\frac{1}{c_i}|H_{ij}(t^*)||u_j(t^*)| + \frac{1}{c_i}|I_i(t^*)| \\ &\le \sum_{j=1}^{n}\frac{1}{c_i}a_{ij}^{+}p_j|x_j| + \sum_{j=1}^{n}\frac{1}{c_i}\alpha_{ij}^{+}p_j|x_j| \\ &\quad + \sum_{j=1}^{n}\frac{1}{c_i}\beta_{ij}^{+}p_j|x_j| + \sum_{j=1}^{n}\frac{1}{c_i}(a_{ij}^{+}+\alpha_{ij}^{+}+\beta_{ij}^{+})q_j \\ &\quad + \bigwedge_{j=1}^{n}\frac{1}{c_i}T_{ij}^{+}u_j^{+} + \bigvee_{j=1}^{n}\frac{1}{c_i}H_{ij}^{+}u_j^{+} + \frac{1}{c_i}I_i^{+} \\ &= \sum_{j=1}^{n} d_{ij}|x_j| + G_i \le \sum_{j=1}^{n} d_{ij}\overline{\xi}_j + G_i.\end{aligned}$$

It follows that $((E_n - D)\overline{\xi})_i \le G_i$, which contradicts $(E_n - D)\overline{\xi} > G$. Therefore (17) holds, i.e., the condition (b) of Lemma 3 is satisfied.

Next, we define a continuous function $\Phi : \Omega \cap \operatorname{Ker} L \times [0,1] \to X$ by

$$\Phi(u,\rho) = \rho\,\mathrm{diag}(-c_1,-c_2,\ldots,-c_n)u + (1-\rho)QNu, \tag{18}$$

for all $u = (x_1,x_2,\ldots,x_n)^T \in \Omega \cap \operatorname{Ker} L = \Omega \cap \mathbb{R}^n$ and $\rho \in [0,1]$. If $u(t) = (x_1(t),x_2(t),\ldots,x_n(t))^T \in \partial\Omega \cap \operatorname{Ker} L$, then $u(t)$ is a constant vector in $\mathbb{R}^n$, and there exists some $i \in \{1,2,\ldots,n\}$ such that $|x_i| = \overline{\xi}_i$. It follows that

$$\begin{aligned}&(\Phi(u,\rho))_i \\ &\quad = -c_i x_i + (1-\rho)\Big[\sum_{j=1}^{n} f_j(x_j)\frac{1}{\omega}\int_0^{\omega} a_{ij}(t)\,\mathrm{d}t \\ &\qquad + \bigwedge_{j=1}^{n} f_j(x_j)\frac{1}{\omega}\int_0^{\omega}\alpha_{ij}(t)\,\mathrm{d}t + \frac{1}{\omega}\int_0^{\omega} I_i(t)\,\mathrm{d}t \\ &\qquad + \bigvee_{j=1}^{n} f_j(x_j)\frac{1}{\omega}\int_0^{\omega}\beta_{ij}(t)\,\mathrm{d}t \\ &\qquad + \bigvee_{j=1}^{n}\frac{1}{\omega}\int_0^{\omega} T_{ij}(t)u_j(t)\,\mathrm{d}t \\ &\qquad + \bigwedge_{j=1}^{n}\frac{1}{\omega}\int_0^{\omega} H_{ij}(t)u_j(t)\,\mathrm{d}t\Big].\end{aligned} \tag{19}$$

We claim that

$$|(\Phi(u,\rho))_i| > 0. \tag{20}$$

If this is not true, then $|(\Phi(u,\rho))_i| = 0$. Indeed,

$$0 = -c_i x_i + (1-\rho)\Bigg[\sum_{j=1}^n f_j(x_j)\frac{1}{\omega}\int_0^\omega a_{ij}(t)\,\mathrm{d}t + \bigwedge_{j=1}^n f_j(x_j)\frac{1}{\omega}\int_0^\omega \alpha_{ij}(t)\,\mathrm{d}t + \frac{1}{\omega}\int_0^\omega I_i(t)\,\mathrm{d}t + \bigvee_{j=1}^n f_j(x_j)\frac{1}{\omega}\int_0^\omega \beta_{ij}(t)\,\mathrm{d}t + \bigvee_{j=1}^n \frac{1}{\omega}\int_0^\omega T_{ij}(t)u_j(t)\,\mathrm{d}t + \bigwedge_{j=1}^n \frac{1}{\omega}\int_0^\omega H_{ij}(t)u_j(t)\,\mathrm{d}t\Bigg].$$

Therefore there exists some $t^{**} \in [0,\omega]$ such that

$$0 = -c_i x_i + (1-\rho)\Bigg[\sum_{j=1}^n a_{ij}(t^{**})f_j(x_j) + \bigwedge_{j=1}^n \alpha_{ij}(t^{**})f_j(x_j) + I_i(t^{**}) + \bigvee_{j=1}^n \beta_{ij}(t^{**})f_j(x_j) + \bigvee_{j=1}^n T_{ij}(t^{**})u_j(t^{**}) + \bigwedge_{j=1}^n H_{ij}(t^{**})u_j(t^{**})\Bigg].$$

Thus

$$\begin{aligned}\overline{\xi}_i &= |x_i| \\ &\le (1-\rho)\Bigg[\sum_{j=1}^n \frac{1}{c_i}|a_{ij}(t^{**})||f_j(x_j)| + \frac{1}{c_i}|I_i(t^{**})| + \bigwedge_{j=1}^n \frac{1}{c_i}|\alpha_{ij}(t^{**})||f_j(x_j)| + \bigvee_{j=1}^n \frac{1}{c_i}|\beta_{ij}(t^{**})||f_j(x_j)| \\ &\quad + \bigwedge_{j=1}^n \frac{1}{c_i}|T_{ij}(t^{**})||u_j(t^{**})| + \bigvee_{j=1}^n \frac{1}{c_i}|H_{ij}(t^{**})||u_j(t^{**})|\Bigg] \\ &\le \sum_{j=1}^n \frac{1}{c_i}a_{ij}^+ p_j|x_j| + \sum_{j=1}^n \frac{1}{c_i}\alpha_{ij}^+ p_j|x_j| + \sum_{j=1}^n \frac{1}{c_i}\beta_{ij}^+ p_j|x_j| + \sum_{j=1}^n \frac{1}{c_i}(a_{ij}^+ + \alpha_{ij}^+ + \beta_{ij}^+)q_j \\ &\quad + \bigwedge_{j=1}^n \frac{1}{c_i}T_{ij}^+ u_j^+ + \bigvee_{j=1}^n \frac{1}{c_i}H_{ij}^+ u_j^+ + \frac{1}{c_i}I_i^+ \\ &= \sum_{j=1}^n d_{ij}|x_j| + G_i \le \sum_{j=1}^n d_{ij}\overline{\xi}_j + G_i.\end{aligned}$$

This implies that $((E_n - D)\overline{\xi})_i \le G_i$, which contradicts $(E_n - D)\overline{\xi} > G$. Therefore (20) holds, which means that

$$\Phi(x_1, x_2, \dots, x_n, \rho) \ne (0, 0, \dots, 0)^T,$$

$\forall (x_1, x_2, \dots, x_n) \in \partial\Omega \cap \operatorname{Ker} L$, $\rho \in [0,1]$. Using the homotopy invariance theorem, we have

$$\begin{aligned}&\deg\{QN, \Omega \cap \operatorname{Ker} L, (0,0,\dots,0)^T\} \\ &= \deg\{(-c_1x_1, -c_2x_2, \dots, -c_nx_n)^T, \\ &\quad \Omega \cap \operatorname{Ker} L, (0,0,\dots,0)^T\} \ne 0.\end{aligned}$$

To summarize, we have proved that Ω satisfies all the conditions of Lemma 3. Thus, by Lemma 3, it follows that $Lx = Nx$ has at least one solution in X, namely, the system (1) has at least one ω-periodic solution. The proof is complete. ■

3. Global exponential stability of periodic solutions

In this section, we will construct some suitable Lyapunov function to study the global exponential stability of the periodic solution of the system (1).

Theorem 2. *If Assumptions (A1) and (A3) are satisfied, then the system (1) has exactly one ω-periodic solution, which is globally exponentially stable.*

Proof. From Theorem 1, the system (1) has at least one ω-periodic solution $z^*(t) = (x_1^*(t), \dots, x_n^*(t))^T$. Suppose that $z(t) = (x_1(t), \dots, x_n(t))^T$ is an arbitrary solution of (1). Then from the system (1) it follows that, for $i = 1, 2, \dots, n$,

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}(x_i(t) - x_i^*(t)) \\ &= -c_i(x_i(t) - x_i^*(t)) \\ &\quad + \sum_{j=1}^n a_{ij}(t)(f_j(x_j(t-\tau_{ij}(t))) - f_j(x_j^*(t))) \\ &\quad + \bigwedge_{j=1}^n \alpha_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) - \bigwedge_{j=1}^n \alpha_{ij}(t)f_j(x_j^*(t)) \\ &\quad + \bigvee_{j=1}^n \beta_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) - \bigvee_{j=1}^m \beta_{ij}(t)f_j(x_j^*(t)).\end{aligned}$$

By (A1) and Lemma 2, we have

$$\begin{aligned} D^-|x_i(t)-x_i^*(t)| &\le -c_i|x_i(t)-x_i^*(t)| \\ &+\sum_{j=1}^n |a_{ij}(t)||f_j(t-\tau_{ij}(t))-f_j(x_j^*(t))| \\ &+\Big|\bigwedge_{j=1}^n \alpha_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) \\ &-\bigwedge_{j=1}^n \alpha_{ij}(t)f_j(x_j^*(t))\Big| \\ &+\Big|\bigvee_{j=1}^n \beta_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) \\ &-\bigvee_{j=1}^n \beta_{ij}(t)f_j(x_j^*(t))\Big| \\ \le -c_i|x_i(t)-x_i^*(t)| &+\sum_{j=1}^n (a_{ij}^+ +\alpha_{ij}^+ +\beta_{ij}^+) \\ &\times p_j|x_j(t-\tau_{ij}(t))-x_j^*(t)|, \end{aligned} \tag{21}$$

where D^- denotes the upper left derivative. If we let $y_i(t)=x_i(t)-x_i^*(t)$, then (21) becomes

$$\begin{aligned} &D^-|y_i(t)| \\ &\le -c_i|y_i(t))|+\sum_{j=1}^n (a_{ij}^+ +\alpha_{ij}^+ +\beta_{ij}^+)p_j \sup_{t-\tau\le s\le t}|y_j(s)| \\ &= -c_i|y_i(t))|+\sum_{j=1}^n (a_{ij}^+ +\alpha_{ij}^+ +\beta_{ij}^+)p_j\overline{y}_j(t), \end{aligned} \tag{22}$$

where $\overline{y}_j(t) = \sup_{t-\tau\le s\le t}|y_j(s)|$. From (A3) and Lemma 1, we obtain that there exists a vector $\eta = (\eta_1,\eta_2,\dots,\eta_n)^T > (0,0,\dots,0)^T$ such that

$$(E_n-D)\eta > (0,0,\dots,0)^T.$$

Indeed, for $i=1,2,\dots,n$,

$$\begin{aligned} &\eta_i-\sum_{j=1}^n d_{ij}\eta_j \\ &\qquad = \eta_i-\sum_{j=1}^n \frac{1}{c_i}(a_{ij}^+ +\alpha_{ij}^+ +\beta_{ij}^+)p_j\eta_j > 0, \end{aligned}$$

which implies that

$$-c_i\eta_i+\sum_{j=1}^n (a_{ij}^+ +\alpha_{ij}^+ +\beta_{ij}^+)p_j\eta_j < 0. \tag{23}$$

We can choose a small positive constant $\lambda<1$ such that, for $i=1,2,\dots,n$,

$$\lambda\eta_i+\left[-c_i\eta_i+\sum_{j=1}^n (a_{ij}^+ +\alpha_{ij}^+ +\beta_{ij}^+)p_j\eta_j e^{\lambda\tau}\right] < 0. \tag{24}$$

We can choose a constant $\gamma>1$ such that

$$\gamma\eta_i e^{-\lambda t} > 1, \quad \forall t\in[-\tau,0]. \tag{25}$$

For each $\varepsilon>0$, let

$$Y_i(t)=\gamma\eta_i\left[\sum_{j=1}^n \overline{y}_j(0)+\varepsilon\right]e^{-\lambda t}, \quad i=1,2,\dots,n. \tag{26}$$

From (24) and (26), it follows that

$$\begin{aligned} &D_-Y_i(t) \\ &= -\lambda\gamma\eta_i\left[\sum_{j=1}^n \overline{y}_j(0)+\varepsilon\right]e^{-\lambda t} \\ &> \left[-c_i\eta_i+\sum_{j=1}^n (a_{ij}^+ +\alpha_{ij}^+ +\beta_{ij}^+)p_j\eta_j e^{\lambda\tau}\right]\gamma \\ &\quad\times\left[\sum_{j=1}^n \overline{y}_j(0)+\varepsilon\right]e^{-\lambda t} \\ &= -c_i\gamma\eta_i\left[\sum_{j=1}^n \overline{y}_j(0)+\varepsilon\right]e^{-\lambda t} \\ &\quad+\sum_{j=1}^n \Big[(a_{ij}^+ +\alpha_{ij}^+ +\beta_{ij}^+)p_j\eta_j\gamma \\ &\quad\times\left(\sum_{j=1}^n \overline{y}_j(0)+\varepsilon\right)e^{-\lambda(t-\tau)}\Big] \\ &= -c_iY_i(t)+\sum_{j=1}^n (a_{ij}^+ +\alpha_{ij}^+ +\beta_{ij}^+)p_j\overline{Y}_j(t), \end{aligned} \tag{27}$$

where $\overline{Y}_j(t) = \sup_{t-\tau\le s\le t}Y_j(s)$. From (25) and (26), we have, for $t\in[-\tau,0]$,

$$\begin{aligned} Y_i(t) &= \gamma\eta_i\left[\sum_{j=1}^n \overline{y}_j(0)+\varepsilon\right]e^{-\lambda t} \\ &> \sum_{j=1}^n \overline{y}_j(0)+\varepsilon > |y_i(t)|. \end{aligned} \tag{28}$$

We claim that

$$|y_i(t)| < Y_i(t), \quad \forall\, t>0, \quad i=1,2,\dots,n. \tag{29}$$

If not, there must exist some $i\in\{1,2,\dots,n\}, t_i>0$ such that, for $j=1,2,\dots,n, t\in[-\tau,t_i)$,

$$|y_i(t_i)| = Y_i(t_i), \quad |y_j(t)| < Y_j(t). \tag{30}$$

Indeed, for $j=1,2,\dots,n, t\in[-\tau,t_i)$,

$$|y_i(t_i)|-Y_i(t_i)=0, \quad |y_j(t)|-Y_j(t)<0. \tag{31}$$

It follows that

$$
\begin{aligned}
0 &\le D^-(|y_i(t_i) - Y_i(t_i))\\
&= \limsup_{h\to 0^-} \frac{1}{h}\Big\{[|y_i(t_i+h)| - Y_i(t_i+h)]\\
&\quad - [|y_i(t_i)| - Y_i(t_i)]\Big\}\\
&\le \limsup_{h\to 0^-} \frac{|y_i(t_i+h)| - |y_i(t_i)|}{h}\\
&\quad - \liminf_{h\to 0^-} \frac{Y_i(t_i+h) - Y_i(t_i)}{h}\\
&= D^-|y_i(t_i)| - D_-Y_i(t_i).
\end{aligned} \tag{32}
$$

From (22), (27) and (30), we obtain

$$
\begin{aligned}
&D^-|y_i(t_i)|\\
&\le -c_i|y_i(t_i)| + \sum_{j=1}^{n}(a_{ij}^+ + \alpha_{ij}^+ + \beta_{ij}^+)p_j|\overline{y}_j(t_i)|\\
&= -c_iY_i(t_i) + \sum_{j=1}^{n}(a_{ij}^+ + \alpha_{ij}^+ + \beta_{ij}^+)p_j|\overline{y}_j(t_i)|\\
&\le -c_iY_i(t_i) + \sum_{j=1}^{n}(a_{ij}^+ + \alpha_{ij}^+ + \beta_{ij}^+)p_j|\overline{Y}_j(t_i)|\\
&\le D_-Y_i(t_i)
\end{aligned} \tag{33}
$$

which contradicts (32). Therefore (29) holds. Let $\varepsilon \to 0^+$ and $M = n\max_{1\le i\le n}\{\gamma\eta_i + 1\}$. From (26) and (29) it follows that

$$
\begin{aligned}
|x_i(t) - x_i^*(t)| = |y_i(t)| &\le \gamma\eta_i \sum_{j=1}^{n}\overline{y}_j(0)e^{-\lambda t}\\
&\le n\gamma\eta_i\|\varphi - \varphi^*\|e^{-\lambda t}\\
&\le M\|\varphi - \varphi^*\|e^{-\lambda t}, \quad i = 1,2,\dots,n
\end{aligned}
$$

for $t > 0$. This completes the proof. ■

4. Illustrative example

Consider the following fuzzy cellular neural network with time-varying delays:

$$
\begin{aligned}
x_i'(t) &= -x_i(t) + \sum_{j=1}^{3} a_{ij}(t)f_j(x_j(t-\tau_{ij}(t)))\\
&\quad + \bigwedge_{j=1}^{3} \alpha_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) + I_i(t)\\
&\quad + \bigvee_{j=1}^{3} \beta_{ij}(t)f_j(t-\tau_{ij}(t)) + \bigwedge_{j=1}^{3} T_{ij}(t)u_j(t)\\
&\quad + \bigvee_{j=1}^{3} H_{ij}(t)u_j(t), \quad i = 1,2,3,
\end{aligned} \tag{34}
$$

where

$$
\begin{aligned}
&a_{11}(t) = \alpha_{11}(t) = \beta_{11}(t) = \frac{1}{4}\sin t,\\
&a_{12}(t) = \alpha_{12}(t) = \beta_{12}(t) = \frac{1}{9}\cos t,\\
&a_{13}(t) = \alpha_{13}(t) = \beta_{13}(t) = \frac{1}{4}\sin t,\\
&a_{21}(t) = \alpha_{21}(t) = \beta_{21}(t) = \frac{1}{2}\sin t,\\
&a_{22}(t) = \alpha_{22}(t) = \beta_{22}(t) = \frac{1}{6}\cos t,\\
&a_{23}(t) = \alpha_{23}(t) = \beta_{23}(t) = \frac{1}{2}\sin t,\\
&a_{31}(t) = \alpha_{31}(t) = \beta_{31}(t) = \sin t,\\
&a_{32}(t) = \alpha_{32}(t) = \beta_{32}(t) = \frac{1}{6}\cos t,\\
&a_{33}(t) = \alpha_{33}(t) = \beta_{33}(t) = \frac{1}{9}\cos t,\\
&\tau_{11}(t) = \tau_{12}(t) = \tau_{13}(t) = \cos t,\\
&\tau_{21}(t) = \tau_{22}(t) = \tau_{23}(t) = \sin t,\\
&\tau_{31}(t) = \tau_{32}(t) = \tau_{33}(t) = \frac{1}{2}\sin t,\\
&\ I_1(t) = \cos t, \quad I_2(t) = \sin t, \quad I_3(t) = 2\cos t,\\
&T_{ij}(t) = H_{ij}(t) = \sin t, \quad K_{ji}(t) = N_{ji}(t) = \cos t,\\
&\ u_i(t) = u_j(t) = 2\sin t, \quad (i,j = 1,2).
\end{aligned}
$$

Take $f_j(x) = \frac{1}{2}(|x+1| - |x-1|)$ $(j = 1,2,3)$. We have $p_i = 1$ $(i = 1,2,3)$. By simple computation, we get

$$
a_{11}^+ = \alpha_{11}^+ = \beta_{11}^+ = \frac{1}{4}, \quad a_{12}^+ = \alpha_{12}^+ = \beta_{12}^+ = \frac{1}{9},
$$

$$
a_{13}^+ = \alpha_{13}^+ = \beta_{13}^+ = \frac{1}{4}, \quad a_{21}^+ = \alpha_{21}^+ = \beta_{21}^+ = \frac{1}{2},
$$

$$
a_{22}^+ = \alpha_{22}^+ = \beta_{22}^+ = \frac{1}{6}, \quad a_{23}^+ = \alpha_{23}^+ = \beta_{23}^+ = \frac{1}{2},
$$

$$
a_{31}^+ = \alpha_{31}^+ = \beta_{31}^+ = 1, \quad a_{32}^+ = \alpha_{32}^+ = \beta_{32}^+ = \frac{1}{6},
$$

$$
a_{33}^+ = \alpha_{33}^+ = \beta_{33}^+ = \frac{1}{9}.
$$

Then we have

$$
\begin{aligned}
D &= (c_i^{-1}(a_{ij}^+ + \alpha_{ij}^+ + \beta_{ij}^+)p_j)_{3\times 3}\\
&= \begin{pmatrix} \frac{3}{4} & \frac{1}{3} & \frac{3}{4}\\ \frac{3}{2} & \frac{1}{2} & \frac{3}{2}\\ 3 & \frac{1}{2} & \frac{1}{3} \end{pmatrix}.
\end{aligned}
$$

Therefore,

$$E_3 - D = \begin{pmatrix} \frac{1}{4} & -\frac{1}{3} & -\frac{3}{4} \\ -\frac{3}{2} & \frac{1}{2} & -\frac{3}{2} \\ -3 & -\frac{1}{2} & \frac{2}{3} \end{pmatrix}.$$

Hence it follows that all the conditions needed in Theorem 2 are satisfied. Therefore, according to Theorem 2, the system (34) has one 2π-periodic solution which is globally exponentially stable.

5. Conclusion

In this paper, we use the continuation theorem of coincidence degree theory and the Lyapunov function to study the existence and global exponential stability of a periodic solution for fuzzy cellular neural networks with time-varying delays. The sufficient conditions for the existence and global stability of the periodic solution are independent of time delays. Moreover, an example is given to illustrate the effectiveness of the new results.

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