STATIC AND DYNAMIC ANALYSIS OF A PUMP IMPELLER WITH A BALANCING DEVICE PART I: STATIC ANALYSIS

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This part of the work presents the design and static analysis of an impeller for a single-stage pump. The impeller is directly connected with a balancing device. The impeller needs to have a properly designed system of longitudinal and lateral clearances on both sides. With the simplifying assumptions concerning the flow and distribution of pressure in the longitudinal and lateral clearances, the static analysis involved deriving relationships between the impeller geometry and the basic performance parameters of the pump. A numerical example was used to show the calculation procedure of static characteristics for the predetermined parameters.

Key words: pump impeller, balance device, static analysis.

1. Introduction

One of the major tasks during the design of impeller pumps, especially multi-stage pumps, is to properly select and correctly design the balancing device or system responsible for balancing and reducing the longitudinal forces that act along the axis of the shaft. In multi-stage pumps the balancing systems comprise properly designed balancing drums and discs, or hydrostatic thrust bearings. In single-stage pumps, where longitudinal forces are relatively small, balancing vanes or balancing holes in the impeller are generally used. Another solution is to apply external systems of rolling-element bearings, as discussed in (e.g. Jędral, 2001; Korczak, 2005).

Balancing systems of impeller pumps have been studied both theoretically and experimentally, for example, in Korczak (2005), Korczak *et al.* (2005), Pavlenko (2008). Different design solutions have been considered for single-stage and multi-stage pumps. Some of them were patented, for example, (Martsinkovsky *et al.*, 1995; Kubota, 1999) and (Chiba *et al.*, 2007), respectively.

It can be assumed that hydrostatic and hydrodynamic forces occurring in the longitudinal and lateral clearances of the balancing system ensure the right position of the impeller relative to the pump casing and stable operation of the pump. The objective of this study was to analyze an impeller of a single-stage pump

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(Martsinkovsky *et al.*, 1995) directly connected with the balancing device with specially designed balancing rings. The balancing device acts as both the longitudinal and lateral bearings of the single-stage centrifugal pump. The paper focuses on the static analysis of the system.

2. Impeller design

Impellers directly connected with a balancing device were first patented in the 1990s, for instance (Korczak *et al*., 2005; Pavlenko, 2008). The distinctive feature of the design of the single-stage pump impeller described in the patent (Martsinkovsky *et al.*, 1995) is the lack of a classic drive shaft linked to the rolling element bearings. Instead of a bearing node, there are longitudinal and lateral seal clearances of the impeller and the balancing device. The hydrodynamic forces and moments generated in these clearances position the impeller relative to the pump casing. The impeller is driven by a flexible shaft with a relatively small diameter. The small diameter of the shaft results from the torque passed from the motor via a ball joint to the impeller.
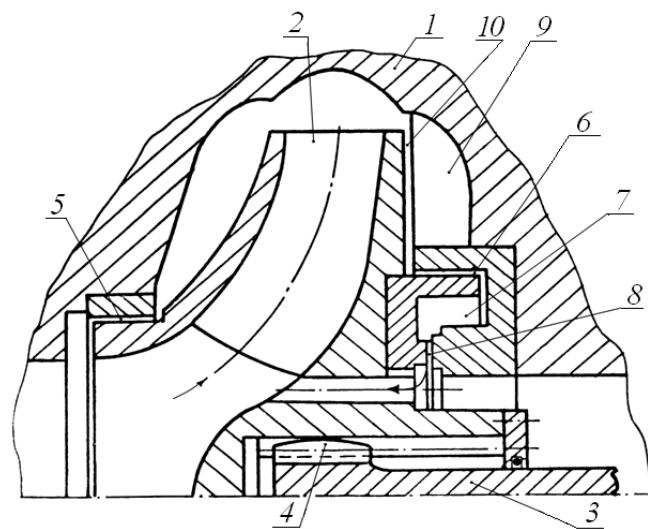

Fig.1.   Pump impeller with a balance device: 1- pump casing, 2- impeller, 3- shaft; 4 – ball joint; 5, 6 – longitudinal seals of the impeller; 7 – annular chamber; 8- face seal; 9- radial vanes; 10 – lateral clearance of the impeller shroud.

Figure 1 shows a diagram of a single-stage centrifugal pump with the impeller, 2, driven by a flexible shaft, 3, via a ball joint, 4. The longitudinal clearances 5 and 6, sealing wear-ring clearances, and the lateral clearance, 8, with the annular chamber, 7, play the role of lateral and longitudinal self-adjusting hydrostatic bearings of the impeller. In the pump casing, 1, behind the rear shroud of the impeller, there are radial vanes, 9, which suppress the rotation of the fluid (generated by the impeller) and prevent a loss of pressure in the centripetal direction.

The flexible shaft allows radial, angular and axial misalignments of the impeller. Exposed to hydrodynamic forces and moments and a gyroscopic moment during the pump operation, the impeller aligns itself and assumes optimal (static) positions. These external excitations cause a decrease in the amplitude of the impeller vibrations. The system of clearances sealing the impeller shrouds, shown in Fig.1, operates in a similar way as the balancing device in a multi-stage centrifugal pump.

3. Static analysis of the system

The static characteristics of the design node show the relationship between the width (height) of the lateral clearance, 8, the rate of the fluid flow through this clearance, and external excitations. The characteristics are determined on the basis of the equation of equilibrium of axial forces acting on the shroud and from the condition of continuity (balance) of flow through the seal clearances 6 and 8 (Fig.2). For simplicity, Fig.2 shows the quantities describing the impeller geometry.
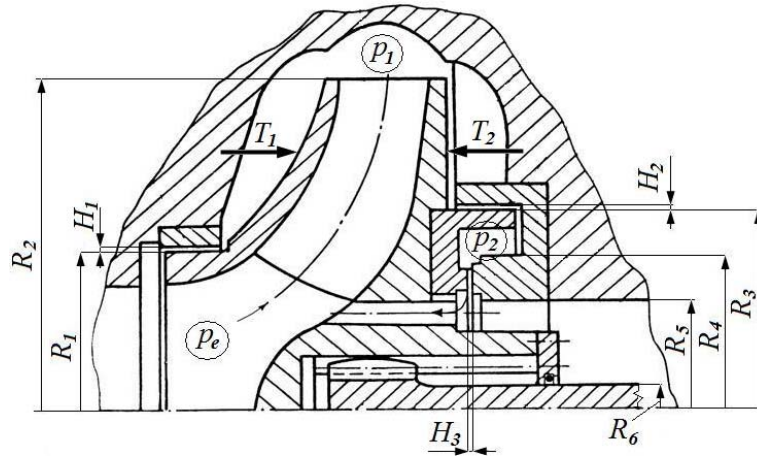

Fig.2. Geometry of the impeller with a balancing device.

The equation of axial forces generated by the pressure acting on the surfaces of the impeller shrouds is written as

$$T_1 + \left(A'_e + A_3 \right) p_e - T_2 - A'_2 p_2 - 0.5 A_c \left(p_2 + p_e \right) = 0 \tag{3.1}$$

where

$$A'_e = \pi \left(R_1^2 - R_5^2 \right), \qquad A_c = \pi \left(R_4^2 - R_5^2 \right), \qquad A'_2 = \pi \left(R_3^2 - R_4^2 \right), \qquad A_3 = \pi R_6^2 \tag{3.2}$$

A'_e, A_c, A_3 - surface areas of the impeller (Fig.2).

Forces T_1, T_2, acting on the outer surfaces of the impeller shrouds, A_A and A_B, are calculated by integrating the radial distribution of pressure (Jędral, 2001; Antoszewski *et al.*, 2008)

$$T_1 = A_A \left(p_1 - p_{A^*} \right); \qquad T_2 = A_B \left(p_1 - p_{B^*} \right) \tag{3.3}$$

where
$$p_{A*} = \frac{\rho \omega^2}{2\pi} \kappa_1^2 A_A; \qquad p_{B*} = \frac{\rho \omega^2}{2\pi} \kappa_2^2 A_B$$

p_{A^*}, p_{B^*} - pressures at the inlet to the longitudinal clearances 5 and 6 (along radii R_1 and R_3) (Fig.2); κ - coefficient taking into account the geometry of the impeller and the pump casing. $\kappa \approx \left(0.7 \div 0.9 \right)$.

After substituting and arranging the components, we obtain the following condition of equilibrium

$$A_l p_l + A_e p_e - T_* - A_2 p_2 = 0 \qquad (3.4)$$

where

$$A_l = A_A - A_B, \qquad A_e = A'_e + A_3 - 0.5 A_c, \qquad A_2 = A'_2 + 0.5 A_c, \qquad (3.5)$$

$$T_* = p_{A*} A_A - p_{B*} A_B = \frac{\rho \omega^2}{2\pi} \left(A_A^2 \kappa_l^2 - A_B^2 \kappa_2^2 \right).$$

The last term of Eq.(3.4) (on the left) comprises pressure in the annular chamber 7 (Fig.2), which is partly dependent on the width of the face clearance 11.

The inlet pressure (p_e) and the outlet pressure (p_l) of the impeller and its angular velocity ω represent the external forces (excitations). To make the interpretation of results easier, the analysis will be conducted using dimensionless quantities. After dividing the components in Eq.(3.4) by the product $A_A p_n$, we obtain the following dimensionless quantities

$$\overline{A}_l = \frac{A_l}{A_A}, \qquad \overline{A}_2 = \frac{A_2}{A_A}, \qquad \overline{A}_e = \frac{A_e}{A_A},$$

$$\psi_l = \frac{p_l}{p_n}, \qquad \psi_2 = \frac{p_2}{p_n}, \qquad \psi_e = \frac{p_e}{p_n}, \qquad (3.6)$$

$$K_1 = \frac{\rho \omega_n^2}{2\pi A_A p_n} A_A^2 \kappa_l^2, \qquad K_2 = \frac{\rho \omega_n^2}{2\pi A_A p_n} A_B^2 \kappa_2^2, \qquad \Omega = \frac{\omega}{\omega_n}, \qquad u = \frac{z}{H_3}.$$

Thus, Eq.(3.4) takes the following form

$$\left[\overline{A}_l \psi_l + \overline{A}_e \psi_e - \left(K_l - K_2 \right) \Omega^2 \right] \Big/ \overline{A}_2 = \psi_2 . \qquad (3.7)$$

The controlled dimensionless quantity ψ_2 is the response to the external excitations ψ_l, ψ_e, Ω.

To obtain a static characteristic in a dimensionless form, it is necessary to make the quantity ψ_2 dependent on the dimensionless width of the lateral (face) clearance: $u = z/H_3$. For a steady state, we assume the continuity of fluid flow $(Q_2 = Q_3)$ through the longitudinal (6) and lateral (face) (8) clearances, closing the annular chamber 10, where the pressure is p_2 (ψ_2). For a turbulent flow, the flow rates can be determined as follows

$$Q_2 = g_2 \sqrt{p_l - p_{B*} - p_2}, \qquad Q_3 = g_3 \sqrt{p_2 - p_e} . \qquad (3.8)$$

If the local hydraulic losses are omitted, the capacities of the longitudinal clearance with eccentricity ε and the lateral clearance are defined by the following relationship (Kubota, 1999)

$$g_2 = g_{2n}\left(1+0.19\varepsilon^2\right), \qquad g_{2n} = \frac{4\pi R_3 H_2^{1.5}}{\sqrt{\lambda_2 \rho l_2}}; \qquad g_3 = g_{3n}u^{1.5}, \qquad g_{3n} = \frac{4\pi R_4 H_3^{1.5}}{\sqrt{\lambda_3 \rho l_3}} . \tag{3.9}$$

where $\lambda_2 \approx 0.04, \lambda_3 \approx 0.06$ - coefficients of the continuous hydraulic loss for self-similar turbulent flows in longitudinal and lateral clearances. If the squares of the flow rates are equal $Q_2^2 = Q_3^2$, we have

$$\psi_2 = \frac{\alpha_{23}\left(1+0.19\varepsilon^2\right)^2\left(\psi_1 - \psi_{B*}\right) + u^3\psi_e}{\alpha_{23}\left(1+0.19\varepsilon^2\right)^2 + u^3} \tag{3.10}$$

where

$$\alpha'_{23} = \frac{g_{2n}^2}{g_{3n}^2} = \frac{3R_3^2 l_3}{2R_4^2 l_2} \cdot \frac{H_2^3}{H_3^3}, \qquad\qquad \alpha_{23} = \frac{g_2^2}{g_{3n}^2} = \alpha'_{23}\left(1+0.19\varepsilon^2\right)^2 . \tag{3.11}$$

The derivative of the dimensionless pressure (quantity ψ_2) relative to the dimensionless width of the lateral clearance defines the "hydrostatic rigidity" of the node considered (the design node) of the impeller–clearance seals system

$$\frac{\partial \psi_2}{\partial u} = -\frac{3u^2 \alpha_{23}\left(1+0.19\varepsilon^2\right)^2}{\left[\alpha_{23}\left(1+0.19\varepsilon^2\right)^2 + u^3\right]^2}\left(\psi_1 - \psi_{B*} - \psi_e\right) .$$

The negative value of this quantity indicates the stability of the impeller.
By equating relationship (3.7) to (3.10), we find a static characteristic of the node considered

$$u = \left\{\alpha_{23}\frac{\left(\overline{A}_2 - \overline{A}_1\right)\psi_1 - \overline{A}_e\psi_e + \left(K_1 - K_2\right)\Omega^2 - \overline{A}_2\psi_{B*}}{\overline{A}_1\psi_1 - \left(\overline{A}_2 - \overline{A}_e\right)\psi_e - \left(K_1 - K_2\right)\Omega^2}\right\}^{1/3} . \tag{3.12}$$

The drop in pressure ψ_{B*} along the radius of the load-bearing shroud can be expressed by a dimensionless angular velocity of the impeller

$$\psi_{B*} = K_2\frac{A_A}{A_B}\Omega^2 .$$

After substituting the relationship into formula (3.12), we obtain the final form of the dimensionless static characteristic

$$u = \left\{ \alpha'_{23} \left(1 + 0.19\varepsilon^2 \right)^2 \frac{\left(\overline{A}_2 - \overline{A}_1 \right)\psi_1 - \overline{A}_e\psi_e + \left(K_1 - K_2 - \overline{A}_2 K_2\, A_A/A_B \right)\Omega^2}{\overline{A}_1\psi_1 - \left(\overline{A}_2 - \overline{A}_e \right)\psi_e - \left(K_1 - K_2 \right)\Omega^2} \right\}^{1/3} . \tag{3.13}$$

The relationship can be used to determine the width of the lateral clearance depending on external forces $\left(\psi_1, \psi_e, \Omega \right)$ and the geometry of the design node. It should be noted that the axial displacement of the impeller is due to its eccentricity.

The range of the operational parameters is limited by the pumping pressure at which the width of the lateral clearance does not exceed the admissible values: $u_{\min} < u < u_{\max}$. The clearance opens completely when $u \to \infty$; thus, the denominator in formula (3.13) decreases to zero, which occurs at the following pressure

$$\psi_{1*} = \frac{1}{\overline{A}_1} \left[\left(\overline{A}_2 - \overline{A}_e \right)\psi_e + \left(K_1 - K_2 \right)\Omega^2 \right]. \tag{3.14}$$

The other boundary condition is the closure of the clearance: $u = 0$. This condition occurs when the value of the numerator in relationship (3.13) decreases to zero, which determines the pumping pressure

$$\psi_{1**} = \frac{1}{\overline{A}_2 - \overline{A}_1} \left[\overline{A}_e\psi_e - \left(K_1 - K_2 - \overline{A}_2 \frac{A_A}{A_B} K_2 \right)\Omega^2 \right]. \tag{3.15}$$

The range of the pumping pressure at which the width of the lateral clearance is equal to or greater than zero is: $\psi_{1*} < \psi_1 < \psi_{1**}$.

The design features of the pump analyzed here need to be selected in such a way that, with the nominal operational parameters, i.e., $\psi_{1n} = 1, \Omega_n = 1$, the width of the lateral clearance has a nominal value, i.e., $z = z_n, u = 1$. Equating expression (3.13) to unity for the nominal operational parameters (for the condition that $u = 1$), we obtain an equation that can be used to find the clearance capacities α_{23}^* (3.11)

$$\alpha'^*_{23} = \frac{\overline{A}_1 - \left(\overline{A}_2 - \overline{A}_e \right)\psi_e - \left(K_1 - K_2 \right)}{\overline{A}_2 - \overline{A}_1 - \overline{A}_e\psi_e + K_1 - K_2 - \overline{A}_2 K_2\, A_A/A_B}. \tag{3.16}$$

Assuming initially that the radii R_3, R_4, R_5 and the clearance lengths l_1, l_2 result from the design of the flow system of the pump and that the shaft diameter $d_w = 2R_6$ results from the condition of torsional strength, we can use formula (3.11) to find the ratio of the widths of the longitudinal clearances H_3/H_2

$$\frac{H_3}{H_2} = \left(\frac{2}{3} \alpha'^*_{23} \frac{l_2 R_4^2}{l_3 R_3^2} \right)^{-1/3} . \tag{3.17}$$

In centrifugal pumps, the pumping pressure is proportional to the square of the angular velocity of the impeller: $p_1 = B\omega^2$, where B – generalized parameter characterizing the geometry of the basic flow system of the pump and maintaining a constant value at different rotational velocities

$$\mathrm{B} = \frac{p_n}{\omega_n^2} = \mathrm{const}; \qquad p_1 = p_n\,\omega^2/\omega_n^2, \qquad \psi_1 = \Omega^2 \tag{3.18}$$

where ω_n, $p_n = p_{1n}$ - nominal angular velocity of the impeller and the corresponding nominal pumping pressure.

In many a case, especially in pumps used in the power industry, the inlet pressure, because of the susceptibility to cavitation, is increased by the intake axial impeller mounted on the common shaft. As a result, the inlet and outlet pressures of the centrifugal impeller are proportional to the square of the angular velocity of the shaft

$$p_e = C\omega^2, \qquad C = p_{en}/\omega_n^2, \qquad p_e = p_{en}\Omega^2, \qquad \psi_e = \psi_{en}\Omega^2 \;. \tag{3.19}$$

Substituting quantities (3.18) and (3.19) into the relationship describing the static characteristic (3.13) and dividing it by Ω^2, we obtain a relationship from which it follows that the width of the lateral clearance is not dependent on the external forces and remains constant

$$u = \left\{ \alpha_{23} \frac{\overline{A}_2 - \overline{A}_1 - \overline{A}_e\psi_{en} + \left(K_1 - K_2 - \overline{A}_2 K_2\, A_A/A_B\right)}{\overline{A}_1 - \left(\overline{A}_2 - \overline{A}_e\right)\psi_{en} - \left(K_1 - K_2\right)} \right\}^{\!1/3} = \mathrm{const}\,. \tag{3.20}$$

When the inlet pressure does not depend on the angular velocity of the impeller, relationship (3.13) assumes the form

$$u = \left\{ \alpha_{23} \frac{\left(\overline{A}_2 - \overline{A}_1 + K_1 - K_2 - \overline{A}_2 K_2\, A_A/A_B\right)\Omega^2 - \overline{A}_e\psi_e}{\left[\overline{A}_1 - \left(K_1 - K_2\right)\right]\Omega^2 - \left(\overline{A}_2 - \overline{A}_e\right)\psi_e} \right\}^{\!1/3}. \tag{3.21}$$

These relationships for static characteristics are simplified, for the condition that $p_e \ll p_1$.

The next characteristic determined at static equilibrium is the rate of flow through the rear clearances: longitudinal and lateral, which is calculated using one of the formulas (3.8). For instance

$$Q_3^2 = g_{3n}^2 u^3 \left(p_2 - p_e\right) = g_{3n}^2 p_n u^3 \left(\psi_2 - \psi_e\right);$$

$$\overline{Q}^2 = Q_3^2/Q_n^2 = u^3\left(\psi_2 - \psi_e\right); \qquad Q_n^2 = g_{3n}^2 p_n\,. \tag{3.22}$$

After substituting expressions (3.8) and (3.10), we obtain

$$\overline{Q} = \left\{ \frac{\alpha_{23}}{\overline{A}_2} \left[\left(\overline{A}_2 - \overline{A}_1\right)\psi_1 - \overline{A}_e\psi_e + \left(K_1 - K_2 - \overline{A}_2 K_2\, \frac{\overline{A}_A}{A_B}\right)\Omega^2 \right] \right\}^{\!0,5}. \tag{3.23}$$

With the nominal value of the lateral clearance height, relationship (3.23) is simplified to

$$\bar{Q}(u=1) = \frac{\alpha_{23}}{1+\alpha_{23}}\left(\psi_1 - \psi_e - K_2\frac{A_A}{A_B}\Omega^2\right). \tag{3.24}$$

The final relationship can be used for initial evaluation of the rate of flow through (volumetric loss in) the rear clearances: longitudinal and lateral.

4. Numerical example

The calculations of the static and dynamic characteristics were performed for an impeller of a modernized high-speed single-stage centrifugal pump. The pump modernization involved applying an impeller with a system of seal clearances, shown in Fig.1.

1.Input parameters.

Let us assume the following geometrical dimensions of the pump impeller

$R_1=56mm$, $R_2=75mm$, $R_3=65mm$, $R_4=58mm$,

$R_5=51mm$, $R_6=15mm$, $l_1=l_2=15mm$,

$H_1=H_2=0.2mm$ (with symbols being the same as those in Fig.2).

The assumptions concerning the pump performance parameters are:

$p_n = 3.5\,MPa$; $p_e = 1.25\,MPa$; $\omega_n = 1500s^{-1}$;

$\rho = 10^3\,kg/m^3$; $\kappa_1 = 0.8$; $\kappa_2 = 0.6$.

2. Calculation of the surface areas of the impeller, based on Eqs (3.2), (3.5) and (3.6), and the moments of inertia.

$$A_A = \pi\left(R_2^2 - R_1^2\right) = 7.58\cdot10^{-3}\,m^2, \qquad A_B = \pi\left(R_2^2 - R_3^2\right) = 4.08\cdot10^{-3}\,m^2,$$

$$A_2' = \pi\left(R_3^2 - R_4^2\right) = 2.76\cdot10^{-3}\,m^2, \qquad A_c = \pi\left(R_4^2 - R_5^2\right) = 2.40\cdot10^{-3}\,m^2,$$

$$A_2 = A_2' + 0.5A_c = 3.96\cdot10^{-3}\,m^2, \qquad A_e' = \pi\left(R_1^2 - R_5^2\right) = 1.68\cdot10^{-3}\,m^2,$$

$$A_3 = \pi R_6^2 = 7.07\cdot10^{-4}\,m^2, \qquad A_e = A_e' + A_3 - 0.5A_c = 1.19\cdot10^{-3}\,m^2;$$

$$\bar{A}_1 = \left(A_A - A_B\right)/A_A = 0.437; \qquad \bar{A}_2 = A_2/A_A = 0.45; \qquad \bar{A}_e = A_e/A_A = 0.152.$$

Coefficients (3.6): $K_1 = 0.312$; $K_2 = 0.056$.

3. Range of admissible values of the pumping pressure.

From Eqs (3.14) and (3.15), we obtain: $\psi_{1*} = 0.615$; $\psi_{1**} = 3.38$. The range of these values is much larger than that resulting from the characteristics of the pump prior to its modernization

4. Determination of the nominal value of the lateral clearance.

From relationships (3.16) and (3.17) we have $\alpha_{23}^* = 0.67$ and $H_3 = 0.28 \, mm$.

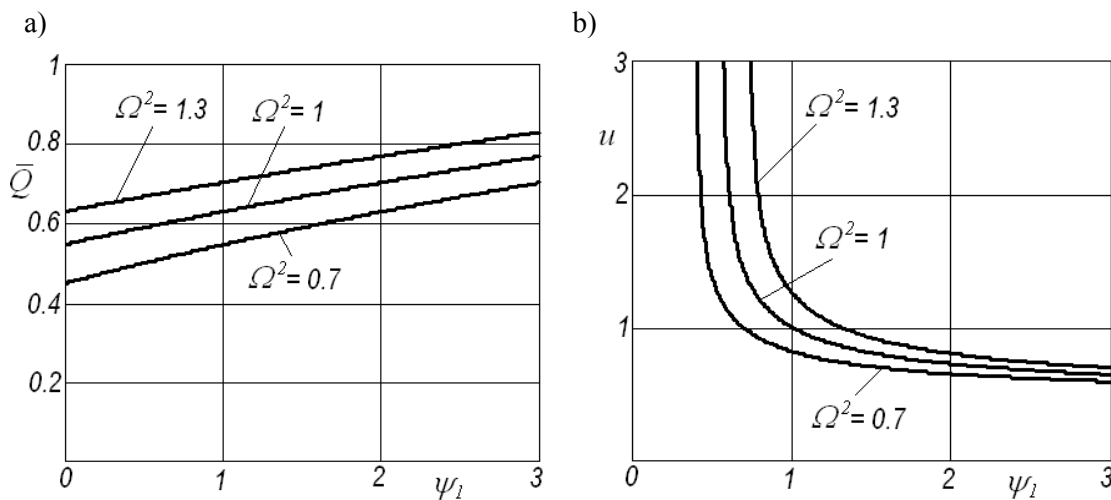5. Calculation and graphical representation of the static and flow characteristics.

Fig.3. Dimensionless static characteristics: a) lateral clearance width u, b) volumetric flow rate Q versus the pumping pressure ψ_1 and the impeller angular velocity, Ω .

Figure 3a shows dimensionless characteristics of the lateral clearance width versus the pumping pressure, based on relationship (3.13), for three values of rotational velocity. The corresponding flow characteristics, based on relationship (3.23), are shown in Fig.3b.

From the static characteristics it is evident that the width of the lateral clearance decreases with an increase in the pumping pressure, and increases with an increase in the angular velocity. When the pumping parameters are nominal $(\psi_1 = \Omega = 1)$, the width of the lateral clearance assumes the predetermined value, $z = H_3$. The rate of the flow through the longitudinal and then lateral clearances is slightly dependent on the pumping pressure; it increases with an increase in the pump angular velocity.

Conclusions

The static analysis, which was performed for simplified assumptions of flow and distribution of pressure in the longitudinal and lateral clearances, required deriving relationships between the geometry of the impeller and the basic parameters of the pump performance. The equation of flow continuity was used to determine the coefficient of hydrostatic rigidity, the negative value of which constitute the condition of static

balance of the impeller. The static characteristics derived in this study can be used for designing impellers with a balancing device.

Nomenclature

A_e', A_c, A_3 — surface areas of the impeller of Eq.(3.2) $[m^2]$

A_A, A_B — outer surfaces area of the impeller shrouds $[m^2]$

$\bar{A}_1, \bar{A}_2, \bar{A}_e$ — dimensionless surface areas of Eq.(3.6)

B, C — parameters of Eqs (3.18) and (3.19)

g_2, g_3 — capacities of the longitudinal and lateral clearances of Eq.(3.9)

g_{2n}, g_{3n} — coefficients in Eq.(3.9)

H_1, H_2 — width of the longitudinal clearances $[m]$

H_3 — width of the lateral clearance $[m]$

K_1, K_2 — dimensionless coefficients of Eq.(3.6)

p_e, p_1 — inlet pressure and outlet pressure of the impeller $[N/m^2]$

p_2 — pressure in the annular chamber of a balance device $[N/m^2]$

p_n — nominal pumping pressure $[N/m^2]$

R_1, R_2 — radii of the impeller $[m]$

R_3, R_4, R_5 — radii of a balance device $[m]$

T_1, T_2 — axial forces acting on the impeller shrouds $[N]$

Q_2, Q_3 — fluid flow through the longitudinal and lateral clearances of Eq.(3.8) $[m^3/s]$

$\bar{Q}$ — dimensionless fluid flow of Eq.(3.23)

$u = z/H_3$ — dimensionless width of the lateral (face) clearance

z — axial displacement of the impeller $[m]$

$\alpha_{23}, \alpha'_{23}$ — coefficients of Eq.(3.11)

ε — relative eccentricity of the longitudinal clearance

λ_2, λ_3 — coefficients of the hydraulic losses

ρ — fluid density $[kg/m^3]$

κ_1, κ_2 — coefficients of the geometry of the impeller in Eq.(3.5)

ψ_1, ψ_e, ψ_2 — dimensionless pressures of Eq.(3.6)

Ω — dimensionless angular velocity of Eq.(3.6)

ω — angular velocity of the impeller $[1/s]$

ω_n — nominal angular velocity $[1/s]$

References

Antoszewski B., Evin E. and Audy J. (2008): *Study of the effect of electro-spark coatings on friction in pin-on-disc testing*. – Journal of Tribology-Transactions of the ASME, No.3, pp.253- 262.

Chiba Y., Ishikawa Ch., Katsura H. and Abe D. (2007): *Single-shaft multistage pump*. – Patent US7198457B2.

Jędral W. (2001): *Centrifugal Pumps*. – Warsaw: PWN.

Korczak A. (2005): *Investigations of systems counterbalancing the arial thrust in multi-stage cetrifugal pumps*. – Scientific Book of the Silesian University of Technology, No.1679, Gliwice.

Korczak A., Peczkis G. and Martsinkovsky V.A. (2005): *Using the locking hydraulic device for rotor balancing*. – Bulletin of Sumy State University. Series Engineering, No.1, pp.68-76.

Kubota Y. (1999): *Thrust-balancing device*. – Patent EP0952352A2.

Martsinkovsky V.A., Demin S.I., Gulyj A.N. and Bachkina A.A. (1995): *Centrifugal pump*. – Patent SU1771248.

Pavlenko I. (2008): *Static analysis of the locking automatic balancing device of the centrifugal pumps*. – Scientific Book of the Kielce University of Technology, Exploitation and Mechanical Engineering, No.10, tom 2, pp.165–172.