DE GRUYTER OPEN

Int. J. of Applied Mechanics and Engineering, 2014, vol.19, No.2, pp.347-363
DOI: 10.2478/ijame-2014-0023

INTERACTION DUE TO MECHANICAL SOURCE IN GENERALIZED THERMO MICROSTRETCH ELASTIC MEDIUM

R. SINGH*
Department of Mathematics
S.G.A.D. Govt. College
Punjab, INDIA
E-mail: kalsi_ranjit@yahoo.com

V. KUMAR
Department of Mathematics
Lovely Professional University
Punjab, INDIA
E-mail: varun.kumar@hotmail.co.in

The eigen value approach, following the Laplace and Hankel transformation has been employed to find a general solution of the field equations in a generalized thermo microstretch elastic medium for an axisymmetric problem. An infinite space with the mechanical source has been applied to illustrate the utility of the approach. The integral transformations have been inverted by using a numerical inversion technique to obtain normal displacement, normal force stress, couple stress and microstress in the physical domain. Numerical results are shown graphically.

Key words: generalized thermo-microstretch, eigen value, Laplace transform, Hankel transform, mechanical source.

1. Introduction

The classical theory of elasticity successfully explains the behavior of construction materials (various sorts of steel, aluminum, concrete) provided the stresses do not exceed the elastic limit and no stress concentration occurs. But it is inadequate to model the modern engineering components which possess internal structure such as polycrystalline materials and materials with fibrous or coarse grain structure. Eringen (1966) coined a term micropolar elasticity and used this theory to explain the deformation of elastic media with such oriented particles. A micropolar continuum is a collection of interconnected particles that are made up of dipole atoms or dumb-bell molecules and are subjected to surface and body couples which is capable of translational as well as rotational motion.

The governing equations of thermoelasticity in the usual framework of linear coupled thermoelasticity consists of the wave type (hyperbolic) equations of motion and the diffusion type (parabolic) equation of heat conduction. But it was observed that if an isotropic, homogeneous, elastic material is subjected to thermal or mechanical disturbances, the effects in the temperature and displacement field are felt immediately at an infinite distance from the source of disturbance. This implies that a part of the disturbance has an infinite velocity of propagation which is physically impossible. To overcome these discrepancies two generalizations to the coupled theory were introduced. The first is due to Lord and

---

* To whom correspondence should be addressed

Shulman (1967), who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law. The second generalization is known as the theory of thermoelasticity with two relaxation times or the theory of temperature–rate-dependent thermoelasticity. Green and Lindsay obtained an explicit version of the constitutive equations in 1972.

Nowacki (1966) and Eringen (1970) extended the linear theory of micropolar continua to include the thermal effect and formulated the micropolar thermoelasticity theory. The linear theory of elastic materials with stretch is one of the generalizations of the classical theory of elasticity. Eringen (1971) developed the theory of micropolar elastic solid with stretch which included the effect of axial stretch during the rotation of molecules. Microstretch solids are capable of stretching and contraction independent of their translation and rotation. Thus, in these solids, the motion is characterized by seven degrees of freedom, namely three for translation, three for rotation and one for stretch. Porous media whose pores are filled with gas inviscid liquid, asphalt and composite fibrous materials are some examples of microstretch elastic solids. Eringen (1990) also developed a continuum theory of thermo-microstretch elastic solids. Green and Naghdi (1993) proposed the theory of thermoelasticity without energy dissipation and presented the derivation of a complete set of governing equations of the linearized version of the theory for homogeneous and isotropic materials in terms of displacement and temperature fields and proved the uniqueness of the solution of the corresponding initial mixed boundary value problem. Iesan and Neppa (1995) contributed to this field by studying a problem on extension and bending of a microstretch elastic circular cylinder. A problem of bending of microstretch elastic plates was investigated by Ciarletta (1999). Chandrasekharaiah and Srinath (2000) studied the problem of thermoelastic waves without energy disspation in an unbounded body having a spherical cavity. Aouadi (2008) studied the linear theory of microstretch thermoelastic bodies with microtemperature and proved the existence of coupling of microrotation vector field with the microtemperatures for isotropic bodies. Kumar and Partap (2009) investigated the propagation of axisymmetric free vibrations in microstretch thermoelastic homogeneous isotropic solids which were subjected to stress free thermally insulated and isothermal conditions. Othman and Lotfy (2010) applied the normal mode analysis on the general model of the equations of generalized thermo-microstretch for a homogeneous isotropic elastic half space of different theories. Othman *et al*. (2013) analyzed the effect of gravity on the same model for generalized thermo-microstretch for a homogenous isotropic elastic half-space solid subjected to a Mode-I crack problem in the context of Green Naghdi theory.

2. Formulation and solution of the problem

We consider a homogeneous, isotropic generalized thermo-microstretch elastic medium of infinite extent pointing vertically into the medium. Field equations and the constitutive relations without body forces, body couples, heat sources and stretch force are given by Eringen (1990), Lord and Shulman (1967) and Green and Lindsay (1972) as

$$\begin{gathered}\lambda_0 \nabla\varphi^* + (\lambda + 2\mu + K)\nabla(\nabla\cdot\boldsymbol{u}) - (\mu + K)\nabla\times\nabla\times\boldsymbol{u} + \\ + K\nabla\times\boldsymbol{\varphi} - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)\nabla T = \rho\frac{\partial^2\boldsymbol{u}}{\partial t^2},\end{gathered} \tag{2.1}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla\cdot\boldsymbol{\varphi}) - \gamma\nabla\times\nabla\times\boldsymbol{\varphi} + K\nabla\times\boldsymbol{u} - 2K\boldsymbol{\varphi} = \rho j\frac{\partial^2\boldsymbol{u}}{\partial t^2}, \tag{2.2}$$

$$\rho\, C^*\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) + \nu T_0\left(\frac{\partial}{\partial t} + \Xi\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla\cdot\boldsymbol{u} + \nu_1 T_0\frac{\partial\varphi^*}{\partial t} = K^*\nabla^2 T, \tag{2.3}$$

$$\alpha_0 \nabla^2 \varphi^* + \frac{1}{3}\nu_1 \varphi^* - \frac{\lambda_0}{3}\nabla\cdot\boldsymbol{u} = \frac{3}{2}\rho j \frac{\partial^2 \varphi^*}{\partial t^2}, \tag{2.4}$$

$$t_{ij} = \lambda\ u_{r,r}\ \delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \in_{ijr}\ \varphi_r\right) - \left(T + \tau_0 \frac{\partial T}{\partial t}\right)\delta_{ij}, \tag{2.5}$$

$$m_{ij} = \alpha\varphi_{r,r}\delta_{ij} + \beta\varphi_{i,j} + \gamma\varphi_{j.i}, \tag{2.6}$$

$$\lambda_i = \alpha_0 \varphi_{j,i}^{\ *} \tag{2.7}$$

where

$$\nu = \left(3\lambda + 2\mu + K\right)\alpha_{t_1}, \qquad \nu_1 = \left(3\lambda + 2\mu + K\right)\alpha_{t_2}.$$

Since we are considering a two-dimensional axisymmetric problem, so we assume the components of the displacement vector $\boldsymbol{u}$ and microrotation vector φ are of the form

$$\boldsymbol{u} = \left(u_r, 0, u_z\right), \qquad \varphi = \left(0, \varphi_\theta, 0\right). \tag{2.8}$$

Here due to symmetry about the z-axis all the quantities are independent of θ, so that $\frac{\partial}{\partial\theta} \equiv 0$. With these considerations and using Eqs (2.8), the system of Eqs (2.1) to (2.7) reduces to

$$\begin{gathered}(\lambda+\mu)\left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_z}{\partial r \partial z}\right] + (\mu+K)\left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \\ -K\frac{\partial\varphi_\theta}{\partial z} - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial r} + \lambda_0\frac{\partial\varphi^*}{\partial r} = \rho\frac{\partial^2 u_r}{\partial t^2},\end{gathered} \tag{2.9}$$

$$\begin{gathered}(\lambda+\mu)\left[\frac{\partial^2 u_r}{\partial z^2} + \frac{1}{r}\frac{\partial u_r}{\partial z} + \frac{\partial^2 u_z}{\partial z \partial r}\right] + (\mu+K)\left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{\partial^2 u_z}{\partial z^2}\right] + \\ -\frac{K}{r}\frac{\partial\left(r\varphi_\theta\right)}{\partial r} - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} + \lambda_0\frac{\partial\varphi^*}{\partial z} = \rho\frac{\partial^2 u_z}{\partial t^2},\end{gathered} \tag{2.10}$$

$$\gamma\left[\frac{\partial^2\varphi_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi_\theta}{\partial r} - \frac{\varphi_\theta}{r^2} + \frac{\partial^2\varphi_\theta}{\partial z^2}\right] - 2K\varphi_\theta + K\left[\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right] - K\frac{\partial\varphi_\theta}{\partial z} - = \rho j\frac{\partial^2\varphi_\theta}{\partial t^2}, \tag{2.11}$$

$$\begin{gathered}K^*\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right] = \rho C^*\left[\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right] + \\ +\nu T_0\left[\frac{\partial}{\partial t} + \Xi\tau_0\frac{\partial^2}{\partial t^2}\right]\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial r}\right) + \nu T_0\frac{\partial\varphi^*}{\partial t},\end{gathered} \tag{2.12}$$

$$\alpha_0\left[\frac{\partial^2\varphi^*}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi^*}{\partial r}+\frac{\partial^2\varphi^*}{\partial z^2}\right]+\frac{1}{3}\nu_1 T-\frac{1}{3}\lambda_1\varphi^*+$$
$$-\frac{1}{3}\lambda_0\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial r}\right)=\frac{3}{2}\rho j\frac{\partial^2\varphi^*}{\partial t^2} \tag{2.13}$$

where $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$.

Introducing dimensionless quantities as

$$z'=\frac{\omega^* z}{C_1},\qquad t'=\omega^* t\ ,\qquad \tau_0'=\omega^*\tau_0\ ,\qquad \tau_1'=\omega^*\tau_1\ ,$$

$$T'=\frac{T}{T_0},\qquad u_r'=\frac{\rho\omega^* C_1 u_r}{\nu T_0},\qquad u_z'=\frac{\rho\omega^* C_1 u_z}{\nu T_0},$$

$$\varphi_\theta'=\frac{\rho\, C_1^2}{\nu T_0}\varphi_\theta,\qquad t_{zz}'=\frac{t_{zz}}{\nu T_0},\qquad t_{zr}'=\frac{t_{zr}}{\nu T_0},\qquad m_{z\theta}'=\frac{\omega^*}{C_1\nu T_0}m_{z\theta}, \tag{2.14}$$

$$\omega^*=\frac{C^*\left(\lambda+2\mu\right)}{K^*},\qquad C_1^2=\frac{\lambda+2\mu}{\rho},\qquad \varphi^{*'}=\frac{\rho c_1^2}{\nu T_0}\varphi^*,\qquad \lambda_z'=\frac{\omega^*}{c_1\nu T_0}\lambda_z.$$

After suppressing the primes for convenience Eqs (2.9)-(2.13) reduce to

$$m_1\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}\right]+m_2\frac{\partial^2 u_z}{\partial r\partial z}+m_3\frac{\partial^2 u_r}{\partial z^2}+$$
$$-m_4\frac{\partial\varphi_\theta}{\partial z}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial r}+m_5\frac{\partial\varphi^*}{\partial z}=\frac{\partial^2 u_r}{\partial t^2}, \tag{2.15}$$

$$m_1\frac{\partial^2 u_r}{\partial r^2}+m_2\left(\frac{\partial^2 u_z}{\partial r\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right)+m_3\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right)+$$
$$-m_4\frac{1}{r}\frac{\partial(r\varphi_\theta)}{\partial r}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}+m_5\frac{\partial\varphi^*}{\partial z}=\frac{\partial^2 u_z}{\partial t^2}, \tag{2.16}$$

$$\left[\frac{\partial^2\varphi_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi_\theta}{\partial r}-\frac{\varphi_\theta}{r^2}+\frac{\partial^2\varphi_\theta}{\partial z^2}\right]-2m_6\varphi_\theta+m_6\left[\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r}\right]=m_7\frac{\partial^2\varphi_\theta}{\partial t^2}, \tag{2.17}$$

$$\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right]-\left[\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right]=$$
$$=\varepsilon\left[\frac{\partial}{\partial t}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial r}\right)+\Xi\tau_0\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial r}\right)\right]+\varepsilon m_8\frac{\partial\varphi^*}{\partial t}, \tag{2.18}$$

$$\frac{\partial^2\varphi^*}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi^*}{\partial r}+\frac{\partial^2\varphi^*}{\partial z^2}+Tm_9-m_{10}\varphi^*-m_{11}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial r}\right)=m_{12}\frac{\partial^2\varphi^*}{\partial t^2} \tag{2.19}$$

where

$$m_1=\frac{\lambda+2\mu+K}{\rho C_1^2},\qquad m_2=\frac{\lambda+\mu}{\rho C_1^2},\qquad m_3=\frac{\mu+K}{\rho C_1^2},\qquad m_4=\frac{K}{\rho C_1^2},$$

$$m_5=\frac{\lambda_0}{\rho C_1^2},\qquad m_6=\frac{KC_1^2}{\gamma\omega^{*2}},\qquad m_7=\frac{\rho jC_1^2}{\gamma},\qquad m_8=\frac{\nu_1}{\nu},$$

$$m_9=\frac{\rho\nu_1 c_1^4}{3\nu\alpha_0\omega^{*2}},\qquad m_{10}=\frac{\lambda_1 c_1^2}{3\alpha_0\omega^{*2}},\qquad m_{11}=\frac{\lambda_0 c_1^2}{3\alpha_0\omega^{*2}},\qquad m_{12}=\frac{3\rho jc_1^2}{2\alpha_0}.$$

Applying the Laplace and Hankel transforms on Eqs (2.15)-(2.19) defined by

$$\overline{f}(r,z,p)\;=L\{f(r,z,t)\}=\int_0^\infty f(r,z,t)\,\exp(-pt)\,dt\,, \tag{2.20}$$

$$\tilde{f}(\xi,z,p)\;=H\left\{\overline{f}(x,z,p)\right\}=\int_0^\infty r\overline{f}(x,z,p)\,J_n(\xi r)dr\,, \tag{2.21}$$

we obtain

$$D^2\tilde{u}_r\;=\;\frac{1}{m_3}\left[\left(m_1\xi^2+p^2\right)\tilde{u}_r-\iota\,\xi\,m_2 D\tilde{u}_z+m_4 D\tilde{\varphi}_\theta+\iota\xi\left(1+\tau_1 p\right)\,\tilde{T}+m_5\xi\tilde{\varphi}^*\right], \tag{2.22}$$

$$D^2\tilde{u}_z\;=\;\frac{1}{m_1}\left[\left(m_1\xi^2+p^2\right)\tilde{u}_z-\,\xi\,m_2 D\tilde{u}_r-\xi m_4 D\tilde{\varphi}_\theta+\left(1+\tau_1 p\right)\,D\tilde{T}+m_5\tilde{\varphi}^{*\prime}\right], \tag{2.23}$$

$$D^2\tilde{\varphi}_\theta\;=\;-\;m_6\;D\tilde{u}_r+\;\xi\;m_6\tilde{u}_z+\left(2m_6+\xi^2+m_7 p^2\right)\tilde{\varphi}_\theta, \tag{2.24}$$

$$D^2\tilde{T}=\;\varepsilon\,p\left(1+\tau_0 p\Xi\right)\xi\,\tilde{u}_r+\varepsilon\,p\left(1+\tau_0 p\Xi\right)\,D\tilde{u}_z+\left\{\xi^2+p\left(1+\tau_0 p\right)\tilde{T}\;\right\}+\varepsilon\,pm_8\tilde{\varphi}^*, \tag{2.25}$$

$$D^2\tilde{\varphi}^*=\xi\,m_{11}\tilde{u}_r+m_{11}D\tilde{u}_z-m_9\tilde{T}+\left(m_{12}p^2+\xi^2+m_{10}\right)\tilde{\varphi}^*. \tag{2.26}$$

The system of Eqs (2.22)-(2.26) can be written as

$$\frac{dW(\xi,z,p)}{dz} = A(\xi,p)W(\xi,z,p) \tag{2.27}$$

where

$$W = \begin{bmatrix} U \\ U' \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & I \\ A_2 & A_1 \end{bmatrix}, \qquad U = \begin{bmatrix} \tilde{u}_r & \tilde{u}_z & \tilde{\varphi}_\theta & \tilde{T} & \tilde{\varphi}^* \end{bmatrix}^t,$$

$$A_1 = \begin{bmatrix} 0 & \dfrac{\xi m_2}{m_3} & \dfrac{m_4}{m_3} & 0 & 0 \\ -\dfrac{\xi m_2}{m_1} & 0 & 0 & \dfrac{1+\tau_1 p}{m_1} & -\dfrac{m_5}{m_1} \\ -m_6 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon\, p\left(1+\tau_0 p\Xi\right) & 0 & 0 & 0 \\ 0 & m_{11} & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} \dfrac{\left(m_1\xi^2+p^2\right)}{m_3} & 0 & 0 & -\dfrac{\xi\left(1+\tau_1 p\right)}{m_3} & \dfrac{m_5\xi}{m_3} \\ 0 & \dfrac{\left(m_3\xi^2+p^2\right)}{m_1} & -\dfrac{m_4\xi}{m_1} & 0 & 0 \\ 0 & -m_6\xi & \left(2m_6+\xi^2+m_7 p^2\right) & 0 & 0 \\ \varepsilon\, p\left(1+\tau_0 p\Xi\right)\xi & 0 & 0 & \xi^2+p\left(1+\tau_0 p\right) & \varepsilon\, p m_8 \\ \xi m_{11} & 0 & 0 & -m_9 & \left(m_{12}p^2+\xi^2+m_{10}\right) \end{bmatrix}$$

where O is the null matrix, I is the unit matrix of order 4, $[]^t$ is the transpose of matrix [] and $D = \dfrac{d}{dz}$.

To solve Eq.(2.27), we take

$$W(\xi,z,p) = W(\xi,p)e^{qz}, \tag{2.28}$$

for some parameter q so that

$$A(\xi,p)W(\xi,z,p) = qW(\xi,z,p), \tag{2.29}$$

which leads to the eigen value problem. The characteristic equation corresponding to the matrix A is given by

$$\det\left(A-qI\right)=0. \tag{2.30}$$

On expanding Eq.(2.30), we get

$$q^{10} - \sigma_1 q^8 + \sigma_2 q^6 - \sigma_3 q^4 + \sigma_4 q^2 + \sigma_5 = 0 \tag{2.31}$$

where $\sigma_i's$ ($i=1,2,..,5$) are functions of ξ and p.

The eigen values of the matrix A are characteristic roots of Eq.(2.31). The eigen vectors $X(\xi, p)$ corresponding to the eigen value q_s can be determined by solving the system of homogenous equations

$$[A - qI]X(\xi, p) = 0. \tag{2.32}$$

The set of eigen vectors $X_s(\xi, p); s=1,2,\ldots,10$ may be defined as

$$X_s(\xi, p) = \begin{bmatrix} X_{s1}(\xi, p) \\ X_{s2}(\xi, p) \end{bmatrix} \tag{2.33}$$

where

$$X_{i1} = \begin{bmatrix} a_i q_i & b_i & -\xi & c_i & d_i \end{bmatrix}^t, \qquad X_{i2} = \begin{bmatrix} a_i q_i^2 & b_i q_i & -\xi q_i & c_i q_i & d_i q_i \end{bmatrix}^t,$$

$$q = q_i; \qquad i = 1,2,3,4,5,$$

$$X_{j1} = \begin{bmatrix} -a_i q_i & b_i & -\xi & c_i & d_i \end{bmatrix}^t, \qquad X_{j2} = \begin{bmatrix} a_i q_i^2 & -b_i q_i & \xi q_i & -c_i q_i & -d_i q_i \end{bmatrix}^t,$$

$$j = i + 5; \qquad q = -q_i;$$

for $\quad i=1,2,3,4,5,$

$$\Delta_s = m_6\left(F_1 - F_2 q_s^2\right), \qquad a_s = \frac{q_s}{\Delta_s}\left(m_6 g_{11} F_3 + F_2 H_3\right), \qquad b_s = \frac{-1}{\Delta_s}\left(m_6 g_{11} F_3 q_s^2 + F_1 H_3\right),$$

$$c_s = \frac{q_s\left[\varepsilon\, p m_8\left(a_s H_1 + \xi\, b_s g_{10} - \xi\, g_{11}\right) - \xi\, g_7 g_9\left(b_s + \xi\, a_s\right)\right]}{\xi\left(\varepsilon\, p m_8 g_6 + g_7 H_4\right)}, \quad d_s = \frac{\left[c_s m_9 - m_{11} q_s\left(b_s + \xi a_s\right)\right]}{H_5},$$

$$F_1 = a^* k H_1 - b^* k \xi^2 + b^* f \xi^2 H_5 + g H_1 H_2 H_3 - a^* f h \xi^2 - g h \xi^2 H_4,$$

$$F_2 = a^* k c^* - b^* k + b^* f H_5 + g c^* H_4 H_5 - a^* f h - g h H_4, \qquad F_3 = a^* k + g H_4 H_5,$$

$$H_1 = g_1 - q^2, \qquad H_2 = g_2 - q^2, \qquad H_3 = g_3 - q^2, \qquad H_4 = g_4 - q^2, \qquad H_5 = g_5 - q^2,$$

$$a^* = \in p m_8, \qquad b^* = g_7 g_9, \qquad c^* = g_{10}, \qquad f = g_6, \qquad g = g_7,$$

$$h = g_7 m_{11}, \qquad k = g_7 m_9, \qquad f_1 = \frac{m_1 \xi^2 + p^2}{m_3}, \qquad f_2 = \frac{m_2 \xi^2 + p^2}{m_1},$$

$$f_3 = 2m_6 + \xi^2 + m_7 p^2, \qquad f_4 = m_{10} + \xi^2 + m_{12} p^2, \qquad f_5 = p + \xi^2 + \tau_0 p^2,$$

$$f_6 = f_1 f_2, \qquad f_7 = f_1 f_3, \qquad f_8 = f_1 f_4, \qquad f_9 = f_1 f_5,$$

$$f_{10} = f_2 f_3, \qquad f_{11} = f_2 f_4, \qquad f_{12} = f_2 f_5, \qquad f_{13} = f_3 f_4,$$

$$f_{14} = f_3 f_5, \qquad f_{15} = f_4 f_5, \qquad g_1 = \sum_{l=1}^{5} f_l, \qquad g_2 = \sum_{l=6}^{15} f_l,$$

$$g_3 = \sum_{l=16}^{25} f_l, \qquad g_4 = \sum_{l=26}^{30} f_l, \qquad g_5 = f_1 f_2 f_3 f_4 f_5, \qquad g_6 = \frac{\varepsilon p \left(1 + p\tau_0\right)}{m_1},$$

$$g_7 = \frac{m_5 m_{11}}{m_1 m_3}, \qquad g_8 = \xi^2 \left(m_2 - m_1 + m_3\right) g_7, \qquad g_9 = \xi^2 \left(f_1 + f_3 + f_5 + m_4 m_6\right) g_7,$$

$$g_{10} = \frac{m_4 m_6 \left(1 - g_6\right)}{m_3}, \qquad g_{11} = \frac{\xi^2 \left(m_2 - m_1\right) g_6}{m_3}.$$

Thus the solution of Eq.(2.27) as given by Sharma and Chand (1992) is

$$W\left(\xi, z, q\right) = \sum_{i=1}^{5} \left[B_i X_i \exp\left(q_i z\right) + B_{i+5} X_{i+5} \exp\left(-q_i z\right) \right] \tag{2.34}$$

where $B_i's$ $(i=1,2,\ldots,10)$ are arbitrary constants. Equation (2.34) represents the solution of the generalized thermo microstretch elastic medium for the axisymmetric case and gives displacement, microrotation, temperature distribution and scalar microstretch in the transformed domain.

3. Application

We consider an infinite generalized thermo microstretch elastic space in which a concentrated force is $F = -F_0 \dfrac{\delta(r)\delta(t)}{2\pi r}$ where F_0 is the magnitude of the force, acting in the direction of the z-axis at the origin of the cylindrical polar co-ordinate system as shown in the Fig.1.

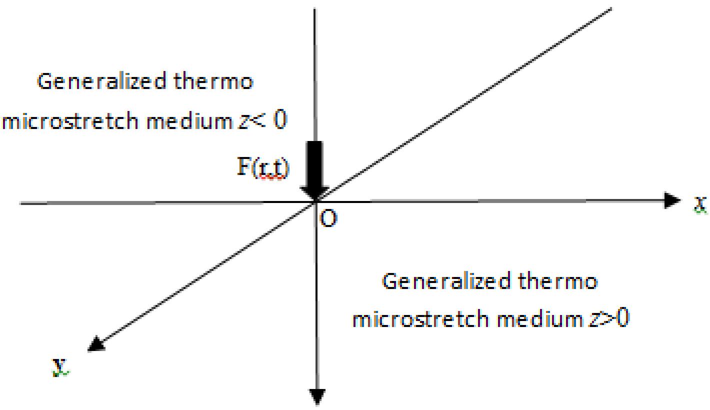

Fig.1. Geometry of the problem.

The boundary conditions for the plane $z=0$ are given by,

$$u_r\left(r,0^+,t\right)\ -u_r\left(r,0^-,t\right)\ =0,\qquad u_z\left(r,0^+,t\right)\ -u_z\left(r,0^-,t\right)\ =0, \tag{3.1}$$

$$\varphi_\theta\left(r,\ 0^+,\ t\right)-\varphi_\theta\left(r,\ 0^-,\ t\right)=0,\quad \varphi^*\left(r,\ 0^+,\ t\right)-\varphi^*\left(r,\ 0^-,\ t\right)=0 \tag{3.2}$$

$$T\left(r,0^+,t\right)T\left(r,0^-,t\right)\ =0,\ =0,\quad \frac{\partial T}{\partial z}\left(r,\ 0^+,\ t\right)-\frac{\partial T}{\partial z}\left(r,\ 0^-,\ t\right)=0, \tag{3.3}$$

$$t_{zr}\left(r,0^+,t\right)\ -t_{zr}\left(r,0^-,t\right)\ =0,\qquad t_{zz}\left(r,0^+,t\right)\ -t_{zz}\left(r,0^-,t\right)=-F_0\frac{\delta(r)\delta(t)}{2\pi r}, \tag{3.4}$$

$$m_{z\theta}\left(r,\ 0^+,\ t\right)-m_{z\theta}\left(r,\ 0^-,\ t\right)=0,\quad \lambda_z\left(r,\ 0^+,\ t\right)-\lambda_z\left(r,\ 0^-,\ t\right)=0. \tag{3.5}$$

For $z>0$: Making use of Eqs (2.8) and (2.14) on Eqs (2.5)-(2.7) and $F_0'=\dfrac{F_0}{K}$, we get the stresses in the non-dimensional form with primes. After suppressing the primes and applying the Laplace and Hankel transforms defined by Eqs (2.20) and (2.21) on the resulting equations and using boundary conditions Eqs (3.1)-(3.5), we get the transformed components of displacement, microrotation, scalar microstretch, temperature distribution, tangential force stress, normal force stress, tangential couple stress and microstress for $z>0$, given by

$$\tilde{u}_r\left(\xi,z,p\right)=-\sum_{s=1}^{5} a_s q_s B_{s+5} e^{-q_s z}, \tag{3.6}$$

$$\tilde{u}_z\left(\xi,z,p\right)=\sum_{s=1}^{5} b_s B_{s+5} e^{-q_s z}, \tag{3.7}$$

$$\tilde{\varphi}_\theta\left(\xi,z,p\right)=-\xi\sum_{s=1}^{5}B_{s+5}e^{-q_s z}, \tag{3.8}$$

$$\tilde{\varphi}^*\left(\xi,z,p\right)=\sum_{s=1}^{5}c_s B_{s+5}e^{-q_s z}, \tag{3.9}$$

$$\tilde{T}\left(\xi,z,p\right)=\sum_{s=1}^{5}d_s B_{s+5}e^{-q_s z}, \tag{3.10}$$

$$\tilde{t}_{zr}\left(\xi,z,p\right)=\sum_{s=1}^{5}\left(m_3 a_s q_s^2-\xi\, b_s m_{14}+\xi\, m_4\right)B_{s+5}e^{-q_s z}, \tag{3.11}$$

$$\tilde{t}_{zz}\left(\xi,z,p\right)=-\sum_{s=1}^{5}\left[\xi m_{15}a_s q_s+m_1\, b_s q_s-\, m_{15}c_s+\left(1+\tau_0 p\right)d_s\right]B_{s+5}e^{-q_s z}, \tag{3.12}$$

$$\tilde{m}_{z\theta}\left(\xi,z,p\right)=\xi\, m_{13}\sum_{s=1}^{5}q_s B_{s+5}e^{-q_s z}, \tag{3.13}$$

$$\tilde{\lambda}_z(\xi,z,p)=-m_{16}\sum_{s=1}^{5}c_s q_s B_{s+5}e^{-q_s z}. \tag{3.14}$$

For *z<0*: the above expressions get suitably modified, e.g.,

$$\tilde{u}_r\left(\xi,z,p\right)=\sum_{s=1}^{5}a_s q_s B_s e^{q_s z} \tag{3.15}$$

where

$$m_{13}=\frac{\gamma\omega^{*2}}{\rho C_1^4},\qquad m_{14}=\frac{\mu}{\rho C_1^2},\qquad m_{15}=\frac{\lambda}{\rho C_1^2},\qquad m_{16}=\frac{\alpha\omega^{*2}}{\rho C_1^4}.$$

Making use of the transformed displacement, microrotation, temperature distribution, scalar microstretch and stress components given by Eqs (3.6)-(3.14) in region $z>0$ and equations for the region $z<0$ in the boundary conditions, we obtain ten linear relations between $B_i's$ ($i=1,2,..,10$) which on solving give

$$\begin{aligned}B_1=B_6=\frac{F_0}{4\pi q_1\Delta_1^*}&\Big[\left(a_3-a_1\right)\left\{\left(c_4-c_2\right)\left(d_5-d_2\right)-\left(d_4-d_2\right)\left(c_5-c_2\right)\right\}+\\&+\left(a_4-a_2\right)\left\{\left(c_1-c_2\right)\left(d_3-d_2\right)-\left(d_5-d_2\right)\left(c_3-c_2\right)\right\}+\\&+\left(a_5-a_2\right)\left\{\left(c_3-c_2\right)\left(d_4-d_2\right)-\left(d_3-d_2\right)\left(c_4-c_2\right)\right\}\Big],\end{aligned}$$

$$B_2 = B_7 = \frac{F_0}{4\pi q_2 \Delta_1^*}\Big[(a_3 - a_1)\{(c_5 - c_1)(d_4 - d_1) - (d_5 - d_1)(c_4 - c_1)\} +$$
$$+(a_4 - a_1)\{(c_3 - c_1)(d_5 - d_1) - (d_3 - d_1)(c_5 - c_1)\} +$$
$$+(a_5 - a_1)\{(c_4 - c_1)(d_3 - d_1) - (d_4 - d_1)(c_3 - c_1)\}\Big],$$

$$B_3 = B_8 = \frac{F_0}{4\pi q_3 \Delta_1^*}\Big[(a_2 - a_1)\{(c_5 - c_1)(d_5 - d_1) - (d_4 - d_1)(c_5 - c_1)\} +$$
$$+(a_4 - a_1)\{(c_5 - c_1)(d_2 - d_1) - (d_5 - d_1)(c_2 - c_1)\} +$$
$$+(a_5 - a_1)\{(c_2 - c_1)(d_4 - d_1) - (d_2 - d_1)(c_4 - c_1)\}\Big],$$

$$B_4 = B_9 = \frac{F_0}{4\pi q_4 \Delta_1^*}\Big[(a_2 - a_1)\{(c_5 - c_1)(d_3 - d_1) - (c_3 - c_1)(d_5 - d_1)\} +$$
$$+(a_3 - a_1)\{(c_2 - c_1)(d_5 - d_1) - (c_5 - c_1)(d_2 - d_1)\} +$$
$$+(a_5 - a_1)\{(c_3 - c_1)(d_2 - d_1) - (c_2 - c_1)(d_3 - d_1)\}\Big],$$

$$B_5 = B_{10} = \frac{F_0}{4\pi q_5 \Delta_1^*}\Big[(a_2 - a_1)\{(c_3 - c_1)(d_4 - d_1) - (d_3 - d_1)(c_4 - c_1)\} +$$
$$+(a_3 - a_1)\{(c_4 - c_1)(d_2 - d_1) - (d_4 - d_1)(c_2 - c_1)\} +$$
$$+(a_4 - a_1)\{(c_2 - c_1)(d_3 - d_1) - (d_2 - d_1)(c_3 - c_1)\}\Big]$$

where

$$\Delta_1^* = m_1\Big[(c_2 - c_5)\{a_1(d_3 b_4 - d_4 b_3) + a_3(d_4 b_1 - d_1 b_4) + a_4(d_1 b_3 - d_3 b_1)\} +$$
$$+(c_2 - c_3)\{a_1(d_4 b_5 - d_5 b_4) + a_4(d_5 b_1 - d_1 b_5) + a_5(d_1 b_4 - d_4 b_1)\} +$$
$$+(c_4 - c_5)\{a_1(d_2 b_3 - d_3 b_2) + a_2(d_3 b_1 - d_1 b_3) + a_3(d_1 b_2 - d_2 b_1)\} +$$
$$+(c_2 - c_4)\{a_1(d_5 b_3 - d_3 b_5) + a_3(d_1 b_5 - d_5 b_1) + a_5(d_3 b_1 - d_1 b_3)\} +$$
$$+(c_4 - c_3)\{a_1(d_5 b_2 - d_2 b_5) + a_2(d_1 b_5 - d_5 b_1) + a_5(d_2 b_1 - d_1 b_2)\} +$$
$$+(c_5 - c_3)\{a_1(d_2 b_4 - d_4 b_2) + a_4(d_1 b_2 - d_2 b_1) + a_2(d_4 b_1 - d_1 b_4)\} +$$
$$+(c_3 - c_1)\{a_2(d_4 b_5 - d_5 b_4) + a_4(d_5 b_2 - d_2 b_5) + a_5(d_2 b_4 - d_4 b_2)\} +$$
$$+(c_5 - c_1)\{a_2(d_3 b_4 - d_4 b_3) + a_3(d_4 b_2 - d_2 b_4) + a_4(d_2 b_3 - d_3 b_2)\} +$$
$$+(c_4 - c_1)\{a_2(d_5 b_3 - d_3 b_5) + a_3(d_2 b_5 - d_5 b_2) + a_5(d_3 b_2 - d_2 b_3)\} +$$
$$+(c_2 - c_1)\{a_3(d_5 b_4 - d_4 b_5) + a_4(d_3 b_5 - d_5 b_3) + a_5(d_4 b_3 - d_3 b_4)\}.$$

Thus functions $\tilde{u}_r, \tilde{u}_z, \tilde{\varphi}_\theta, \tilde{T}, \tilde{t}_{zr}, \tilde{t}_{zz}, \tilde{m}_{z\theta}, \tilde{\varphi}^*$ and $\tilde{\lambda}_z$ have been determined in the transform domain and these enable us to find the displacements, microrotation, temperature distribution field, stresses, scalar microstretch and microstress.

4. Method for the inversion of transforms

The transformed solutions are functions of the form $\tilde{f}(\xi, z, p)$ and to get the function $f(r, z, p)$, first we invert the Hankel transform by using

$$\overline{f}(\xi, z, p) = \int_0^\infty \xi \tilde{f}(\xi, z, p) J_n(\xi r) d\xi. \tag{4.1}$$

The expression Eq.(4.1) gives us the Laplace transform $\overline{f}(\xi, z, p)$ of the function $\overline{f}(\xi, z, p)$. Now for the fixed values of r and z the function $\overline{f}(\xi, z, p)$ can be considered as the Laplace transform $\overline{g}(p)$ of some function $g(t)$. Following Honig and Hirdes (1984), the Laplace transformed function $\overline{g}(p)$ can be inverted numerically as given below.

The function $g(t)$ can be obtained from $\overline{g}(p)$ by using the inversion formula

$$g(t) = \frac{1}{2\pi i} \int_{c+\iota\infty}^{c+\iota\infty} e^{pt} \overline{g}(p) d\xi \tag{4.2}$$

where C is an arbitrary real number greater than all the real parts of the singularities of $\overline{g}(p)$. The actual procedure to invert the Laplace transform consists of Eq.(4.2) together with the ε-algorithm. The values of C and L are chosen according to the criteria outlined by Honig and Hirdes (1984).

The last step is to calculate the integral in Eq.(4.1). The method for evaluating this integral is described by Press *et al.* (1986), which involves the use of Romberg's integration with an adaptive step size. It also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

5. Numerical results and discussion

Following Eringen (1984), we take the following values of relevant parameters for a magnesium crystal as

$\lambda = 9.4 \times 10^{10}\ N/m^2$, $\mu = 4 \times 10^{10}\ N/m^2$, $K = 1 \times 10^{10}\ N/m^2$,

$\rho = 1.74 \times 10^3\ kg/m^3$, $\gamma = 0.779 \times 10^{-9}\ N$, $j = 0.2 \times 10^{-19}\ m^2$,

$K^* = 1.1753 \times 10^{-19}\ m^2$, $\omega^* = 0.0787 \times 10^{-1}\ N\sec/m^2$, $\tau_0 = 6.131 \times 10^{-13} s$,

$\tau_1 = 8.765 \times 10^{-13} s$, $\varepsilon = 0.073$, $T_0 = 296K$, $\lambda_0 = 0.5 \times 10^{10}\ N/m^2$,

$\lambda_1 = 0.5 \times 10^{10}\ N/m^2$, $\alpha_0 = 0.779 \times 10^{-9}\ N$, $C^* = 3.525 J\ Kg^{-1} K^{-1}$.

The variations of the non-dimensional normal displacement $U_z \left(= 4\pi u_z / F_0\right)$, non-dimensional normal stress $T_{zz} \left(= 4\pi t_{zz} / F_0\right)$, non-dimensional tangential couple stress $M_{z\theta} \left(= 4\pi m_{z\theta} / F_0\right)$, non-dimensional microstress $\lambda_z^*\left(=4\pi\lambda_z/F_0\right)$ and non-dimensional temperature distribution $T^*\left(=4\pi T/F_0\right)$ with the non-dimensional radial distance '*r*' at the plane *z=1*, $h=10^{-10}\ m$ and the coupling coefficient $\varepsilon=0.073$ have been shown in Figs 2-6 for (a) generalized thermo microstretch elastic (GTMSE) medium (b) generalized thermo micropolar elastic (GTME) medium; (c) generalized thermoelastic (GTE) for time *t=0.1*, *0.125* and *0.5*.

The behaviour of displacement for both theories (L-S and G-L) in all three media (GTMSE, GTMS, GTE) is similar, whereas due to the stretch effect, the value of normal displacement in the GTMSE medium is slightly different as compared to those in the GTME medium for L-S and G-L theories as shown in Fig.2.

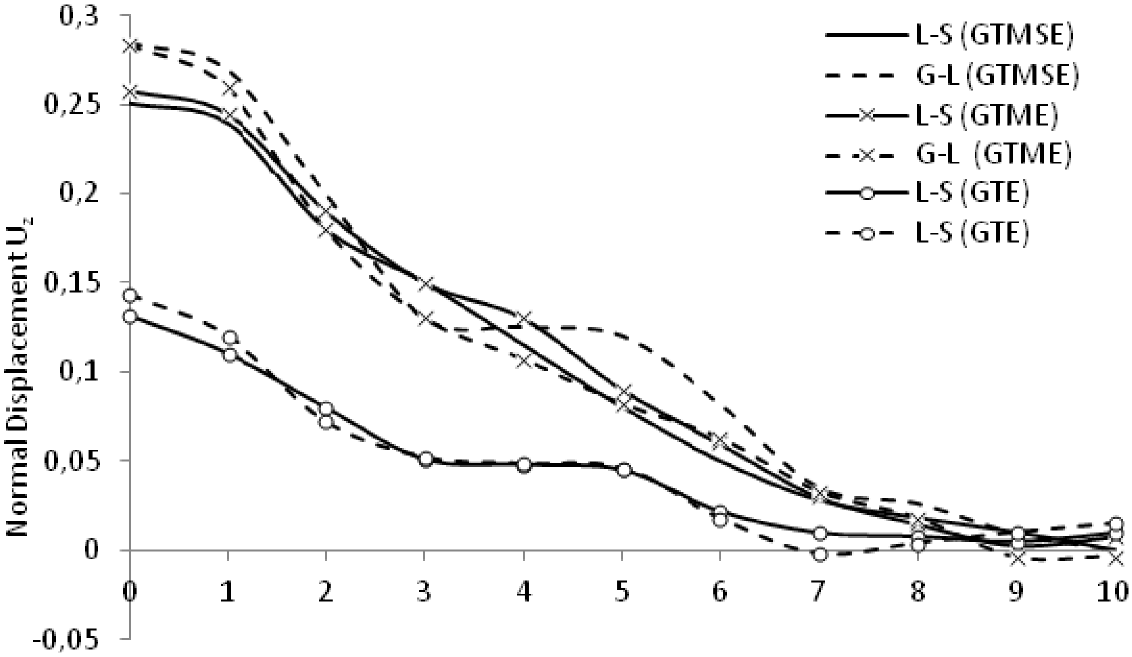

Fig.2. Variations of normal displacement $U_z\left(=4\pi u_z/F_0\right)$.

The value of normal stress in the GTMSE medium is smaller as compared to those in GTME in the ranges *0<r<1.8* and *2.8<r<4.6* but it is larger in the ranges *1.8<r<2.8* and *4.6<r<6*. The value of normal stress in the GTE is very small as compared to those in the GTMSE and GTME medium in the ranges *0<r<1.5* and *3.6<r<4.8*, whereas the reverse happens in other ranges. The distribution of normal stress for both the theories in all the three media has been shown in Fig.3.

The stretch effect on tangential couple stress can be observed in Fig.4, where the value of the tangential couple stress in the GTMSE medium is large in the ranges *0<r<1.5* and *4<r<5.5*; small in the ranges *1.5<r<4* and *5.5<r<6* as compared to those in the GTME medium for both the theories.

The behaviour of microstress in the GTMSE medium is similar for both theories, as shown in Fig.5, whereas the value for the G-L theory is large in comparison with those of the L-S theory in the ranges *0<r<1.75* and *3<r<5*, but is small in the range *1.75<r<3*, while the values are same for both the theories for *r>5*.

The value of the temperature field is large in the GTMSE medium as compared to those in the GTME and GTE media for both theories as depicted in Fig.6, where the value of the temperature field in the GTME medium are multiplied by 10^2 and 10^3 for L-S and G-L theories, respectively and in the GTE medium by 10^2 for both the theories, to show the behavior simultaneously.

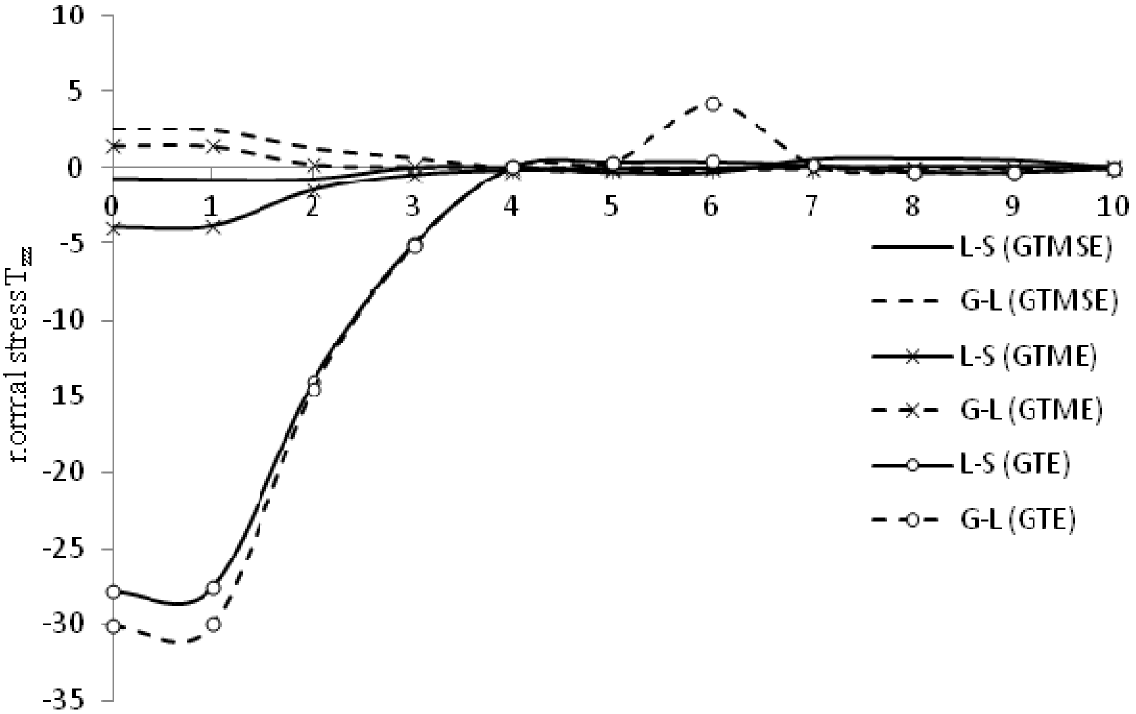

Fig.3. Variations of normal stress $T_{zz} = 4\pi t_{zz}/F_0$.

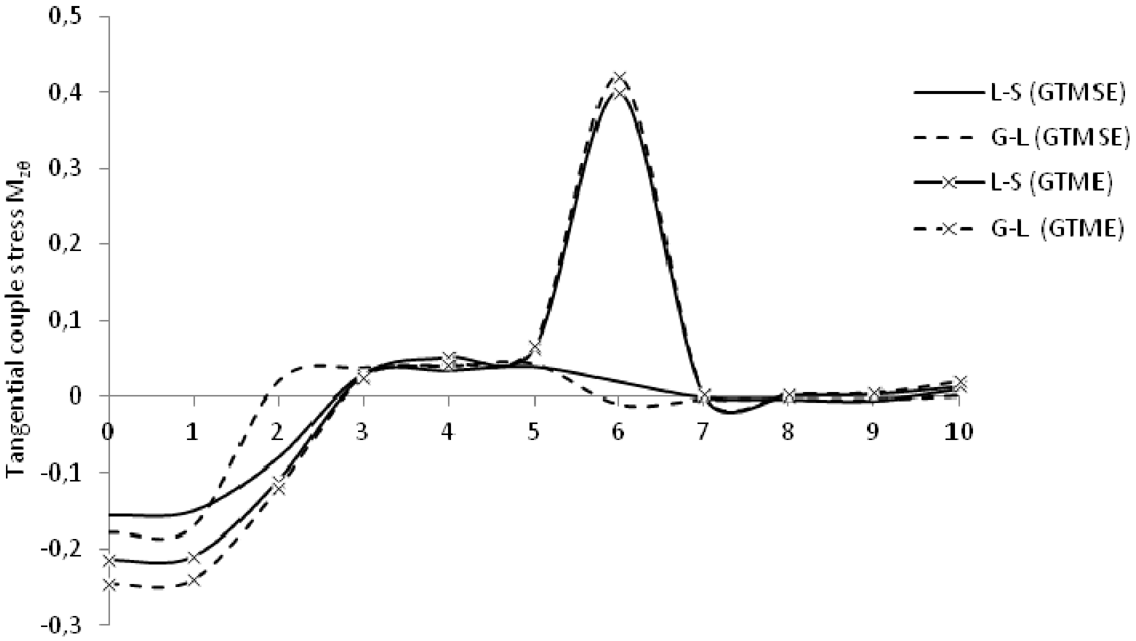

Fig.4. Variations of tangential couple stress $M_{z\theta}\left(= 4\pi m_{z\theta}/F_0\right)$.

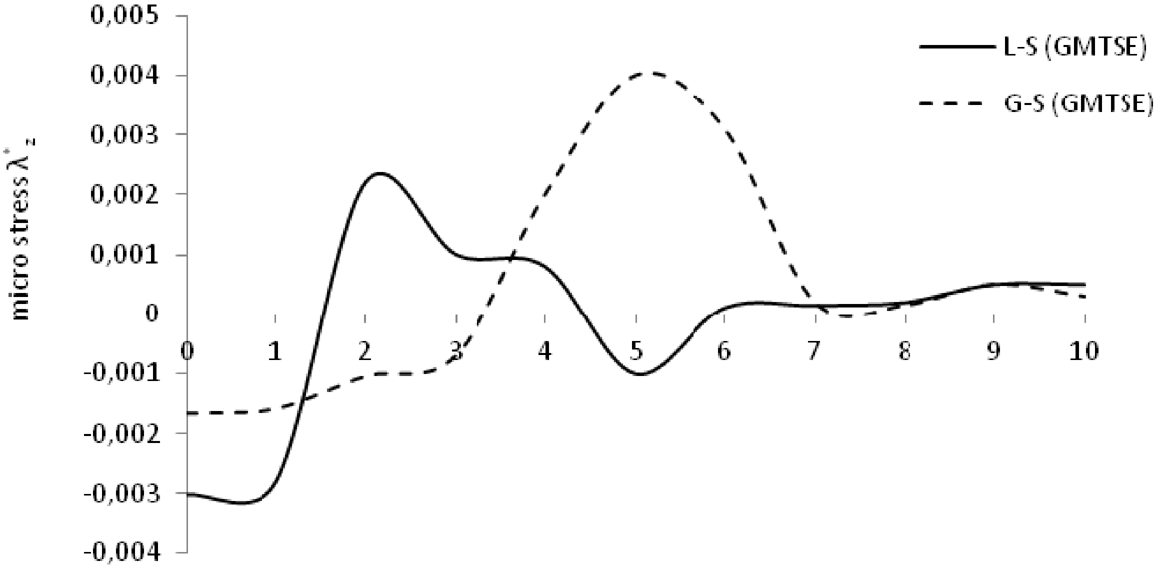

Fig.5. Variations of microstress $\lambda^*_z \left(= 4\pi\lambda_z / F_0\right)$.

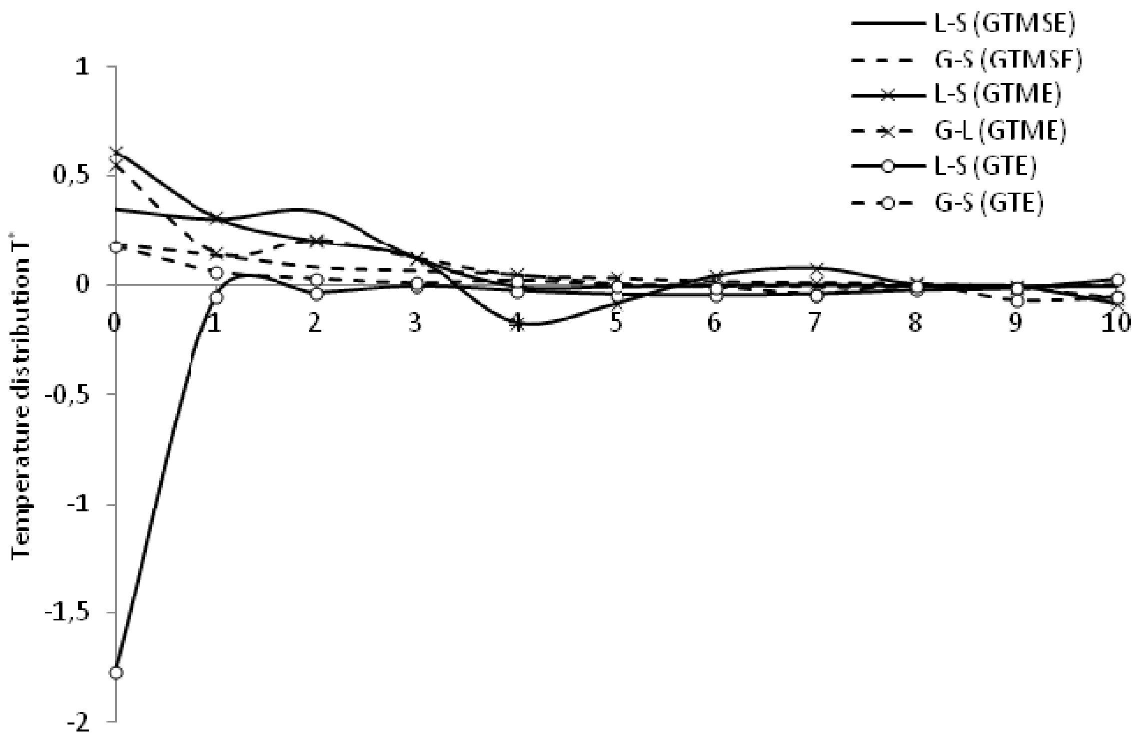

Fig.6. Variations of temperature distribution $T^* \left(= 4\pi T / F_0\right)$.

6. Conclusion

Hence we conclude that the effect of microstretch on displacement, normal stress, tangential couple stress, microstress and temperature distribution depends upon the radial distance r. Also, for a mechanical source this effect is inversely proportional to the radial distance. Using these results, it is possible to investigate the disturbance caused by a more general source for practical applications.

Nomenclature

C^* – specific heat at constant strain
j – micro-inertia
K^* – coefficient of thermal conductivity
m_{ij} – couple stress tensor
T – temperature change
t_{ij} – force stress tensor
$\boldsymbol{u}$ – displacement vector
$\nu,\ \nu_1$ – mechanical and thermal constant
α, β, γ, K – micropolar material constants
$\alpha_{t_1}, \alpha_{t_2}$ – coefficient of linear expansion
Δ – gradient operator
δ_{ij} – Kronecker delta
ε_{ijr} – alternating tensor
λ, μ – Lame's constants
λ_z – microstress component
ρ – density
τ_0 – thermal relaxation time
τ_1 – thermal relaxation time
φ – microrotation vector
φ^* – scalar microstretch

References

Aouadi M. (2008): *Some theorems in the isotropic theory of microstretch thermoelasticity with microtemperatures.* – J. of Thermal Stresses, vol.31, No.7, pp.649-662.

Chandrasekharaiah D.S. and Srinath K.S. (2000): *Thermoelastic wave without energy dissipation in an unbounded body with a spherical cavity*. – Int. J. Math. and Math. Sci., vol.23, pp.555-562.

Ciarletta M. (1999): *A theory of micropolar thermoelasticity without energy dissipation*. – J. Thermal Stresses, vol.22, pp.581-594.

Eringen A.C. (1966): *Linear theory of micropolar elasticity*. – J. Math. Mech., vol.15, pp.909-923.

Eringen A.C. (1970): *Foundation of micropolar thermoelasticity.* – Course of Lectures No.23. Verlag Springer.

Eringen A.C. (1971): *Micropolar elastic solids with stretch*. – Ari. Kitabevi Matbassi, vol.24, pp.1-18.

Eringen A.C. (1984): *Plane waves in non-local micropolar elasticity*.– Int. J. Eng. Sci., vol.22, pp.1113-1121.

Eringen A.C. (1990): *Theory of thermo-microstretch elastic solids*. – Int. J. Eng. Sci., vol.28, pp.1291-1301.

Green A.E. and Lindsay K.A. (1972): *Thermoelasticity*. – J. Elasticity, vol.2, pp.1-7.

Green A.E. and Naghdi P.M. (1993): *Thermoelasticity without energy dissipation*. – J. Elasticity, vol.31, pp.189-208.

Honig G. and Hirdes V. (1984): *A method for the numerical inversion of the Laplace transforms*. – J. of Computational and App. Math., vol.10, pp.113–132.

Iesan D. and Nappa L. (1995): *Extension and bending of microstretch elastic circular cylinders*. – Int. J. Eng. Sci., vol.33, pp.1139-1151.

Kumar R. and Partap G. (2009): *Axisymmetric free vibrations in microstretch thermoelastic homogenous isotropic plate.* – Int. J. of App. Mech. and Eng., vol.14, No.1, pp.211-237.

Lord H.W. and Shulman Y. (1967): *A generalized dynamical theory of thermoelasticity*. – J. Mech. Phys. Solid, vol.15, pp.299-309.

Nowacki W. (1966): *Couple stresses in the theory of thermoelasticity*. – Proc. ITUAM Symposia, Vienna, Springer–Verlag, pp.259-278.

Othman M.I.A., Lotfy Kh. and Farouk R.M. (2010): *Generalized thermo-microstretch elastic medium with temperature dependent properties for different theories*. – Eng. Analysis with Boundary Elements, vol.34, No.3, pp.229-237.

Othman M.I.A., Atwa S.Y., Jahangir A. and Khan A. (2013): *Gravitational effect on plane waves in generalized thermo-microstretch elastic solid under Green Naghdi*. – Appl. Math. Inf. Sci. Lett.1, vol.2, pp.25-38.

Press W.H., Teukolsky and Vellerling (1986): *Numerical Recipes in FORTRAN 2nd Edition.* – Cambridge: Cambridge University Press.

Sharma J.N. and Chand D. (1992): *On the axisymmetric and plane strain problems of generalized thermoelasticity.* – Int. J. Eng. Sci., vol.33, pp.223-230.

Received: October 17, 2013

Revised: January 23, 2014