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ANTI – PLANE CRACK EMANATING FROM THE INTERFACE IN A BOUNDED SMART PEMO- ELASTIC STRUCTURE

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The magnetoelectroelastic analysis of two bonded dissimilar piezo-electro-magneto-elastic ceramics with a crack perpendicular to and terminating at the interface is made. By using the Fourier integral transform (in perpendicular directions in each materials), the mixed boundary conditions and continuity conditions are transformed to a singular integral equation with generalized Cauchy kernel, the solution of which has been well studied, and classical methods are directly applicable here to obtain the closed form solution. The results are presented for a permeable crack under anti-plane shear loading and in-plane electric and magnetic loadings, as prescribed electric displacement and magnetic inductions or electric and magnetic fields. The results indicate that the magnetoelectroelastic field near the crack tip in the homogeneous PEMO- elastic ceramic is dominated by a traditional inverse square-root singularity, while the coupled field near the crack tip at the interface exhibits the singularity of the power law $r^{-\alpha}$, r being the distance from the interface crack tip and α depending on the material constants of a bimaterial. In particular, electric and magnetic fields have no singularity at the crack tip in a homogeneous solid, whereas they are singular around the interface crack tip. Numerical results are given graphically to show the effects of the material properties on the singularity order, field intensity factors and energy release rates. The results presented in this paper should have potential applications to the design of multilayered magnetoelectroelastic structures.

Key words: interface, anti-plane shear crack, singularity of power law, field intensity factors, magnetoelectroelastic behaviour, Fourier transform, Cauchy singular integral equation, exact solution.

1. Introduction

The newly emerging materials named magnetoelectroelasticity, which exhibit piezoelectric, piezomagnetic and electromagnetic properties, have found increasingly wide engineering applications, particularly in aerospace and automotive industries. Magnetoelectroelastic solids have been widely used as transducer, sensors and actuators in smart structures. Because of the brittleness of PEMO – elastic materials, a high possibility of material debonding and cracking or sliding of the interface exists. Consequently, this problem has been the subject of research and discussion in the literature on elasticity theory of coupled fields. Li and Kardomateas (2006) investigated the mode III interface crack problem for dissimilar piezoelectromagnetoelastic bimaterial media. The extended Stroh's theory and analytic principle of complex analysis have been used to obtain the solution for interfacial cracks between two dissimilar magnetoelectroelastic half – planes by Li and Kardomateas (2007). The problem for an anti – plane interface crack between two dissimilar PEMO – elastic layers was analyzed by Wang and Mai (2006). Gao *et al.* (2003) derived the exact solution for a permeable interface crack between two dissimilar magnetoelectroelastic solids under general applied loads. Gao *et al.* (2004) derived also the static solution related to anti – plane crack problem. The anti – plane shear cracks are a class of simple problems. But, for the case of a crack perpendicular to the interface the problem becomes more complicated. This problem has been the subject of research in the classical literature of elasticity theory. Cook and Erdogan (1972) and Erdogan and Cook (1974) were apparently the first to publish the solution of this problem for two bonded dissimilar isotropic half – planes. For piezoelectric bi – ceramics an arbitrarily oriented plane crack terminating at the interface was extended by Qin and Yu (1997). The anti – plane shear crack normal to and terminating at the interface of two piezoelectric ceramics was extended later by Li and Wang (2007). Although the above studies deal strictly with piezoelectric, it is reasonable to assume that the extension of the findings to electromagnetoelastic material is valid.

To the best of the author's knowledge, the behaviour of interfacial cracks normal to and terminating at the interface of two bonded piezoelectromagnetoelastic materials has not been addressed yet. Motivated by these considerations, the author investigates the anti – plane deformations and in – plane electric and magnetic fields of a PEMO – elastic bimaterial with Mode – III interface crack normal to and terminating at the interface.

The crack is assumed to be electrically and magnetically permeable. Under an applied electric, magnetic and mechanical loading, electric, magnetic and elastic behaviours near both crack tips are obtained. Two kinds of loading conditions are adopted. By using the Fourier integral transform, in perpendicular directions in each materials, the associated boundary value problem is transformed to a singular integral equation with generalized Cauchy kernel. Similar types of equations have been studied, and classical methods of their solutions are directly applicable here to obtain the solution in a closed form. The results indicate that the magnetoelectroelastic field near the crack tip in a homogeneous PEMO – elastic ceramic exhibits an inverse square – root singularity, while the singular field near the interface crack tip is dominated by a singularity of the power law. The singularity order is dependent on relevant 2×6 material constants of two ceramics. The effects of magneto – electro – mechanical parameters on the field intensity factors are evaluated by numerical analysis, which could be of particular interest to the analysis and design of smart sensors / actuators constructed from magnetoelectroelastic composite laminates.

2. Formulation of the problem

2.1. Basic equations

For a linearly magnetoelectroelastic medium under anti – plane shear coupled with in-plane electric and magnetic fields there is only the nontrivial anti – plane displacement w

$$u_x = 0\,, \qquad u_y = 0\,, \qquad u_z = w(x,y), \tag{2.1}$$

strain components γ_{xz} and γ_{yz}

$$\gamma_{xz} = \frac{\partial w}{\partial x}\,, \qquad \gamma_{yz} = \frac{\partial w}{\partial y}, \tag{2.2}$$

stress components τ_{xz} and τ_{yz}, in-plane electrical and magnetic potentials φ and ψ, which define electric and magnetic field components E_x, E_y, H_x and H_y

$$E_x = -\frac{\partial\varphi}{\partial x}\,, \qquad E_y = -\frac{\partial\varphi}{\partial y}\,, \qquad H_x = -\frac{\partial\psi}{\partial x}\,, \qquad H_y = -\frac{\partial\psi}{\partial y}, \tag{2.3}$$

and electrical displacement components D_x, D_y, and magnetic induction components B_x, B_y with all field quantities being the functions of coordinates x and y.

The relations Eqs (2.2) and (2.3) have the form

$$\gamma_{\alpha z} = w_{,\alpha}\,, \qquad E_\alpha = -\varphi_{,\alpha}\,, \qquad H_\alpha = -\psi_{,\alpha} \tag{2.4}$$

where $\alpha = x, y$ and $w_{,\alpha} = \partial w / \partial \alpha$.

For a linearly magnetoelectroelastic medium the coupled constitutive relations can be written in the matrix form as follows

$$\left[\tau_{\alpha z}, D_\alpha, B_\alpha\right]^T = C\left[\gamma_{\alpha z}, -E_\alpha, -H_\alpha\right]^T \tag{2.5}$$

where the superscript T denotes the transpose of a matrix and

$$C = \begin{bmatrix} c_{44} & e_{15} & q_{15} \\ e_{15} & -\varepsilon_{11} & -d_{11} \\ q_{15} & -d_{11} & -\mu_{11} \end{bmatrix} \tag{2.6}$$

where c_{44} is the shear modulus along the z – direction, which is the direction of poling and is perpendicular to the isotropic plane (x, y), ε_{11} and μ_{11} are dielectric permittivity and magnetic permeability coefficients, respectively, e_{15}, q_{15} and d_{11} are piezoelectric, piezomagnetic and magneto-electric coefficients, respectively.

The mechanical equilibrium equation (called the Euler equation), the charge and current conservation equations (called Maxwell equations), in the absence of the body force electric and magnetic charge densities, can be written as

$$\tau_{z\alpha,\alpha} = 0, \qquad D_{\alpha,\alpha} = 0, \qquad B_{\alpha,\alpha} = 0, \qquad \alpha = x, y. \tag{2.7}$$

Subsequently, the Euler and Maxwell equations take the following form

$$C\left[\nabla^2 w, \nabla^2 \varphi, \nabla^2 \psi\right]^T = [0,0,0]^T \tag{2.8}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator.

Since $|C| \neq 0$, one can decouple Eq.(2.8) as follows

$$\nabla^2 w = 0, \qquad \nabla^2 \varphi = 0, \qquad \nabla^2 \psi = 0. \tag{2.9}$$

If we introduce, for convenience of mathematics in some boundary value problems, two unknown functions

$$\left[\chi - e_{15} w, \eta - q_{15} w\right]^T = C_0 \left[\varphi, \psi\right]^T \tag{2.10}$$

where

$$C_0 = \begin{bmatrix} -\varepsilon_{11} & -d_{11} \\ -d_{11} & -\mu_{11} \end{bmatrix}, \tag{2.11}$$

then

$$\left[\varphi, \psi\right]^T = C_0^{-1}\left[\chi - e_{15} w, \eta - q_{15} w\right]^T \tag{2.12}$$

where

$$C_0^{-1} = \frac{1}{\varepsilon_{11}\mu_{11} - d_{11}^2}\begin{bmatrix} -\mu_{11} & d_{11} \\ d_{11} & -\varepsilon_{11} \end{bmatrix} = \begin{bmatrix} e_1 & e_2 \\ e_2 & e_3 \end{bmatrix}. \tag{2.13}$$

The governing field variables are

$$
\begin{aligned}
&\tau_{zk} = \tilde{c}_{44} w_{,k} - \alpha D_k - \beta B_k, \\
&\varphi = \alpha w + e_1 \chi + e_2 \eta, \\
&\psi = \beta w + e_2 \chi + e_3 \eta, \\
&D_k = \chi_{,k}, \\
&B_k = \eta_{,k}, \qquad k = x, y,
\end{aligned}
\tag{2.14}
$$

$$
\nabla^2 w = 0, \qquad \nabla^2 \chi = 0, \qquad \nabla^2 \eta = 0 \tag{2.15}
$$

where

$$
\begin{aligned}
&\tilde{c}_{44} = c_{44} + \alpha e_{15} + \beta q_{15}, \\
&\alpha = \frac{\mu_{11} e_{15} - d_{11} q_{15}}{\varepsilon_{11} \mu_{11} - d_{11}^2} = -\left(e_1 e_{15} + e_2 q_{15}\right), \\
&\beta = \frac{\varepsilon_{11} q_{15} - d_{11} e_{15}}{\varepsilon_{11} \mu_{11} - d_{11}^2} = -\left(e_3 q_{15} + e_2 e_{15}\right).
\end{aligned}
\tag{2.16}
$$

Note that $\tilde{c}_{44}$ is the piezo – electro – magnetically stiffened elastic constant.

Note also that the inverse of a matrix C is defined by parameters α, β, $\tilde{c}_{44}$ and e_1, e_2, e_3 as follows

$$
C^{-1} = \frac{1}{\tilde{c}_{44}} \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \alpha^2 + \tilde{c}_{44} e_1 & \alpha\beta + \tilde{c}_{44} e_2 \\ \beta & \alpha\beta + \tilde{c}_{44} e_2 & \beta^2 + \tilde{c}_{44} e_3 \end{bmatrix}. \tag{2.17}
$$

These material parameters will appear in our solutions.

2.2. Boundary conditions

Consider a crack terminating at the interface of two bonded dissimilar PEMO – elastic ceramics polarized in the z – direction. For convenience, we denote the PEMO – elastic ceramics occupying the right and left half – planes $x \geq 0$ and $x \leq 0$ as piezoceramics I and II, respectively, shown in Fig.1.

Let a crack be perpendicular to the interface and be situated at $[0, a]$ ($a > 0$) in the positive x – direction in ceramic I. For an anti – plane shear crack having no thickness (so-called "mathematical crack"), the crack surfaces contact each other, in reality; so the crack is electrically and magnetically contacted. Consequently, the electric and magnetic boundary conditions at the crack surfaces can be described according to so – called double permeable conditions, namely

$$
\begin{aligned}
&D_y\left(x, 0^+\right) = D_y\left(x, 0^-\right), \qquad B_y\left(x, 0^+\right) = B_y\left(x, 0^-\right), \\
&\varphi\left(x, 0^+\right) = \varphi\left(x, 0^-\right), \qquad \psi\left(x, 0^+\right) = \psi\left(x, 0^-\right).
\end{aligned}
\tag{2.18}
$$

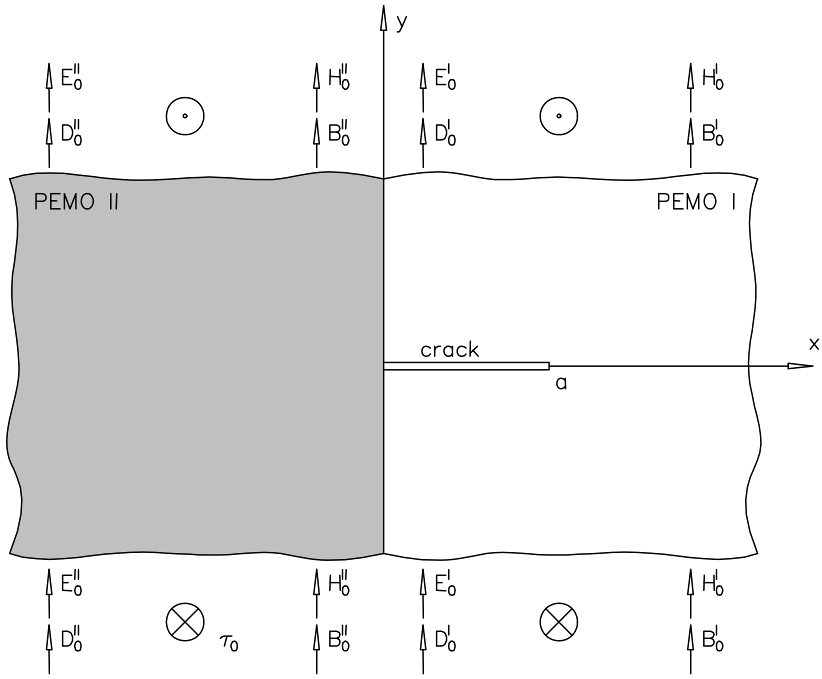

Fig.1. Two bonded dissimilar PEMO – elastic ceramics with a crack perpendicular to and terminating at the interface

Note that besides the crack surfaces, the above conditions, in fact, certainly hold at the crack-absent parts of the crack plane. Using the relations Eq.(2.14) it can be shown that the condition Eq.(2.18) may be replaced by conditions as follows

$$\chi_{,y}\left(x,0^{+}\right)=\chi_{,y}\left(x,0^{-}\right), \qquad \eta_{,y}\left(x,0^{+}\right)=\eta_{,y}\left(x,0^{-}\right), \tag{2.19a}$$

$$\chi=e_{15}w, \qquad \eta=q_{15}w \qquad \text{for} \qquad x,y=0\pm. \tag{2.19b}$$

Let the constant mechanical loads and uniform electric displacement and magnetic induction or electric field and magnetic field be applied at infinity (two cases of electric and magnetic loads), and the following

$$\begin{gathered}\tau_{yz}^{I}\left(x,y\right)=\tau_{0}^{I}, \quad D_{y}^{I}\left(x,y\right)=D_{0}^{I}, \quad B_{y}^{I}\left(x,y\right)=B_{0}^{I} \quad \text{or} \quad E_{y}^{I}\left(x,y\right)=E_{0}^{I},\\ H_{y}^{I}\left(x,y\right)=H_{0}^{I}, \quad x>0, \quad y\rightarrow\pm\infty,\\ \tau_{yz}^{II}\left(x,y\right)=\tau_{0}^{II}, \quad D_{y}^{II}\left(x,y\right)=D_{0}^{II}, \quad B_{y}^{II}\left(x,y\right)=B_{0}^{II} \quad \text{or} \quad E_{y}^{II}\left(x,y\right)=E_{0}^{II},\\ H_{y}^{II}\left(x,y\right)=H_{0}^{II}, \quad x<0, \quad y\rightarrow\pm\infty\end{gathered} \tag{2.20}$$

where $\tau_0^I\left(\tau_0^{II}\right)$, $D_0^I\left(D_0^{II}\right)$ and $B_0^I\left(B_0^{II}\right)$ or $E_0^I\left(E_0^{II}\right)$ and $H_0^I\left(H_0^{II}\right)$, are prescribed constants, a quantity with superscribes I or II that specifies the one in the PEMO – ceramic I or II, respectively. To solve the crack problem in linear elastic solids, the superposition technique is usually used. Thus we first solve the magnetoelectroelastic field problem without the cracks in the medium under electric, magnetic and mechanical loads. This elementary solution is the following

$$\tau_{yz}^{J} = \tau_{0}^{J},$$

$$D_y^J = D^J = \begin{cases} D_0^J, & \text{case I,} \\ \left[\dfrac{e_{15}}{c_{44}}\tau_0 + \left(\varepsilon_{11} + \dfrac{e_{15}^2}{c_{44}} \right) E_0 + \left(d_{11} + \dfrac{e_{15} q_{15}}{c_{44}} \right) H_0 \right]^J, & \text{case II} \end{cases}$$

$$B_y^J = B^J = \begin{cases} B_0^J, & \text{case I,} \\ \left[\dfrac{q_{15}}{c_{44}}\tau_0 + \left(d_{11} + \dfrac{e_{15} q_{15}}{c_{44}} \right) E_0 + \left(\mu_{11} + \dfrac{q_{15}^2}{c_{44}} \right) H_0 \right]^J, & \text{case II,} \end{cases} \tag{2.21}$$

with $J = I, II.$

In addition the crack surfaces are traction – free, that is,

$$\tau_{yz}^{I}(x, y) = 0; \qquad y = 0\pm, \qquad 0 < x < a, \tag{2.22}$$

and owing to the symmetry one can directly write the following conditions

$$w^I(x,0) = 0, \qquad x > a, \qquad w^{II}(x,0) = 0, \qquad x < 0. \tag{2.23}$$

We further consider the situation when the interface under consideration is perfectly bonded, across which the displacement, stress, electric and magnetic potentials, electric displacement and magnetic induction are continuous

$$w^I(0,y) = w^{II}(0,y), \qquad \tau_{xz}^I(0,y) = \tau_{xz}^{II}(0,y); \qquad -\infty < y < \infty, \tag{2.24}$$

$$\varphi^I(0,y) = \varphi^{II}(0,y), \qquad D_x^I(0,y) = D_x^{II}(0,y); \qquad -\infty < y < \infty, \tag{2.25}$$

$$\psi^I(0,y) = \psi^{II}(0,y), \qquad B_x^I(0,y) = B_x^{II}(0,y); \qquad -\infty < y < \infty. \tag{2.26}$$

3. Method of solution

From the symmetry of the problem, it is sufficient to consider the upper half-plane of the bi-ceramic. Consequently, for $y \geq 0$ it is easily found that an appropriate solution of the problem, which satisfies the boundary conditions Eqs (2.19a) and (2.20), takes the following form

$$\begin{bmatrix} w^I(x,y) \\ \chi^I(x,y) \\ \eta^I(x,y) \end{bmatrix} = \begin{bmatrix} \gamma^I \\ D^I \\ B^I \end{bmatrix} y + \begin{bmatrix} 1 \\ e_{15}^I \\ q_{15}^I \end{bmatrix} \int_0^\infty A_1(\xi) e^{-y\xi} \cos(\xi x) d\xi + \int_0^\infty \begin{bmatrix} B_1(\xi) \\ C_1(\xi) \\ D_1(\xi) \end{bmatrix} e^{-\xi x} \sin(\xi y) d\xi, \tag{3.1}$$

for $x \geq 0$ and

$$\begin{bmatrix} w^{II}(x,y) \\ \chi^{II}(x,y) \\ \eta^{II}(x,y) \end{bmatrix} = \begin{bmatrix} \gamma^{II} \\ D^{II} \\ B^{II} \end{bmatrix} y + \begin{bmatrix} 1 \\ e_{15}^{II} \\ q_{15}^{II} \end{bmatrix} \int_0^\infty A_2(\xi) e^{-y\xi} \cos(\xi x) d\xi + \int_0^\infty \begin{bmatrix} B_2(\xi) \\ C_2(\xi) \\ D_2(\xi) \end{bmatrix} e^{-\xi x} \sin(\xi y) d\xi, \tag{3.2}$$

for $x \leq 0$ where A_j, B_j, C_j and D_j ($j = 1, 2$) are unknowns to be determined from given boundary conditions and where

$$\gamma^J = \frac{\tau_0^J + \alpha^J D^J + \beta^J B^J}{\tilde{c}_{44}^J}; \qquad J = I, II. \tag{3.3}$$

Furthermore, with the aid of Eqs (2.14) one can give the components of stress, electric displacement, magnetic induction and electric and magnetic potentials

$$\begin{aligned} \begin{bmatrix} \tau_{yz}^I(x,y) \\ D_y^I(x,y) \\ B_y^I(x,y) \end{bmatrix} &= \begin{bmatrix} \tau_0^I \\ D^I \\ B^I \end{bmatrix} - \begin{bmatrix} c_{44}^I \\ e_{15}^I \\ q_{15}^I \end{bmatrix} \int_0^\infty \xi A_1(\xi) e^{-y\xi} \cos(\xi x) d\xi + \\ &+ \int_0^\infty \xi \begin{bmatrix} \tilde{c}_{44}^I B_1(\xi) - \alpha^I C_1(\xi) - \beta^I D_1(\xi) \\ C_1(\xi) \\ D_1(\xi) \end{bmatrix} e^{-\xi x} \cos(\xi y) d\xi, \end{aligned} \tag{3.4}$$

$$\begin{bmatrix} \varphi^I(x,y) \\ \psi^I(x,y) \end{bmatrix} = C_I^{-1} \begin{bmatrix} \tau_0^I \\ D^I \\ B^I \end{bmatrix} y + 0 + \begin{bmatrix} \alpha & e_1 & e_2 \\ \beta & e_2 & e_3 \end{bmatrix}^I \int_0^\infty \xi \begin{bmatrix} B_1(\xi) \\ C_1(\xi) \\ D_1(\xi) \end{bmatrix} e^{-\xi x} \sin(\xi y) d\xi, \tag{3.5}$$

for $x \geq 0$ and

$$\begin{aligned} \begin{bmatrix} \tau_{yz}^{II}(x,y) \\ D_y^{II}(x,y) \\ B_y^{II}(x,y) \end{bmatrix} &= \begin{bmatrix} \tau_0^{II} \\ D^{II} \\ B^I \end{bmatrix} - \begin{bmatrix} c_{44}^{II} \\ e_{15}^{II} \\ q_{15}^I \end{bmatrix} \int_0^\infty \xi A_2(\xi) e^{-y\xi} \cos(\xi x) d\xi + \\ &+ \int_0^\infty \xi \begin{bmatrix} \tilde{c}_{44}^{II} B_1(\xi) - \alpha^{II} C_2(\xi) - \beta^{II} D_2(\xi) \\ C_2(\xi) \\ D_2(\xi) \end{bmatrix} e^{-\xi x} \cos(\xi y) d\xi, \end{aligned} \tag{3.6}$$

$$\begin{bmatrix}\varphi^{II}(x,y)\\ \psi^{II}(x,y)\end{bmatrix}=C_{II}^{-1}\begin{bmatrix}\tau_0^{II}\\ D^{II}\\ B^{II}\end{bmatrix}y+0+\begin{bmatrix}\alpha & e_1 & e_2\\ \beta & e_2 & e_3\end{bmatrix}^{II}\int_0^\infty \xi\begin{bmatrix}B_2(\xi)\\ C_2(\xi)\\ D_2(\xi)\end{bmatrix}e^{\xi x}\sin(\xi y)\,d\xi, \tag{3.7}$$

for $x \le 0$ and

$$\begin{aligned}\begin{bmatrix}\tau_{xz}^{I}(x,y)\\ D_x^{I}(x,y)\\ B_x^{I}(x,y)\end{bmatrix}=&-\begin{bmatrix}c_{44}^{I}\\ e_{15}^{I}\\ q_{15}^{I}\end{bmatrix}\int_0^\infty \xi A_1(\xi)e^{-y\xi}\sin(\xi x)\,d\xi+\\ &-\int_0^\infty \xi\begin{bmatrix}\tilde{c}_{44}^{I}B_1(\xi)-\alpha^I C_1(\xi)-\beta^I D_1(\xi)\\ C_1(\xi)\\ D_1(\xi)\end{bmatrix}e^{-\xi x}\sin(\xi y)\,d\xi,\end{aligned} \tag{3.8}$$

for $x \ge 0$ and

$$\begin{aligned}\begin{bmatrix}\tau_{xz}^{II}(x,y)\\ D_x^{II}(x,y)\\ B_x^{II}(x,y)\end{bmatrix}=&-\begin{bmatrix}c_{44}^{II}\\ e_{15}^{II}\\ q_{15}^{II}\end{bmatrix}\int_0^\infty \xi A_2(\xi)e^{-y\xi}\sin(\xi x)\,d\xi+\\ &+\int_0^\infty \xi\begin{bmatrix}\tilde{c}_{44}^{II}B_2(\xi)-\alpha^{II} C_2(\xi)-\beta^{II} D_2(\xi)\\ C_2(\xi)\\ D_2(\xi)\end{bmatrix}e^{-\xi x}\sin(\xi y)\,d\xi,\end{aligned} \tag{3.9}$$

for $x \le 0$.

Now, the application of the continuity conditions Eqs (2.24), (2.25) and (2.26) at the interface $x = 0$ to Eqs (3.1) to (3.9) yields

$$\frac{\tau_0^I+\alpha^I D^I+\beta^I B^I}{\tilde{c}_{44}^{I}}=\frac{\tau_0^{II}+\alpha^{II} D^{II}+\beta^{II} B^{II}}{\tilde{c}_{44}^{II}}, \tag{3.10}$$

$$C_I^{-1}\begin{bmatrix}\tau_0^{I}\\ D^{I}\\ B^{I}\end{bmatrix}=C_{II}^{-1}\begin{bmatrix}\tau_0^{II}\\ D^{II}\\ B^{II}\end{bmatrix},\qquad C_J^{-1}=\frac{1}{\tilde{c}_{44}^{J}}\begin{bmatrix}\alpha & \alpha^2+\tilde{c}_{44}^{I}e_1 & \alpha\beta+\tilde{c}_{44}^{I}e_2\\ \beta & \alpha\beta+\tilde{c}_{44}^{I}e_2 & \beta^2+\tilde{c}_{44}^{I}e_3\end{bmatrix}^J, \tag{3.11}$$

and

$$\begin{gathered}-\left[\tilde{c}_{44}^{I}B_1(\xi)-\alpha^I C_1(\xi)-\beta^I D_1(\xi)\right]=\left[\tilde{c}_{44}^{II}B_2(\xi)-\alpha^{II} C_2(\xi)-\beta^{II} D_2(\xi)\right],\\ C_1(\xi)=-C_2(\xi),\qquad D_1(\xi)=-D_2(\xi),\end{gathered} \tag{3.12}$$

$$\begin{bmatrix}\alpha^{II} & e_1^{II} & e_2^{II}\\ \beta^{II} & e_2^{II} & e_3^{II}\end{bmatrix}\begin{bmatrix}B_2(\xi)\\ C_2(\xi)\\ D_2(\xi)\end{bmatrix}=\begin{bmatrix}\alpha^{I} & e_1^{I} & e_2^{I}\\ \beta^{I} & e_2^{I} & e_3^{I}\end{bmatrix}\begin{bmatrix}B_1(\xi)\\ C_1(\xi)\\ D_1(\xi)\end{bmatrix}. \tag{3.13}$$

The first two equations, that is, Eqs (3.10) and (3.11), give three constraints for applied remote electro-magneto-mechanical loadings, from which we may determine the loadings of ceramics II, namely τ_0^{II} , D^{II} and B^{II} by means of loadings of ceramics I, namely τ_0^{I} , D^{I} and B^{I}. In other words, in order to guarantee the continuity of all physical quantities at the perfectly bonded interface, applied electro-magneto-mechanical loadings must obey the relations Eqs (3.10) and (3.11). The five Eqs (3.12) and (3.13) give the constraints with respect to unknown functions, that is, the disturbed field, due to the presence of a crack, should to satisfy those equations.

From the condition Eq.(2.23)$_2$ along with Eq.(3.2) one gets

$$A_2(\xi) = 0. \tag{3.14}$$

Continuity of $w(x,y)$ at the interface $x = 0$ requires

$$\int_0^\infty \left[B_2(\xi) - B_1(\xi)\right]\sin(\xi y)\,d\xi = \int_0^\infty A_1(\xi)e^{-\xi y}\,d\xi, \tag{3.15}$$

so that

$$B_2(\xi) - B_1(\xi) = \frac{2}{\pi}\int_0^\infty A_1(\eta)\frac{\xi}{\xi^2+\eta^2}\,d\eta, \tag{3.16}$$

since

$$\int_0^\infty e^{-\eta y}\sin(\xi y)\,dy = \frac{\xi}{\xi^2+\eta^2}. \tag{3.17}$$

The result Eq.(3.16) in connection with Eqs (3.12) and (3.13) yields

$$\begin{aligned}
B_1(\xi) = &-\frac{2}{\pi}I\frac{\tilde{c}_{44}^{II}\Delta + \left(e_3^{II}+e_3^{I}\right)\alpha^{II}\left(\alpha^{II}-\alpha^{I}\right) + \left(e_1^{II}+e_1^{I}\right)\beta^{II}\left(\beta^{II}-\beta^{I}\right)}{\left(\tilde{c}_{44}^{I}+\tilde{c}_{44}^{II}\right)\Delta + \left(e_3^{II}+e_3^{I}\right)\left(\alpha^{II}-\alpha^{I}\right)^2 + \left(e_1^{II}+e_1^{I}\right)\left(\beta^{II}-\beta^{I}\right)^2} + \\
&+\frac{2}{\pi}I\frac{\left(e_2^{II}+e_2^{I}\right)\left(\alpha^{I}\beta^{II}+\beta^{I}\alpha^{II}+2\alpha^{II}\beta^{II}\right)}{\left(\tilde{c}_{44}^{I}+\tilde{c}_{44}^{II}\right)\Delta + \left(e_3^{II}+e_3^{I}\right)\left(\alpha^{II}-\alpha^{I}\right)^2 + \left(e_1^{II}+e_1^{I}\right)\left(\beta^{II}-\beta^{I}\right)^2}, \\
B_2(\xi) = &\frac{2}{\pi}I\frac{\tilde{c}_{44}^{I}\Delta - \left(e_3^{II}+e_3^{I}\right)\alpha^{I}\left(\alpha^{II}-\alpha^{I}\right) - \left(e_1^{II}+e_1^{I}\right)\beta^{I}\left(\beta^{II}-\beta^{I}\right)}{\left(\tilde{c}_{44}^{I}+\tilde{c}_{44}^{II}\right)\Delta + \left(e_3^{II}+e_3^{I}\right)\left(\alpha^{II}-\alpha^{I}\right)^2 + \left(e_1^{II}+e_1^{I}\right)\left(\beta^{II}-\beta^{I}\right)^2} + \\
&+\frac{2}{\pi}I\frac{\left(e_2^{II}+e_2^{I}\right)\left(\alpha^{I}\beta^{II}+\beta^{I}\alpha^{II}+2\alpha^{II}\beta^{II}\right)}{\left(\tilde{c}_{44}^{I}+\tilde{c}_{44}^{II}\right)\Delta + \left(e_3^{II}+e_3^{I}\right)\left(\alpha^{II}-\alpha^{I}\right)^2 + \left(e_1^{II}+e_1^{I}\right)\left(\beta^{II}-\beta^{I}\right)^2},
\end{aligned} \tag{3.18}$$

$$C_1(\xi) = B_1(\xi)\frac{\beta^{I}\left(e_2^{II}+e_2^{I}\right)-\alpha^{I}\left(e_3^{II}+e_3^{I}\right)}{\Delta} - B_2(\xi)\frac{\beta^{II}\left(e_2^{II}+e_2^{I}\right)-\alpha^{II}\left(e_3^{II}+e_3^{I}\right)}{\Delta},$$

$$D_1(\xi) = -B_1(\xi)\frac{\beta^{I}\left(e_1^{II}+e_1^{I}\right)-\alpha^{I}\left(e_2^{II}+e_2^{I}\right)}{\Delta} + B_2(\xi)\frac{\beta^{II}\left(e_1^{II}+e_1^{I}\right)-\alpha^{II}\left(e_2^{II}+e_2^{I}\right)}{\Delta}$$

where

$$\Delta=\left(e_1^{II}+e_1^{I}\right)\left(e_3^{II}+e_3^{I}\right)-\left(e_2^{II}+e_2^{I}\right)^2, \tag{3.19}$$

$$I=\int_0^\infty A_1(\eta)\frac{\xi}{\xi^2+\eta^2}d\eta. \tag{3.20}$$

In the special cases we obtain that:
for both piezoelectric materials

$$\begin{aligned}
&B_1(\xi)=-\frac{2}{\pi}I\frac{c_{44}^{II}\left(\varepsilon_{11}^{I}+\varepsilon_{11}^{II}\right)+e_{15}^{II}\left(e_{15}^{I}+e_{15}^{II}\right)}{\left(c_{44}^{I}+c_{44}^{II}\right)\left(\varepsilon_{11}^{I}+\varepsilon_{11}^{II}\right)+\left(e_{15}^{I}+e_{15}^{II}\right)^2},\\
&B_2(\xi)=\frac{2}{\pi}I\frac{c_{44}^{I}\left(\varepsilon_{11}^{I}+\varepsilon_{11}^{II}\right)+e_{15}^{I}\left(e_{15}^{I}+e_{15}^{II}\right)}{\left(c_{44}^{I}+c_{44}^{II}\right)\left(\varepsilon_{11}^{I}+\varepsilon_{11}^{II}\right)+\left(e_{15}^{I}+e_{15}^{II}\right)^2},\\
&C_1(\xi)=-\frac{2}{\pi}I\frac{e_{15}^{I}\left(c_{44}^{II}\varepsilon_{11}^{II}+\left(e_{15}^{II}\right)^2\right)+e_{15}^{II}\left(c_{44}^{I}\varepsilon_{11}^{I}+\left(e_{15}^{I}\right)^2\right)}{\left(c_{44}^{I}+c_{44}^{II}\right)\left(\varepsilon_{11}^{I}+\varepsilon_{11}^{II}\right)+\left(e_{15}^{I}+e_{15}^{II}\right)^2},\\
&D_1(\xi)=0,
\end{aligned} \tag{3.21}$$

for both piezomagnetic materials

$$\begin{aligned}
&B_1(\xi)=-\frac{2}{\pi}I\frac{c_{44}^{II}\left(\mu_{11}^{I}+\mu_{11}^{II}\right)+q_{15}^{II}\left(q_{15}^{I}+q_{15}^{II}\right)}{\left(c_{44}^{I}+c_{44}^{II}\right)\left(\mu_{11}^{I}+\mu_{11}^{II}\right)+\left(q_{15}^{I}+q_{15}^{II}\right)^2},\\
&B_2(\xi)=\frac{2}{\pi}I\frac{c_{44}^{I}\left(\mu_{11}^{I}+\mu_{11}^{II}\right)+q_{15}^{I}\left(q_{15}^{I}+q_{15}^{II}\right)}{\left(c_{44}^{I}+c_{44}^{II}\right)\left(\mu_{11}^{I}+\mu_{11}^{II}\right)+\left(q_{15}^{I}+q_{15}^{II}\right)^2},\\
&C_1(\xi)=0,\\
&D_1(\xi)=-\frac{2}{\pi}I\frac{q_{15}^{I}\left(c_{44}^{II}\mu_{11}^{II}+\left(q_{15}^{II}\right)^2\right)+q_{15}^{II}\left(c_{44}^{I}\mu_{11}^{I}+\left(q_{15}^{I}\right)^2\right)}{\left(c_{44}^{I}+c_{44}^{II}\right)\left(\mu_{11}^{I}+\mu_{11}^{II}\right)+\left(q_{15}^{I}+q_{15}^{II}\right)^2}.
\end{aligned} \tag{3.22}$$

The formulae Eq.(3.21) are equivalent to these derived by Li and Wang (2007) who solved the problem of two bonded dissimilar piezoelectric media with an anti-plane shear crack perpendicular to and terminated at the interface. Next, we denote that

$$g(x) = \frac{\partial w^I(x,0^+)}{\partial x}. \tag{3.23}$$

From the boundary conditions Eq.(2.23), $g(x)$ should satisfy the single-value displacement constraint condition, that is

$$\int_0^a g(x)dx = 0. \tag{3.24}$$

Utilizing Eq.(3.1)$_1$ in Eq.(2.23) leads to

$$w^I(x,0) = \int_0^\infty A_1(\xi)\cos(\xi x)d\xi = 0, \qquad x > a, \tag{3.25}$$

from which together with Eq.(3.23), by use of the inverse Fourier transform, can be deduced

$$A_1(\xi) = -\frac{2}{\pi\xi}\int_0^a g(t)\sin(\xi t)dt. \tag{3.26}$$

Now, we calculate the following

$$\int_0^\infty \frac{\xi}{\xi^2+\eta^2}A_1(\eta)d\eta = -\frac{2}{\pi}\xi\int_0^a g(t)dt\int_0^\infty \frac{\sin(\eta t)}{\eta(\xi^2+\eta^2)}d\eta. \tag{3.27}$$

Using the result

$$\int_0^\infty \frac{\sin(\eta t)}{\eta(\xi^2+\eta^2)}d\eta = \frac{\pi(1-e^{-\xi t})}{2\xi^2}, \tag{3.28}$$

we find with the use of Eq.(3.24) that

$$\int_0^\infty \frac{\xi}{\xi^2+\eta^2}A_1(\eta)d\eta = \int_0^a \frac{e^{-\xi t}}{\xi}g(t)dt. \tag{3.29}$$

Substitution of Eq.(3.29) into Eq.(3.18) yields the expressions for $B_1(\xi)$, $B_2(\xi)$, $C_1(\xi)$ and $D_1(\xi)$ in terms of $g(x)$.

From the fraction free condition Eq.(2.22) from Eq.(3.4)$_1$ one can derive

$$\int_0^\infty \xi\left[c_{44}^I A_1(\xi)\cos(\xi x) - \left(\tilde{c}_{44}^I B_1(\xi) - \alpha^I C_1(\xi) - \beta^I D_1(\xi)\right)e^{-\xi x}\right]d\xi = \tau_0^I. \tag{3.30}$$

Substituting Eqs (3.26) and (3.18) with the use of Eq.(3.29), into Eq.(3.30) we have with the help of known integrals

$$\frac{2}{\pi}\int_0^\infty \sin(\xi t)\cos(\xi x)\,d\xi = \frac{1}{\pi}\left(\frac{1}{t-x}+\frac{1}{t+x}\right),$$
$$\int_0^\infty e^{-\xi(t+x)}d\xi = \frac{1}{t+x}; \qquad t+x>0, \tag{3.31}$$

the following singular integral equation with the generalized Cauchy kernel for $g(t)$

$$\frac{1}{\pi}\int_0^a \left(\frac{1}{t-x}+\frac{\lambda}{t+x}\right)g(t)\,dt = -\frac{\tau_0^I}{c_{44}^I}; \qquad 0<x<a \tag{3.32}$$

where

$$\lambda = 1-2\frac{\tilde{c}_{44}^{II}\Delta+\left(e_3^{II}+e_3^I\right)\alpha^{II}\left(\alpha^{II}-\alpha^I\right)+\left(e_1^{II}+e_1^I\right)\beta^{II}\left(\beta^{II}-\beta^I\right)-\left(e_2^{II}+e_2^I\right)\left(\alpha^I\beta^{II}+\beta^I\alpha^{II}+2\alpha^{II}\beta^{II}\right)}{\left(\tilde{c}_{44}^I+\tilde{c}_{44}^{II}\right)\Delta+\left(e_3^{II}+e_3^I\right)\left(\alpha^{II}-\alpha^I\right)^2+\left(e_1^{II}+e_1^I\right)\left(\beta^{II}-\beta^I\right)^2}\times$$
$$\times\left[\frac{\tilde{c}_{44}^I}{c_{44}^I}-\frac{\left(e_3^{II}+e_3^I\right)\alpha^I\left(\alpha^{II}-\alpha^I\right)+\left(e_1^{II}+e_1^I\right)\beta^I\left(\beta^{II}-\beta^I\right)-\left(e_2^{II}+e_2^I\right)\left(\alpha^I\beta^{II}+\beta^I\alpha^{II}-2\alpha^I\beta^I\right)}{c_{44}^I\Delta}\right]+$$
$$+2\frac{\left(e_3^{II}+e_3^I\right)\alpha^I\alpha^{II}+\left(e_1^{II}+e_1^I\right)\beta^I\beta^{II}-\left(e_2^{II}+e_2^I\right)\left(\alpha^I\beta^{II}+\beta^I\alpha^{II}\right)}{c_{44}^I\Delta}. \tag{3.33}$$

For both piezoelectric materials λ is obtained as follows

$$\lambda = \frac{\left(c_{44}^I-c_{44}^{II}\right)\left(\varepsilon_{11}^I+\varepsilon_{11}^{II}\right)+\left(e_{15}^I\right)^2-\left(e_{15}^{II}\right)^2+2e_{15}^I\left(e_{15}^{II}-e_{15}^I\frac{c_{44}^{II}}{c_{44}^I}\right)}{\left(c_{44}^I+c_{44}^{II}\right)\left(\varepsilon_{11}^I+\varepsilon_{11}^{II}\right)+\left(e_{15}^I+e_{15}^{II}\right)^2}. \tag{3.34}$$

The value of λ for both piezomagnetic materials is obtained from formula Eq.(3.34) if we replace ε_{11} by μ_{11} and e_{15} by q_{15}. It is noted that in a usual integral equation with the Cauchy kernel, other kernels except the Cauchy kernel are continuous over the entire interval involved. In addition to the singularity of the Cauchy kernel terms $1/(t-x)$ as $t\to x$ in Eq.(3.32) the other term $\lambda/(t+x)$ is also unbonded as $t, x\to 0$ simultaneously. Particularly for two elastic dielectric materials, meaning $e_{15}=0$, and dimagnetic, meaning $q_{15}=0$, the elastic field and electric field, and elastic field and magnetic field are not coupled as well as when $d_{11}=0$ the electromagnetic field does not occur. In this case λ reduces to

$$\lambda = \frac{c_{44}^I-c_{44}^{II}}{c_{44}^I+c_{44}^{II}}. \tag{3.35}$$

Then the integral equation is simplified to

$$\frac{1}{\pi}\int_0^a \left(\frac{1}{t-x}+\frac{\lambda}{t+x}\right)g(t)\,dt = -\frac{\tau_0}{c_{44}}. \tag{3.36}$$

This equation is equivalent to that derived by Cook and Erdogan (1972) and Erdogan and Cook (1974), who were apparently the first to publish the solution of an anti – plane shear crack terminating at the interface of two joined purely elastic media.

4. Magnetoelectroelastic field

4.1. Solution of the singular integral equation

Based on the result derived by Bueckner (1966), the desired solution for $g(t)$ of Eq.(3.32) subjected to Eq.(3.24) can be obtained as follows

$$g(x)=\frac{\tau_0^I}{2c_{44}^I \sin\left(\frac{\pi\alpha}{2}\right)}\left[\left(\frac{x}{a+\sqrt{a^2-x^2}}\right)^{\alpha}\left(\frac{\alpha a}{\sqrt{a^2-x^2}}+1\right)+\right.$$
$$\left.+\left(\frac{x}{a+\sqrt{a^2-x^2}}\right)^{-\alpha}\left(\frac{\alpha a}{\sqrt{a^2-x^2}}-1\right)\right], \tag{4.1}$$

for $0<x<a$ with

$$\cos(\pi\alpha)=-\lambda \tag{4.2}$$

where $0< \alpha <1$.

Once $g(t)$ is determined the crack tearing displacement can be obtained by the followings integrations

$$w^I\left(x,0^+\right)=\int_0^x g(x)dx=-\frac{\tau_0^I x}{2c_{44}^I \sin\left(\frac{\pi\alpha}{2}\right)}\left[\left(\frac{x}{a+\sqrt{a^2-x^2}}\right)^{\alpha}-\left(\frac{x}{a+\sqrt{a^2-x^2}}\right)^{-\alpha}\right], \tag{4.3}$$

$0\le x\le a$.

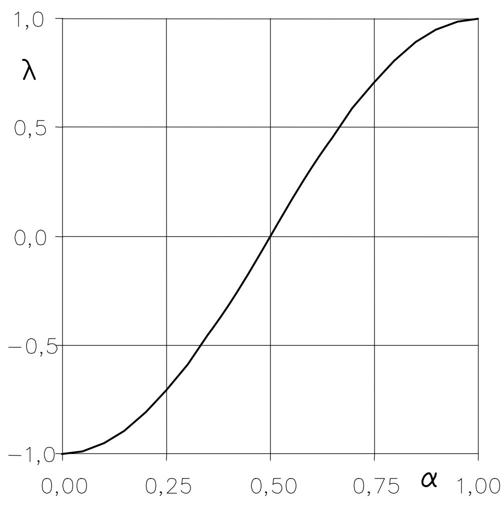

Fig.2. The curve $\lambda = -\cos(\pi\alpha)$; λ is the bi–material parameter and α is singularity order parameter.

4.2. Crack tearing displacement

Expanding the expression Eq.(4.3) near the crack tips yields the asymptotic crack tearing displacement as

$$w^I(x,0) = \frac{\tau_0^I}{c_{44}^I}\frac{\alpha}{\sin\left(\frac{\pi\alpha}{2}\right)}\sqrt{2a(a-x)} + O(r); \qquad r = a - x \approx 0, \tag{4.4}$$

$$w^I(x,0) = \frac{\tau_0^I a^\alpha}{2c_{44}^I \sin\left(\frac{\pi\alpha}{2}\right)} x^{1-\alpha} + O(r); \qquad r = x \approx 0, \tag{4.5}$$

at the right and left crack tip.

Here $O(r)$ denotes the infinitesimal terms compared to r, r being the distance from the crack tip. Only for $\alpha = 1/2$ the behaviours of the crack tearing displacement for both tips are the same.

4.3. Asymptotic crack – tip field

Anti-plane shear crack and in-plane electric displacement and magnetic induction may be deduced by evaluating the followings integrals

$$\tau_{yz}^I(x,0) = \frac{1}{\pi}c_{44}^I\int_0^a\left(\frac{1}{t-x} + \frac{\lambda}{t+x}\right)g(t)\,dt + \tau_0^I,$$

$$D_y^I(x,0) = \frac{1}{\pi}e_{15}^I\int_0^a\left(\frac{1}{t-x} + \frac{1-2\lambda_D}{1+t}\right)g(t)\,dt + D^I, \tag{4.6}$$

$$B_y^I(x,0) = \frac{1}{\pi}q_{15}^I\int_0^a\left(\frac{1}{t-x} + \frac{1-2\lambda_B}{1+t}\right)g(t)\,dt + B^I,$$

for $x > a$ and

$$\tau_{yz}^{II}(x,0) = \frac{1-\lambda}{\pi}c_{44}^I\int_0^a\frac{g(t)}{t-x}dt + \tau_0^{II},$$

$$D_y^{II}(x,0) = \frac{2\lambda_D}{\pi}e_{15}^I\int_0^a\frac{g(t)}{t-x}dt + D^{II}, \tag{4.7}$$

$$B_y^{II}(x,0) = \frac{2\lambda_B}{\pi}q_{15}^I\int_0^a\frac{g(t)}{t-x}dt + B^{II},$$

for $x < 0$, where $1-\lambda$ is defined by Eq.(3.33) and

$$
\begin{gathered}
e_{15}^{I}\lambda_{D}=\left\{\Delta\left[\left(\tilde{c}_{44}^{II}\alpha^{I}+\tilde{c}_{44}^{I}\alpha^{II}\right)\left(e_{3}^{II}+e_{3}^{I}\right)-\left(\tilde{c}_{44}^{II}\beta^{I}+\tilde{c}_{44}^{I}\beta^{II}\right)\left(e_{2}^{II}+e_{2}^{I}\right)\right]+\right.\\
+\left(e_{3}^{II}+e_{3}^{I}\right)\left(e_{2}^{II}+e_{2}^{I}\right)\left(\alpha^{II}-\alpha^{I}\right)\left(\alpha^{I}\beta^{II}+\beta^{I}\alpha^{II}+2\alpha^{II}\beta^{II}\right)+\\
+\left(e_{3}^{II}+e_{3}^{I}\right)\left(e_{1}^{II}+e_{1}^{I}\right)\left(\beta^{II}-\beta^{I}\right)\left(\alpha^{I}\beta^{II}-\beta^{I}\alpha^{II}\right)+\\
+\left(e_{2}^{II}+e_{2}^{I}\right)^{2}\left(\beta^{II}-\beta^{I}\right)\left(\alpha^{I}\beta^{II}+\beta^{I}\alpha^{II}+2\alpha^{II}\beta^{II}\right)\times\\
\times\left\{\Delta\left[\left(\tilde{c}_{44}^{I}+\tilde{c}_{44}^{II}\right)\Delta+\left(e_{3}^{II}+e_{3}^{I}\right)\left(\alpha^{II}-\alpha^{I}\right)^{2}+\left(e_{1}^{II}+e_{1}^{I}\right)\left(\beta^{II}-\beta^{I}\right)^{2}\right]\right\}^{-1},\\
q_{15}^{I}\lambda_{B}=\left\{\Delta\left[\left(\tilde{c}_{44}^{II}\beta^{I}+\tilde{c}_{44}^{I}\beta^{II}\right)\left(e_{1}^{II}+e_{1}^{I}\right)-\left(\tilde{c}_{44}^{II}\alpha^{I}+\tilde{c}_{44}^{I}\alpha^{II}\right)\left(e_{2}^{II}+e_{2}^{I}\right)\right]+\right.\\
-\left(e_{1}^{II}+e_{1}^{I}\right)\left(e_{2}^{II}+e_{2}^{I}\right)\left(\beta^{II}-\beta^{I}\right)\left(\alpha^{I}\beta^{II}+\beta^{I}\alpha^{II}+2\alpha^{II}\beta^{II}\right)+\\
-\left(e_{2}^{II}+e_{2}^{I}\right)\left(e_{1}^{II}+e_{1}^{I}\right)\left(\beta^{II}-\beta^{I}\right)\left(\alpha^{I}\beta^{II}-\beta^{I}\alpha^{II}\right)+\\
-\left(e_{2}^{II}+e_{2}^{I}\right)^{2}\left(\alpha^{II}-\alpha^{I}\right)\left(\alpha^{I}\beta^{II}+\beta^{I}\alpha^{II}+2\alpha^{II}\beta^{II}\right)\times\\
\times\left\{\Delta\left[\left(\tilde{c}_{44}^{I}+\tilde{c}_{44}^{II}\right)\Delta+\left(e_{3}^{II}+e_{3}^{I}\right)\left(\alpha^{II}-\alpha^{I}\right)^{2}+\left(e_{1}^{II}+e_{1}^{I}\right)\left(\beta^{II}-\beta^{I}\right)^{2}\right]\right\}^{-1}.
\end{gathered}
\tag{4.8}
$$

For both piezoelectric or piezomagnetic materials Eq.(4.8) gives

$$
e_{15}^{I}\lambda_{D}=\frac{e_{15}^{I}\left(c_{44}^{II}\varepsilon_{11}^{II}+\left(e_{15}^{II}\right)^{2}\right)+e_{15}^{II}\left(c_{44}^{I}\varepsilon_{11}^{I}+\left(e_{15}^{I}\right)^{2}\right)}{\left(c_{44}^{I}+c_{44}^{II}\right)\left(\varepsilon_{11}^{I}+\varepsilon_{11}^{II}\right)+\left(e_{15}^{I}+e_{15}^{II}\right)^{2}},\qquad q_{15}^{I}\lambda_{B}=0,
\tag{4.9}
$$

or

$$
e_{15}^{I}\lambda_{D}=0,\qquad q_{15}^{I}\lambda_{B}=\frac{q_{15}^{I}\left(c_{44}^{II}\mu_{11}^{II}+\left(q_{15}^{II}\right)^{2}\right)+q_{15}^{II}\left(c_{44}^{I}\mu_{11}^{I}+\left(q_{15}^{I}\right)^{2}\right)}{\left(c_{44}^{I}+c_{44}^{II}\right)\left(\mu_{11}^{I}+\mu_{11}^{II}\right)+\left(q_{15}^{I}+q_{15}^{II}\right)^{2}}.
\tag{4.10}
$$

The analytical expressions for physical quantities may be obtained by substituting the solution Eq.(4.1) into Eqs (4.6) and (4.7). We omit the full solution and pay our attention to the asymptotic crack – tip field. This is very interesting from the view point of fracture mechanics. From Eq.(4.1), one can write out the singular behaviour of the function $g(x)$ near the point $x = 0$ and $x = a$ by the following asymptotic expressions

$$
g(x)=-\frac{\tau_{0}^{I}}{2c_{44}^{I}}\frac{\alpha}{\sin\left(\frac{\pi\alpha}{2}\right)}\sqrt{\frac{2a}{a-x}}+0(1);\qquad x\approx a-0,
\tag{4.11}
$$

$$g(x) = -\frac{\tau_0^I}{2c_{44}^I}\frac{\alpha-1}{\sin\left(\frac{\pi\alpha}{2}\right)}\left[\frac{2(a-x)}{x}\right]^{\alpha} + 0(1); \qquad x \approx 0+0 \tag{4.12}$$

where $0(1)$ stands for nonsingular terms.

Now we define the intensity factor at the right crack tip in the homogeneous solid and the left crack tip at the interface of a bi – medium as

$$K_{\text{hom}}^{q} = \underset{x\to a^{+}}{1}\sqrt{2\pi(x-a)}q^{I}\left(x,0^{+}\right), \tag{4.13}$$

$$K_{\text{int}}^{q} = \underset{x\to 0^{-}}{1}\left(-2\pi x\right)^{\alpha} q^{II}\left(x,0^{+}\right), \tag{4.14}$$

respectively, where q stands for one of τ_{yz}, γ_{yz}, D_y, B_y, E_y and H_y.

4.3.1. Magnetoelectroelastic field near the crack tip in the homogeneous PEMO – elastic ceramics

Using the integral

$$\frac{1}{\pi}\int_0^a \frac{1}{(t-x)\sqrt{a-t}}dt = -\frac{2}{\pi\sqrt{x-a}}\tan^{-1}\sqrt{\frac{a}{x-a}}, \qquad x > a, \tag{4.15}$$

we obtain from Eq.(4.6)$_1$

$$\tau_{yz}^{I}(x,0) = \frac{1}{\pi}c_{44}^{I}\int_0^a \frac{g(t)}{t-x}dt + 0(1) = \frac{K_{\text{hom}}^{\tau}}{\sqrt{2\pi(x-a)}} \tag{4.16}$$

where

$$K_{\text{hom}}^{\tau} = \frac{\alpha}{\sin(\pi\alpha/2)}\tau_o^I\sqrt{\pi a}, \tag{4.17}$$

denotes the stress intensity factor at the right crack tip. Other field intensity factors are related to K_{hom}^{τ} as follows

$$K_{\text{hom}}^{\gamma} = \frac{1}{c_{44}^I}K_{\text{hom}}^{\tau}, \qquad K_{\text{hom}}^{D} = \frac{e_{15}^I}{c_{44}^I}K_{\text{hom}}^{\tau}, \qquad K_{\text{hom}}^{B} = \frac{q_{15}^I}{c_{44}^I}K_{\text{hom}}^{\tau},$$
$$K_{\text{hom}}^{\varphi} = K_{\text{hom}}^{\psi} = K_{\text{hom}}^{E} = K_{\text{hom}}^{H} = 0. \tag{4.18}$$

For the crack tip in the homogeneous PEMO – elastic medium the elastic, electric and magnetic fields still exhibit an inverse square – root singularity at the crack tip. The application of electric and magnetic fields does not alter the stress intensity factors. The stress intensity factor depends on the material properties of two PEMO – elastic ceramics involved, since it is governed by Eq.(4.17) and α by Eq.(4.2). The intensity factors K^{γ}_{hom}, K^{D}_{hom} and K^{B}_{hom} are related to K^{τ}_{hom} and also depend on the material properties, as shown in Eq.(4.18).

4.3.2. Magnetoelectroelastic field near the crack tip at the interface

Using the known result (Tricomi, 1985)

$$\frac{1}{\pi}\int_{0}^{a}\frac{1}{(t-x)}\left(\frac{a-t}{t}\right)^{\alpha}dt=\frac{1}{\sin(\pi\alpha)}\left[\left(\frac{x-a}{x}\right)^{\alpha}-1\right],\qquad x<0, \tag{4.19}$$

putting Eq.(4.12) into Eq.(4.7) and using Eq.(4.19), we obtain the asymptotic expressions for the anti – plane shear stress and in – plane electric displacement and magnetic induction, as well as elastic strain, electric and magnetic field, ahead on the left crack tip at the interface as follows

$$\begin{gathered}\left[K^{\tau}_{\text{int}};\;K^{D}_{\text{int}};\;K^{B}_{\text{int}};\;K^{\gamma}_{\text{int}};\;K^{E}_{\text{int}};\;K^{H}_{\text{int}}\right]=\\=\frac{\sqrt{2}(1-\alpha)}{(1+\lambda)\sqrt{1-\lambda}}\frac{\tau^{I}_{0}}{c^{I}_{44}}(4\pi a)^{\alpha}\left[c^{I}_{44}\frac{1-\lambda}{2};\;e^{I}_{15}\lambda_{D};\;q^{I}_{15}\lambda_{B};\;\lambda_{\gamma};\;\lambda_{E};\;\lambda_{H}\right]\end{gathered} \tag{4.20-4.25}$$

where the identity is used as follows

$$\sin(\pi\alpha)\sin\left(\frac{\pi\alpha}{2}\right)=(1+\lambda)\sqrt{\frac{1-\lambda}{2}},$$

and where

$$\begin{aligned}\lambda_{\gamma}&=\Big[\tilde{c}^{I}_{44}\Delta-\left(e^{II}_{3}+e^{I}_{3}\right)\alpha^{I}\left(\alpha^{II}-\alpha^{I}\right)-\left(e^{II}_{1}+e^{I}_{1}\right)\beta^{I}\left(\beta^{II}-\beta^{I}\right)+\\&+\left(e^{II}_{2}+e^{I}_{2}\right)\left(\alpha^{I}\beta^{II}+\beta^{I}\alpha^{II}+2\alpha^{II}\beta^{II}\right)\Big]\times\\&\times\left[\left(\tilde{c}^{I}_{44}+\tilde{c}^{II}_{44}\right)\Delta+\left(e^{II}_{3}+e^{I}_{3}\right)\left(\alpha^{II}-\alpha^{I}\right)^{2}+\left(e^{II}_{1}+e^{I}_{1}\right)\left(\beta^{II}-\beta^{I}\right)^{2}\right]^{-1},\\ \lambda_{E}&=\Big\{\lambda_{\gamma}\Big[\alpha^{I}\Delta+\left(\alpha^{II}-\alpha^{I}\right)\left(e^{I}_{1}\left(e^{II}_{3}+e^{I}_{3}\right)-e^{I}_{2}\left(e^{II}_{2}+e^{I}_{2}\right)\right)-\left(\beta^{II}-\beta^{I}\right)\left(e^{I}_{1}e^{II}_{2}-e^{I}_{2}e^{II}_{1}\right)\Big]+\\&-\alpha^{II}\left(e^{I}_{1}\left(e^{II}_{3}+e^{I}_{3}\right)-e^{I}_{2}\left(e^{II}_{2}+e^{I}_{2}\right)\right)+\beta^{II}\left(e^{I}_{1}e^{II}_{2}-e^{I}_{2}e^{II}_{1}\right)\Big\}\Delta^{-1},\\ \lambda_{H}&=\Big\{\lambda_{\gamma}\Big[\beta^{I}\Delta+\left(\beta^{II}-\beta^{I}\right)\left(e^{I}_{3}\left(e^{II}_{1}+e^{I}_{1}\right)-e^{I}_{2}\left(e^{II}_{2}+e^{I}_{2}\right)\right)-\left(\alpha^{II}-\alpha^{I}\right)\left(e^{I}_{2}e^{II}_{3}-e^{I}_{3}e^{II}_{2}\right)\Big]+\\&-\beta^{II}\left(e^{I}_{2}\left(e^{II}_{2}+e^{I}_{2}\right)-e^{I}_{3}\left(e^{II}_{1}+e^{I}_{1}\right)\right)+\alpha^{II}\left(e^{I}_{2}e^{II}_{3}-e^{I}_{3}e^{II}_{2}\right)\Big\}\Delta^{-1},\end{aligned} \tag{4.26}$$

for the PEMO – elastic bi-material and

$$\lambda_\gamma = \frac{c_{44}^I\left(\varepsilon_{11}^I + \varepsilon_{11}^{II}\right) + e_{15}^I\left(e_{15}^I + e_{15}^{II}\right)}{\left(c_{44}^I + c_{44}^{II}\right)\left(\varepsilon_{11}^I + \varepsilon_{11}^{II}\right) + \left(e_{15}^I + e_{15}^{II}\right)^2},$$

$$\lambda_E = \frac{c_{44}^{II} e_{15}^I - c_{44}^I e_{15}^{II}}{\left(c_{44}^I + c_{44}^{II}\right)\left(\varepsilon_{11}^I + \varepsilon_{11}^{II}\right) + \left(e_{15}^I + e_{15}^{II}\right)^2}, \tag{4.27}$$

$$\lambda_H = 0,$$

for the piezoelectric bi-material and

$$\lambda_\gamma = \frac{c_{44}^I\left(\mu_{11}^I + \mu_{11}^{II}\right) + q_{15}^I\left(q_{15}^I + q_{15}^{II}\right)}{\left(c_{44}^I + c_{44}^{II}\right)\left(\mu_{11}^I + \mu_{11}^{II}\right) + \left(q_{15}^I + q_{15}^{II}\right)^2},$$

$$\lambda_E = 0, \tag{4.28}$$

$$\lambda_H = \frac{c_{44}^{II} q_{15}^I - c_{44}^I q_{15}^{II}}{\left(c_{44}^I + c_{44}^{II}\right)\left(\mu_{11}^I + \mu_{11}^{II}\right) + \left(q_{15}^I + q_{15}^{II}\right)^2},$$

for the piezomagnetic bi-material.

Note that for the piezoelectric bi – material we have

$$\lambda_\tau = \frac{1-\lambda}{2} = \frac{c_{44}^I\left(\varepsilon_{11}^I + \varepsilon_{11}^{II}\right) + \left(e_{15}^{II}\right)^2 + \left(e_{15}^I\right)^2 \dfrac{c_{44}^{II}}{c_{44}^I}}{\left(c_{44}^I + c_{44}^{II}\right)\left(\varepsilon_{11}^I + \varepsilon_{11}^{II}\right) + \left(e_{15}^I + e_{15}^{II}\right)^2}. \tag{4.29}$$

The material parameters for the piezoelectric ceramics coincide, in general, with the ones derived by Li and Wang (2007). But in λ, defined exactly by Eq.(3.34), the fourth term in numerator of Eq.(3.34) is omitted in Eq.(3.20) of Li and Wang paper. In consequence, the conclusions in Tab.2 of Li and Wang paper that λ vanishes also in the case of ceramics poled in an opposite direction are incorrect. The formula Eq.(3.34) shows that only for two bonded piezoelectric ceramics with c_{44} unchanged poled in the same direction (not opposite) the field singularity at the interface crack tip maintains the inverse square root singularity, since in this case $\lambda = 0$ and $\alpha = 1/2$. The parameter λ_E in this paper has an opposite sign to that presented by Wang and Li. This gives that for $\rho_c > 1$ $\left(c_{44}^{II} > c_{44}^I\right)$ meaning that piezoelectric ceramic II is stiffer than piezoelectric ceramic I $\left(e_{15}^{II} = e_{15}^I\right)$, in this case $\lambda_E > 0$, so stands also $K_{\text{int}}^E > 0$ and K_{int}^E increases with ρ_c. Also it is seen that K_{int}^E decreases with the ratio ρ_e of e_{15}^{II} to e_{15}^I. In the paper of Li and Wang (2007) the conclusions, associated with K_{int}^E, are inverse. The presented conclusions are consistent with physical considerations. The field intensity factors must satisfy the constitutive equations

$$K^{\tau}=c_{44}^{II}K^{\gamma}-e_{15}^{II}K^{E}, \qquad K^{D}=e_{15}^{II}K^{\gamma}+\varepsilon_{11}^{II}K^{E}, \tag{4.30}$$

or material parameters must satisfy the equivalent equations

$$c_{44}^{I}\lambda_{\tau}=c_{44}^{II}\lambda_{\gamma}-e_{15}^{II}\lambda_{E}, \qquad e_{15}^{I}\lambda_{D}=e_{15}^{II}\lambda_{\gamma}+\varepsilon_{11}^{II}\lambda_{E}. \tag{4.31}$$

It is easily verified that both constitutive relations Eq.(4.31) are satisfied by the coefficients defined by Eqs (4.9), (4.27) and (4.29). In general, for the magnetoelectroelastic ceramic the field intensity factors must satisfy the constitutive equations

$$\begin{bmatrix} K_{\text{int}}^{\tau} \\ K_{\text{int}}^{\varphi} \\ K_{\text{int}}^{\psi} \end{bmatrix}=\begin{bmatrix} \tilde{c}_{44} & -\alpha & -\beta \\ \alpha & e_{1} & e_{2} \\ \beta & e_{2} & e_{3} \end{bmatrix}\begin{bmatrix} K^{w} \\ K^{D} \\ K^{B} \end{bmatrix}, \tag{4.32}$$

as Eq.(2.14) shows. Of course, we have $K_{\text{int}}^{\varphi}=-K_{\text{int}}^{E}$ and $K_{\text{int}}^{\psi}=-K_{\text{int}}^{H}$.

4.4. The energy release rate

For the magnetoelectrically permeable crack, the energy release rates are very important to evaluate the behaviours of crack tips. In accordance with the definition of the energy release rate proposed by Pak (1990) (the virtual crack closure integral) the energy release rate can finally be derived as

$$G=\frac{1}{2c_{44}^{I}}\left[\left(K_{\text{hom}}^{\tau}\right)^{2}+\left(\frac{\pi a}{2}\right)^{1-2\alpha}\left(K_{\text{int}}^{\tau}\right)^{2}\right]=G_{\text{hom}}\frac{1}{2}\left[\left(k_{\text{hom}}^{\tau}\right)^{2}+\left(k_{\text{int}}^{\tau}\right)^{2}\right] \tag{4.33}$$

where

$$G_{\text{hom}}=\frac{\pi a\left(\tau_{0}^{I}\right)^{2}}{2c_{44}^{I}}, \tag{4.34}$$

$$k_{\text{hom}}^{\tau}=\frac{K_{\text{hom}}^{\tau}}{\tau_{0}^{I}\sqrt{\dfrac{\pi a}{2}}}, \tag{4.35}$$

$$k_{\text{int}}^{\tau}=\frac{K_{\text{int}}^{\tau}}{\tau_{0}^{I}\left(\dfrac{\pi a}{2}\right)^{\alpha}}, \tag{4.36}$$

are the energy release rate for homogeneous material (no bi-material) and normalized stress intensity factors at the right and left crack tip. One interesting observation from Eq.(4.33) is that though the energy release rate, G, is independent of the applied electric-magnetic load, it is affected by electric-magnetic properties of two constituents of the bi-material media.

4.5. Electric displacement and magnetic induction inside the crack

Since the medium inside the crack (usually air or vacuum) allows some penetrations of some electric and magnetic fields, these fields may not be zero. Suppose the normal components of the electric displacement and magnetic induction inside the crack are d_0 and b_0, respectively. Then from permeable crack boundary conditions Eq.(2.18) and solutions Eqs (4.20) – (4.25) it follows that the quantities d_0 and b_0 are as follows

$$d_0 = \begin{cases} D_0^I - \dfrac{e_{15}^I \tau_0^I}{c_{44}^I} \dfrac{2\lambda_D}{1-\lambda}, & \text{case I} \\ \dfrac{e_{15}^I \tau_0^I}{c_{44}^I}\left(1 - \dfrac{2\lambda_D}{1-\lambda}\right) + \left(\varepsilon_{11}^I + \dfrac{\left(e_{15}^I\right)^2}{c_{44}^I}\right) E_0^I + \left(d_{11}^I + \dfrac{e_{15}^I q_{15}^I}{c_{44}^I}\right) H_0^I, & \text{case II} \end{cases}$$

$$b_0 = \begin{cases} B_0^I - \dfrac{q_{15}^I \tau_0^I}{c_{44}^I} \dfrac{2\lambda_B}{1-\lambda}, & \text{case I} \\ \dfrac{q_{15}^I \tau_0^I}{c_{44}^I}\left(1 - \dfrac{2\lambda_B}{1-\lambda}\right) + \left(\mu_{11}^I + \dfrac{\left(q_{15}^I\right)^2}{c_{44}^I}\right) H_0^I + \left(d_{11}^I + \dfrac{e_{15}^I q_{15}^I}{c_{44}^I}\right) E_0^I, & \text{case II.} \end{cases} \tag{4.37}$$

Then using Eqs (2.21), we obtain

$$D^I - d_0 = \frac{e_{15}^I \tau_0^I}{c_{44}^I} \frac{2\lambda_D}{1-\lambda},$$

$$B^I - b_0 = \frac{q_{15}^I \tau_0^I}{c_{44}^I} \frac{2\lambda_B}{1-\lambda}, \tag{4.38}$$

in both cases of loading conditions.

The electric displacement and magnetic induction intensity factors are proportional to $D_0^I - d_0$ and $B_0^I - b_0$, respectively, (Rogowski, 2011) and the following relations hold

$$K_{\text{int}}^D = K_{\text{int}}^\tau \frac{e_{15}^I}{c_{44}^I} \frac{2\lambda_D}{1-\lambda},$$

$$K_{\text{int}}^B = K_{\text{int}}^\tau \frac{q_{15}^I}{c_{44}^I} \frac{2\lambda_B}{1-\lambda}, \tag{4.39}$$

which are in agreement with the solutions Eqs (4.20)-(4.25). For piezoelectric bi-materials or piezomagnetic bi-materials we have, for instance

$$K_{\mathrm{int}}^{D}=K_{\mathrm{int}}^{\tau}\frac{c_{44}^{II}\varepsilon_{11}^{II}e_{15}^{I}+c_{44}^{I}\varepsilon_{11}^{I}e_{15}^{II}+e_{15}^{I}e_{15}^{II}\left(e_{15}^{I}+e_{15}^{II}\right)}{c_{44}^{II}c_{44}^{I}\left(\varepsilon_{11}^{I}+\varepsilon_{11}^{II}\right)+c_{44}^{I}\left(e_{15}^{II}\right)^{2}+c_{44}^{II}\left(e_{15}^{I}\right)^{2}},$$

$$K_{\mathrm{int}}^{B}=K_{\mathrm{int}}^{\tau}\frac{c_{44}^{II}\mu_{11}^{II}q_{15}^{I}+c_{44}^{I}\mu_{11}^{I}q_{15}^{II}+q_{15}^{I}q_{15}^{II}\left(q_{15}^{I}+q_{15}^{II}\right)}{c_{44}^{II}c_{44}^{I}\left(\mu_{11}^{I}+\mu_{11}^{II}\right)+c_{44}^{I}\left(q_{15}^{II}\right)^{2}+c_{44}^{II}\left(q_{15}^{I}\right)^{2}}. \tag{4.40}$$

In particular, for a fully permeable crack considered here, and two identical magneto– or electro – elastic planes polarized in opposite directions we have (from Eq.(4.40))

$$K_{\mathrm{int}}^{D}=K_{\mathrm{int}}^{B}=0. \tag{4.40a}$$

Note that the crack tip electric displacement K_{int}^{D} and the electric displacement inside the crack d_0 exist only in the piezoelectric plane. Alternatively, the crack tip magnetic induction intensity factor K_{int}^{B} and the magnetic induction inside the crack b_0 exist only in the piezomagnetic plane. All quantities occur in the PEMO – elastic bi – material.

5. Results and discussions

In studying the fracture behaviour of the PEMO – elastic material the field intensity factors are of significance. In this section, examples are given to illustrate the effects of material properties on the field intensity factor and the order of singularity.

5.1. Effect of material constants on the singularity order

We now consider the dependence of the singularity order on 2×6 constituent independent piezoelectromagnetoelastic constants. Although an analytical evaluation of the relative sensitivities is possible, on the basis of the results presented above, it is rather cumbersome. Therefore the sensitivity is evaluated here in another way.

Firstly, we assume that both materials are piezoelectric and $c_{44}^{II}=\rho_c c_{44}^{I}$, $e_{15}^{II}=\rho_e e_{15}^{I}$ and $\varepsilon_{11}^{II}=\rho_\varepsilon \varepsilon_{11}^{I}$, and analyze the situations

(a) ρ_c changes and $\rho_e=\rho_\varepsilon=1$, that is, no change,

(b) ρ_e changes and $\rho_c=\rho_\varepsilon=1$,

(c) ρ_ε changes and $\rho_c=\rho_e=1$.

This states that the right half – plane is fixed and the left one contains a fictitious material with only changing ρ_c or ρ_e or ρ_ε.

Then:

a) The changes of the ratio ρ_c of c_{44}^{II} to c_{44}^{I} : we have

$$\lambda=\frac{(1-\rho_c)(1+m)}{1+\rho_c+2m},\quad m=\left(\frac{(e_{15})^{2}}{c_{44}\varepsilon_{11}}\right)^{I},\quad |\lambda|<1,\quad \rho_c<3+\frac{2}{m}\quad \text{for}\quad \rho_e=\rho_\varepsilon=1, \tag{5.1}$$

or

$$\lambda = \frac{1-\rho_c}{1+\rho_c} - m, \qquad \lambda = 0 \qquad \text{for} \qquad \rho_c = \frac{1-m}{1+m},$$

$$\rho_c < \frac{2}{m} - 1, \qquad 0 < m < 1 \qquad \text{for} \qquad \rho_e = -1, \qquad \rho_\varepsilon = 1. \tag{5.2}$$

Figure 3 shows the effects of the varying elastic stiffness ρ_c on λ and α with unchanging piezoelectric and piezomagnetic constants $\rho_e = \rho_\varepsilon = 1$ or $\rho_e = -1$ and $\rho_\varepsilon = 1$. Note that $\lambda = 0$ and $\alpha = 1/2$ for $\rho_e = -1$, $\rho_\varepsilon = 1$ and if $\left(c_{44}^I - c_{44}^{II}\right)c_{44}^I \varepsilon_{11}^I = \left(c_{44}^I + c_{44}^{II}\right)\left(e_{15}^I\right)^2$, $e_{15}^{II} = -e_{15}^I$ or if $\rho_c = 1$ and $\rho_e = \rho_\varepsilon = 1$. Note also that $\lambda\left(\rho_e = 1, \rho_\varepsilon = 1\right) > \lambda\left(\rho_e = -1, \rho_\varepsilon = 1\right)$ for all of ρ_c.

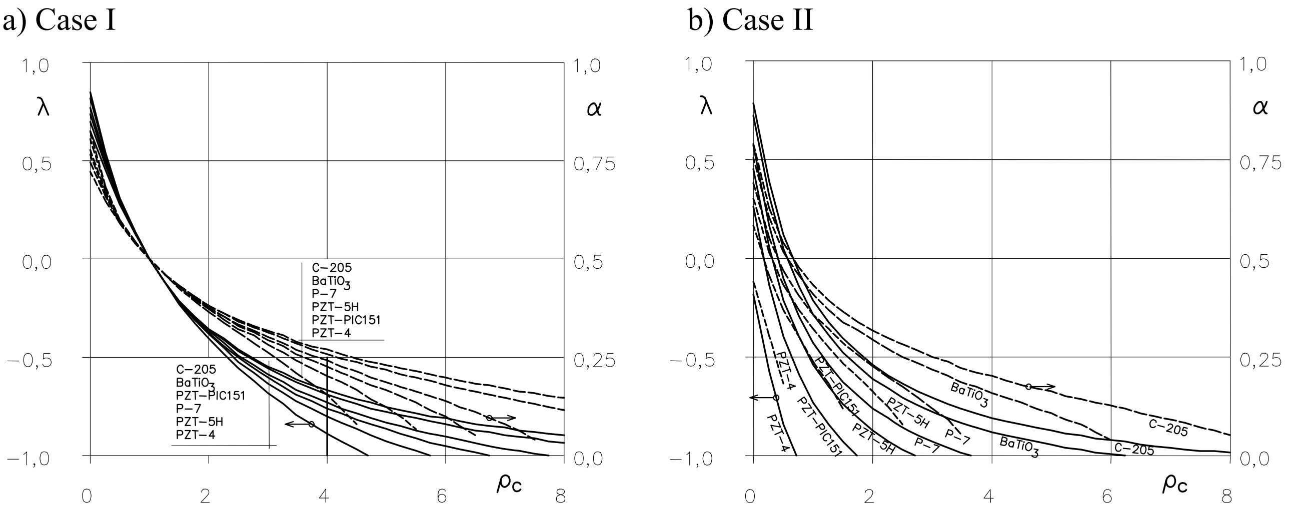

Fig.3. The effect of ρ_c on λ and α with $\rho_e = \rho_\varepsilon = 1$ (Case I) and $\rho_e = -1$, $\rho_\varepsilon = 1$ (Case II).

The singularity order α is larger for the same two ceramics poled in opposite directions together since $\alpha\left(\rho_e = 1, \rho_\varepsilon = 1\right) < \alpha\left(\rho_e = -1, \rho_\varepsilon = 1\right)$.

We take six kinds of particular piezoelectric ceramics as representatives, the relevant material constants and parameters *m*, and $1/m$ which are listed in Tab.1 (with materials poling axes aligned in the positive *z* – direction).

Table 1. Relevant material properties (Wang and Yu, 2001; Gu *et al.*, 2002) and values of material parameters *m* and $1/m$.

	c_{44} $\left[10^9\,N/m^2\right]$	e_{15} $\left[C/m^2\right]$	ε_{11} $\left[10^{-9}\,C/Vm\right]$	m	$1/m$
$BaTiO_3$	43.0	11.60	11.20	0.279	4.348
PZT-5H	35.3	17.00	15.10	0.542	1.844
PZT-4	25.6	13.44	6.00	1.175	0.851
P-7	25.0	13.50	17.10	0.430	2.325
C-205	87.0	13.59	7.95	0.210	4.761
PZT-PIC151	20.0	12.00	9.82	0.733	1.364

b) The changes of the ratio ρ_e of e_{15}^{II} to e_{15}^{I} : we have

$$\lambda = -\frac{\left(1-\rho_e\right)^2}{\frac{4}{m}+\left(1+\rho_e\right)^2}, \qquad \rho_e > -\frac{1}{m} = -\left(\frac{c_{44}\varepsilon_{11}}{\left(e_{15}\right)^2}\right)^I,$$

$$\lambda_{\max} = 0 \quad \text{for} \quad \rho_e = 1, \quad \lambda_{\min} = -1-m \quad \text{for} \quad \rho_e = -1-\frac{2}{m} < -\frac{1}{m}, \tag{5.3}$$

$$\lambda = -1 \quad \text{for} \quad \rho_e = -\frac{1}{m}, \quad \lambda = -\frac{m}{4+m} \quad \text{for} \quad \rho_e = 0, \quad \lambda = -m \quad \text{for} \quad \rho_e = -1.$$

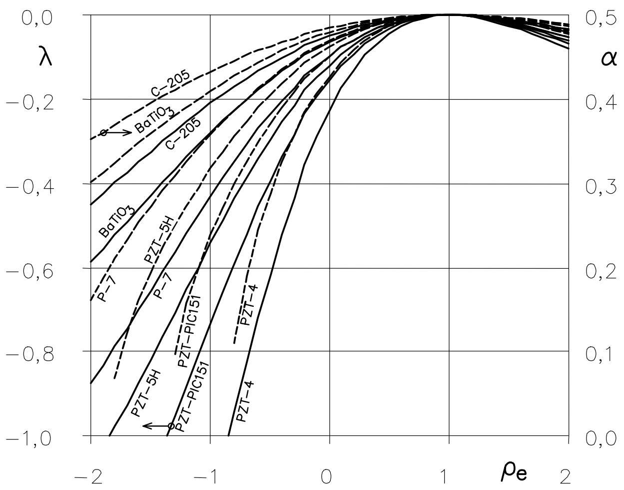

Fig.4. Effects of ρ_e on λ and α with $\rho_c = \rho_\varepsilon = 1$ ($\rho_e > 0$ or $\rho_e < 0$ denote piezoelectric ceramics poled parallel to or anti – parallel to the z – axis, respectively, i.e., $\rho_e = -1$ denotes that $e_{15}^{I} = -e_{15}^{II} = e_{15}$).

For $-1/m < \rho_e < 1$ the singularity parameter λ increases from -1 to maximum $\lambda = 0$ and for $\rho_e > 1$ declines to -1. Then the singularity parameter α varies between (0, 0.5), respectively. If both poling directions are opposite i.e., one is in the z – direction and the other is in the ($-z$) - direction, then to satisfy the condition $\lambda > -1$, $\left|e_{15}^{II(-)}\right| / e_{15}^{I(+)} < 1/m$ or $\left|e_{15}^{II(-)}\right| e_{15}^{I(+)} < c_{44}^{I}\varepsilon_{11}^{I}$ must hold. If the selection of e_{15}^{II} violates the condition $|\lambda| < 1$, then the electroelastic field near the interface crack tip is dominated by either logarithmic singularity or is bonded.

c) For ρ_ε varying and other parameter unchanged it is easily found that $\lambda = 0$ and $\alpha = 0.5$ for $\rho_c = \rho_\varepsilon = 1$ and varying ρ_ε. But if $\rho_e = -1$, then

$$\lambda = -\frac{2m}{1+\rho_\varepsilon}; \qquad m = \left(\frac{\left(e_{15}\right)^2}{c_{44}\varepsilon_{11}}\right)^I. \tag{5.4}$$

Figure 5 shows the variation of λ and α with the ratio ρ_ε for $\dfrac{e_{15}^{II(-)}}{e_{15}^{I(+)}} = -1.$

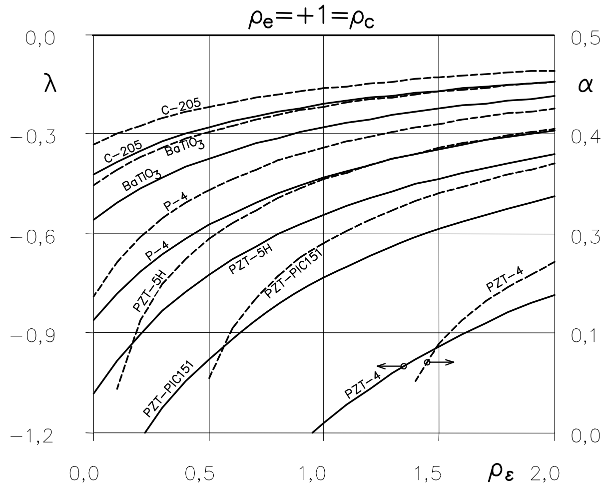

Fig.5. Effects of ρ_ε on λ and α with $\rho_c = 1$ when $\rho_e = \dfrac{e_{15}^{II(-)}}{e_{15}^{I(+)}} = -1$; for $\rho_e = 1$ we have $\lambda = 0$ and $\alpha = 0.5$.

The parameter λ assumes negative values, increases from $-2m$ to zero with $\rho_\varepsilon > 0$. The singularity parameter α is positive and increases from $(1/\pi)\arccos(2m)$ to $1/2$ with $\rho_\varepsilon > 0$. Note that $2m$ must be less than unity if ρ_ε tends to zero or $m < 1$ for $\rho_\varepsilon > 1$. Some materials shown in Tab.1 limit the range of ρ_ε, for example PZT-4 has $m = 1.175$ and ρ_ε must be larger 1.35 to ensure that $\lambda < -1$. Of course, this situation is addressed to two piezoelectrics poled in opposite directions.

For piezomagnetic materials the parameter m is

$$m = \left(\frac{(q_{15})^2}{c_{44}\mu_{11}} \right)^I, \tag{5.5}$$

and for the magnetostrictive material $CoFe_2O_4$ assumes the value $m = 0.0113$.

For $CoFe_2O_4$ we have

$$c_{44} = 45.3GPa, \qquad q_{15} = 550N/Am, \qquad \mu_{11} = 590 \times 10^{-6}\, N/A^2. \tag{5.6}$$

The “relative sensitivity” analysis includes three cases:

(a) The changes of the ratio ρ_c of c_{44}^{II} to c_{44}^{I}: we have

$$\lambda = \frac{1.0113(1-\rho_c)}{1.0226+\rho_c}, \qquad \rho_c < 20.7, \qquad \rho_q = \rho_\mu = 1 \qquad \text{or,} \tag{5.7}$$

$$\lambda = \frac{1-\rho_c}{1+\rho_c} - 0.0113, \qquad \rho_c < 16.7, \qquad \rho_q = -1, \qquad \rho_\mu = 1.$$

Approximately

$$\lambda = \frac{1-\rho_c}{1+\rho_c}, \qquad \rho_c < 16. \tag{5.8}$$

For the magnetoelectroelastic composite $BaTiO_3$ – $CoFe_2O_4$ ($V_f = 0{,}5$) $\left(q_{15}\right)^2 / c_{44}\mu_{11} = 0.005$ and $\left(e_{15}\right)^2 / c_{44}\varepsilon_{11} \approx 0.135$.

Figure 6 shows the effect of ρ_c on λ and α for the $CoFe_2O_4$ magnetostrictive ceramic

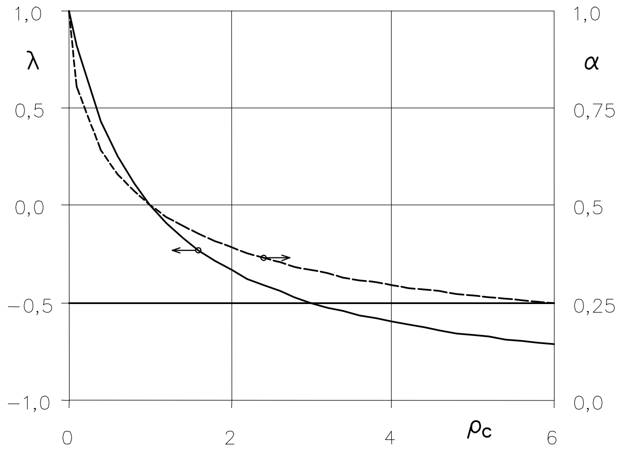

Fig.6. Effects of ρ_c on λ and α for $CoFe_2O_4$, $\rho_\mu = 1$ and $\rho_q = 1$ or $\rho_q = -1$.

For both poling directions the values of λ and α are the same.

(b) The changes of the ratio ρ_q of q_{15}^{II} to q_{15}^{I} :we have

$$\lambda = -\frac{\left(1-\rho_q\right)^2}{35.4+\left(1+\rho_q\right)^2}, \qquad \rho_q > -8.85,$$

$$\lambda_{\max} = 0 \quad \text{for} \quad \rho_q = 1, \qquad \lambda = -1 \quad \text{for} \quad \rho_q = -8.85. \tag{5.9}$$

Figure 7 shows the effect of ρ_q on λ and α for the $CoFe_2O_4$ ceramic.

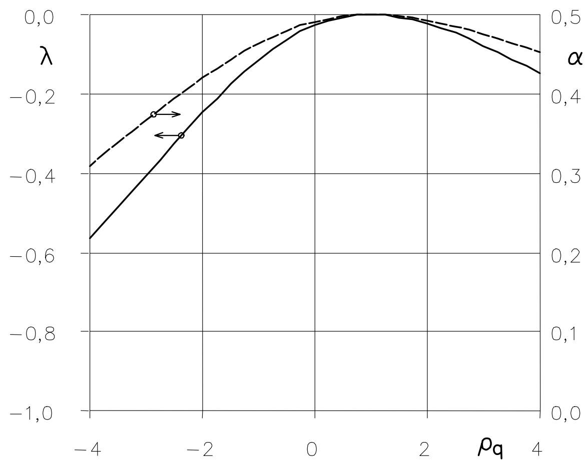

Fig.7. Effects of ρ_q on λ and α for $CoFe_2O_4$, $\rho_c = \rho_\mu = 1$.

(c) The changes of the ratio ρ_μ of μ_{15}^{II} to μ_{15}^{I} : we have

$$\lambda = -\frac{0.0226}{1+\rho_\mu} \quad \text{for} \quad \rho_q = -1 \quad \text{and} \quad \lambda = 0 \quad \text{for} \quad \rho_q = 1, \quad \text{always} \quad \rho_c = 1. \tag{5.10}$$

Figure 8 shows the effect of ρ_μ on λ and α for $CoFe_2O_4$.

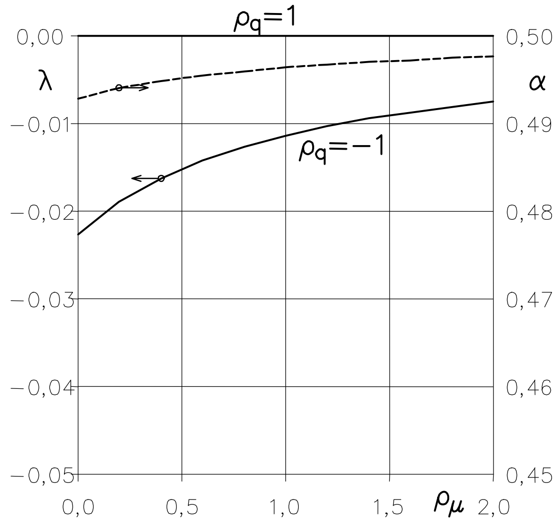

Fig.8. Effects of ρ_μ on λ and α for $CoFe_2O_4$, $\rho_q = 1$ or $\rho_q = -1$ and $\rho_c = 1$.

5.2. Effect of material constants on the field intensity factors

The material constants also affect the intensity factors. Figure 9 presents the variation of normalized SIFs $k_{\text{hom}}^{\tau} < k_{\text{int}}^{\tau}$ and k_{int}^{τ} defined by Eqs (4.35) and (4.36) which depend on α and λ

$$k_{\text{hom}}^{\tau} = \frac{2\alpha}{\sqrt{1+\lambda}}, \tag{5.11}$$

$$k_{\text{int}}^{\tau} = 2^{3\alpha - \frac{1}{2}} \frac{1-\alpha}{1+\lambda} \sqrt{1-\lambda}. \tag{5.12}$$

For $0 < \alpha < 1$ k_{hom}^{τ} increases monotonously from $2\sqrt{2}/\pi$ through 1 to $\sqrt{2}$ as α tends to zero, equals $1/2$ and 1, respectively. From Figs 8 and 2 one can observe that the effect of ρ_c on k_{int}^{τ} is more evident than that on k_{hom}^{τ}. Moreover, ρ_c increases the singularity parameter α that decreases (see Fig.3) and k_{int}^{τ} rises suddenly, while k_{hom}^{τ} falls down slightly. For $\rho_c < 1$ and $\rho_e = 1$ or $\rho_c < (1-m)/(1+m)$ and $\rho_e = -1$ we have $\alpha > 1/2$. This means that if piezoelectric II is more elastically complaisant than piezoelectric ceramic I, in this case $k_{\text{hom}}^{\tau} > k_{\text{int}}^{\tau}$. On the other hand, for $\rho_c > 1$, which gives $\alpha < 1/2$, meaning that piezoelectric II is stiffer than piezoelectric I in this case $k_{\text{hom}}^{\tau} < k_{\text{int}}^{\tau}$. From Figs 3, 4 and 5 we see that: the range $0<\alpha<1/2$ corresponds to $\rho_c>1$ or $\rho_c>(1-m)/(1+m)$ (in the case $\rho_e=-1$), $\rho_e>-1/m$ and $\rho_\varepsilon>0$. Then always $k_{\text{int}}^{\tau}>k_{\text{hom}}^{\tau}$. The range $1/2<\alpha<1$ is for $0<\rho_c<1$ or $0<\rho_c<(1-m)/(1+m)$ (in the case $\rho_e=-1$). Then $k_{\text{int}}^{\tau}<k_{\text{hom}}^{\tau}$ for all of α.

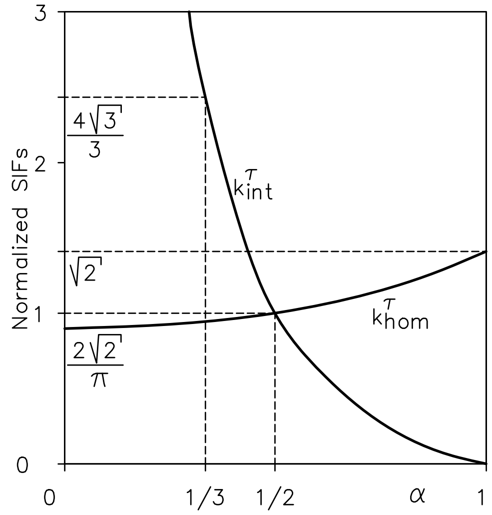

Fig.9. Normalized SIFs as a function of α.

Note that the case $\alpha=1$, $\lambda=1$ gives the limiting values $k_{\text{int}}^{\tau}=0$ and $k_{\text{hom}}^{\tau}=\sqrt{2}$ which gives

$$K_{\text{int}}^{\tau}=0, \qquad K_{\text{hom}}^{\tau}=\tau_0^I\sqrt{\pi a}. \tag{5.13}$$

This is the solution for the edge crack of length a.

The normalized intensity factors for strain, electric displacement, magnetic induction, electric field and magnetic field at the interface crack tip are defined by Eqs (4.20) to (4.25) and by formula

$$k_{\text{int}}^{q}=\frac{c_{44}^{I}}{\tau_o^{I}}\frac{K_{\text{int}}^{q}}{(\pi a/2)^{\alpha}} \tag{5.14}$$

where q stands for one of γ, D, B, E and H.

Then we have

$$\begin{gathered}\left[k_{\text{int}}^{\gamma};\ k_{\text{int}}^{D};\ k_{\text{int}}^{B};\ k_{\text{int}}^{E};\ k_{\text{int}}^{H}\right]= \\ =\frac{\sqrt{2}(1-\alpha)8^{\alpha}}{(1+\lambda)\sqrt{1-\lambda}}\left[\lambda_\gamma;\ e_{15}^{I}\lambda_D;\ q_{15}^{I}\lambda_B;\ \lambda_E;\ \lambda_H\right],\end{gathered} \tag{5.15}$$

respectively.

Of course, the normalized intensity factors satisfy the constitutive Eqs (2.5), that is,

$$\left[k_{\text{int}}^{\tau};\ k_{\text{int}}^{D};\ k_{\text{int}}^{B}\right]=C^{II}\left[k_{\text{int}}^{\gamma};\ -k_{\text{int}}^{E};\ -k_{\text{int}}^{H}\right], \tag{5.16}$$

with the matrix Eq.(2.6) or inverse form with the use of matrix $\left(C^{II}\right)^{-1}$, defined by Eq.(2.17).

The analysis above implies that for the magnetically (or electrically) permeable interfacial cracks, the applied magnetic (or electric) loadings have no influence on the fracture behaviours of the crack tips.

Figures 10 and 11 illustrate the variation of k_{int}^{τ} and k_{hom}^{τ}.

a) Case I b) Case II

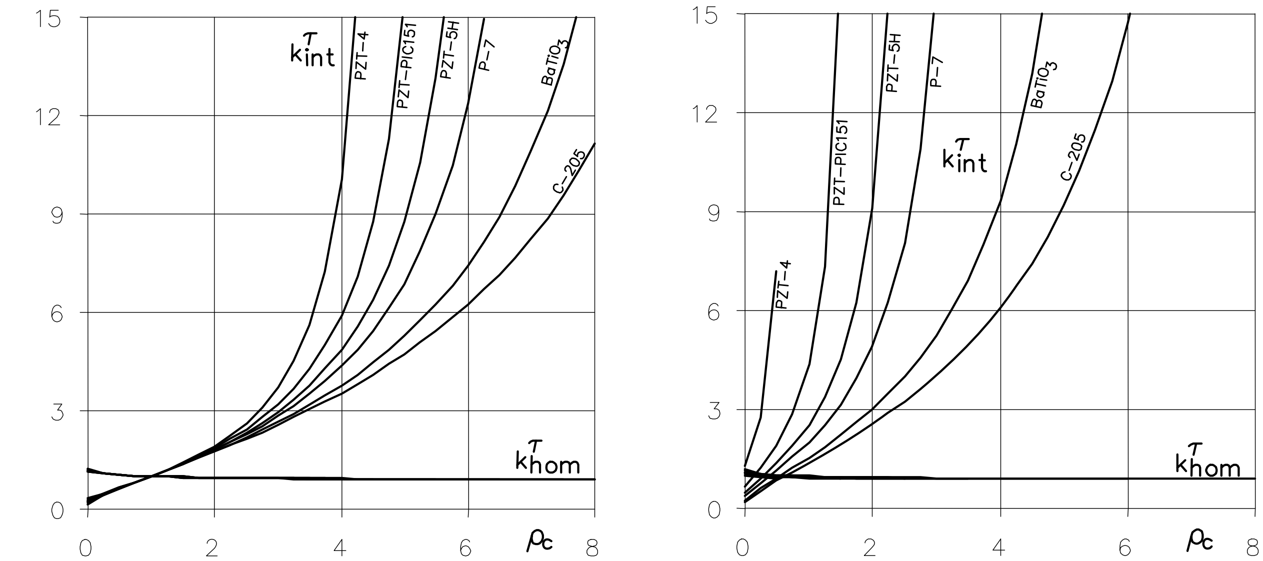

Fig.10. Normalized SIFs as a function of ρ_c with $\rho_e = \rho_\varepsilon = 1$ (Case I) and $\rho_e = -1$, $\rho_\varepsilon = 1$ (Case II).

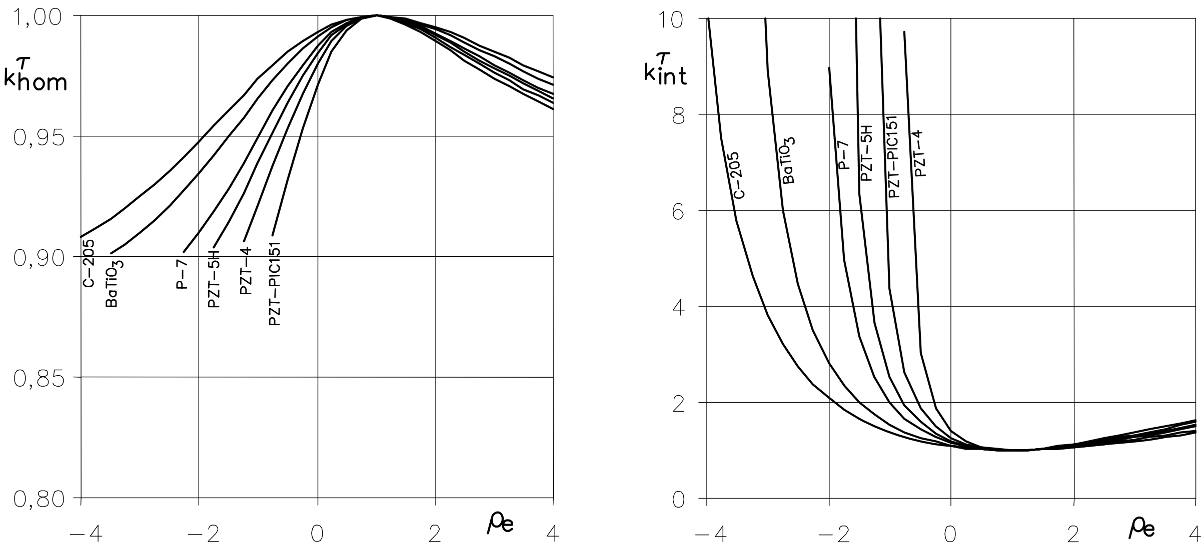

Fig.11. Normalized SIFs as a function of ρ_e with $\rho_c = \rho_\varepsilon = 1$.

We have

$$k_{\text{hom}}^{\tau} = \frac{2\sqrt{2}}{\pi} \quad \text{for} \quad \rho_c = 3 + \frac{2}{m}\left(\rho_e = \rho_\varepsilon = 1\right) \quad \text{or} \quad \rho_c = \frac{2}{m} - 1\left(\rho_e = -1, \rho_\varepsilon = 1\right). \tag{5.17}$$

The figures show that the normalized stress intensity factor in a homogeneous solid is only weakly dependent on the elastic constants and dielectric permeabilities. In contrast, k_{int}^{τ} strongly depends on ρ_c and ρ_e. This is consistent with physical considerations: for a large difference of piezocoefficients $\rho_e < 0$ or $\rho_e > 1$ the k_{int}^{τ} are larger than k_{hom}^{τ} (Fig.11). From Fig.10 it can be seen that piezoelectric ceramic II is more complaisant than piezoelectric ceramic I ($\rho_c < 1$), then $k_{\text{hom}}^{\tau} > k_{\text{int}}^{\tau}$. In contrast, if $\rho_c > 1$ means that piezoelectric ceramic II is stiffer than piezoelectric ceramic I, in this case $k_{\text{hom}}^{\tau} < k_{\text{int}}^{\tau}$.

Other normalized field intensity factors are presented in Figs 12 and 13.

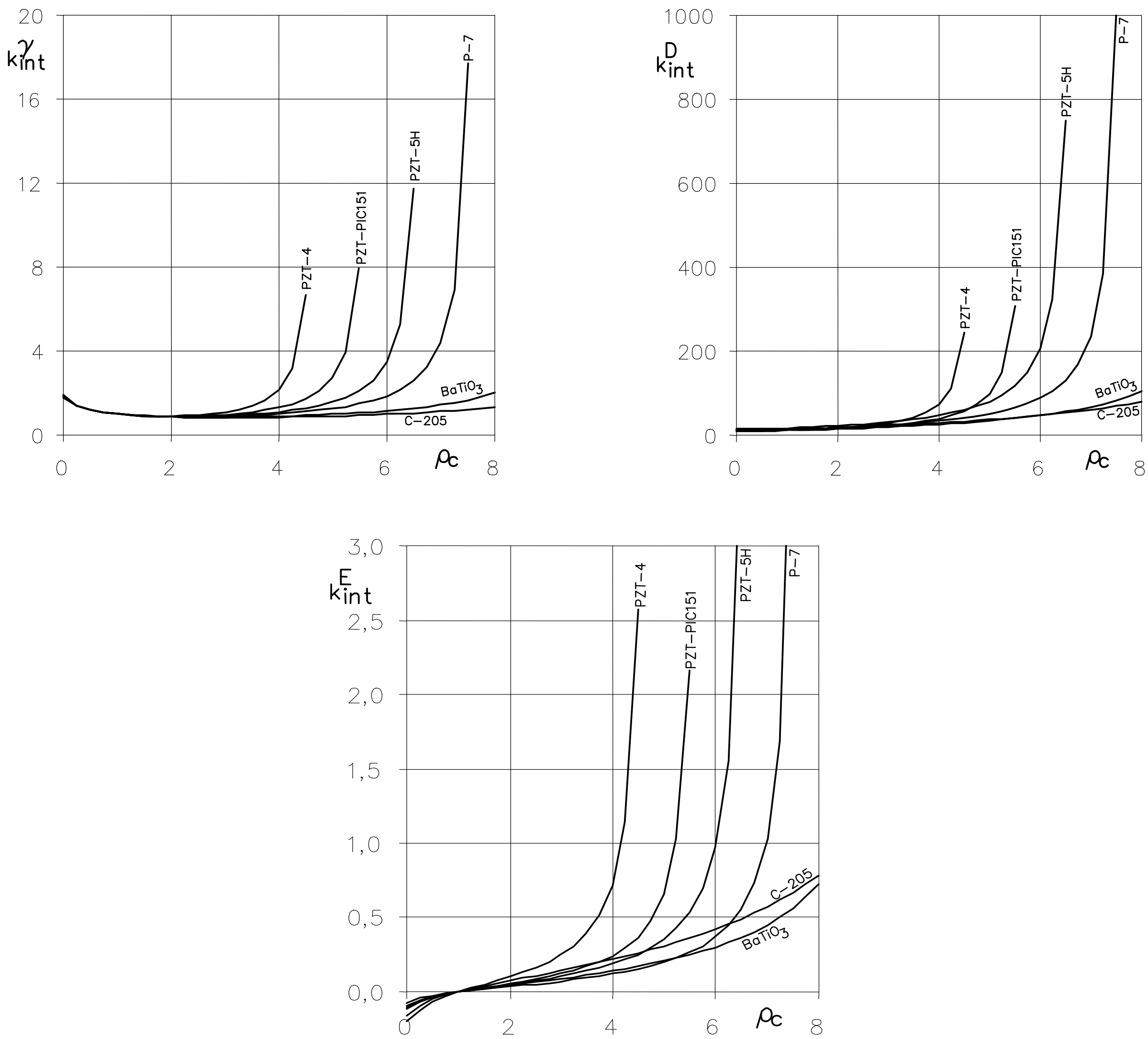

Fig.12. Variation of k_{int}^{γ}, k_{int}^{D} $\left(\text{in } C/m^2\right)$ and k_{int}^{E} $\left(\text{in } 10^6\, kV/m\right)$ against ρ_c with $\rho_e = \rho_\varepsilon = 1$.

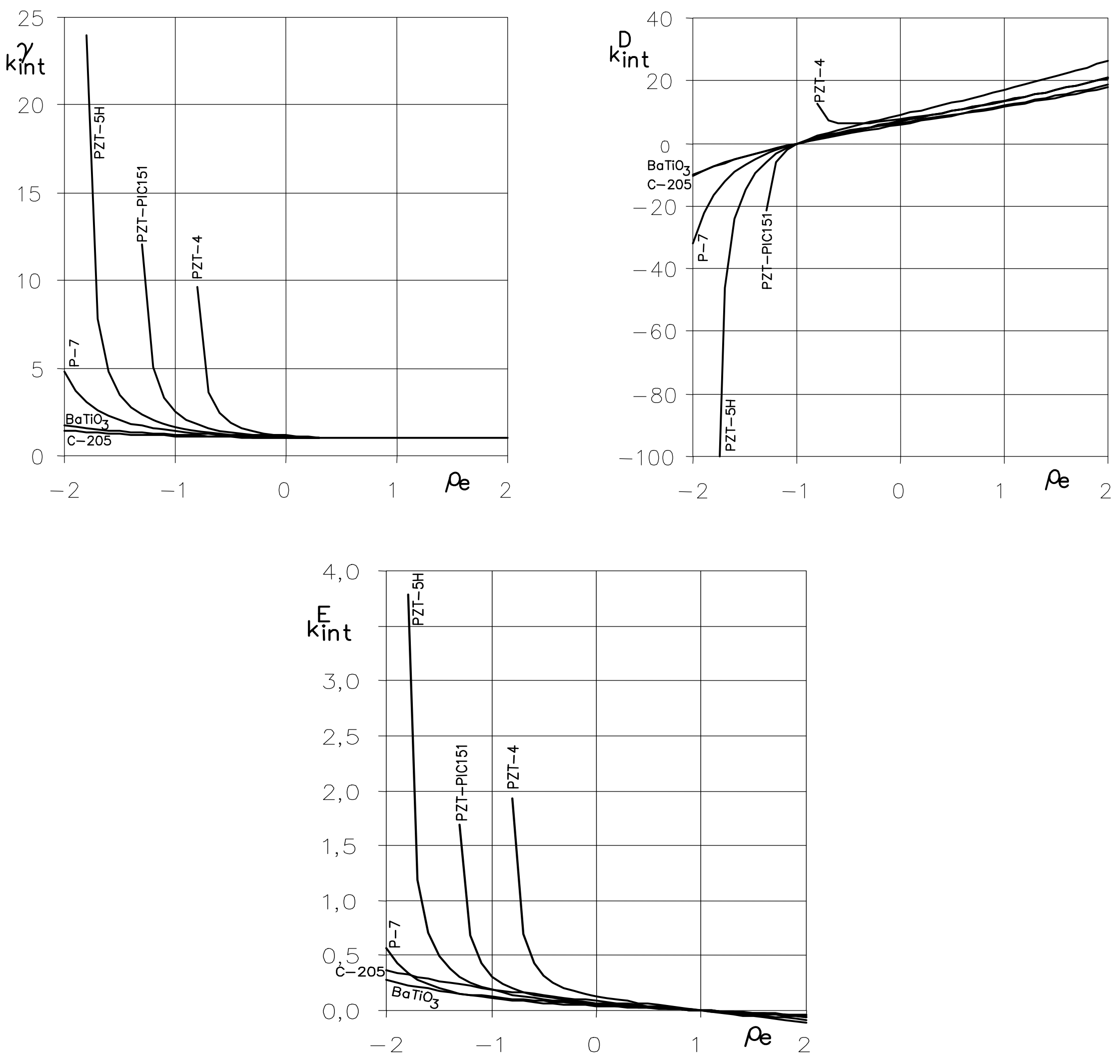

Fig.13. Variation of k_{int}^{γ} , k_{int}^{D} $\left(\text{in } C/m^2\right)$ and k_{int}^{E} $\left(\text{in } 10^6\, kV/m\right)$ against ρ_e with $\rho_c = \rho_\varepsilon = 1$.

k_{int}^{E} is equal to zero for $\rho_c = 1$ (Fig.12) and for $\rho_e = 1$ (Fig.13). From Eq.(4.27) one finds that $k_{\text{int}}^{E} = 0$ occurs only when $c_{44}^{II}/c_{44}^{I} = e_{15}^{II}/e_{15}^{I}$. In Fig.13 we see that ρ_e has a strong influence on k_{int}^{D} and k_{int}^{E} and k_{int}^{γ} if $\rho_e < 1$. When $\rho_e = 1$, $k_{\text{int}}^{\gamma} = 1$, $k_{\text{int}}^{E} = 0$ and $k_{\text{int}}^{D} = e_{15}^{I}$, as expected.

Figure 14 presents the variation of normalized ERRs, G / G_{hom} obtained from Eq.(4.33) with the use of Eqs (5.1) and (5.2).

There are two states where $G = G_{\text{hom}}$. The first state, in which $\alpha = 1/2$ and $\lambda = 0$, that is, $c_{44}^{II} = c_{44}^{I}$, corresponds to a crack in the monolithic medium (no bi – material). The second state, in which α and λ tend to unity, corresponds to the edge crack problem (the second material is air). For $\alpha > 1/2$ ERRs decrease weakly from 1 to 0.69 for $\alpha = 3/4$ and later increase to unity for $\alpha \to 1$. In this case piezoelectric ceramic II is more elastically complaisant. The range $0 < \alpha < 1/2$ corresponds to the following cases: $\rho_c > 1$ or $\rho_c > (1-m)/(1+m)$ (in the case $\rho_e = -1$); $\rho_e > -1/m$ and $\rho_\varepsilon > 0$ (for any ε). Then always $G > G_{\text{hom}}$

and piezoelectric II is stiffer than piezoelectric I. Similar conclusions may be formulated for the magnetostrictive material, changing material parameters e_{15} and ε_{11} by q_{15} and μ_{11}, respectively.

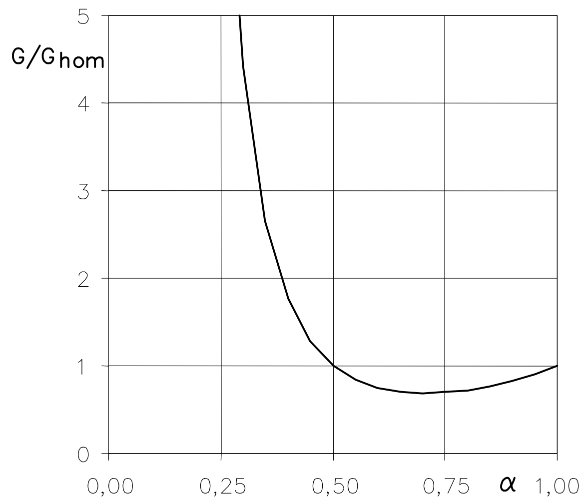

Fig.14. Normalized ERRs, $\frac{G}{G_{\mathrm{hom}}}$ as a function of α : $\frac{G}{G_{\mathrm{hom}}}=\frac{2\alpha^2}{1+\lambda}\left[1+8^{2\alpha-1}\left(1-\frac{1}{\alpha}\right)^2\frac{1-\lambda}{1+\lambda}\right]$, $\lambda=-\cos(\pi\alpha)$

$\alpha=\frac{3}{4}$, $\lambda=\frac{1}{\sqrt{2}}$, $\frac{G}{G_{\mathrm{hom}}}=\frac{11\sqrt{2}-10}{8}$, $\alpha=\frac{1}{4}$, $\lambda=-\frac{1}{\sqrt{2}}$, $\frac{G}{G_{\mathrm{hom}}}=\frac{67+47\sqrt{2}}{16}$.

A crack between a piezoelectric material and a piezeomagnetic material

Magneto-electro-elastic materials usually comprise an alternating piezoelectric medium and piezomagnetic medium. Here, we consider a special case, namely, the right medium I is a piezoelectric and the left medium II is a piezomagnetic (Case I) or inversely (Case II). The material constants of the piezoelectric medium (No.I) and piezomagnetic medium (No.II) have the following values (Huang and Kuo, 1997; Annigeri *et al*., 2007; Song and Sih, 2003):
$BaTiO_3$ – piezoelectric (barium titanate)

$$c_{44}^{I}=43\times10^{9}\,Nm^{-2},\qquad e_{15}^{I}=11.6Cm^{-2},\qquad q_{15}^{I}=0,$$
$$\varepsilon_{11}^{I}=11.2\times10^{-9}\,CV^{-1}m^{-1},\qquad d_{11}^{I}=0,\qquad \mu_{11}^{I}=5.0\times10^{-6}\,NA^{-2}. \tag{5.18}$$

$CoFe_2O_4$ – piezomagnetic (cobalt iron oxide):

$$c_{44}^{II}=45.3\times10^{9}\,Nm^{-2},\qquad e_{15}^{II}=0,\qquad q_{15}^{II}=550NA^{-1}m^{-1},$$
$$\varepsilon_{11}^{II}=0.08\times10^{-9}\,CV^{-1}m^{-1},\qquad 11^{II}=0,\qquad \mu_{11}^{II}=590\times10^{-6}\,NA^{-2}. \tag{5.19}$$

The material parameter Eq.(3.33) assumes the values

$$\lambda = 1 - \frac{2}{\dfrac{c_{44}^J}{c_{44}^I + \dfrac{\left(e_{15}^I\right)^2}{\varepsilon_{11}^I + \varepsilon_{11}^{II}}} + \dfrac{c_{44}^J}{c_{44}^{II} + \dfrac{\left(q_{15}^{II}\right)^2}{\mu_{11}^I + \mu_{11}^{II}}}}; \qquad J = I, II \tag{5.20}$$

where c_{44}^J is the shear modulus of the cracked material, for Case I and Case II, respectively. We have

$$\lambda = \begin{cases} -0.1618, & \text{Case I} \\ -0.1028, & \text{Case II} \end{cases}, \qquad \alpha = \begin{cases} 0.4483, & \text{Case I} \\ 0.4672, & \text{Case II} \end{cases}. \tag{5.21}$$

The energy release rates are obtained as follows

$$G = \begin{cases} \pi a(15.0) \times 10^{-12} \left(\tau_0^I\right)^2 \times \left[m^2/N\right], & \text{Case I} \\ \pi a(12.9) \times 10^{-12} \left(\tau_0^I\right)^2 \times \left[m^2/N\right], & \text{Case II.} \end{cases} \tag{5.22}$$

For the “homogenous” composite $BaTiO_3$ / $CoFe_2O_4$ with the ratio roughly 50 : 50 we have with the use of arithmetic mean: $c_{44} = 44.15 \times 10^9\, Nm^{-2}$ and G_{hom} assumes the value

$$G_{\text{hom}} = \pi a \left(11.4 \times 10^{-12} \left(\tau_0^I\right)^2\right) \times \left[m^2/N\right]. \tag{5.23}$$

We see that ERRs for bi – materials cannot be determined by the mixture rule since it is a significant new feature in the interface crack problem considered in this paper.

Obviously for piezoelectric/piezomagnetic composite (I / II) is $\mu_{11}^I << \mu_{11}^{II}$ and $\varepsilon_{11}^{II} << \varepsilon_{11}^I$ and Eq.(5.20) reduces to the formula

$$\lambda = 1 - \frac{\tilde{c}_{44}^*}{c_{44}^J}; \qquad J = I, II \tag{5.24}$$

where $\tilde{c}_{44}^*$ is the harmonic mean of the piezoelectric and piezomagnetic stiffened elastic constants $\tilde{c}_{44}^I$ and $\tilde{c}_{44}^{II}$ is defined as follows

$$\frac{1}{\tilde{c}_{44}^*} = \frac{1}{2}\left(\frac{1}{\tilde{c}_{44}^I} + \frac{1}{\tilde{c}_{44}^{II}}\right) \tag{5.25}$$

where

$$\tilde{c}_{44}^I = c_{44}^I + \frac{\left(e_{15}^I\right)^2}{\varepsilon_{11}^I}, \qquad \tilde{c}_{44}^{II} = c_{44}^{II} + \frac{\left(q_{15}^{II}\right)^2}{\mu_{11}^{II}}. \tag{5.26}$$

Using Eqs (5.24) to (5.26) we obtain

$$\lambda = \begin{cases} -0.1626, & \text{Case I} \\ -0.1036, & \text{Case II} \end{cases}, \qquad \alpha = \begin{cases} 0.4480, & \text{Case I} \\ 0.4670, & \text{Case II} \end{cases}$$

$$G = \begin{cases} \pi a (15.0) \times 10^{-12} \left(\tau_0^I\right)^2 \times \left[m^2/N\right], & \text{Case I} \\ \pi a (12.9) \times 10^{-12} \left(\tau_0^I\right)^2 \times \left[m^2/N\right], & \text{Case II.} \end{cases} \tag{5.27}$$

6. Conclusions

A crack perpendicular to and terminating at the interface of two bonded dissimilar piezoelectromagnetoelastic media is studied in this paper. Analytical solutions and numerical simulations suggest the following conclusions:

(i) A closed form solution has been obtained for a crack between two dissimilar magneto electro – elastic ceramics. The crack is localized in one materials and its one tip lies on the interface. Expressions for the crack – tip field intensity factors, the electromagnetic fields inside the crack are given for electrically and magnetically permeable crack assumptions.

(ii) The energy release rate can be explicitly expressed in terms of the intensity factors. It is affected by electric – magnetic properties of the constituents of the bi-material media. The normalized energy release rate is unity for the homogeneous medium ($\rho_c = 1$) and for the edge crack ($\rho_c = 0$) and assumes a minimum value 0.69 for $\rho_c = 3 - 2\sqrt{2} = 0.18$. If ρ_c tends to infinity, also this quantity tends to infinity (the interface is clamped).

(iii) For two identical magneto-electro-elastic planes polarized in opposite directions we have $K_{\text{int}}^D = 0 = K_{\text{int}}^B$.

(iv) At the interface we have $K_{\text{int}}^E = 0$ when $c_{44}^{II}/c_{44}^{I} = e_{15}^{II}/e_{15}^{I}$, while $K_{\text{int}}^H = 0$ if $c_{44}^{II}/c_{44}^{I} = q_{15}^{II}/q_{15}^{I}$.

(v) An application of electric and magnetic fields does not alter the stress intensity factors; they depend on the elastic, electric and magnetic constants of bi – material ceramic.

(vi) The coupling between electromagnetic fields and mechanical field leads to existing electric displacement and magnetic induction of intensity factors at the crack tip, which respond to the applied stress intensity factor.

(vii) If magnetic effects are neglected the result of the stress intensity factors is the same as the solution for the piezoelectric materials given by Li and Wang (2007), but k_{int}^E differs in sign.

The results could be of particular interest to the analysis and design of smart sensors and actuators constructed from magneto-electro-elastic composite laminates. Nowadays, electro – magneto – elastic coupled multiphase composites have a wide range of applications in science and engineering such as space planes, supersonic air planes, rockets, missiles nuclear fusion, reactors and submarines.

Nomenclature

a – radius of the penny – shaped crack
b_0 – magnetic induction supported by the crack gap
B_x , B_y – magnetic induction components
c – material property matrix
C^{-1} – material compliances matrix
c_{44} – shear modulus
$\tilde{c}_{44}$ – piezoelectromagnetically stiffened elastic constant

d_0	– electric displacement supported by the crack gap
D_x , D_y	– electric displacement components
B_0, D_0, τ_0 or E_0, H_0, τ_0	– magnetic, electrical and mechanical loading
E_x , E_y	– electric field components
G	– energy release rate
H_x , H_y	– magnetic field components
d_{11}	– magnetoelectric constant
e_{15}	– piezoelectric constant
J	– index $J = I$ denotes PEMO – ceramic I, $J = II$ PEMO – ceramic II
K^{τ}_{hom}	– mode III stress intensity factor for the crack tip in homogeneous material
K^{D}_{hom}	– electric displacement intensity factor in homogeneous material
K^{B}_{hom}	– magnetic induction intensity factor in homogeneous material
K^{τ}_{int}	– mode III stress intensity factor for the crack tip located at interface
K^{D}_{int}	– electric displacement intensity factor at interface
K^{B}_{int}	– magnetic induction intensity factor at interface
k^{τ}_{hom}	– normalized stress intensity factor at “homogeneous” crack tip
k^{τ}_{int}	– normalized stress intensity factor at “interface” crack tip
q_{15}	– piezomagnetic constant
$w(x,y)$	– anti – plane displacement vector
(x,y)	– plane coordinate system
α	– singularity order parameter
λ	– bimaterial parameter
ε_{11}	– dielectric constant (permittivity)
μ_{11}	– magnetic constant (permeability)
ρ_c , ρ_e , ρ_ε	– parameters of sensitivity of piezoelectric material
ρ_c , ρ_q , ρ_μ	– parameters of sensitivity of piezomagnetic material

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