

# FIRST ORDER CHEMICAL REACTION EFFECTS ON EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE MASS DIFFUSION IN THE PRESENCE OF THERMAL RADIATION

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Effects of transfer of mass and free convection on the flow field of an incompressible viscous fluid past an exponentially accelerated vertical plate with variable surface temperature and mass diffusion are studied. Results for velocity, concentration, temperature are obtained by solving governing equations using the Laplace transform technique. It is observed that the velocity increases with decreasing values of the chemical reaction parameter or radiation parameter. But the trend is just reversed with respect to the time parameter. The skin friction is also studied.

Key words: exponential, radiation, isothermal, vertical plate, heat and mass transfer.

#### 1. Introduction

The effect of a chemical reaction depends on whether the reaction is homogeneous or heterogeneous. This depends on whether the reaction occurs at an interface or as a single phase volume reaction. In well-mixed systems, if it takes place at an interface, the reaction is heterogeneous; and homogeneous if it takes place in solution. Chambre and Young (1958) analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das *et al.* (1994) studied the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on a moving isothermal vertical plate in the presence of a chemical reaction was studied by Das *et al.* (1996).

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircrafts, missiles, satellites and space vehicles. England and Emery (1969) studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. The radiation effect on mixed convection along a isothermal vertical plate was studied by Hossain and Takhar(1996). Das *et al.* (1999) analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

Gupta (1979) studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using the perturbation method. Kafousias and Raptis (1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar (1984). The

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skin friction for accelerated vertical plate was studied analytically by Hossain and Shayo (1986). Jha *et al.* (1991) analyzed mass transfer effects on an exponentially accelerated infinite vertical plate with a constant heat flux and uniform mass diffusion. The first order chemical reaction on an exponentially accelerated vertical plate with mass diffusion was studied by Muthucumaraswamy and Valliammal (2009).

However, the study of thermal radiation effects on unsteady flow past an exponentially accelerated vertical plate in the presence of chemical reaction of first order with variable heat and mass diffusion has not been studied in the literature. It is proposed to study the chemical reaction effects on unsteady flow past an exponentially accelerated vertical plate with variable mass diffusion, in the presence of thermal radiation. Such a study is found useful in energy storage, food processing, freezing.

#### 2. Mathematical analysis

The fluid is assumed to be in the direction of the x'-axis which is taken along the vertical plate in the upward direction. The y'-axis is taken to be normal to the plate. Initially, the temperature of the plate and the fluid is assumed to be the same. Initially, the temperature of the plate is  $T'_{\infty}$  and the concentration level in the fluid is assumed to be  $C'_{\infty}$ . At time t' > 0, the plate temperature is raised to  $T'_{w}$  and the concentration level in the fluid is raised to  $C'_{w}$ .

Inertia terms are negligible as we are considering an infinite vertical plate. There is a first order chemical reaction between the diffusing species and the fluid. Under usual Boussinesq's approximation, the unsteady free convective flow of an incompressible fluid in a dimensional form is given by

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_{\infty}) + g\beta^* (C' - C'_{\infty}) + v \frac{\partial^2 u'}{\partial {y'}^2}, \qquad (2.1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'}, \qquad (2.2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K_l (C' - C'_{\infty}), \qquad (2.3)$$

with the following initial and boundary conditions

$$t' \leq 0: \quad u' = 0, \qquad T' = T_{\infty}, \qquad C = C_{\infty} \text{ for all } y'$$

$$t' > 0: \quad u' = u_0 \exp(a't'), \quad T' = T_{\infty} + (T_W - T_{\infty})At', \quad C = C_{\infty} + (C_W - C_{\infty})At' \quad \text{at } y' = 0 \qquad (2.4)$$

$$u' = 0, \qquad T' \to T_{\infty}, \qquad C \to C_{\infty} \quad \text{as } y' \to \infty, \quad \text{where } A = \frac{u_0^2}{v}.$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_{\mathbf{r}}}{\partial y'} = -4a^* \sigma \left( T_{\infty}'^{4} - T'^{4} \right). \tag{2.5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that  $T'^4$  may be expressed as a linear function of the temperature. This is accomplished by expanding  $T'^4$  in a Taylor series about  $T'_{\infty}$  and neglecting higher-order terms, thus

$$T'^{4} \cong 4T_{\infty}'^{3}T' - 3T_{\infty}'^{4}$$
. (2.6)

By using Eqs (2.5) and (2.6), Eq.(2.2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + 16a^* \sigma T_{\infty}'^3 (T_{\infty}' - T').$$
(2.7)

On introducing the following non-dimensional quantities

$$U = \frac{u'}{u_0}, \quad t = \frac{t'u_0^2}{v}, \quad Y = \frac{y'u_0}{v}, \quad \theta = \frac{T' - T'_{\infty}}{T'_W - T'_{\infty}},$$
  

$$Gr = \frac{g\beta v \left(T'_W - T'_{\infty}\right)}{u_0^3}, \quad C = \frac{C' - C'_{\infty}}{C'_W - C'_{\infty}}, \quad Gc = \frac{vg\beta^* \left(C'_W - C'_{\infty}\right)}{u_0^3},$$
  

$$R = \frac{16a^* v^2 \sigma T'_{\infty}^3}{ku_0^2}, \quad \Pr = \frac{\mu Cp}{k}, \quad Sc = \frac{v}{D}, \quad K = \frac{vK_l}{u_0^2}, \quad a = \frac{a'v}{u_0^2}$$
(2.8)

in Eq.(2.1) to Eqs(2.4), leads to

$$\frac{\partial U}{\partial t} = \operatorname{Gr} \theta + \operatorname{Gc} C + \frac{\partial^2 U}{\partial Y^2}, \qquad (2.9)$$

$$\frac{\partial \theta}{\partial t} = \frac{I}{\Pr} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr} \theta , \qquad (2.10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{\mathrm{Sc}} \frac{\partial^2 C}{\partial Y^2} - K C.$$
(2.11)

The initial and boundary conditions in a non-dimensional form are

 $U = 0, \qquad \theta = 0, \qquad C = 0, \quad \text{for all} \quad Y, t \le 0,$  $t \ge 0: \qquad U = \exp(\text{at}), \quad \theta = t, \qquad C = t, \quad \text{at} \qquad Y = 0,$  $U = 0, \qquad \theta \to 0, \quad C \to 0 \quad \text{as} \qquad Y \to \infty.$  (2.12)

All the physical variables are defined in the nomenclature.

The solutions are obtained for the flow field in the presence of a first order chemical reaction and radiation. Equations (2.9) to (2.11), subject to the boundary conditions (2.12), are solved by the usual Laplace-transform technique and the expressions for temperature, velocity and concentration are as follows

$$\begin{aligned} \theta &= \frac{I}{2} \left[ \exp(2\eta\sqrt{bt} \Pr) \operatorname{erte}(\eta\sqrt{\Pr} + \sqrt{bt}) + \exp(-2\eta\sqrt{bt} \Pr) \operatorname{erte}(\eta\sqrt{\Pr} - \sqrt{bt}) \right] + \\ &= \frac{\eta\sqrt{\Pr}T}{2\sqrt{b}} \left[ \exp(-2\eta\sqrt{bt} \Pr) \operatorname{erte}(\eta\sqrt{\Pr} - \sqrt{bt}) - \exp(2\eta\sqrt{bt} \Pr) \operatorname{erte}(\eta\sqrt{\Pr} + \sqrt{bt}) \right], \end{aligned} \tag{2.13} \end{aligned}$$

$$\begin{aligned} & C &= \frac{I}{2} \left[ \exp(2\eta\sqrt{Kt} \operatorname{Sc}) \operatorname{erte}(\eta\operatorname{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{Kt} \operatorname{Sc}) \operatorname{erte}(\eta\sqrt{\operatorname{Sc}} - \sqrt{Kt}) \right] + \\ &= \frac{\sqrt{Sc}\sqrt{m}}{2\sqrt{K}} \left\{ \exp(-2\eta\sqrt{Kt} \operatorname{Sc}) \operatorname{erte}(\eta\sqrt{\operatorname{Sc}} - \sqrt{Kt}) - \exp(2\eta\sqrt{Kt} \operatorname{Sc}) \operatorname{erte}(\eta\sqrt{\operatorname{Sc}} + \sqrt{Kt}) \right\}, \end{aligned} \tag{2.14} \end{aligned}$$

$$\begin{aligned} & U &= \frac{\exp(at)}{2} \left[ \exp(2\eta\sqrt{at}) \operatorname{erte}(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at}) \operatorname{erte}(\eta - \sqrt{at}) \right] + \\ &+ \left[ \frac{\operatorname{Gr}}{2\sqrt{K}} \left\{ \exp(2\eta\sqrt{at}) \operatorname{erte}(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at}) \operatorname{erte}(\eta - \sqrt{at}) \right] + \\ &+ \left[ \frac{\operatorname{Gr}}{2c^2(1 - \operatorname{Pr})} + \frac{\operatorname{Gc}}{d^2(1 - \operatorname{Sc})} \right] \operatorname{erte}(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at}) \operatorname{erte}(\eta - \sqrt{at}) \right] + \\ &- \frac{\operatorname{Gc}}{2d^2(1 - \operatorname{Pr})} \left\{ \exp(2\eta\sqrt{At}) \operatorname{erte}(\eta\sqrt{\operatorname{Pr}} + \sqrt{bt}) + \exp(-2\eta\sqrt{At}) \operatorname{erte}(\eta - \sqrt{At}) \right] \right\} + \\ &- \frac{\operatorname{Gr}}{2c^2(1 - \operatorname{Pr})} \left\{ \exp(2\eta\sqrt{At}) \operatorname{erte}(\eta\sqrt{\operatorname{Pr}} + \sqrt{bt}) + \exp(-2\eta\sqrt{At}) \operatorname{ertc}(\eta - \sqrt{At}) \right\} \right\} + \\ &- \frac{\operatorname{Gr}}{2c^2(1 - \operatorname{Pr})} \left\{ \exp(2\eta\sqrt{At}) \operatorname{ertc}(\eta\sqrt{\operatorname{Pr}} + \sqrt{bt}) + \exp(-2\eta\sqrt{At}) \operatorname{ertc}(\eta\sqrt{\operatorname{Pr}} + \sqrt{bt}) \right\} \right\} + \\ &+ \frac{\operatorname{Gr}}{2c^2(1 - \operatorname{Pr})} \left\{ \exp(2\eta\sqrt{Rt}) \operatorname{ertc}(\eta\sqrt{\operatorname{Pr}} - \sqrt{bt}) - \exp(2\eta\sqrt{Rt}) \operatorname{ertc}(\eta\sqrt{\operatorname{Pr}} + \sqrt{bt}) \right\} + \\ &+ \frac{\operatorname{Gr}}{2c^2(1 - \operatorname{Pr})} \left\{ \exp(ct) \left[ \exp(2\eta\sqrt{Rt}) \operatorname{ertc}(\eta\sqrt{\operatorname{Pr}} - \sqrt{bt}) - \exp(2\eta\sqrt{Rt}) \operatorname{ertc}(\eta\sqrt{\operatorname{Pr}} + \sqrt{bt}) \right\} \right\} + \\ &+ \frac{\operatorname{Gr}}{2c^2(1 - \operatorname{Pr})} \left\{ \exp(ct) \left[ \exp(2\eta\sqrt{Kt} \operatorname{Sc}) \operatorname{ertc}(\eta\sqrt{\operatorname{Sc}} - \sqrt{Kt}) + \\ &+ \exp(-2\eta\sqrt{Kt} \operatorname{Sc}) \operatorname{ertc}(\eta\sqrt{\operatorname{Sc}} - \sqrt{Kt}) + \\ &+ \exp(-2\eta\sqrt{Kt} \operatorname{Sc}) \operatorname{ertc}(\eta\sqrt{\operatorname{Sc}} - \sqrt{Kt}) \right\} \right\} + \\ &+ \frac{\operatorname{Ge}}{2d^2(1 - \operatorname{Sc})} \left\{ \exp(dt) \left[ \exp(2\eta\sqrt{Kt} \operatorname{Sc}) \operatorname{ertc}(\eta\sqrt{\operatorname{Sc}} - \sqrt{Kt}) \right] + \\ &+ \frac{\operatorname{Ge}}{2d^2(1 - \operatorname{Sc})} \left\{ \exp(dt) \left[ \exp(2\eta\sqrt{Kt} \operatorname{Sc}) \operatorname{ertc}(\eta\sqrt{\operatorname{Sc}} - \sqrt{Kt}) \right] \right\} \right\}$$

complementary error function.

# 3. Skin friction

We now study skin-friction from velocity. It is given in a non-dimensional form as

$$\tau = -\frac{dU}{d\eta}\Big|_{\eta=0}.$$
(3.1)

From Eqs (2.15) and (3.1), we have

$$\begin{split} \tau &= \frac{\exp(at)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{at})\operatorname{erfc}(\eta + \sqrt{at}) +}{+\exp(-2\eta\sqrt{at})\operatorname{erfc}(\eta - \sqrt{at})} \Biggr] + (B+D)\operatorname{erfc}(\eta) + \\ &+ (At+Ct) \Biggl[ (I+2\eta^2)\operatorname{erfc}(\eta) - \frac{2\eta\exp(-\eta^2)}{\sqrt{\pi}} \Biggr] - \frac{B\exp(ct)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{ct})\operatorname{erfc}(\eta + \sqrt{ct}) +}{+\exp(-2\eta\sqrt{ct})\operatorname{erfc}(\eta - \sqrt{ct})} \Biggr] + \\ &- \frac{D\exp(dt)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{dt})\operatorname{erfc}(\eta + \sqrt{dt}) +}{+\exp(-2\eta\sqrt{dt})\operatorname{erfc}(\eta - \sqrt{dt})} \Biggr] - \frac{B(I+ct)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{R})\operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{bt}) +}{+\exp(-2\eta\sqrt{Rt})\operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{bt})} \Biggr] + \\ &+ \frac{\operatorname{APr}\eta\sqrt{t}}{2\sqrt{R}} \Biggl[ \frac{\exp(-2\eta\sqrt{R})\operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{bt}) +}{+\exp(2\eta\sqrt{Rt})\operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{bt})} \Biggr] + \\ &+ \frac{B\exp(ct)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{ScKt})\operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{(b+c)t}) +}{+\exp(-2\eta\sqrt{ScKt})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})} \Biggr] + \\ &- \frac{D(I+dt)}{2} \Biggl[ \exp(2\eta\sqrt{ScKt})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) + \\ &+ \exp(2\eta\sqrt{ScKt})\operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Kt}) + \\ &+ \frac{\exp(ct)}{\sqrt{K}} \Biggl[ \frac{\exp(2\eta\sqrt{ScKt})\operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Kt}) +}{+\exp(-2\eta\sqrt{ScKt})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})} \Biggr] + \\ &+ \frac{D\exp(dt)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{ScKt})\operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Kt}) +}{+\exp(-2\eta\sqrt{ScKt})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})} \Biggr] + \\ &+ \frac{D\exp(dt)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) +}{+\exp(-2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})} \Biggr] + \\ &+ \frac{\exp(ct)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{ScKt})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) +}{+\exp(-2\eta\sqrt{ScKt})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})} \Biggr] + \\ &+ \frac{\exp(ct)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) +}{+\exp(-2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})} \Biggr] + \\ &+ \frac{\exp(ct)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) +}{+\exp(-2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})} \Biggr] + \\ &+ \frac{\exp(ct)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) +}{+\exp(-2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})} \Biggr] + \\ &+ \frac{\exp(ct)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) +}{+\exp(-2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})} \Biggr] + \\ &+ \frac{\exp(ct)}{2} \Biggl[ \frac{\exp(2\eta\sqrt{Sc(K+d)t})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) +}{+\exp(-2\eta\sqrt{Sc(K+d)t})} \Biggr] \Biggr]$$

where 
$$b = \frac{R}{\Pr}$$
,  $c = \frac{R}{l-\Pr}$ ,  $d = \frac{K \operatorname{Sc}}{l-\operatorname{Sc}}$ ,  $A = \frac{\operatorname{Gr}}{c(l-\Pr)}$ ,  $B = \frac{\operatorname{Gr}}{c^2(l-\Pr)}$ ,  $C = \frac{\operatorname{Gc}}{d(l-\operatorname{Sc})}$ ,  $D = \frac{\operatorname{Gc}}{d^2(l-\operatorname{Sc})}$ 

#### 4. Discussion of results

#### 4.1. Analysis of temperature, concentration and velocity profiles

The numerical values of the velocity, temperature and concentration are computed for different physical parameters such as the thermal radiation parameter, chemical reaction parameter, Schmidt number, thermal Grashof number and mass Grashof number. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of the Prandtl number Pr is 0.71 which corresponds to air. The purpose of the calculations given here is to assess the effects of the parameters a, R, K, Gr, Gc and Sc upon the nature of the flow and transport. Skin friction is also calculated for different values of a, R, K, Gr, Gc and Sc.

The temperature profiles for different values of the thermal radiation parameter R = 0.2, 2, 5, 10 in the presence of air at time t = 0.4 are shown in Fig.1. The effect of the thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter. This shows there is a drop in the temperature due to higher thermal radiation.



Fig.1. Temperature profiles for different values of *R*.

Figure 2, demonstrates the effect of temperature for different values of time t = 0.2, 0.4, 0.6, 1. It is observed that the wall concentration increases with increasing values of t.



Fig.2. Temperature profiles for different values of t.

The concentration profiles for different values of the Schmidt number Sc = 0.16, 0.3, 0.6, 2.01 and K = 2 at time t = 0.2 are shown in Fig.3. The effect of the Schmidt number is important in the concentration field. As expected, the concentration increases with decreasing values of the Schmidt number. The numerical values of the Schmidt number and the corresponding species are listed in the following Tab.1

Table	1
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Species	Schmidt number	Name of the chemical species
$H_2$	0.16	Hydrogen
Не	0.3	Helium
$H_2O$	0.6	Water Vapor
$C_6H_5CH_2CH_3$	2.01	Ethyl Benzene



Fig.3. Concentration profiles for different values of Sc.

Figure 4, illustrates the effect of concentration for different values of the chemical reaction parameter K = 0.2, 2, 5, 10 at t = 0.8 The effect of the chemical reaction parameter is important in the concentration field. The profiles have the common feature that the concentration decreases in a monotonic fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with decreasing values of the chemical reaction parameter.

The effect of velocity for different values of the radiation parameter R=0.2, 5, 10, a=0.9, K=20, Gr=Gc=10 and t=0.4 is shown in Fig.5. The trend shows that the velocity increases with decreasing values of the radiation parameter.

Figure 6, illustrates the effect of the velocity for different values of the chemical reaction parameter K = 0.2, 8, 20, R = 10, Gr = 10, Gc = 10, a = 0.9 and t = 0.6. The trend shows that the velocity increases with decreasing values of the chemical reaction parameter.

The velocity profiles for different time t = 0.2, 0.3, 0.4, R = 2, a = 0.9, Gr = Gc = 5 and K = 0.2 are shown in Fig.7. This shows that the velocity increases gradually with respect to time t.

Figure 8 demonstrates the effect of the velocity for different values of thermal Grashof number Gr = 5, 10 and mass Grashof number Gc = 10, 5, K = 10, a = 0.9, R = 0.2 and t = 0.6. It is clear that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

The velocity profiles for different values of a = 0.2, 0.5, 0.9, Gr = Gc = 5 and R = 0.2 at t = 0.2 are studied and presented in Fig.9. It is observed that the velocity increases with increasing values of a.



Fig.4. Concentration profiles for different values of *K*.



Fig.5. Velocity profiles for different values of *R*.



Fig.6. Velocity profile for different values of *K*.



Fig.7. Velocity profiles for different values of *t*.



Fig.9. Velocity profiles for different values of *a*.

#### 4.2. Analysis of skin friction

Figure 10 shows the skin friction for different values of "a" when the thermal Grashof number and mass Grashof number are positive (heating of the plate). The values of skin friction are given for different values of "a" and Gr, Gc when Sc = 0.6, Pr = 0.71. It is observed that skin friction increases for the increasing values of "a".

The skin friction values for different values of Gr, Gc when Sc = 0.6, Pr = 0.71, a = 0.5, R = 2, K = 5 are shown in Fig.11. The skin friction decreases for increasing values of Gr, Gr when Gr, Gc are positive (heating of the plate).

Skin friction values for different values of the Schmidt number (when Pr = 0.71, a = 0) are shown in Fig.12. The observations show that the skin friction increases for increasing values of Sc when the thermal Grashof number and mass Grashof number are positive.

Figure 13 shows the skin friction for different values of the chemical reaction parameter. Skin friction for different values of chemical reaction parameter when Sc = 0.6, Pr = 0.71. It is observed that the skin friction increases for decreasing values of *K* when the thermal Grashof number and mass Grashof number are positive.

Figure 14 shows the skin friction for different values of the Prandtl number. Skin friction for different values of the Prandtl number are given when the Schmidt number is 0.6. It is observed that the skin friction increases for increasing values of the Prandtl number when Gr, Gc are positive.



Fig.10. Skin friction for different values of a.







Fig.12. Skin friction for different values of Sc.



Fig.14. Skin friction for different values of Pr.

### 5. Conclusion

An exact analysis of thermal radiation effects on unsteady flow past an exponentially accelerated infinite vertical plate with variable mass diffusion, in the presence of a chemical reaction of first order has been studied. The dimensionless equations are solved using the Laplace transform technique. The effects of velocity, temperature and concentration for different parameters like a, R, K, Gr, Gc, Sc and t are studied. The following conclusions are drawn:

- (1) The temperature increases with decreasing values of the thermal radiation parameter R and the trend is reversed with respect to time.
- (2) The concentration increases with decreasing values of the Schmidt number and chemical reaction parameter K.
- (3) The velocity increases with decreasing values of the chemical reaction parameter K and thermal radiation parameter R. Also, velocity increases with increasing values of a, time and the thermal and mass Grashof number.
- (4) The skin friction increases with increasing values of the Schmidt number, a and Prandtl number.
- (5) The skin friction increases with decreasing values of the chemical reaction parameter K and thermal and mass Grashof number.

# Nomenclature

- C dimensionless concentration
- C' species concentration in the fluid
- $C_p$  specific heat at constant pressure
- D mass diffusion coefficient
- erfc complementary error function
- Gc mass Grashof number
- Gr thermal Grashof number
- g acceleration due to gravity
- $\tilde{K}$  thermal conductivity
- Pr Prandtl number
- Sc Schmidt number
- T temperature of the fluid near the plate
- t' time
- u velocity of the fluid in the x-direction
- $u_0$  velocity of the plate
- u dimensionless velocity
- Y dimensionless coordinate axis normal to the plate
- y coordinate axis normal to the plate
- $\beta$  volumetric coefficient of thermal expansion
- $\beta^*$  volumetric coefficient of expansion with concentration
- $\theta$  dimensionless temperature
- $\eta$  similarity parameter
- $\mu$  coefficient of viscosity
- v kinematic viscosity
- $\rho$  density of the fluid
- $\tau \quad \text{ dimensionless skin-friction}$

#### Subscripts

- w conditions at the wall
- $\infty$  free stream conditions

#### References

- Chambre P.L. and Young J.D. (1958): On the diffusion of a chemically reactive species in a laminar boundary layer *flow.* The Physics of Fluids, vol.1, pp.48-54.
- Das U.N., Deka R. and Soundalgekar V.M. (1994): *Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction.* Forschung im Ingenieurwesen, vol.60, pp.284-287.
- Das U.N., Deka R. and Soundalgekar V.M. (1996): *Radiation effects on flow past an impulsively started vertical infinite plate.* J. Theo. Mech., vol.1, pp.111-115.
- Das U.N., Deka R. and Soundalgekar V.M. (1999): *Effects of mass transfer on flow past an impulsively started infinite vertical plate with chemical reaction.* The Bulletin, GUMA, vol.5, pp.13-20.
- England W.G. and Emery A.F. (1969): *Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas.* J. Heat Transfer, vol.91, pp.37-44.
- Gupta A.S., Pop I. and Soundalgekar V.M. (1979): Convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid. Rev. Roum. Sci. Techn.-Mec. Apl., vol.24, pp.561-568.
- Hossain M.A. and Shayo L.K. (1986): *The skin friction in the unsteady free convection flow past an accelerated plate.* Astrophysics and Space Science, vol.125, pp.315-324.
- Hossain M.A. and Takhar H.S. (1996): Radiation effect on mixed convection along a vertical plate with uniform surface temperature. Heat and Mass Transfer, vol.31, pp.243-248.
- Jha B.K., Prasad R. and Rai D. (1991): Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux. Astrophysics and Space Science, vol.181, pp.125-134.
- Kafousias N.G. and Raptis A.A. (1981): Mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection. Rev. Roum. Sci. Techn.-Mec. Apl., vol.26, pp.11-2.
- Muthucumaraswamy R. and Valliammal V. (2009): *First order chemical reaction on exponentially accelerated isothermal vertical plate with mass diffusion.* Annals of Faculty of Hunedoara, Journal of Engineering-Tome VII, Fascicule I.
- Singh A.K. and Kumar N. (1984): Free convection flow past an exponentially accelerated vertical plate. Astrophysics and Space Science, vol.98, pp.245-258.

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