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PROPAGATION OF PLANE WAVES IN A ROTATING TRANSVERSELY ISOTROPIC TWO TEMPERATURE GENERALIZED THERMOELASTIC SOLID HALF-SPACE WITH VOIDS

R. BIJARNIA
Government College, Bhuna, Fatehabad
Haryana, INDIA
E-mail: rupeekuk@gmail.com

B. SINGH*
Post Graduate Government College, Sector-11
Chandigarh, INDIA
E-mail: bsinghgc11@gmail.com

The paper is concerned with the propagation of plane waves in a transversely isotropic two temperature generalized thermoelastic solid half-space with voids and rotation. The governing equations are modified in the context of Lord and Shulman theory of generalized thermoelasticity and solved to show the existence of four plane waves in the $x-z$ plane. Reflection of these plane waves from thermally insulated stress free surface is also studied to obtain a system of four non-homogeneous equations. For numerical computations of speed and reflection coefficients, a particular material is modelled as transversely isotropic generalized thermoelastic solid half-space. The speeds of plane waves are computed against the angle of propagation to observe the effects of two temperature and rotation. Reflection coefficients of various reflected waves are also computed against the angle of incidence to observe the effects of various parameters.

Key words: anisotropy, generalized thermoelasticity, voids, plane waves, reflection, reflection coefficients.

1. Introduction

The classical dynamical coupled theory of thermoelasticity was extended to generalized thermoelasticity theories by Lord and Shulman [1] and Green and Lindsay [2]. These theories consider heat propagation as a wave phenomenon rather than a diffusion phenomenon and predict a finite speed of heat propagation. Ignaczak and Ostoja-Starzewski [3] presented the analysis of above two theories in their book on "Thermoelasticity with Finite Wave Speeds". The representative theories in the range of generalized thermoelasticity are reviewed by Hetnarski and Ignaczak [4]. Plane wave propagation in thermoelasticity has many applications in various engineering fields. Problems of wave propagation in in coupled or generalized thermoelasticity have been studied by various researchers [5-12]. Chen and Gurtin [13] and Chen *et al*. [14, 15] formulated a theory of thermoelasticity which depends on two distinct temperatures, the conductive temperature Φ and the thermodynamic temperature T. Boley and Tolins [16] showed that the two temperatures T and Φ, and the strain are represented in the form of a travelling wave plus a response, which occurs instantaneously throughout the body. Warren and Chen [17] studied the wave propagation in the two-temperature theory of coupled thermoelasticity. Puri and Jordan [18] discussed the propagation of harmonic plane waves in the two temperature theory. Youssef [19] formulated a theory of two-temperature generalized thermoelasticity. Kumar and Mukhopadhyay [20] extended the work of Puri and Jordan [18] in the context of

* To whom correspondence should be addressed

the linear theory of two-temperature generalized thermoelasticity formulated by Youssef [19]. The theory of linear elastic materials with voids is an important generalization of the classical theory of elasticity. This theory is used for investigating various types of geological and biological materials for which the classical theory of elasticity is not adequate. The theory of linear elastic materials with voids deals with materials with a distribution of small pores or voids, where the volume of void is included among the kinematics variables. This theory reduces to the classical theory in the limiting case of the volume of void tending to zero. The nonlinear theory of elastic materials with voids was developed by Nunziato and Cowin [21]. Cowin and Nunziato [22] developed a theory of linear elastic materials with voids to study mathematically the mechanical behavior of porous solids. Puri and Cowin [23] studied the behaviour of plane waves in a linear elastic materials with voids.

Iesan [24] developed the linear theory of thermoelastic materials with voids. Ciarletta and Scalia [25] developed the nonlinear theory of non simple thermoelastic materials with voids. Ciarletta and Scarpetta [26] studied some results on thermoelasticity for dielectric materials with voids. Dhaliwal and Wang [27] developed a heat flux dependent theory of thermoelasticity with voids. Marin [28, 29] studied uniqueness and domain of influence results in thermoelastic bodies with voids. Chirita and Scalia [30] studied the spatial and temporal behavior in linear thermoelasticity of materials with voids. Pompei and Scalia [31] studied the asymptotic spatial behavior in linear thermoelasticity of materials with voids. Recently Singh [32, 33] studied the plane wave propagation in an isotropic thermoelastic solid with voids. The present investigation is motivated by the well established theories given by Lord and Shulman [1], Iesan [24] and Dhaliwal and Sherief [34], where we derived the governing equations for homogeneous transversely isotropic rotating generalized thermoelastic solid with voids. These governing equations are then solved for the *x-z* plane to show the existence of four coupled plane waves. The boundary conditions at stress-free thermally insulated surface are solved to obtain a system of four nonhomogeneous equations for the incidence of plane waves. A numerical example is given in the last section to discuss and visualize the dependence of speeds and reflection coefficients of plane waves upon the angle of incidence of striking wave, void and rotation parameters, and other material parameters.

2. Basic equations

Following Lord and Shulman [1], Iesan [24] and Dhaliwal and Sherief [34], the governing equations for an anisotropic two-temperature generalized thermoelastic solid with voids in the absence of body forces and heat sources are

(i) The equation of motion

$$\rho \ddot{u} = \sigma_{ji,j} . \tag{2.1}$$

(ii) The stress-strain-temperature-voids relation

$$\sigma_{ij} = c_{ijkl} e_{kl} + \gamma_{ij}\phi - \beta_{ij} T . \tag{2.2}$$

(iii) The displacement-strain relation

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{2.3}$$

(iv) The energy equation

$$q_{i,i} = \rho T_0 \dot{\eta} . \tag{2.4}$$

(v) The modified Fourier law

$$K_{ij}T_{,j} = q_i + \tau_0 \dot{q}_i . \tag{2.5}$$

(vi) The entropy-strain-temperature-voids relation

$$\rho\eta = \frac{\rho c_E}{T_0} T + \beta_{ij} e_{ij} + m\phi . \tag{2.6}$$

(vii) The equilibrated force balance equation

$$\rho\chi\ddot{\phi} - h_{i,i} = -\gamma_{ij} e_{ij} - \xi\phi + mT . \tag{2.7}$$

(viii) The relation between equilibrated stress and volume fraction gradient

$$h_i = \alpha_{ij}\phi_{,i} . \tag{2.8}$$

(ix) The relation between two temperatures

$$\Phi - T = a^* \Phi_{,ii} . \tag{2.9}$$

3. Analytical 2D solution

We consider a homogeneous and transversely isotropic two temperature thermoelastic rotating medium with voids of an infinite extent with the Cartesian coordinates system (x, y, z), which is previously at uniform temperature. We assume that the medium is transversely isotropic in such a way that the planes of isotropy are perpendicular to the $z-$ axis. The origin is taken on the plane surface and the $z-$ axis is taken normally into the medium $(z \geq 0)$. The surface $z = 0$ is assumed stress free and thermally insulated. The present study is restricted to the plane strain parallel to the $x - z$ plane, with the displacement vector $\boldsymbol{u} = (u_1, 0, u_3)$. With the help of Eqs (2.1)-(2.9), we obtain the following equations in the $x - z$ plane

$$c_{11}u_{1,11} + (c_{13} + c_{44})u_{3,13} + c_{44}u_{1,33} - \beta_1 T_{,1} + \gamma_1 \phi_{,1} = \rho\left\{\ddot{u}_1 - \left(\Omega^2 u_1 - 2\Omega\dot{u}_3\right)\right\}, \tag{3.1}$$

$$c_{44}u_{3,11} + (c_{13} + c_{44})u_{1,13} + c_{33}u_{3,33} - \beta_3 T_{,3} + \gamma_3 \phi_{,3} = \rho\left\{\ddot{u}_3 - \left(\Omega^2 u_3 + 2\Omega\dot{u}_1\right)\right\}, \tag{3.2}$$

$$\begin{aligned} K_1 T_{,11} + K_3 T_{,33} = \rho c_E \left(\dot{T} + \tau_0 \ddot{T}\right) + \beta_1 T_0 \left(\dot{u}_{1,1} + \tau_0 \ddot{u}_{1,1}\right) + \\ + \beta_3 T_0 \left(\dot{u}_{3,3} + \tau_0 \ddot{u}_{3,3}\right) + m T_0 \left(\dot{\phi} + \tau_0 \ddot{\phi}\right), \end{aligned} \tag{3.3}$$

$$\alpha_1 \phi_{,11} + \alpha_3 \phi_{,33} - \xi\phi - \left(\gamma_1 u_{1,1} + \gamma_3 u_{3,3}\right) + mT = \rho\chi\ddot{\phi} , \tag{3.4}$$

$$\Phi - T = a^* \left(\Phi_{,11} + \Phi_{,33}\right) . \tag{3.5}$$

Now, we seek the two dimensional solutions of the Eqs (3.1)-(3.5) in the $x-z$ plane, as

$$\left\{u_1,u_3,\Phi,\phi\right\}=\left\{Ad_1Ad_3,k\bar{\Phi},k\bar{\phi}\right\}e^{ik\left(xp_1+zp_3-Vt\right)} \tag{3.6}$$

where $A,\bar{\Phi},\bar{\phi}$ are arbitrary constants.

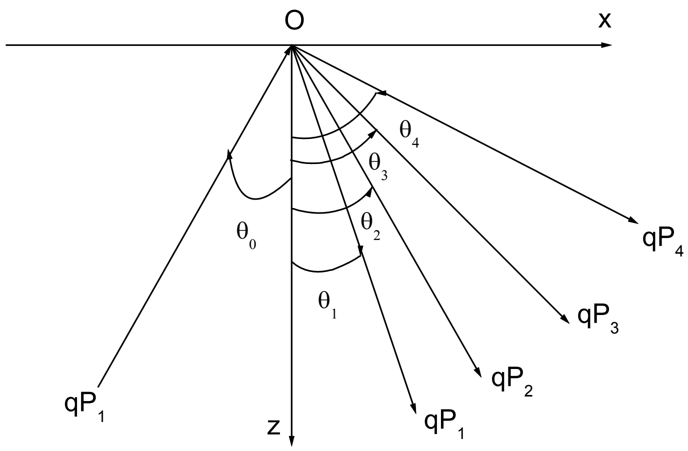

Fig.1. Reflection of plane wave at the free surface.

With the help of Eq.(3.6), Eqs (3.1)-(3.5) admit a non trivial solution if the following biquadratic equation holds

$$L\zeta^4+M\zeta^3+N\zeta^2+O\zeta+P=0 \tag{3.7}$$

where

$$\zeta=\rho V^2,\quad L=\left(1-\bar{\Omega}^2\right)^2,\quad M=-\left(1-\bar{\Omega}^2\right)^2 s_1-\left(1+\bar{\Omega}^2\right)\left\{\delta_1+\delta_3+\varepsilon\left(p_1^2+\bar{\beta}^2p_3^2\right)\right\},$$

$$N=\left(1-\bar{\Omega}^2\right)^2\delta_5\delta_6+\left(1+\bar{\Omega}^2\right)\left\{s_1\left(\delta_1+\delta_3\right)-\varepsilon\left(s_4p_1^2+s_2\bar{\beta}p_3^2\right)+\bar{\gamma}_1\left(s_5p_1^2+s_3\bar{\gamma}p_3^2\right)\right\}+$$
$$+\delta_1\delta_3-\delta_2^2+\varepsilon\left(\delta_3p_1^2+\delta_1\bar{\beta}^2p_3^2-2\delta_2\bar{\beta}p_1p_3\right)+i2\bar{\Omega}p_1p_3\left\{\varepsilon\left(\bar{\beta}s_4-s_2\right)+\bar{\gamma}_1\left(s_3-\bar{\gamma}s_5\right)\right\},$$

$$O=\left(1+\bar{\Omega}^2\right)\left\{-\delta_5\delta_6\left(\delta_1+\delta_3\right)+\delta_5\bar{\gamma}_1\left(p_1^2+\bar{\gamma}^2p_3^2\right)\right\}-s_1\left(\delta_1\delta_3-\delta_2^2\right)+\varepsilon\left\{\delta_3s_4p_1^2+\delta_1\bar{\beta}s_2p_3^2+\right.$$
$$\left.-\delta_2p_1p_3\left(s_2+\bar{\beta}s_4\right)\right\}-\bar{\gamma}_1\left\{\delta_3s_5p_1^2+\delta_1s_3\bar{\gamma}p_3^2-\delta_2p_1p_3\left(s_3+\bar{\gamma}s_5\right)\right\}+$$
$$+i2\bar{\Omega}\bar{\gamma}_1\delta_5p_1p_3\left(1-\bar{\gamma}\right)+\varepsilon\bar{\gamma}_1\left(\bar{\gamma}-\bar{\beta}\right)^2p_1^2p_3^2,$$

$$P=\delta_5\delta_6\left(\delta_1\delta_3-\delta_2^2\right)-\bar{\gamma}_1\delta_5\left\{\delta_3p_1^2+\delta_1\bar{\gamma}^2p_3^2-\delta_2\left(1+\bar{\gamma}\right)p_1p_3\right\},$$

$$s_1=\delta_5+\delta_6-\varepsilon\bar{\gamma}_1\bar{m}^2,\quad s_2=\overline{\gamma\gamma}_1\bar{m}-\bar{\beta}\delta_6,\quad s_3=\varepsilon\bar{\beta}\bar{m}-\bar{\gamma},\quad s_4=\bar{\gamma}_1\bar{m}-\delta_6,\quad s_5=\varepsilon\bar{m}-1,$$

$$\bar{\Omega}=\frac{\Omega}{\omega},\quad \delta_1=c_{11}p_1^2+c_{44}p_3^2,\quad \delta_2=(c_{13}+c_{44})p_1p_3,\quad \delta_3=c_{44}p_1^2+c_{33}p_3^2,$$

$$\delta_4=K_1p_1^2+K_3p_3^2,\quad \delta_5=\delta_4/c_E\tau_0^*b^*,\quad \delta_6=\frac{1}{\chi^*}\left(\alpha_1^*p_1^2+\alpha_3^*p_3^2+\xi\right),$$

$$\alpha_1^*,\alpha_3^*,\chi^*=(\alpha_1,\alpha_3,\chi)k^2,\quad \tau_0^*=\tau_0+i\omega^{-1},\quad \bar{\beta}=\beta_3/\beta_1,\quad \bar{\gamma}=\gamma_3/\gamma_1,$$

$$\bar{m}=\frac{m}{\beta_1\gamma_1},\quad \varepsilon=\frac{\beta_1^2T_0}{\rho c_E},\quad \bar{\gamma}_1=\frac{\gamma_1^2}{\chi^*}.$$

The bi-quadratic Eq.(3.7) with complex coefficients is solved numerically by using Ferrari's method. The roots $\zeta_1=\rho V_1^2$, $\zeta_2=\rho V_2^2$, $\zeta_3=\rho V_3^2$ and $\zeta_4=\rho V_4^2$, correspond to four quasi plane waves, namely, quasi $P_1\,(qP_1)$, quasi $P_2\,(qP_2)$, quasi $P_3\,(qP_3)$ and quasi $P_4\,(qP_4)$ waves. If we write $V_j^{-1}=v_j^{-1}-i\omega^{-1}q_j$ $(j=1,2,.,4)$, then v_j and q_j are speeds of propagation and the attenuation coefficients of the waves.

4. Limiting cases of Eq.(3.7)

Case I: In the absence of rotation $(\bar{\Omega}\to 0)$, Eq.(3.7) reduces to

$$L'\zeta^4+M'\zeta^3+N'\zeta^2+O'\zeta+P'=0 \tag{4.1}$$

where

$$L'=1,\quad M'=-s_1-\left\{\delta_1+\delta_3+\varepsilon\left(p_1^2+\bar{\beta}^2p_3^2\right)\right\},$$

$$N'=\delta_5\delta_6+\left\{s_1(\delta_1+\delta_3)-\varepsilon\left(s_4p_1^2+s_2\bar{\beta}p_3^2\right)+\bar{\gamma}_1\left(s_5p_1^2+s_3\bar{\gamma}p_3^2\right)\right\}+$$
$$+\delta_1\delta_3-\delta_2^2+\varepsilon\left(\delta_3p_1^2+\delta_1\bar{\beta}^2p_3^2-2\delta_2\bar{\beta}p_1p_3\right),$$

$$O'=\left\{-\delta_5\delta_6(\delta_1+\delta_3)+\delta_5\bar{\gamma}_1\left(p_1^2+\bar{\gamma}^2p_3^2\right)\right\}-s_1\left(\delta_1\delta_3-\delta_2^2\right)+\varepsilon\left\{\delta_3s_4p_1^2+\delta_1\bar{\beta}s_2p_3^2+\right.$$
$$\left.-\delta_2p_1p_3\left(s_2+\bar{\beta}s_4\right)\right\}-\bar{\gamma}_1\left\{\delta_3s_5p_1^2+\delta_1s_3\bar{\beta}p_3^2-\delta_2p_1p_3\left(s_3+\bar{\gamma}s_5\right)\right\}+\varepsilon\bar{\gamma}_1\left(\bar{\gamma}-\bar{\beta}\right)^2p_1^2p_3^2,$$

$$P'=\delta_5\delta_6\left(\delta_1\delta_3-\delta_2^2\right)-\bar{\gamma}_1\delta_5\left\{\delta_3p_1^2+\delta_1\bar{\gamma}p_3^2-\delta_2\left(1+\bar{\gamma}\right)p_1p_3\right\}.$$

Case II: In the absence of void parameters ($\bar{\gamma}_1,\bar{\gamma},\bar{m}$ and $\delta_6\to 0$), Eq.(3.7) is

$$L''\zeta^3+M''\zeta^3+N''\zeta^2+O''\zeta+P''=0 \tag{4.2}$$

where

$$L'' = L, \quad M'' = M, \quad N'' = \left(1+\bar{\Omega}^2\right)s_1\left(\delta_1+\delta_3\right)+\delta_1\delta_3-\delta_2^2+\varepsilon\left(\delta_3 p_1^2+\delta_1\bar{\beta}^2 p_3^2-2\delta_2\bar{\beta}p_1p_3\right),$$

$$O'' = -s_1\left(\delta_1\delta_3-\delta_2^2\right), \quad P''=0,$$

$$s_1=\delta_5, \quad s_2=0, \quad s_3=-1, \quad s_4=0, \quad s_5=-1.$$

In the absence of rotation, Eq.(4.2) reduces to the velocity equation as obtained by Singh and Bijarnia [35].

Case III: In the absence of transverse isotropy $c_{11}=c_{33}=\lambda+2\mu, c_{13}=\lambda, c_{44}=\mu$, $\bar{\beta}=1, \bar{\gamma}=1$, $K_1=K_3=K, \alpha_1=\alpha_3=\alpha$, Eq.(3.7) reduces to

$$L'''\zeta^3+M'''\zeta^3+N'''\zeta^2+O'''\zeta+P'''=0 \tag{4.3}$$

where

$$L'''=L, \quad M'''=-\left(1-\bar{\Omega}^2\right)^2 s_1-\left(1+\bar{\Omega}^2\right)\left(\delta_1+\delta_3+\varepsilon\right),$$

$$N'''=\left(1-\bar{\Omega}^2\right)^2\delta_5\delta_6+\left(1+\bar{\Omega}^2\right)\left\{s_1\left(\delta_1+\delta_3\right)-\varepsilon s_2+\bar{\gamma}_1 s_3\right\}+\mu\left(\lambda+2\mu+\varepsilon\right),$$

$$O'''=\left(1+\bar{\Omega}^2\right)\left\{-\delta_5\delta_6\left(\lambda+2\mu\right)+\delta_5\bar{\gamma}_1\right\}-\left\{s_1\left(\lambda+2\mu\right)+s_2\varepsilon-s_3\bar{\gamma}_1\right\}\mu,$$

$$P'''=\delta_5\mu\left\{\delta_6\left(\lambda+2\mu\right)-\bar{\gamma}_1\right\},$$

$$s_2=s_4=\bar{\gamma}_1\bar{m}-\delta_6, \qquad s_3=s_5=\varepsilon\bar{m}-1,$$

$$\delta_1=\left(\lambda+\mu\right)p_1^2+\mu, \quad \delta_2=\left(\lambda+\mu\right)p_1p_3, \quad \delta_3=\left(\lambda+\mu\right)p_3^2+\mu,$$

$$\delta_4=K, \quad \delta_5=K/c_E\tau_0^* b^*, \quad \delta_6=\frac{1}{\chi^*}\left(\alpha^*+\xi\right),$$

$$\alpha^*=\alpha k^2, \qquad \bar{\beta}=1, \quad \bar{\gamma}=1,$$

$$\bar{m}=\frac{m}{\beta\gamma}, \quad \varepsilon=\frac{\beta^2 T_0}{\rho c_E}, \quad \bar{\gamma}_1=\frac{\gamma^2}{\chi^*}.$$

5. Boundary conditions

The boundary conditions at $z=0$ are given by
(i) Vanishing of the normal stress component

$$\sigma_{33}^{(\alpha)} = 0 . \tag{5.1}$$

(ii) Vanishing of the tangential stress component

$$\sigma_{31}^{(\alpha)} = 0 . \tag{5.2}$$

(iii) Vanishing of the normal heat flux component

$$\frac{\partial \Phi^{(\alpha)}}{\partial z} = 0 . \tag{5.3}$$

(iv) Vanishing of the normal equilibrated stress component

$$\frac{\partial \phi^{(\alpha)}}{\partial z} = 0 \tag{5.4}$$

where

$$\sigma_{33}^{(\alpha)} = c_{33} u_{3,3}^{(\alpha)} + c_{13} u_{1,1}^{(\alpha)} - \beta_3 \left\{ \Phi^{(\alpha)} - a^* \left(\Phi_{,11}^{(\alpha)} + \Phi_{,33}^{(\alpha)} \right) \right\} - \gamma_3 \varphi^{(\alpha)} , \tag{5.5}$$

$$\sigma_{31}^{(\alpha)} = c_{44} \left(u_{1,3}^{(\alpha)} + u_{3,1}^{(\alpha)} \right) . \tag{5.6}$$

In view of Eq.(3.6), the appropriate solutions satisfying the boundary conditions (5.1)-(5.4) in haf-space $z \geq 0$ are

$$\left\{ u_1^{(\alpha)}, u_3^{(\alpha)}, \Phi^{(\alpha)}, \phi^{(\alpha)} \right\} = \left\{ d_1^{(\alpha)}, d_3^{(\alpha)}, k_\alpha F^{(\alpha)}, k_\alpha G^{(\alpha)} \right\} A^{(\alpha)} e^{ik_\alpha \left(x p_1^{(\alpha)} + z p_3^{(\alpha)} - v_\alpha t \right)} \tag{5.7}$$

where

$$F^{(\alpha)} = -\frac{\left[\rho v_\alpha^2 \left\{ \left(1 + \bar{\Omega}^2 \right) A_{1\bar{\gamma}}^{(\alpha)} + i 2 \bar{\Omega} A_{2\bar{\gamma}}^{(\alpha)} \right\} - A_{3\bar{\gamma}}^{(\alpha)} - \delta_2^{(\alpha)} A_{4\bar{\gamma}}^{(\alpha)} \right]}{\left(\beta_1 \bar{\gamma} - \beta_3 \right) b^{*(\alpha)} p_1^{(\alpha)} p_3^{(\alpha)}} ,$$

$$G^{(\alpha)} = \frac{\left[\rho v_\alpha^2 \left\{ \left(1 + \bar{\Omega}^2 \right) A_{1\bar{\beta}}^{(\alpha)} + i 2 \bar{\Omega} A_{2\bar{\beta}}^{(\alpha)} \right\} - A_{3\bar{\beta}}^{(\alpha)} - \delta_2^{(\alpha)} A_{4\bar{\beta}}^{(\alpha)} \right]}{\left(\gamma_1 \bar{\beta} - \gamma_3 \right) p_1^{(\alpha)} p_3^{(\alpha)}} ,$$

$$A_{1\bar{\gamma}}^{(\alpha)} = \bar{\gamma} d_1^{(\alpha)} p_3^{(\alpha)} - d_3^{(\alpha)} p_1^{(\alpha)} , \quad A_{2\bar{\gamma}}^{(\alpha)} = d_1^{(\alpha)} p_1^{(\alpha)} + \bar{\gamma} d_3^{(\alpha)} p_3^{(\alpha)} ,$$

$$A_{3\bar{\gamma}}^{(\alpha)} = \bar{\gamma} \delta_1^{(\alpha)} d_1^{(\alpha)} p_3^{(\alpha)} - \delta_3^{(\alpha)} d_3^{(\alpha)} p_1^{(\alpha)} , \quad A_{4\bar{\gamma}}^{(\alpha)} = \bar{\gamma} d_3^{(\alpha)} p_3^{(\alpha)} - d_1^{(\alpha)} p_1^{(\alpha)} ,$$

$$A_{1\beta}^{(\alpha)} = \bar{\beta} d_1^{(\alpha)} p_3^{(\alpha)} - d_3^{(\alpha)} p_1^{(\alpha)}, \quad A_{2\bar{\beta}}^{(\alpha)} = d_1^{(\alpha)} p_1^{(\alpha)} + \bar{\beta} d_3^{(\alpha)} p_3^{(\alpha)},$$

$$A_{3\bar{\beta}}^{(\alpha)} = \bar{\beta} \delta_1^{(\alpha)} d_1^{(\alpha)} p_3^{(\alpha)} - \delta_3^{(\alpha)} d_3^{(\alpha)} p_1^{(\alpha)}, \quad A_{4\bar{\beta}}^{(\alpha)} = \bar{\beta} d_3^{(\alpha)} p_3^{(\alpha)} - d_1^{(\alpha)} p_1^{(\alpha)} .$$

The extension of Snell's law is

$$k_0 p_1^{(0)} = k_1 p_1^{(1)} = k_2 p_1^{(2)} = k_3 p_1^{(3)} = k_4 p_1^{(4)}, \tag{5.8}$$

and

$$k_0 v_0 = k_1 v_1 = k_2 v_2 = k_3 v_3 = k_4 v_4 . \tag{5.9}$$

From the above equations, we can have

$$\frac{p_1^{(0)}}{v_0} = \frac{p_1^{(1)}}{v_1} = \frac{p_1^{(2)}}{v_2} = \frac{p_1^{(3)}}{v_3} = \frac{p_1^{(4)}}{v_4}. \tag{5.10}$$

The appropriate displacement components, temperature and concentration variables given by Eq.(5.7) satisfy the boundary conditions (5.1)-(5.4), if the extension of Snell's law given by Eqs (5.8)-(5.10) holds and we obtain the following system of four non-homogeneous equations with reflection coefficients

$$\sum_{j=1}^{4} a_{ij} Z_j = b_i \quad \left(i = 1,2,..,4\right) \tag{5.11}$$

where

$$a_{1j} = \left\{\left(c_{13} p_1^{(j)} d_1^{(j)} + c_{33} p_3^{(j)} d_3^{(j)}\right) - \beta_3 b^{*(j)} F^{(j)} + \gamma_3 G^{(j)}\right\} k_j ,$$

$$a_{2j} = c_{44}\left(p_3^{(j)} d_1^{(j)} + p_1^{(j)} d_3^{(j)}\right) k_j ,$$

$$a_{3j} = p_3^{(j)} F^{(j)} k_j^2 ,$$

$$a_{4j} = p_3^{(j)} G^{(j)} k_j^2 ,$$

$$b_1 = -\left\{\left(c_{13} p_1^{(0)} d_1^{(0)} + c_{33} p_3^{(0)} d_3^{(0)}\right) - \beta_3 b^{*(0)} F^{(0)} + \gamma_3 G^{(0)}\right\} k_0 ,$$

$$b_2 = -c_{44}\left(p_3^{(0)} d_1^{(0)} + p_1^{(0)} d_3^{(0)}\right) k_0 ,$$

$$b_3 = -p_3^{(0)} F^{(0)} k_0^2 ,$$

$$b_4 = -p_3^{(0)} G^{(0)} k_0^2 ,$$

and

$$Z_1 = \frac{A^{(1)}}{A^{(0)}}, \quad Z_2 = \frac{A^{(2)}}{A^{(0)}}, \quad Z_3 = \frac{A^{(3)}}{A^{(0)}}, \quad Z_4 = \frac{A^{(4)}}{A^{(0)}} ,$$

are the reflection coefficients of reflected qP_1, qP_2, qP_3 and qP_4 waves, respectively.

6. Numerical results and discussion

To study the effects of transverse isotropy and void parameters numerically on the speeds of propagation, and reflection coefficients, we consider the following physical constants of a single crystal of zinc modelled as a generalized anisotropic thermoelastic solid [36].

$$c_{11} = 1.628 \times 10^{11}\, Nm^{-2}, \qquad c_{33} = 1.562 \times 10^{11}\, Nm^{-2} ,$$

$$c_{13} = 0.508 \times 10^{11}\, Nm^{-2}, \qquad c_{44} = 0.385 \times 10^{11}\, Nm^{-2} ,$$

$$\beta_1 = 5.75 \times 10^{6}\, Nm^{-2} deg^{-1}, \qquad \beta_3 = 5.17 \times 10^{6}\, Nm^{-2} deg^{-1} ,$$

$$K_1 = 1.24 \times 10^{2}\, Wm^{-1} deg^{-1}, \qquad K_3 = 1.34 \times 10^{2}\, Wm^{-1} deg^{-1} ,$$

$$C_e = 3.9 \times 10^{2}\, JKg^{-1} deg^{-1}, \qquad \rho = 7.14 \times 10^{3}\, Kgm^{-3} ,$$

$$T_0 = 296 K, \quad \tau_0 = 0.05 \times 10^{-11} s, \quad \omega = 2.$$

with the following void parameters

$$\chi = 1.753 \times 10^{-15} m^2, \quad \alpha_1 = 3.688 \times 10^{-5} N, \quad \alpha_3 = 3.656 \times 10^{-5} N, \quad \xi = 1.475 \times 10^{10}\, Nm^{-2} ,$$

$$\gamma_1 = 1.13849 \times 10^{10}\, Nm^{-2}, \quad \gamma_3 = 1.11349 \times 10^{10}\, Nm^{-2}, \quad m = 2 \times 10^{6}\, Nm^{-2} deg^{-2} ,$$

and with the following components of propagation and unit displacement vectors

$$p_1^{(0)} = \sin\theta_0, \quad p_3^{(0)} = \cos\theta_0, \quad d_1^{(0)} = \cos\theta_0, \quad d_3^{(0)} = -\sin\theta_0 ,$$

$$p_1^{(1)} = \sin\theta_1, \quad p_3^{(1)} = -\cos\theta_1, \quad d_1^{(1)} = \cos\theta_1, \quad d_3^{(1)} = \sin\theta_1 ,$$

$$p_1^{(2)} = \sin\theta_2, \quad p_3^{(2)} = -\cos\theta_2, \quad d_1^{(2)} = \cos\theta_2, \quad d_3^{(2)} = \sin\theta_2 ,$$

$$p_1^{(3)} = \sin\theta_3, \quad p_3^{(3)} = -\cos\theta_3, \quad d_1^{(3)} = \cos\theta_3, \quad d_3^{(3)} = \sin\theta_3,$$

$$p_1^{(4)} = \sin\theta_4, \quad p_3^{(4)} = -\cos\theta_4, \quad d_1^{(4)} = \cos\theta_4, \quad d_3^{(4)} = \sin\theta_4.$$

The bi-quadratic Eq.(3.7) is solved numerically by Ferrari's method to compute the real values of propagation speeds of qP_1, qP_2, qP_3 and qP_4 waves after using the relation $V_j^{-1} = v_j^{-1} - i\omega^{-1}q_j$, $j = 1,..,4$. The speeds of qP_1, qP_2, qP_3 and qP_4 waves are shown graphically against the angle of propagation in Figs 2 and 3. The speeds of qP_1, qP_2, qP_3 and qP_4 waves are computed for the range $0^o \le \theta^o \le 90^o$ of angle of propagation when $\Omega/\omega = 0,5,10$ and when $a^* = 0.5$. For $\Omega/\omega = 0$, the speed of qP_1 wave is $5.0206 \times 10^5\ ms^{-1}$ at $\theta^o = 0^o$. It increases slowly to its maximum value $5.6067 \times 10^5\ ms^{-1}$ at $\theta^o = 90^o$. The variation is shown in Fig.2a by a solid line with circles. This variation reduces to solid line with triangle for $\Omega/\omega = 5$ and to a solid line with stars for $\Omega/\omega = 10$. The increase in value of rotation rate decreases the speed of qP_1 wave. For $\Omega/\omega = 0$, the speed of qP_2 wave is $1.7411 \times 10^5\ ms^{-1}$ at $\theta^o = 0^o$. It increases slowly to its maximum value $1.74388 \times 10^5\ ms^{-1}$ at $\theta^o = 38^o$. Thereafter, it decreases slowly to its minimum value $1.73840 \times 10^5\ ms^{-1}$ at $\theta^o = 90^o$. The variation is shown in Fig.2b by a solid line with circles. This variation reduces to a solid line with a triangle for $\Omega/\omega = 5$ and to a solid line with stars for $\Omega/\omega = 10$. The increase in the value of rotation rate decreases the speed of qP_2 wave. For $\Omega/\omega = 0$, the speeds of qP_3 and qP_4 waves are $0.62047 \times 10^5\ ms^{-1}$ and $0.11354 \times 10^5\ ms^{-1}$ at $\theta^o = 0^o$. Thereafter, the speed of qP_3 wave increases to its maximum value $0.73768 \times 10^5\ ms^{-1}$ at $\theta^o = 43^o$ and then decreases. The speed of qP_4 wave decreases to its minimum value $0.09879 \times 10^5\ ms^{-1}$ at $\theta^o = 57^o$ and then increases slowly. These variations of speeds of qP_3 and qP_4 waves are shown graphically in Figs 2c and 2d. The increase in the value of rotation rate affects the speeds of these waves significantly at each angle of propagation.

The speeds of qP_1, qP_2, qP_3 and qP_4 waves are computed for the range $0^o \le \theta^o \le 90^o$ of the angle of propagation when $a^* = 0, 0.5, 1.0$ and when $\Omega/\omega = 10$. From a comparison of curves in Fig.3, it is observed that the speeds of qP_1, qP_2, qP_3 and qP_4 waves are affected significantly by the two-temperature parameter, where the effect of the two-temperature parameter is observed least on qP_2 wave.

The variations of the reflection coefficients (amplitude ratios) $|Z_1|, |Z_2|, |Z_3|, |Z_4|$ are shown graphically in Fig.4 against the angle of incidence, when $\Omega/\omega = 0,5,10$ and when $a^* = 0.5$. The increase in the value of rotation rate changes the amplitude ratios of reflected waves at each angle of incidence. The effect of rotation is observed maximum near the normal incidence.

The variations of the amplitude ratios $|Z_1|, |Z_2|, |Z_3|, |Z_4|$ are shown graphically in Figure 5 against the angle of incidence, when $a^* = 0, 0.5, 1.0$ and when $\Omega/\omega = 10$. From a comparison of different curves in Fig.5, it is observed that the increase in the value of the two-temperature parameter changes the amplitude ratios of reflected waves at each angle of incidence. The effect of the two-temperature parameter is different for different reflected waves at each angle of incidence. For example, the effect of the two-temperature parameter is observed least on the amplitude ratio of reflected qP_1 wave.

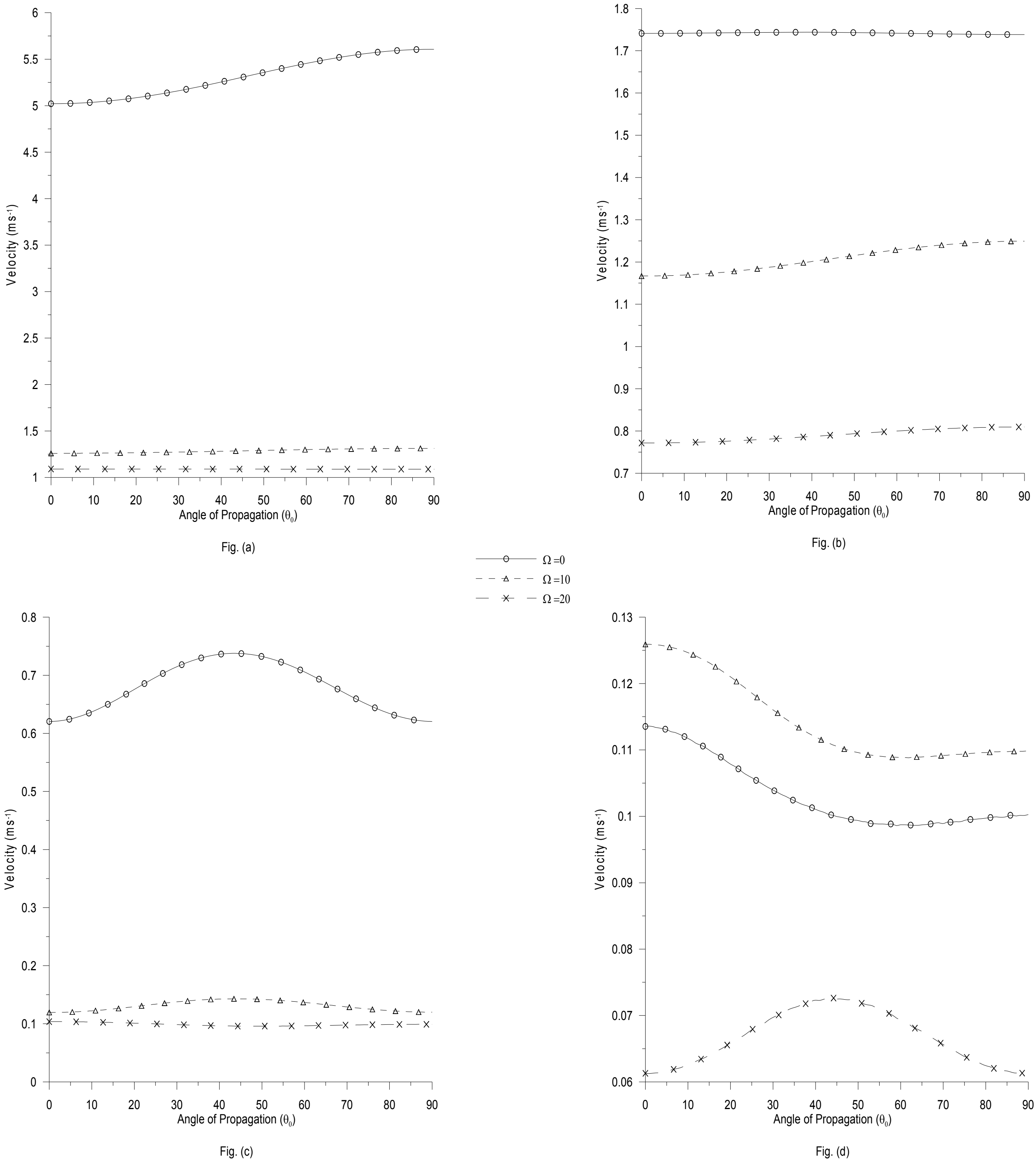

Fig.2. Variations of the speeds of qP_1, qP_2, qP_3, qP_4 with the angle of propagation, when $\Omega = 0, 10, 20$.

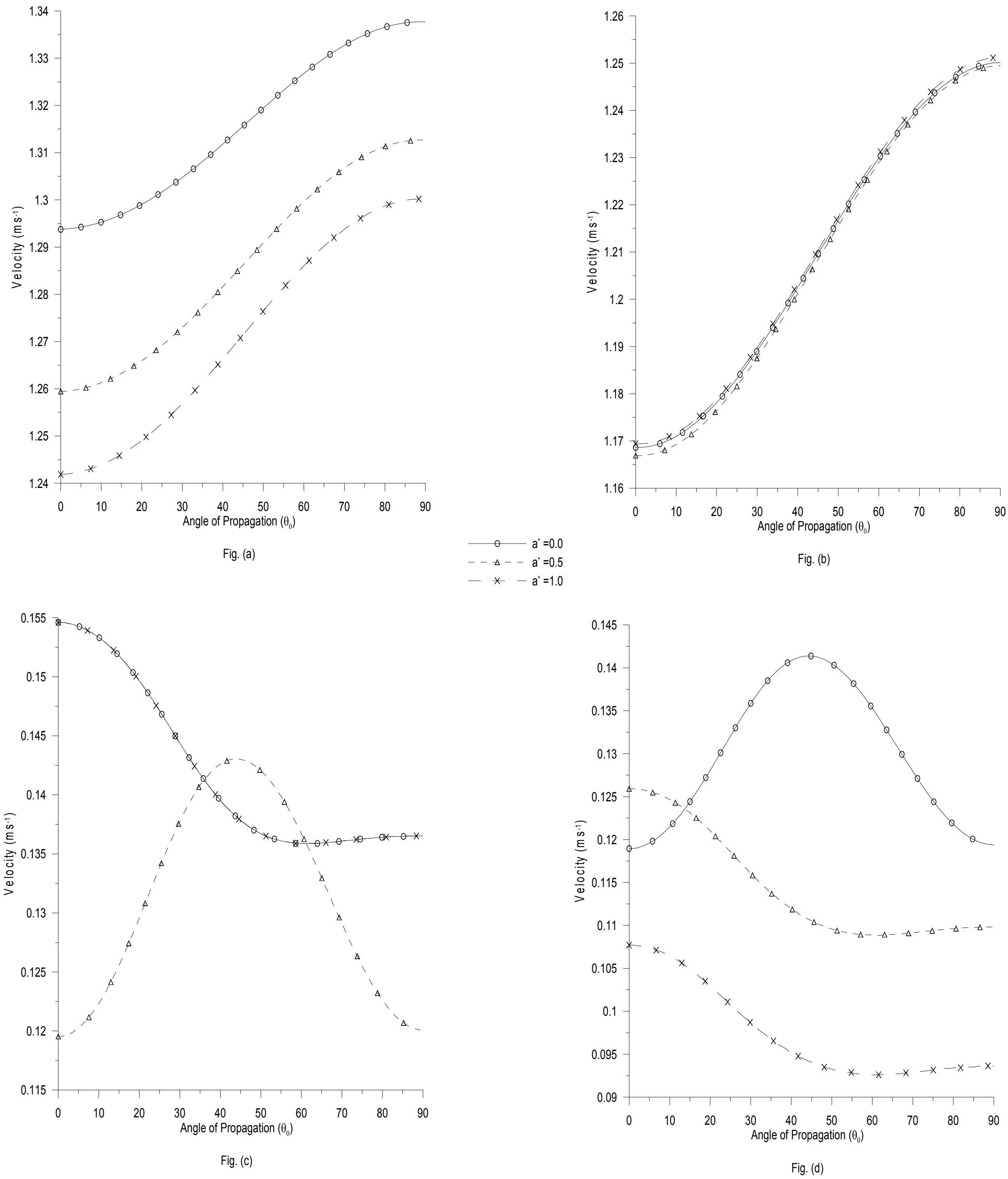

Fig.3. Variations of the speeds of qP_1, qP_2, qP_3, qP_4 with the angle of propagation, when $a^* = 0.0, 0.5, 1.0$.

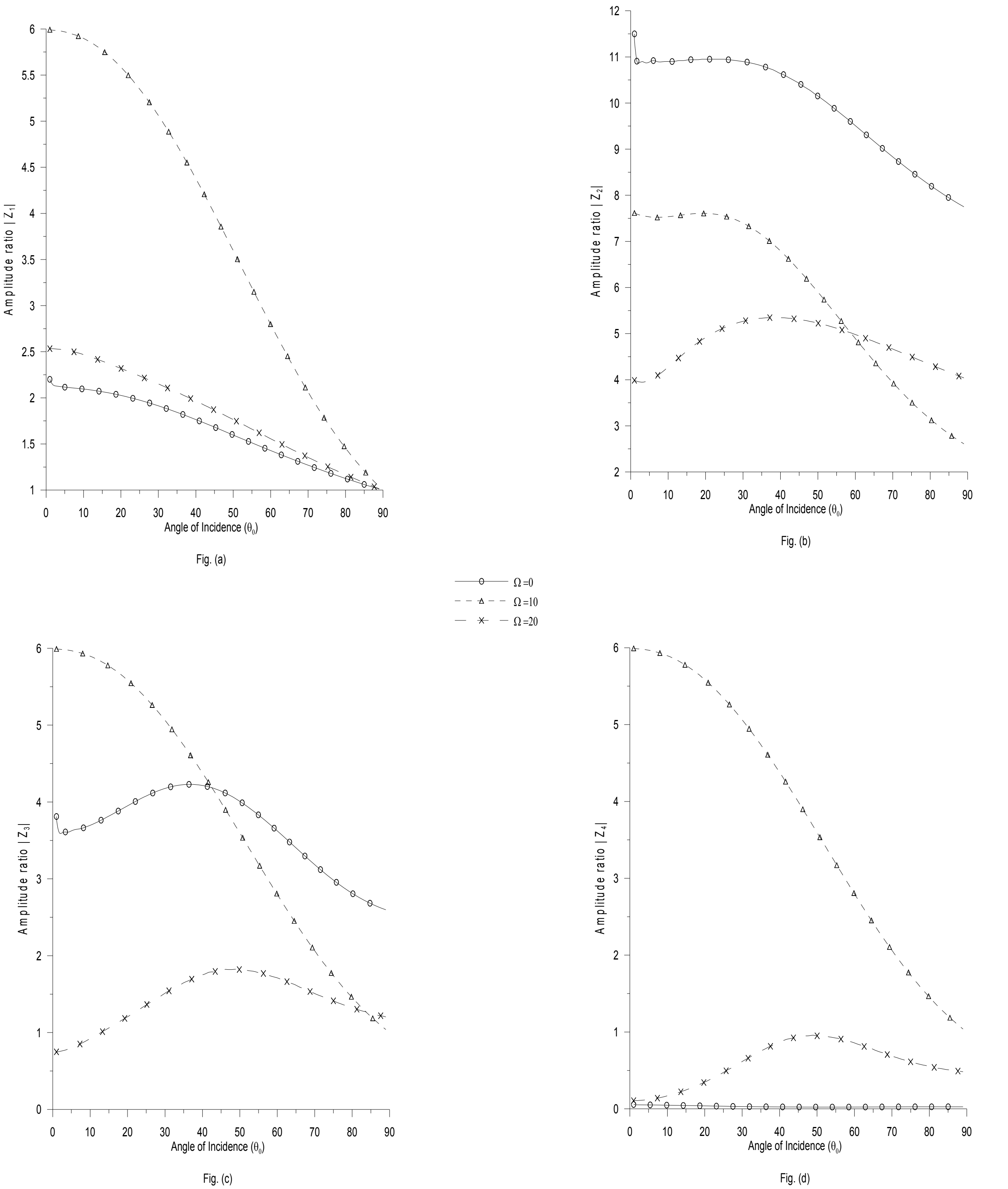

Fig.4. Variations of the amplitude ratios $|Z_1|,|Z_2|,|Z_3|,|Z_4|$ with the angle of incidence, when $\Omega = 0,10,20$.

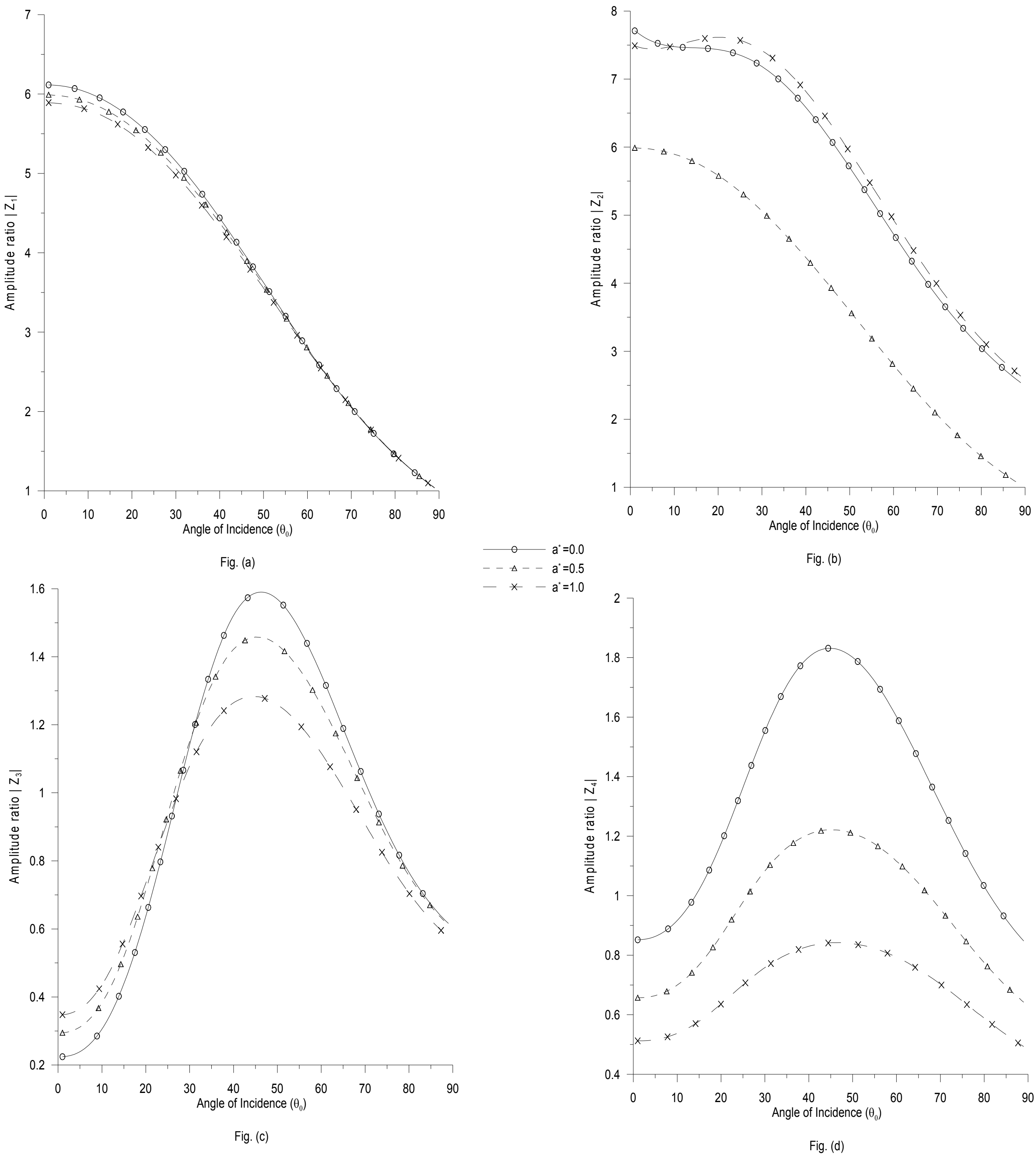

Fig.5. Variations of the amplitude ratios $|Z_1|, |Z_2|, |Z_3|, |Z_4|$ with the angle of incidence, when $a^* = 0.0, 0.5, 1.0$.

7. Conclusion

Two-dimensional plane wave propagation in a rotating two-temperature transversely isotropic thermoelastic solid with voids indicates the existence of four quasi plane waves, namely, qP_1, qP_2, qP_3 and qP_4 waves. Relations between reflection coefficients of various reflected waves are obtained for the incidence of qP_1 wave. The speeds and reflection coefficients of plane waves are computed for a particular material. The speeds and reflection coefficients of plane waves are shown graphically against the angle of propagation for different values of the two-temperature parameter and rotation rate. It is observed that the speeds and amplitude ratios are affected significantly by the presence of rotation and two-temperature in the thermoelastic solid.

Nomenclature

a^* – two-temperature paramater
a_{ij} , b_{ij} – constitutive coefficients
c_E – specific heat at constant strain
c_{ijkl} – tensor of elastic constants
d_1, d_3 – components of displacement vector
e_{ij} – components of the strain tensor
g – intrinsic equilibrated body force
h_i – components of equilibrated stress vector
K_{ij} – thermal conductivity tensor
k – wave number
p_1, p_3 – components of propagation vector
q_i – heat conduction vector
$T = T^* - T_0$ – small temperature increment
T^* – absolute temperature of the medium
T_0 – reference uniform temperature of the body chosen such that $|\theta / T_0| = 1$.
u_i – components of the displacement vector
V – phase velocity
η – entropy per unit mass
ν – volume fractional field
ν_0 – volume distribution function for reference configuration
ρ – mass density
σ_{ij} – components of the stress tensor
τ_0 – relaxation time
$\phi = \nu - \nu_0$, ϕ – change in volume fraction field
χ – equilibrated inertia
Ω – rotation

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