

Brief note

**MASS TRANSFER WITH CHEMICAL REACTION ON FLOW PAST AN
ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE
AND THERMAL RADIATION**

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An exact solution of an unsteady radiative flow past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion is presented here, taking into account the homogeneous chemical reaction of first order. The plate temperature as well as concentration near the plate is raised linearly with time. The dimensionless governing equations are solved using the Laplace-transform technique. The velocity, temperature and concentration fields are studied for different physical parameters such as the thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, radiation parameter, chemical reaction parameter and time. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. But the trend is just reversed with respect to the thermal radiation parameter. It is also observed that the velocity increases with the decreasing chemical reaction parameter.

Key words: linearly accelerated, chemical reaction, radiation, heat and mass transfer.

1. Introduction

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, combustion and furnace design, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. England and Emery (1969) studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Radiation effects on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar (1996). Raptis and Perdikis (1999) studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das *et al* (1996) analyzed radiation effects on the flow past an

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impulsively started infinite isothermal vertical plate. The dimensionless governing equations were solved by the usual Laplace-transform technique.

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in the solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration itself. Chambre and Young (1958) analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das *et al.* (1994) studied the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction were studied by Das *et al.* (1998). The dimensionless governing equations were solved by the usual Laplace-transform technique.

Gupta *et al.* (1979) studied free convection on the flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using the perturbation method. Kafousias and Raptis (1981) extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on the flow past an exponentially accelerated vertical plate were studied by Singh and Naveen (1984). The skin friction for an accelerated vertical plate was studied analytically by Hossain and Shayo (1986). Mass transfer effects on the flow past a uniformly accelerated vertical plate were studied by Soundalgekar (1982). Mass transfer effects on the flow past an accelerated vertical plate with uniform heat flux were analyzed by Singh and Singh (1983). Basant and Prasad (1991) analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources.

Hence, it is proposed to study the first order chemical reaction on unsteady flow past a uniformly accelerated infinite vertical plate with variable temperature and mass transfer in the presence of thermal radiation. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2. Mathematical analysis

Here the unsteady radiative flow of a viscous incompressible fluid past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion in the presence of a chemical reaction of first order is considered. The unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞ . The x -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ . At time $t' > 0$, the plate is accelerated with a velocity $u = \frac{u_0^3 t'}{\nu}$ in its own plane against the gravitational field. The temperature from the plate as well as the concentration level near the plate is raised linearly with time t . It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then under the usual Boussinesq's approximation the unsteady flow is governed by the following equations

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2}, \quad (2.1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \quad (2.2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_l (C' - C'_\infty), \quad (2.3)$$

with the following initial and boundary conditions

$$\begin{aligned} u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y, t' \leq 0 \\ t' > 0: u = \frac{u_0^3 t'}{\nu}, \quad T = T_\infty + (T_w - T_\infty) A t', \\ C' = C'_\infty + (C'_w - C'_\infty) A t' \quad \text{at} \quad y = 0, \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C' \rightarrow C'_\infty \quad \text{as} \quad y \rightarrow \infty. \end{aligned} \quad (2.4)$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma (T_\infty^4 - T^4). \quad (2.5)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \quad (2.6)$$

By using Eqs (2.5) and (2.6), Eq.(2.2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 (T_\infty - T). \quad (2.7)$$

On introducing the following non-dimensional quantities

$$\begin{aligned} U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ \text{Gr} = \frac{g \beta \nu (T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad \text{Gc} = \frac{\nu g \beta^* (C'_w - C'_\infty)}{u_0^3}, \\ R = \frac{16a^* \nu^2 \sigma T_\infty^3}{k u_0^2}, \quad \text{Pr} = \frac{\mu C_p}{k}, \quad \text{Sc} = \frac{\nu}{D}, \quad K = \frac{\nu K_l}{u_0^2}, \end{aligned} \quad (2.8)$$

in Eqs (2.1), (2.3) and (2.7) we get

$$\frac{\partial U}{\partial t} = \text{Gr } \theta + \text{Gc } C + \frac{\partial^2 U}{\partial Y^2} \quad (2.9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\text{Pr}} \theta \quad (2.10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{\text{Sc}} \frac{\partial^2 C}{\partial Y^2} - KC \quad (2.11)$$

The negative sign of K in the last term of Eq.(2.11) indicates that the chemical reaction takes place from the higher level of concentration to the lower level of concentration.

The initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \leq 0 \\ t > 0: \quad U = t, \quad \theta = t, \quad C = t \quad \text{at } Y = 0 \\ U \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } Y \rightarrow \infty \end{aligned} \quad (2.12)$$

The dimensionless governing Eqs (2.9) to (2.11), subject to the initial and boundary conditions (2.12), are solved by the usual Laplace-transform technique and the solutions are derived as follows

$$\begin{aligned} U = t(2bd + 2ce) \left[(1 + 2\eta^2) \cdot \text{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) \right] + 2(d + e)\text{erfc}(\eta) + \\ - d \exp(bt) \left[\exp(2\eta\sqrt{bt}) \cdot \text{erfc}(\eta + \sqrt{bt}) + \exp(-2\eta\sqrt{bt}) \cdot \text{erfc}(\eta - \sqrt{bt}) \right] + \\ - e \exp(ct) \left[\exp(2\eta\sqrt{ct}) \cdot \text{erfc}(\eta + \sqrt{ct}) + \exp(-2\eta\sqrt{ct}) \cdot \text{erfc}(\eta - \sqrt{ct}) \right] + \\ - d(1 + bt) \left[\exp(2\eta\sqrt{Rt}) \cdot \text{erfc}(\eta\sqrt{\text{Pr}} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt}) \cdot \text{erfc}(\eta\sqrt{\text{Pr}} - \sqrt{at}) \right] + \\ + \frac{d\text{PrGm}\eta\sqrt{t}}{\sqrt{R}} \left[\exp(-2\eta\sqrt{Rt}) \cdot \text{erfc}(\eta\sqrt{\text{Pr}} - \sqrt{at}) - \exp(2\eta\sqrt{Rt}) \cdot \text{erfc}(\eta\sqrt{\text{Pr}} + \sqrt{at}) \right] + \\ + d \exp(bt) \left[\exp(-2\eta\sqrt{\text{Pr}(a+b)t}) \cdot \text{erfc}(\eta\sqrt{\text{Pr}} - \sqrt{(a+b)t}) + \right. \\ \left. + \exp(2\eta\sqrt{\text{Pr}(a+b)t}) \cdot \text{erfc}(\eta\sqrt{\text{Pr}} + \sqrt{(a+b)t}) \right] + \\ - e(1 + ct) \left[\exp(-2\eta\sqrt{Kt\text{Sc}}) \cdot \text{erfc}(\eta\sqrt{\text{Sc}} - \sqrt{Kt}) + \exp(2\eta\sqrt{Kt\text{Sc}}) \cdot \text{erfc}(\eta\sqrt{\text{Sc}} + \sqrt{Kt}) \right] + \\ + \frac{ec\eta\sqrt{t}\sqrt{\text{Sc}}}{\sqrt{K}} \left[\exp(-2\eta\sqrt{Kt\text{Sc}}) \cdot \text{erfc}(\eta\sqrt{\text{Sc}} - \sqrt{Kt}) - \exp(2\eta\sqrt{Kt\text{Sc}}) \cdot \text{erfc}(\eta\sqrt{\text{Sc}} + \sqrt{Kt}) \right] + \\ + e \exp(ct) \left[\exp(-2\eta\sqrt{\text{Sc}(K+c)t}) \cdot \text{erfc}(\eta\sqrt{\text{Sc}} - \sqrt{(K+c)t}) + \right. \\ \left. + \exp(2\eta\sqrt{\text{Sc}(K+c)t}) \cdot \text{erfc}(\eta\sqrt{\text{Sc}} + \sqrt{(K+c)t}) \right], \end{aligned} \quad (2.13)$$

$$\theta = \frac{t}{2} \left[\exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} - \sqrt{at}) \right] + \frac{\eta\operatorname{Pr}\sqrt{t}}{2\sqrt{R}} \left[\exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} - \sqrt{at}) \exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} + \sqrt{at}) \right], \tag{2.14}$$

$$C = \frac{t}{2} \left[\exp(2\eta\sqrt{Kt\operatorname{Sc}}) \operatorname{erfc}(\eta\sqrt{\operatorname{Sc}} + \sqrt{Kt}) + \exp(-2\eta\sqrt{Kt\operatorname{Sc}}) \operatorname{erfc}(\eta\sqrt{\operatorname{Sc}} - \sqrt{Kt}) \right] + \frac{\eta\sqrt{\operatorname{Sc}t}}{2\sqrt{K}} \left[\exp(-2\eta\sqrt{Kt\operatorname{Sc}}) \operatorname{erfc}(\eta\sqrt{\operatorname{Sc}} - \sqrt{Kt}) - \exp(2\eta\sqrt{Kt\operatorname{Sc}}) \operatorname{erfc}(\eta\sqrt{\operatorname{Sc}} + \sqrt{Kt}) \right] \tag{2.15}$$

where

$$a = \frac{R}{\operatorname{Pr}}, \quad b = \frac{R}{1 - \operatorname{Pr}}, \quad c = \frac{K\operatorname{Sc}}{1 - \operatorname{Sc}}, \quad d = \frac{\operatorname{Gr}}{2b(1 - \operatorname{Pr})},$$

$$e = \frac{\operatorname{Gc}}{2c^2(1 - \operatorname{Sc})}, \quad \text{and} \quad \eta = \frac{Y}{2\sqrt{t}}.$$

3. Results and discussion

The numerical computations are carried out for different physical parameters $\operatorname{Gr}, \operatorname{Gc}, \operatorname{Sc}, R, \operatorname{Pr}, K$ and t . The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of the Prandtl number Pr represents air ($\operatorname{Pr} = 0.71$). The numerical values of the velocity, temperature and concentration are computed for different physical parameters such as the Prandtl number, thermal Grashof number, mass Grashof number, chemical reaction parameter, radiation parameter, Schmidt number and time and are studied graphically.

Figure 1 the effect of velocity for different values of the radiation parameter ($R = 2, 5, 20$), $\operatorname{Gr} = \operatorname{Gc} = 5$, $K = 7$ and $t = 0.2$. The trend shows that the velocity increases with the decreasing radiation parameter. It is observed that there is a fall in velocity in the presence of high thermal radiation.

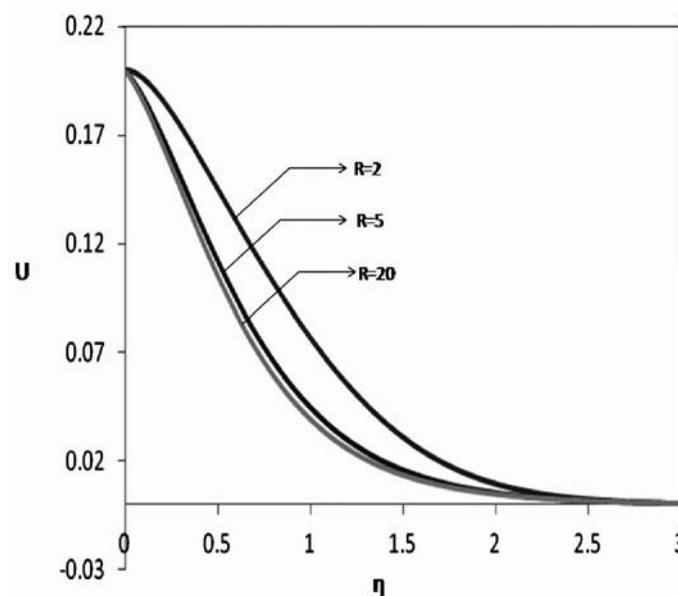


Fig.1. Velocity profiles for different values of R .

The velocity profiles for different values of $(t = 0.2, 0.3, 0.4)$, $K = 7$, $R = 2$ and $Gr = Gc = 5$ are studied and presented in Fig.2. It is observed that the velocity increases with increasing values of the time t . The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream.

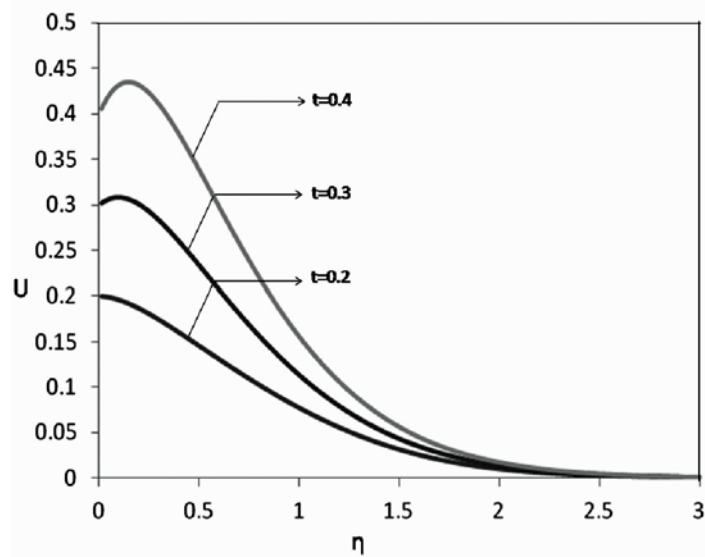


Fig.2. Velocity profiles for different values of t .

Figure 3 illustrates the effect of velocity for different values of the chemical reaction parameter ($K = 0.2, 2, 5$), $Gr = Gc = 5$, $R = 5$ and $t = 0.6$. This shows that the increase in the chemical reaction parameter leads to a fall in the velocity.

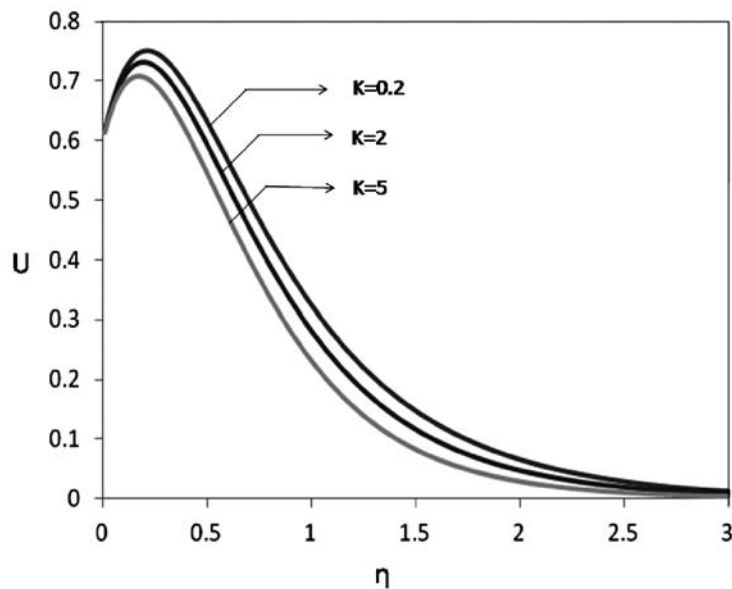


Fig.3. Velocity profiles for different values of K .

Figure 4. demonstrates the effect of velocity fields for different values of the thermal Grashof number ($Gr = 2, 5$), mass Grashof number ($Gc = 5, 10$), $K = 5$, $R = 10$ and $t = 0.6$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

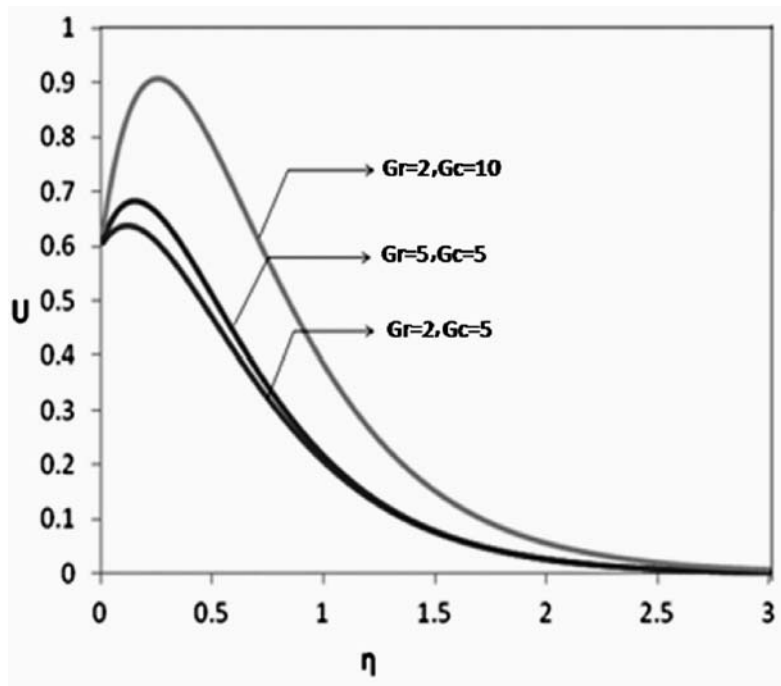


Fig.4. Velocity profiles for different values of Gr, Gc.

4. Conclusions

An exact solution of a radiative flow past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion in the presence of a homogeneous chemical reaction of first order has been studied. The dimensionless governing equations were solved by the usual Laplace-transform technique. The effects of different physical parameters such as the chemical reaction parameter, radiation parameter, thermal Grashof number, mass Grashof number and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t . But the trend is just reversed with respect to the chemical reaction parameter or radiation parameter.

Nomenclature

- A – constant
- C – dimensionless concentration
- C' – species concentration in the fluid
- C_p – specific heat at constant pressure
- C'_w – concentration of the plate
- C'_∞ – concentration of the fluid far away from the plate
- D – mass diffusion coefficient
- erfc – complementary error function
- G – accelerated due to gravity
- Gc – thermal Grashof number
- Gr – mass Grashof number
- K – thermal conductivity

- K – dimensionless chemical reaction parameter
 K_l – chemical reaction parameter
 Pr – Prandtl number
 R – radiation parameter
 Sc – Schmidt number
 T – dimensionless time
 T – temperature of the fluid near the plate
 T_w – concentration of the plate
 T_∞ – concentration of the fluid far away from the plate
 t' – time
 U – dimensionless velocity
 U – velocity of the fluid in the x-direction
 u_0 – velocity of the plate
 x – spatial coordinate along the plate
 y – dimensionless coordinate axis normal to the plate
 y' – coordinate axis normal to the plate
 β – volumetric coefficient of thermal expansion
 β^* – volumetric coefficient of expansion with concentration
 η – similarity parameter
 θ – dimensionless temperature
 μ – coefficient of viscosity
 ρ – density of the fluid
 ν – kinematic viscosity

References

- Basant K.J. Prasad R. and Surendra R. (1991): *Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux.* – *Astrophysics and Space Science*, vol.81, pp.125-134.
- Chambre P.L. and Young J.D. (1958): *On the diffusion of a chemically reactive species in a laminar boundary layer flow.* – *The Physics of Fluids*, vol.1, pp.48-54.
- Das U.N., Deka R.K. and Soundalgekar V.M. (1994): *Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction.* – *Forschung im Ingenieurwesen*, vol.60, pp.284-287.
- Das U.N., Deka R.K. and Soundalgekar V.M. (1996): *Radiation effects on flow past an impulsively started vertical infinite plate.* – *J. Theo. Mech.*, vol.1, pp.111-115.
- Das U.N., Deka R.K. and Soundalgekar V.M. (1999): *Effects of mass transfer on flow past an impulsively started infinite vertical plate with chemical reaction.* – *The Bulletin of Guma.*, vol.5, pp.13-20.
- England W.G. and Emery A.F. (1969): *Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas.* – *J. Heat Transfer*, vol.91, pp.37-44.
- Gupta A.S., PoP I. and Soundalgekar V.M. (1979): *Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid.* – *Rev. Roum. Sci. Techn.- Mec. Apl.*, vol.24, pp.561-568.
- Hossain M.A. and Shayo L.K. (1986): *The skin friction in the unsteady free convection flow past an accelerated plate.* – *Astrophysics and Space Science*, vol.125, pp.315-324.
- Hossain M.A. and Takhar H.S. (1996): *Radiation effect on mixed convection along a vertical plate with uniform surface temperature.* – *Heat and Mass Transfer*, vol.31, pp.243-248.
- Kafousias N.G. and Raptis A.A. (1981): *Mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection.* – *Rev. Roum. Sci. Techn.-Mec. Apl.*, vol.26, pp.11-22.
- Raptis A. and Perdikis C. (1999): *Radiation and free convection flow past a moving plate.* – *Int. J. Appl. Mech. Engg.*, vol.4, pp.817-821.

- Singh A.K. and Naveen K. (1984): Free convection flow past an exponentially accelerated vertical plate. – *Astrophysics and Space Sci.*, vol.98, pp.245-258.
- Singh A.K. and Singh J. (1983): *Mass transfer effects on the flow past an accelerated vertical plate with constant heat flux.* – *Astrophysics and Space Science.*, vol.97, pp.57-61.
- Soundalgekar V.M.(1982): *Effects of mass transfer on flow past a uniformly accelerated vertical plate.* – *Letters in Heat and Mass Transfer*, vol.9, pp.65-72.

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