LINEAR STABILITY EFFECT OF DENSELY DISTRIBUTED POROUS MEDIUM AND CORIOLIS FORCE ON SORET DRIVEN FERROTHERMOHALINE CONVECTION

R. SEKAR[*] and D. MURUGAN
Department of Mathematics, Pondicherry Engineering College
Puducherry - 605 014, INDIA
E-mail: rsekar@pec.edu

The effect of Coriolis force on the Soret driven ferrothermohaline convection in a densely packed porous medium has been studied. A linear stability analysis is carried out using normal mode technique. It is found that stationary convection is favorable for the Darcy model, therefore oscillatory instability is studied. A small thermal perturbation is applied to the basic state and linear stability analysis is used for which the normal mode technique is applied. It is found that the presence of a porous medium favours the onset of convection. The porous medium is assumed to be variable and the effect of the permeable parameter is to destabilize the system. The present work has been carried out both for oscillatory as well as stationary instabilities. The results are depicted graphically.

Key words: Coriolis force, Darcy model, Soret effect, ferrofluid, thermohaline convection.

1. Introduction

A ferrofluid (compound of Latin ferrum, meaning iron, and fluid) is a liquid which becomes strongly magnetized in the presence of a magnetic field. A ferrofluid is a combination of microscopic magnetic particles suspended in a liquid, usually a solvent or water. Ferrofluids are used in loudspeakers to cool the voice coil and reside in the air gap between the voice coil and the speaker's magnet.

Ferrofluids are colloidal liquids made of nanoscale ferromagnetic, or ferromagnetic, particles suspended in a carrier fluid-usually an organic solvent or water. Each tiny particle is thoroughly coated with a surfactant to inhibit clumping. Large ferromagnetic particles can be ripped out of the homogeneous colloidal mixture, forming a separate clump of magnetic dust when exposed to strong magnetic fields. The magnetic attraction of nanoparticles is weak enough so the surfactant's van der Waals repulsion is sufficient to prevent magnetic clumping or agglomeration. Ferrofluids usually do not retain magnetization in the absence of an externally applied field and thus are often classified as "superparamagnets" rather than ferromagnets.

The difference between ferrofluids and magnetorheological fluids (MR fluids) is the size of the particles. The particles in a ferrofluid primarily consist of nanoparticles which are suspended by Brownian motion and generally will not settle under normal conditions. MR fluid particles primarily consist of micrometer-scale particles which are too heavy for Brownian motion to keep them suspended, and thus will settle over time because of the inherent density difference between the particle and its carrier fluid. These two fluids have very different applications as a result.

In ferrofluids, magnetic and liquid states coexist. On the surface, they are deceptively simple systems: a homogeneous phase comprised of three constituents, namely the magnetic particles, surfactant, and the liquid carrier. However, a complex chemistry and a balance of inter-particle forces determine the colloidal stability. The magnetic particles must be in the range of *10nm*. The surfactant tails should be long enough to prevent particle agglomeration. Ferrofluids synthesized about 30 years ago are still stable.

---

[*] To whom correspondence should be addressed

Initially, Finlayson [1] studied the convective instability of a single component ferrofluid layer heated from below in the presence of a vertical uniform magnetic field and explained the concept of thermo mechanical interaction in a ferrofluid. Das Gupta and Gupta [2] showed the stabilizing effect of rotation on setting up of convective instability in ferrofluids. The stability of the magnetic fluid penetration through a porous medium in a high uniform magnetic field oblique to the interface is studied by Zahn and Rosensweig [3]. The effect of the magnetic field along the vertical axis on thermoconvective instability in a ferromagnetic fluid saturating a rotating porous medium with the Darcy model was studied by Sekar *et al.* [4]. The same effect for a Brinkman model was also studied by Sekar *et al.* [5]. The effects of rotation and anisotropy of a porous medium on ferroconvection was analyzed by Vaidyanathan *et al.* [6]. This was extended to a study on the effect of rotation on ferrothermohaline convection saturating a porous medium by Sekar *et al.* [7]. The effect of rotation on ferrothermohaline convection was analyzed and linear theory was used by Sekar *et al.* [8]. Soret-driven thermosolutal convection was studied by Hurle *et al.* [9]. Soret-driven convection in ferrofluids using a non-linear analysis was analyzed by Ryskin *et al.* [10].

Soret-driven ferro thermohaline convection was studied by Vaidyanathan *et al.* [11]. Shevtsova *et al.* [12] carried out a study on the onset of convection in Soret driven instability. The effect of Coriolis force on a Soret driven ferrothermohaline convective system was studied by Sekar *et al.* [13]. Following this, the same analysis in a medium of sparse particle suspension was analyzed by Vaidyanathan *et al.* [14]. The effect of rotation on double diffusive convection in a magnetized ferrofluid with an internal angular momentum was studied by Sunil *et al.* [15]. The effect of Coriolis force on thermal convection in a couple stress fluid saturated rotating rigid porous layer was studied by Shivakumara *et al.* [16]. Paras Ram *et al.* [17-18] discussed the ferrofluid flow with magnetic field dependent viscosity due to a rotating disk with and without a porous medium. More recently, the presence and absence of an anisotropy porous medium on Soret driven ferrothermohaline convection have been investigated by Sekar *et al.* [19-21] using Brinkman and Darcy models. Also, with and without magnetic field dependent (MFD) viscosity on Soret driven ferrothermohaline convection in an anisotropic porous medium was studied by Sekar and Raju [22-23].The effect of temperature dependent viscosity and Coriolis force on Soret driven ferrothermohaline convection in a porous medium and anisotropy effect was studied by Sekar *et al.* [24-25]. Most recently, the presence and absence of Coriolis force on Soret driven ferrothermohaline convection saturating a densely packed anisotropic porous medium was studied by Sekar *et al.* [26-27].

In the present investigation, the convection of Soret-driven ferro thermohaline instability of multi-component fluid layer heated from below and salted from above in a densely packed porous medium with Coriolis force is analysed. Linear stability analysis is used. The conditions for the onset of stationary and oscillatory instabilities are obtained.

2. Formulation of problem

A horizontal layer of an incompressible Boussinesq ferromagnetic fluid of thickness '*d*' saturating a densely packed porous medium with Coriolis force in the presence of a transverse applied magnetic field heated from below and salted from above is considered. The temperature and salinity at the bottom and top surfaces $z = \pm d/2$ are $T_0 \pm \Delta T/2$ and $S_0 \pm \Delta S/2$, respectively. Both the boundaries are taken to be free and perfect conductors of heat and solute. The Soret effect on the temperature gradient is considered. Further, the whole system is assumed to rotate with a uniform constant angular velocity $\Omega=(0, 0, \Omega)$ and the vertical direction is taken as the z axis. The mathematical equations governing the above investigation are as follows:

The continuity equation for an incompressible fluid is

$$\nabla. \boldsymbol{q} = 0 . \tag{2.1}$$

The corresponding momentum equation is

$$\rho_o \frac{D\boldsymbol{q}}{Dt} = -\nabla p + \rho \boldsymbol{g} + \nabla.\left(\boldsymbol{HB}\right) + 2\rho_o \left(\boldsymbol{q}\times\boldsymbol{\Omega}\right) + \frac{\rho_o}{2}\nabla\left(\left|\boldsymbol{\Omega}\times\boldsymbol{r}\right|^2\right) - \frac{\mu}{k}\boldsymbol{q} . \tag{2.2}$$

The temperature equation for an incompressible ferrofluid is

$$\left[\rho_o C_{V,H} - \mu_o \boldsymbol{H} \cdot \left(\frac{\partial \boldsymbol{M}}{\partial \boldsymbol{T}} \right)_{V,H} \right] \frac{dT}{dt} + \mu_o T \left(\frac{\partial \boldsymbol{M}}{\partial \boldsymbol{T}} \right)_{V,H} \cdot \frac{d\boldsymbol{H}}{dt} = K_I \nabla^2 T + \phi . \tag{2.3}$$

The mass flux equation is given by

$$\frac{DS}{Dt} = K_s \nabla^2 S + S_T \nabla^2 T \tag{2.4}$$

where $\rho_o, \boldsymbol{q} = (u, v, w), g = (0, 0, -g), k, t, p, \mu, \boldsymbol{H}, \boldsymbol{B}, C_{V,H}, T, \boldsymbol{M}, K_I, S, K_S, \Omega = (0, 0, \Omega), S_T$ and ϕ are the fluid density, velocity, acceleration due to gravity, permeability of the porous medium, time, pressure, coefficient of viscosity, magnetic field, magnetic induction, heat capacity at constant volume and magnetic field, temperature, magnetization, thermal conductivity, salinity, concentration diffusivity, angular velocity, Soret coefficient and viscous dissipation factor containing second-order terms in velocity, respectively.

Maxwell's equations are

$$\nabla \cdot \boldsymbol{B} = 0, \quad \nabla \times \boldsymbol{H} = 0 . \tag{2.5}$$

Further, $\boldsymbol{B}$, $\boldsymbol{M}$ and $\boldsymbol{H}$ are related by

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{M} + \boldsymbol{H} \right). \tag{2.6}$$

Using Maxwell's equation for non-conducting fluids, one can assume that the magnetization is aligned with the magnetic field and depends on the magnitude of the magnetic field, temperature and salinity, so that

$$\boldsymbol{M} = \frac{\boldsymbol{H}}{H} M \left(H, T, S \right). \tag{2.7}$$

The magnetic equation of state is linearized about the magnetic field H_0, the average temperature T_0 and the average salinity S_0 and so

$$M = M_0 + \chi \left(H - H_0 \right) - K \left(T - T_0 \right) + K_2 \left(S - S_0 \right) \tag{2.8}$$

where $\chi = (\partial M / \partial H)_{H_0, T_0}$ is the susceptibility, $K = -(\partial M / \partial T)_{H_0, T_0}$ is the pyromagnetic coefficient and $K_2 = (\partial M / \partial S)_{H_0, S_0}$ is the salinity magnetic coefficient.

The density equation of state for a Boussinesq two-component fluid is

$$\rho = \rho_0 \left[1 - \alpha_t \left(T - T_0 \right) + \alpha_s \left(S - S_0 \right) \right] \tag{2.9}$$

where α_t is the thermal expansion coefficient and α_s is the solute analog of α_t.

The basic state is assumed to be the quiescent state and taking the components of the magnetization - and magnetic field in the basic state to be $\left[0, 0, M_0(z) \right]$ and $\left[0, 0, H_0(z) \right]$, the basic state quantities obtained are

$$q = q_b = 0, \quad p = p_b(z), \quad \frac{\partial T}{\partial z} = -\beta_t \Rightarrow T_b = T_0 - \beta_t z, \quad \frac{\partial S}{\partial z} = \beta_S \Rightarrow S_b = S_0 + \beta_s z,$$

(2.10)

$$H_b(Z) = \left[H_0 + \frac{K(T_b - T_0)}{1+\chi} - \frac{K_2(S_b - S_0)}{1+\chi} \right]\hat{k}, \quad M_b(Z) = \left[M_0 - \frac{K(T_b - T_0)}{1+\chi} + \frac{K_2(S_b - S_0)}{1+\chi} \right]\hat{k}$$

where β_t and β_S are non-negative constants and $\hat{k} = (0, 0, 1)$ is the unit vector along the vertical direction.

3. Linear stability analysis

The basic state is disturbed by a small thermal perturbation. Consider a perturbed state such that $q = q', \quad p = p_b(z) + p', \quad \mu = \mu_b(z) + \mu', \quad T = T_b(z) + T', \quad H = H_b(z) + H', \quad M = M_b(z) + M'$ where q', p', μ', T', H' and M' are perturbed variables which are assumed to be small. The perturbed state temperature and solute are $T = T_0 - \beta_t z + T'$ and $S = S_0 + \beta_s z + S'$. Let the components of the perturbed magnetization and the magnetic field be $\left(M'_1, M'_2, M_0(z) + M'_3 \right)$ and $\left(H'_1, H'_2, H_0(z) + H'_3 \right)$, respectively.

$$H'_i + M'_i = \left(1 + \frac{M_0}{H_0} \right) H'_i \qquad (i = 1, 2),$$

(3.1)

$$H'_3 + M'_3 = (1+\chi) H'_3 - KT' + K_2 S' + S_T KT'.$$

(3.2)

Let (B_1, B_2, B_3) denote the components of B.

Using Eq.(2.6), one gets the result $B_i = \mu_0 \left(M'_i + H'_i \right)$ and Eqs (3.1) and (3.2) give

$$B_i = \mu_0 \left(1 + \frac{M_0}{H_0} \right) H'_i \qquad (i = 1, 2),$$

(3.3)

$$B_3 = \mu_0 \left[(1+\chi) H'_3 - KT' + K_2 S' + S_T KT' + M_0 + H_0 \right].$$

(3.4)

When the first equation of Eq.(2.5) is used in Eq.(2.1) and the resulting equation is linearized with B_i ($i=1, 2, 3$) given by Eq.(3.3) and Eq.(3.4), we obtain the following components

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu_0 (M_0 + H_0) \frac{\partial H'_1}{\partial z} + 2\rho_0 v\Omega - \frac{\mu}{k} u,$$

(3.5)

$$\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu_0 (M_0 + H_0) \frac{\partial H'_2}{\partial z} - 2\rho_0 u\Omega - \frac{\mu}{k} v,$$

(3.6)

$$\rho_0 \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \mu_0 \left(M_0 + H_0 \right) \frac{\partial H'_3}{\partial z} - \mu_0 H'_3 K\beta_t + \frac{\mu_0 K^2 \beta_t T'}{1+\chi} \left(1 - S_T \right) + \mu_0 H'_3 K_2 \beta_s +$$

$$-\frac{\mu_0 K K_2 \beta_s T'}{1+\chi} \left(1 - S_T \right) - \frac{\mu_0 K K_2 \beta_t S'}{1+\chi} + \frac{\mu_0 K_2^2 \beta_s S'}{1+\chi} + \rho_0 g \alpha_t T' - \rho_0 g \alpha_s S' - \frac{\mu}{k} w. \tag{3.7}$$

Differentiating Eqs (3.5)-(3.7) with respect to x, y and z respectively and adding them the following equation is obtained upon using Eq.(2.1)

$$\nabla^2 p = \mu_0 \left(M_0 + H_0 \right) \frac{\partial}{\partial z} \left(\nabla \cdot \boldsymbol{H}' \right) + 2\rho_0 \Omega \xi + \mu_0 K_2 \beta_s \frac{\partial H'_3}{\partial z} - \mu_0 K\beta_t \frac{\partial H'_3}{\partial z} + \frac{\mu_0 K^2 \beta_t}{1+\chi} \left(1 - S_T \right) \frac{\partial T'}{\partial z} +$$

$$-\frac{\mu_0 K K_2 \beta_s}{1+\chi} \left(1 - S_T \right) \frac{\partial T'}{\partial z} + \frac{\mu_0 K_2^2 \beta_s}{1+\chi} \frac{\partial S'}{\partial z} - \frac{\mu_0 K K_2 \beta_t}{1+\chi} \frac{\partial S'}{\partial z} + \rho_0 g \alpha_t \frac{\partial T'}{\partial z} - \rho_0 g \alpha_s \frac{\partial S'}{\partial z} \tag{3.8}$$

where $\boldsymbol{H}'$ has the components $\left(H'_1 \,, \; H'_2 \,, \; H'_3 \; \right)$. From the second equation of (2.5), $\boldsymbol{H}' = \nabla \phi$ where ϕ is a scalar potential. Elimination of p from Eqs (3.7) and (3.8) then gives

$$\rho_0 \frac{\partial}{\partial t} \left(\nabla^2 w \right) = \mu_0 K_2 \beta_s \nabla_1^2 H'_3 - 2\rho_0 \Omega \frac{\partial \xi}{\partial z} - \mu_0 K \beta_t \nabla_1^2 H'_3 + \frac{\mu_0 K^2 \beta_t}{1+\chi} \left(1 - S_T \right) \nabla_1^2 T' + \rho_0 g \alpha_t \nabla_1^2 T' +$$

$$-\frac{\mu_0 K K_2 \beta_s}{1+\chi} \left(1 - S_T \right) \nabla_1^2 T' + \frac{\mu_0 K_2^2 \beta_s}{1+\chi} \nabla_1^2 S' - \frac{\mu_0 K K_2 \beta_t}{1+\chi} \nabla_1^2 S' - \rho_0 g \alpha_s \nabla_1^2 S' - \frac{\mu}{k} \nabla^2 w \tag{3.9}$$

where $\quad \nabla_1^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ and $\nabla^2 = \nabla_1^2 + \dfrac{\partial^2}{\partial z^2}$.

We now proceed to a normal mode analysis of the above stability problem. Let us take

$$f \left(x,y,z,t \right) = f \left(z,t \right) e^{i \left(k_x x + k_y y \right)}, \quad \phi = \phi \left(z,t \right) e^{i \left(k_x x + k_y y \right)}, \quad w = w \left(z,t \right) e^{i \left(k_x x + k_y y \right)},$$

$$T' = \theta(z,t) e^{i \left(k_x x + k_y y \right)}, \quad S' = S \left(z,t \right) e^{i \left(k_x x + k_y y \right)}, \tag{3.10}$$

with the wave number $k_0^2 = k_x^2 + k_y^2$.

The vertical component of the momentum equation can be calculated as

$$\left(\rho_0 \frac{\partial}{\partial t} + \frac{\mu}{k} \right) \left(\frac{\partial^2}{\partial z^2} - k_0^2 \right) w = \frac{\mu_0 K \beta_t}{1+\chi} \left[\left(1 + \chi \right) \frac{\partial \phi}{\partial z} - K\theta \left(1 - S_T \right) \right] k_0^2 - \rho_0 g \alpha_t k_0^2 \theta +$$

$$-2\rho_0 \Omega \frac{\partial \xi}{\partial z} + \rho_0 g \alpha_s k_0^2 + \frac{\mu_0 K_2 \beta_s}{1+\chi} \left[\left(1 + \chi \right) \frac{\partial \phi}{\partial z} + K_2 S \right] k_0^2 - \frac{\mu_0 K K_2}{1+\chi} \left[\beta_s \left(1 - S_T \right) \theta - \beta_t S \right] k_0^2, \tag{3.11}$$

$$\left(\rho_0 \frac{\partial}{\partial t} + \frac{\mu}{k}\right)\xi = 2\rho_0 \Omega \frac{\partial w}{\partial z} \tag{3.12}$$

where ξ is the z- component of vorticity given by $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$.

The modified Fourier heat conduction equation is

$$\rho_0 C_{V,H} \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z}\right) = K_1\left(\frac{\partial^2}{\partial z^2} - k_0^2\right)\theta + \left[\rho_0 C\beta_t - \frac{\mu_0 K^2 T_0^2 \beta_t}{1+\chi} + \frac{\mu_0 K K_2 T_0 \beta_s}{1+\chi}\right]w \tag{3.13}$$

where $\qquad \rho_0 C = \rho_0 C_{V,H} + \mu_0 K H_0$.

The salinity equation is

$$\frac{\partial S}{\partial t} + \beta_s w = K_s\left(\frac{\partial^2}{\partial z^2} - k_0^2\right)S + S_T\left(\frac{\partial^2}{\partial z^2} - k_0^2\right)\theta. \tag{3.14}$$

Using the analysis similar to Finlayson (1970), one gets

$$(1+\chi)\frac{\partial^2 \phi}{\partial z^2} - \left(1 + \frac{M_0}{H_0}\right)k_0^2 \phi - K\frac{\partial \theta}{\partial z}(1 - S_T) + K_2 \frac{\partial S}{\partial z} = 0. \tag{3.15}$$

The non-dimensional numbers can be written using

$$t^* = \frac{\nu t}{d^2}, \quad w^* = \frac{wd}{\nu}, \quad T^* = \left(\frac{K_1 a R^{1/2}}{\rho_0 C_{V,H} \beta_t \nu d}\right)\theta, \quad \phi^* = \left(\frac{(1+\chi)K_1 a R^{1/2}}{K\rho_0 C_{V,H}\beta_t \nu d^2}\right)\phi, \quad z^* = \frac{z}{d},$$

$$a = k_0 d, \quad D = \frac{\partial}{\partial z^*}, \quad S^* = \left(\frac{K_s a R_s^{1/2}}{\rho_0 C_{V,H}\beta_s \nu d}\right)S, \quad \nu = \frac{\mu}{\rho_0}, \quad \xi^* = \frac{\xi d^2}{\nu}, \quad k^* = \frac{k}{d^2}.$$

Then Eqs (3.11) – (3.15) become

$$\left.\begin{aligned} &\left(\frac{\partial}{\partial t^*} + \frac{1}{k^*}\right)\left(D^2 - a^2\right)w^* = a R^{1/2}\left[M_1 D\phi^* - \left(1 + M_1\left(1 - S_T\right)T^*\right)\right] + a R^{1/2} M_1 M_5 D\phi^* + \\ &-a R^{1/2} M_1 M_5 \left(1 - S_T\right)T^* + a R_S^{1/2}\left[1 + M_4 + \frac{M_4}{M_5}\right]S^* - \left(T_a\right)^{1/2} D\xi^* \end{aligned}\right\} \tag{3.16}$$

$$\left(\frac{\partial}{\partial t^*} + \frac{1}{k^*}\right)\xi^* = \left(T_a\right)^{1/2} Dw^*. \tag{3.17}$$

$$\Pr\left[\frac{\partial T^*}{\partial t^*} - M_2 \frac{\partial}{\partial t^*}(D\phi^*)\right] = \left(D^2 - a^2\right)T^* + aR^{1/2}\left(1 - M_2 - M_2 M_5\right)w^*, \tag{3.18}$$

$$\Pr\frac{\partial S^*}{\partial t^*} = \tau\left(D^2 - a^2\right)S^* - aR_S^{1/2}M_6 w^* + S_T\left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2}\left(D^2 - a^2\right)T^*, \tag{3.19}$$

$$D^2\phi^* - M_3 a^2\phi^* - \left(1 - S_T\right)DT^* + \frac{M_5}{M_6}\left(\frac{R}{R_S}\right)^{1/2}DS^* = 0 \tag{3.20}$$

where the non-dimensional parameters used are

$$M_1 = \frac{\mu_0 K^2 \beta_t}{(1+\chi)\rho_0 g \alpha_t}, \quad M_2 = \frac{\mu_0 K^2 T_0}{(1+\chi)\rho_0 C_{v,H}}, \quad M_3 = \frac{1 + M_0 / H_0}{(1+\chi)},$$

$$M_4 = \frac{\mu_0 K^2 \beta_s}{(1+\chi)\rho_0 g \alpha_s}, \quad M_5 = \frac{K_2 \beta_s}{K\beta_t}, \quad M_6 = \frac{K_S}{K_1}, \quad \tau = \rho_0 C_{v,H}\left(\frac{K_S}{K_1}\right), \tag{3.21}$$

$$\Pr = \frac{\mu C_{v,H}}{K_1}, \quad R_S = \frac{\rho_0 C_{v,H}\beta_s \alpha_s g d^4}{\nu K_S}, \quad R = \frac{\rho_0 C_{v,H}\beta_t \alpha_t g d^4}{\nu K_1},$$

where R_S is the salinity Rayleigh number, R is the thermal Rayleigh number, Pr is the Prandtl number and other parameters represent non-dimensional parameters used appropriately.

4. Analysis of solution at free boundaries

The boundary conditions on velocity, temperature and salinity are

$$w^* = D^2 w^* = T^* = D\phi^* = S^* = \xi^* = D\xi^* = 0 \quad \text{at} \quad z^* = \pm 1/2. \tag{4.1}$$

The exact solutions satisfying Eq. (4.1) are

$$w^* = Ae^{\sigma t^*}\cos\pi z^*, \quad T^* = Be^{\sigma t^*}\cos\pi z^*, \quad S^* = Ce^{\sigma t^*}\cos\pi z^*,$$

$$D\phi^* = Fe^{\sigma t^*}\cos\pi z^*, \quad \phi^* = \frac{F}{\pi}e^{\sigma t^*}\sin\pi z^* \tag{4.2}$$

where A, B, C and F are constants. These functions substituted in the set of Eqs (3.16) – (3.20) give the following four linear homogeneous algebraic equations in A, B, C and F. Removing the asterisks for our convenience, we get

$$\left[\left(\sigma+\frac{1}{k}\right)(\pi^2+a^2)+\frac{T_a\pi^2}{\left(\sigma+\frac{1}{k}\right)}\right]A-aR^{1/2}\left[1+M_1(1-S_T)(1+M_5)\right]B+$$

$$+aR_S^{1/2}\left(1+M_4+M_4M_5^{-1}\right)C+aR^{1/2}M_1(1+M_5)F=0, \tag{4.3}$$

$$aR^{1/2}\left[1-M_2(1+M_5)\right]A-\left(\pi^2+a^2+P_r\sigma\right)B+P_r\sigma M_2E=0, \tag{4.4}$$

$$aR_S^{1/2}M_6A+S_T\left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2}\left(\pi^2+a^2\right)B+\left[\tau\left(\pi^2+a^2\right)+\sigma P_r\right]C=0, \tag{4.5}$$

$$R_S^{1/2}\pi^2\left(1-S_T\right)B-R^{1/2}\pi^2\left(\frac{M_5}{M_6}\right)C-R_S^{1/2}\left(\pi^2+a^2M_3\right)E=0 \tag{4.6}$$

For the existence of non-trivial eigen functions, the determinant of the coefficients of A, B, C and F in Eqs (4.3) – (4.6) must vanish. Following the techniques and analysis of Sekar *et al*. (2007), Eqs (4.3)-(4.6) leads to

$$U\sigma^4+V\sigma^3+W\sigma^2+X\sigma+Y=0 \tag{4.7}$$

where

$$U=\left(\pi^2+a^2\right)\left(\pi^2+a^2M_3\right)P_r^2,$$

$$V=\left(\pi^2+a^2M_3\right)\left[\left(\pi^2+a^2\right)^2(1+\tau)+\frac{2}{k}\left(\pi^2+a^2\right)P_r\right]P_r$$

$$W=\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)\left[\tau\left(\pi^2+a^2\right)^2+\frac{2}{k}\left(\pi^2+a^2\right)P_r(1+\tau)\right]+$$

$$+a^2RP_rM_1(1+M_5)\left[(1-S_T)\left(\pi^2+a^2M_3\right)-\pi^2(1-S_T+M_5)\right]+a^2RP_r\left(\pi^2+a^2M_3\right)+$$

$$+a^2R_sP_r\left(\pi^2+a^2M_3\right)\left(1+M_4+M_4M_5^{-1}\right)M_6+\left(\pi^2+a^2M_3\right)\left[T_a\pi^2+\frac{1}{k^2}\left(\pi^2+a^2\right)\right]P_r^2,$$

$$X=\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)\left[\frac{1}{k^2}(1+\tau)\left(\pi^2+a^2\right)P_r+\pi^2T_aP_r+\frac{2}{k}\tau\left(\pi^2+a^2\right)^2\right]+$$

$$+a^2R\tau\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)\left[1+(1-S_T)M_1(1+M_5)\right]+$$

$$+\frac{1}{k}a^2RP_r\left[\left(\pi^2+a^2M_3\right)\{1+M_1(1+M_5)(1-S_T)\}-\pi^2M_1(1+M_5)\{(1-S_T)+M_5\}\right]+$$

$$-a^2R\left(\pi^2+a^2\right)M_1(1+M_5)\pi^2\left[S_T\left(\frac{M_5}{M_6}\right)^2+\tau(1-S_T)+M_5\right]+$$

$$+a^2R_s\left(\pi^2+a^2M_3\right)\left(1+M_4+M_4M_5^{-1}\right)\left[\frac{1}{k}M_6P_r+\left(\pi^2+a^2\right)\left\{S_T\left(\frac{M_5}{M_6}\right)+M_6\right\}\right],$$

$$Y = \tau\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)^2\left[T_a\pi^2 + \frac{1}{k^2}\left(\pi^2 + a^2\right)\right] +$$

$$+ \frac{1}{k}a^2 R\tau\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left[1 + \left(1 - S_T\right)M_1\left(1 + M_5\right)\right] +$$

$$- \frac{1}{k}a^2 R\left(\pi^2 + a^2\right)M_1\left(1 + M_5\right)\pi^2\left[S_T\left(\frac{M_5}{M_6}\right)^2 + \tau\left(1 - S_T\right) + M_5\right] +$$

$$+ \frac{1}{k}a^2 R_s\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left(1 + M_4 + M_4 M_5^{-1}\right)\left[S_T\left(\frac{M_5}{M_6}\right) + M_6\right].$$

5. Stationary convection

For obtaining stationary instability, the time-independent term $Y=0$. Equation (4.7) helps one to obtain eigen value R^{sc} for which a solution exists

$$R^{sc} = \frac{N_r}{D_r} \tag{5.1}$$

where

$$N_r = \left(\pi^2 + a^2\right)\left[T_a\pi^2 k + \frac{1}{k}\left(\pi^2 + a^2\right)\right] - a^2 R_s\tau^{-1}\left(1 + M_4 + M_4 M_5^{-1}\right)\left[S_T\left(\frac{M_5}{M_6}\right) + M_6\right]$$

and

$$D_r = a^2\left[1 + \left(1 - S_T\right)M_1\left(1 + M_5\right)\right] - \pi^2\left[\frac{a^2 M_1\left(1 + M_5\right)}{\pi^2 + a^2 M_3}\right]\left[S_T\left(\frac{M_5}{M_6}\right)^2\tau^{-1} + \left(1 - S_T\right) + M_5\tau^{-1}\right]$$

For very large M_1, one gets the results for the magnetic mechanism and the critical thermo magnetic Rayleigh number for stationary mode is obtained using

$$N^{sc} = R^{sc}M_1 = \frac{N_r}{D_r} \tag{5.2}$$

where

$$N_r = \left(\pi^2 + a^2\right)\left[T_a\pi^2 k + \frac{1}{k}\left(\pi^2 + a^2\right)\right] - a^2 R_s\tau^{-1}\left(1 + M_4 + M_4 M_5^{-1}\right)\left[S_T\left(\frac{M_5}{M_6}\right) + M_6\right]$$

and

$$D_r = a^2\left[\left(1 - S_T\right)\left(1 + M_5\right)\right] - \pi^2\left[\frac{a^2\left(1 + M_5\right)}{\pi^2 + a^2 M_3}\right]\left[S_T\left(\frac{M_5}{M_6}\right)^2\tau^{-1} + \left(1 - S_T\right) + M_5\tau^{-1}\right]$$

6. Oscillatory convection

The conditions for the onset of oscillatory stabilities are obtained as follows. Taking $\sigma = i\sigma_1$ and $\sigma_1^2 > 0$, following the analysis and techniques of Sekar *et al.* (2007), the critical Rayleigh number for oscillatory mode has been calculated using

$$R^{oc} = \frac{C_2 A_2 + B_2 D_2}{A_2^2 + B_2^2}$$

where

$$A_2 = -U_1\sigma_1^2 + V_1, \quad B_2 = W_1\sigma_1, \quad C_2 = -U_2\sigma_1^4 + W_2\sigma_1^2 - Y_1,$$

$$D_2 = V_2\sigma_1^3 - X_1\sigma_1, \quad \sigma_1^2 = \frac{-B_1 \pm sqrt\left(B_1^2 - 4A_1C_1\right)}{2A_1},$$

$$A_1 = U_2W_1 - U_1V_2, \quad B_1 = V_1V_2 + U_1X_1 - W_1W_2, \quad C_1 = W_1Y_1 - V_1X_1,$$

$$U_1 = a^2 P_r M_1 (1+M_5)\left[(1-S_T)(\pi^2 + a^2 M_3) - \pi^2(1 - S_T + M_5)\right] + a^2 R P_r (\pi^2 + a^2 M_3),$$

$$V_1 = \frac{1}{k}a^2\tau(\pi^2 + a^2 M_3)(\pi^2 + a^2)\left[1 + (1 - S_T)M_1(1+M_5)\right] +$$
$$-\frac{1}{k}a^2(\pi^2 + a^2)M_1(1+M_5)\pi^2\left[S_T\left(\frac{M_5}{M_6}\right)^2 + \tau(1 - S_T) + M_5\right],$$

$$W_1 = a^2\tau(\pi^2 + a^2 M_3)(\pi^2 + a^2)\left[1 + (1 - S_T)M_1(1+M_5)\right] +$$
$$+\frac{1}{k}a^2 P_r\left[(\pi^2 + a^2 M_3)\{1 + M_1(1+M_5)(1 - S_T)\} - \pi^2 M_1(1+M_5)\{(1 - S_T) + M_5\}\right] +$$
$$-a^2(\pi^2 + a^2)M_1(1+M_5)\pi^2\left[S_T\left(\frac{M_5}{M_6}\right)^2 + \tau(1 - S_T) + M_5\right],$$

$$X_1 = (\pi^2 + a^2 M_3)(\pi^2 + a^2)\left[\frac{1}{k^2}(1+\tau)(\pi^2 + a^2)P_r + \pi^2 T_a P_r + \frac{2}{k}\tau(\pi^2 + a^2)^2\right] +$$
$$+a^2 R_s(\pi^2 + a^2 M_3)(1 + M_4 + M_4 M_5^{-1})\left[\frac{1}{k}M_6 P_r + (\pi^2 + a^2)\left\{S_T\left(\frac{M_5}{M_6}\right) + M_6\right\}\right],$$

$$Y_1 = \tau(\pi^2 + a^2 M_3)(\pi^2 + a^2)^2\left[T_a\pi^2 + \frac{1}{k^2}(\pi^2 + a^2)\right] +$$
$$+\frac{1}{k}a^2 R_s(\pi^2 + a^2 M_3)(\pi^2 + a^2)(1 + M_4 + M_4 M_5^{-1})\left[S_T\left(\frac{M_5}{M_6}\right) + M_6\right],$$

$$U_2 = (\pi^2 + a^2)(\pi^2 + a^2 M_3)P_r^2,$$

$$V_2 = (\pi^2 + a^2 M_3)\left[(\pi^2 + a^2)^2(1+\tau) + \frac{2}{k}(\pi^2 + a^2)P_r\right]P_r,$$

$$W_2 = (\pi^2 + a^2 M_3)(\pi^2 + a^2)\left[\tau(\pi^2 + a^2)^2 + \frac{2}{k}(\pi^2 + a^2)(1+\tau)P_r\right] +$$
$$+a^2 R_s P_r(\pi^2 + a^2 M_3)(1 + M_4 + M_4 M_5^{-1})M_6 + (\pi^2 + a^2 M_3)\left[T_a\pi^2 + \frac{1}{k^2}(\pi^2 + a^2)\right]P_r^2.$$

7. Method of solution

The Soret-driven thermoconvective instability of a ferromagnetic fluid heated from below and salted from above rotating in a densely packed porous medium has been analyzed using the Darcy model. The perturbation method is applied and a normal mode analysis is adopted. In the perturbation method, due to the application of a magnetic field, the system is perturbed from the basic state (quiescent state). Accordingly the governing and other equations are modified. Linear stability analysis is considered. Then normal mode analysis is taken. Non-dimensional analysis is carried out and the exact solutions satisfying the appropriate boundary conditions are taken yielding algebraic equations. For getting a non-trivial solution for the system of linear homogeneous equations, the coefficients of the dynamic variables are equated to zero and on simplification, the expression for R^{sc} is obtained. Varying the values of the parameters in the allowable range and getting the corresponding R^{sc} values, we get the stability pattern.

8. Results and discussions

Before discussing the significant results of the convective system, we turn our attention to the possible range of values of various parameters arising in the study. The Prandtl number Pr is assumed to be *0.01*. The Taylor number Ta is assumed to vary from *10* to *10^8*. The Soret parameter S_T is assumed to take values from *-0.002* to *0.002*, the salinity Rayleigh number Ra takes values *-100, 0, 100, 400* and *600*. The non-buoyancy magnetization parameter M_3 is allowed to take values from *5* to *25*. The values of the ratio of mass transport to heat transport τ is assumed to be *0.03, 0.05, 0.07, 0.09* and *0.11*. The buoyancy magnetization parameter M_1 is assumed to be *1000*. For these fluids, M_2 will have a negligible value and hence is taken as zero. M_6 is taken to be *0.1* and M_4 is the effect on magnetization due to salinity. This is allowed to vary from *0.1* to *0.5* taking values less than the non-buoyancy magnetization parameter M_3. M_5 represents the ratio of the salinity effect on the magnetic field and pyromagnetic coefficient. This is varied between *0.1* and *0.5*. The permeability of the porous medium k is assumed to take the values from *0.001, 0.003, 0.005, 0.007* and *0.009* (Darcy number).
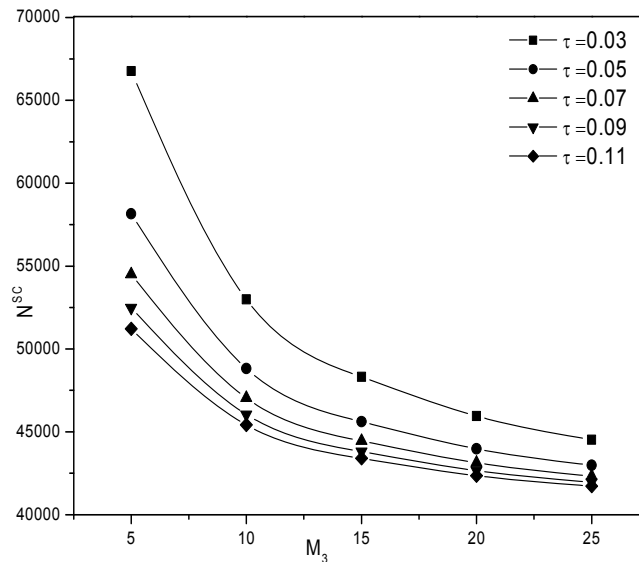

Fig.1. Variation of N^{sc} versus M_3 for different τ with St = *-0.002*, R_S = *100*, k = *0.001* and Ta = *10^5*.

Figure 1 illustrates the variation of the thermal Rayleigh number N^{sc} versus the non-buoyancy magnetization parameter M_3 for Ta=10^5. When the values of the ratio of mass transport to heat transport τ is varied from *0.03* to *0.11*, it is seen that, when M_3 increases from *5* to *25*, N^{sc} decreases indicating the onset of

instability. This is because high magnetization tends to release large energy to the system causing instability to set in earlier. Also, as the ratio of mass transport to heat transport τ increases from *0.03* to *0.11*, there is a fall in the values of N^{sc}. Thus larger values of τ lead to destabilization of the system. The magnetization of the fluid is found to destabilize the system through oscillatory mode, which was discussed by Sekar *et al.* (2007).
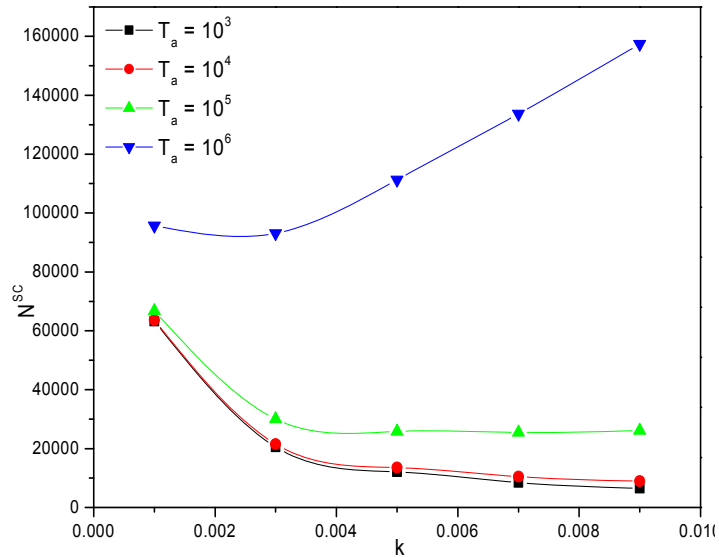

Fig.2. Variation of N^{sc} versus k for different Ta with St = *-0.002*, R_S = *-500*, τ = *0.03* and M_3= *5*.

Figure 2 shows the variation of the thermal Rayleigh number N^{sc} versus the permeability of the porous medium k. It has been observed that as the Taylor number Ta is increased from *10^3* to *10^6*, it is clear that as the permeability k increases from *0.001* to *0.009*, the thermal Rayleigh number N^{sc} values tend to decrease for small values of Ta, whereas for higher values of Ta, N^{sc} decreases for lower values of k, and then increases for higher values of k.
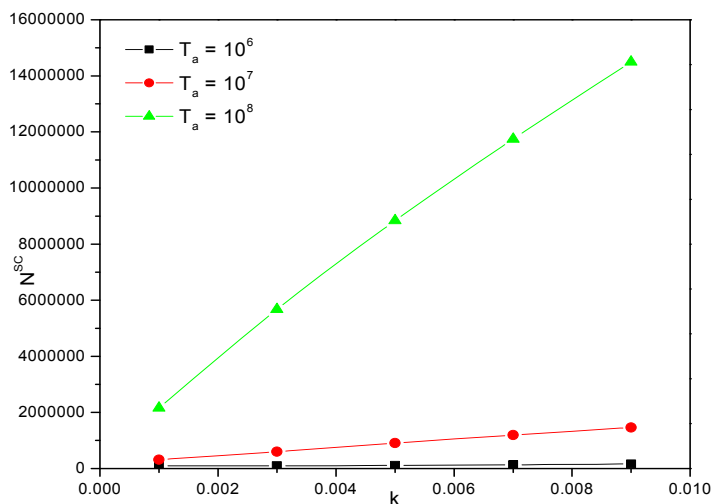

Fig.3. Variation of N^{sc} versus k for different Ta with St = *-0.002*, R_S = *-500*, τ = *0.03* and M_3= *5*.

Figure 3 represents the variation of the thermal Rayleigh number N^{sc} versus permeability of the porous medium k. It has been observed that as the Taylor number Ta is increased from 10^6 to 10^8, it is clear that as the permeability k increases from *0.001* to *0.009*, the thermal Rayleigh number N^{sc} values increases. Therefore rotation leads to stability of the system.
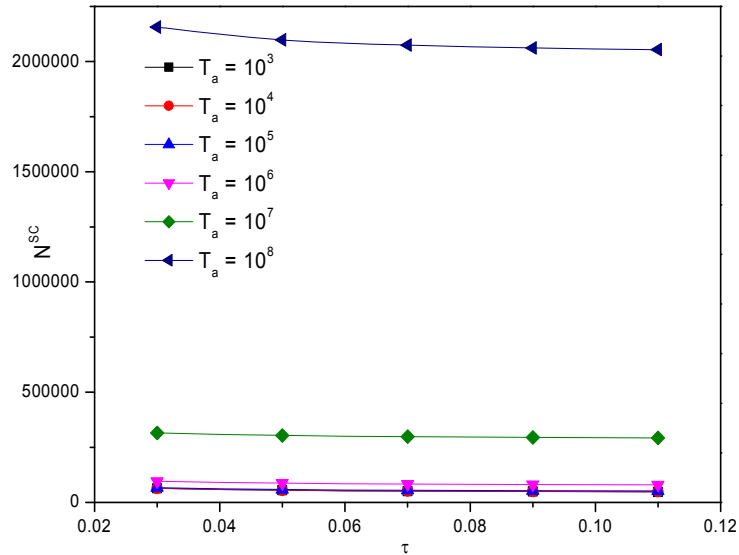

Fig.4. Variation of N^{sc} versus τ for different Ta with St = *-0.002, R_S = -500, k = 0.001*, and *M_3= 5.*

Figure 4 gives the variation of the thermal Rayleigh number N^{sc} versus the ratio of mass transport to heat transport τ (varied from *0.03* to *0.11*). It has been observed that as the Taylor number Ta is increased from *10^3* to *10^8*, there is a notable variation in rotation. As the Taylor number Ta increases, the critical thermal Rayleigh number N^{sc} increases. Therefore, the effect of the Taylor number leads to stability of the system. It is clear that as the ratio of the mass transport to heat transport τ increases from *0.03* to *0.11*, the critical thermal Rayleigh number N^{sc} is almost constant and the system is in equilibrium state due to rotation.
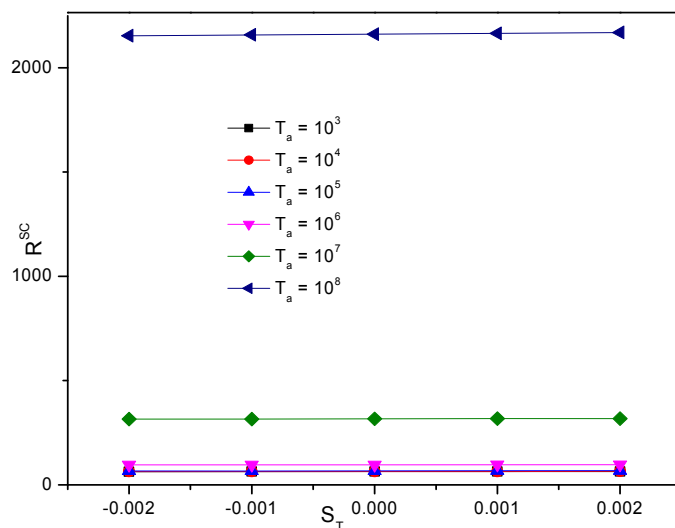

Fig.5. Variation of R^{sc} versus St for different Ta with R_S= *- 500, k = 0.001, τ = 0.03* and *M_3= 5.*

Figure 5 indicates the variation of R^{sc} versus St for different values of Ta. The figure shows a stabilizing behavior which is not much pronounced. The stabilization is minimal when the Taylor number Ta assumes values from 10^3 to 10^6, and then it increases phenomenally. This is indicated by an increase in R^{sc} values.
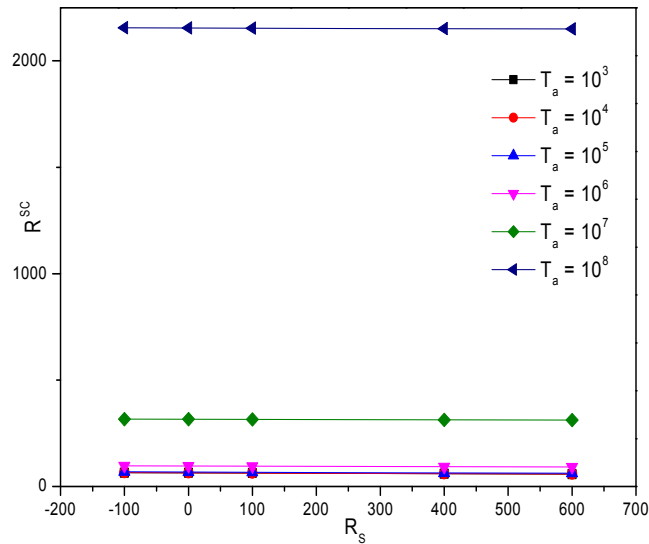

Fig.6. Variation of R^{sc} versus R_S for different Ta with St $= -0.002$, $k = 0.001$, $\tau = 0.03$ and $M_3= 5$.

Figure 6 is a plot of the variation of R^{sc} versus R_S for different values of Ta. This figure shows that as Ta increases, there is an increase in the values of the critical magnetic Rayleigh number R^{sc}. Therefore the Taylor number leads to stability of the system.
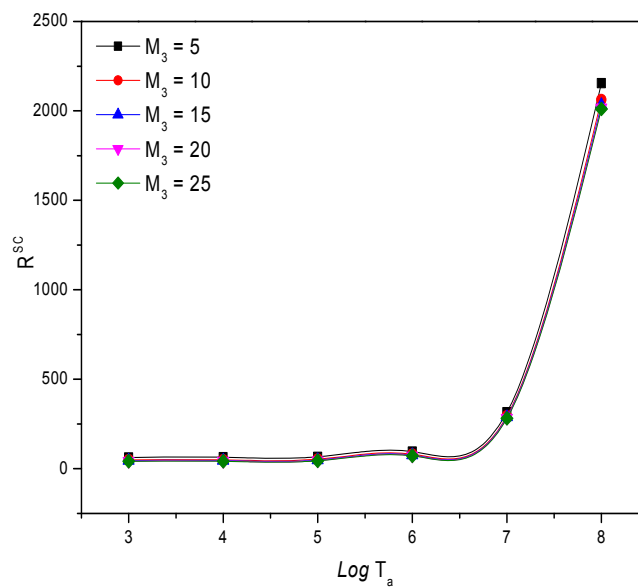

Fig.7. Variation of R^{sc} versus Log Ta for different M_3 with $R_S= 100$, $k = 0.001$, $\tau = 0.03$ and St $= -0.002$.

Figure 7 analyzes the variation of the critical magnetic Rayleigh number R^{sc} versus Log Ta for various values of the non-buoyancy magnetization parameter M_3. When Log Ta increases from *3* to *5*, stabilization is not much pronounced. But when it takes values from *6* to *8*, the increase in R^{sc} values is maximum.
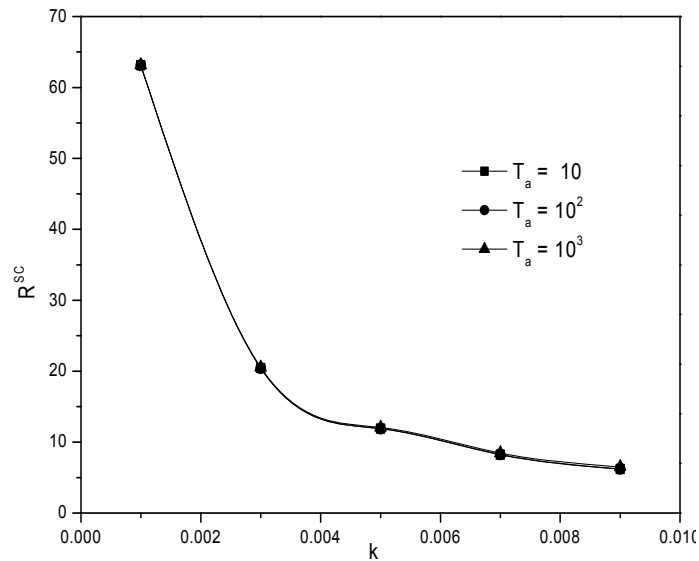

Fig.8. Variation of R^{sc} versus *k* for different Ta with St= *-0.002*, R_S= *100*, τ = *0.03* and M_3= *5*.

Figure 8 represents the variation of the critical magnetic Rayleigh number R^{sc} versus permeability of the porous medium *k* through stationary instability. It has been observed that as the Taylor number Ta is increased from *10* to *10^3*, there is no notable variation. The effect the Taylor number Ta is negligible. For different Ta, no appreciable changes in the curves are noticed. It is clear that as the permeability *k* increases from *0.001* to *0.009*, the critical magnetic Rayleigh number R^{sc} values tend to decrease leading to destabilization. This is due to the fact that an increase in the pore size makes the flow of the fluid easier causing instability to set in earlier.
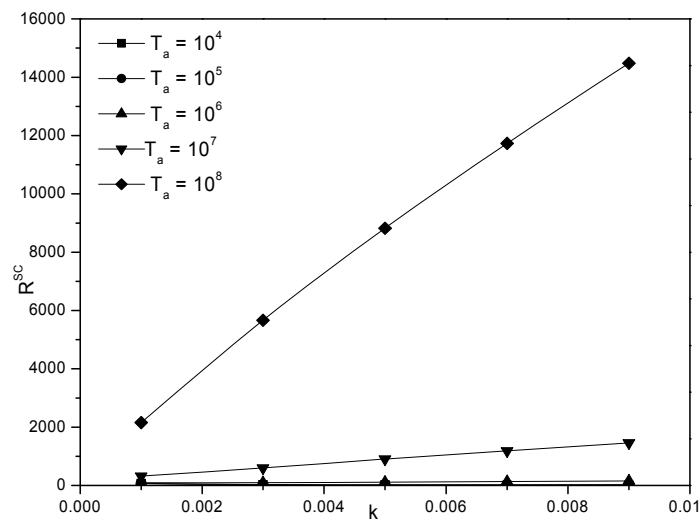

Fig.9. Variation of R^{sc} versus *k* for different Ta with St= *-0.002*, R_S= *100*, τ = *0.03* and M_3= *5*.

Figure 9 gives the variation of the critical magnetic Rayleigh number R^{sc} versus permeability of the porous medium k. It has been observed that as the Taylor number Ta is increased from 10^4 to 10^8, the critical magnetic Rayleigh number R^{sc} values tend to decrease leading to destabilization. But it is clear that as the permeability k increases from *0.001* to *0.009*, the critical magnetic Rayleigh number R^{sc} values tend to increase leading to stabilization.

6. Conclusions

The linear stability of thermohaline convection in a ferrofluid layer heated from below and salted from above saturating a densely packed porous medium subject to a transverse uniform magnetic field has been considered with the effect of rotation. In this investigation, the effect of various parameters such as permeability of the porous medium, non – buoyancy magnetization, buoyancy magnetization, Prandtl number, ratio of mass transport to heat transport, Rayleigh number, Taylor number and salinity Rayleigh number on the onset of convection has been calculated. The thermal critical magnetic Rayleigh numbers for the onset of instability are also determined numerically for sufficiently large values of the buoyancy magnetization parameter M_1 and results are depicted graphically. If $R^{oc} > R^{sc}$, then the system stabilizes through stationary mode. If $R^{oc} < R^{sc}$, then the system stabilizes through oscillatory mode, where R^{oc} and R^{sc} are the critical thermal Rayleigh numbers for the oscillatory and stationary convection system.

In conclusion, Figs 1 to 4 illustrate the critical thermal Rayleigh number N^{sc} versus M_3, k, and τ for different values of τ and Ta. But Figs 5 to 9 indicate the critical magnetic Rayleigh number R^{sc} versus St, R_S, Log Ta, and k for different values of M_3 and Ta. For the oscillatory convection, the Taylor number Ta has a stabilizing effect on various values of k and τ which are studied in Figs 2 – 4. But for various values of M_3, the convective system has a destabilizing effect which is analyzed in Fig.1. For stationary convection, the Taylor number Ta has a stabilizing effect for various values of St and R_S which are studied in Figs 5 – 6. But for various values of k, the convective system has a destabilizing effect which is analyzed in Figs 8 – 9. Figure 7 shows a stabilizing effect for Log Ta and a destabilizing effect for M_3, which are not much pronounced. Thus, rotation tends to stabilize the system.

Furthermore, the principle of exchange of instability is applied to find out the mode of attaining instability. It is found that the system stabilizes through stationary mode for values of Ta $\leq 10^3$ and for oscillatory mode instability is favored for Ta$> 10^3$. Thus convection is favoured in a ferromagnetic fluid by means of spatial variation in magnetization, which is induced when the magnetization of the ferrofluid depends on temperature and salinity.

Nomenclature

B – magnetic induction

$C_{v,H}$ – effective heat capacity at constant volume and magnetic field (kJ/m^3K)

D/Dt – convective derivative $s^{-1} [D/Dt = \partial/\partial t + \boldsymbol{q}.\nabla]$

d – thickness of the fluid layer *m*

$\boldsymbol{g}$ – gravitational acceleration *(0, 0, -g) ms^{-2}*

H – magnetic field *amp/m*

K – mass diffusivity

K – pyromagnetic coefficient $\left[\equiv -(\partial M/\partial T)_{H_0,T_0} \right]$

K_1 – thermal diffusivity *W/mK*

K_2 – salinity magnetic coefficient $\left[\equiv (\partial M/\partial S)_{H_0,T_0} \right]$

K_s – concentration diffusivity *W/mkg*

k – permeability of the porous medium

k_0 – resultant wave number $\left(k_0 = \sqrt{k_x^2 + k_y^2}\right) m^{-1}$

kx, k_y – wave number in the x and y direction m^{-1}

M – magnetization $Ampm^{-1}$

M_0 – mean value of the magnetization at $H = H_0$ and $T = T_0$

p – hydrodynamic pressure (N/m^2)

q – velocity of the ferrofluid (u, v, w) ms^{-1}

S – solute concentration kg

S_T – Soret coefficient

T – temperature K

Ta – Taylor number

t – time s

α_t – coefficient of thermal expansion K^{-1}

α_s – analogous solvent coefficient of expansion K^{-1}

β_t – uniform temperature gradient Km^{-1}

β_s – uniform concentration gradient kgm^{-1}

θ – perturbation in temperature (K)

μ – dynamic viscosity $kgm^{-1}s^{-2}$

μ_0 – magnetic permeability of vacuum

ρ – density of the fluid kgm^{-3}

ρ_0 – mean density of the clean fluid kgm^{-3}

σ – growth rate s^{-1}

φ – viscous dissipation factor containing second order terms in velocity

ϕ – magnetic scalar potential Amp

χ – magnetic susceptibility $\left[\equiv (\partial M / \partial H)_{H_0, T_0}\right]$

Ω – angular velocity

∇ – vector different operator $\left[\equiv \boldsymbol{i}(\partial / \partial x) + \boldsymbol{j}(\partial / \partial y) + \boldsymbol{k}(\partial / \partial z)\right]$

References

[1] Finlayson B.A. (1970): *Convective instability of ferromagnetic fluids.* – J. Fluid Mech., vol.40, pp.753-767.

[2] Das Gupta M. and Gupta A.S. (1979): *Convective instability of a layer of a ferromagnetic fluid rotating about a vertical axis.* – Int. J. Engng. Sci., vol.17, pp.271-277.

[3] Zhan M. and Rosensweig R.E. (1980): *Stability of magnetif fluid penetration through a porous medium with uniform magnetic field oblique to the interface.* – IEEE Transactions of Magnetic, vol.16, pp.275-282.

[4] Sekar R., Vaidyanathan G. and Ramanathan A. (1993): *The ferroconvection in fluids saturating a rotating densely packed porous medium.* – Int. J. of Engng. Sci., vol.31, pp.241-50.

[5] Sekar R. and Vaidyanathan G. (1993): *Convective instability of a magnetized ferrofluid in a rotating porous medium.* – Int. J. of Engng. Sci., vol.31, pp.1139-1150.

[6] Vaidyanathan G., Sekar R. and Ramanathan A. (1998): *Effects of rotation and anisotropy of a porous medium on ferroconvection.* – Ind. J. of Engng. and Mater. Sci., vol.5, pp.436-440.

[7] Sekar R., Vaidyanathan G. and Ramanathan A. (1998): *Effects of rotation on ferrothermohaline convection saturating a porous medium.* – Ind. J. Engng. and Mater. Sci., vol.5, pp.445-452.

[8] Sekar R., Vaidyanathan G. and Ramanathan A. (2000): *Effect of rotation on ferrothermohaline convection.* – J. of Magn. and Magn. Mater., vol.218, pp.266-272.

[9] Hurle D.T.J. and Jakemann E.J. (1971): *Soret-driven thermo solutal convection.* – J. Fluid Mech., vol.47, pp.667-687.

[10] Ryskin A., Muller H.W and Pleiner H. (2003): *Thermo diffusion effects in convection of ferrofluids.* – Magnetohydrodynamics, vol.39, pp.51-55.

[11] Vaidyanathan G., Sekar R., Hemalatha R., Vasanthakumari R. and Senthilnathan S. (2005): *Soret-driven ferrothermohaline convection.* – J. of Magn. and Magn. Mater., vol.288, pp.460-469.

[12] Shevtsova M., Melnikov D.E. and Clegros J. (2006): *Onset of convection in Soret driven instability.* – Phys. Rev., E.73, pp.047302.

[13] Sekar R., Vaidyanathan G., Hemalatha R. and Senthilnathan S. (2007): *Effect of Coriolis force on a Soret driven ferrothermohaline convective system.* - Int. J. of Math. Sci., vol.6 pp.429-441.

[14] Vaidyanathan G., Sekar R. and Hemalatha R. (2007): *Effect of coriolis force on a soret driven ferrothermohaline convection in a medium of sparse particle suspension.* – Ind. J. of Pure and Appl. Phys., vol.45, pp.666-673.

[15] Sunil, Chand P., Mahajan A. and Sharma P. (2011): *Effect of rotation on double diffusive convection in a magnetized ferrofluid with internal angular momentum.* – J. Appl. Fluid Mech., vol.4, No.4, pp.43-52.

[16] Shivakumara I.S., Sureshkumar S. and Devaraju N. (2011): *Coriolis effect on thermal convection in a couple stress fluid saturated rotating rigid porous layer.* – Arch. of Appl. Mech., vol.81, pp.513-530.

[17] Ram P., Kumar V. (2012): *Ferrofluid flow with magnetic field dependent viscosity due to rotating disk in porous medium.* – Int. J. Appl. Mech., vol.4, No.4, pp.1250041.

[18] Ram P. and Sharma K. (2014): *Effect of rotation and MFD viscosity on ferrofluid flow with rotating disk.* – Ind. J. Pure and Appl. Phy., vol.52, pp.87-92.

[19] Sekar R., Vaidyanathan G., Hemalatha R. and Senthilnathan S. (2006): *Effect of sparse distribution pores in a Soret-driven ferro thermohaline convection.* – J. of Magn. and Magn. Mater., vol.302, pp.20-28.

[20] Sekar R., Raju K. and Vasanthakumari R. (2013): *A linear analytical study on Soret-driven ferrothermohaline convection in an anisotropic porous medium.* – J. of Magn. and Magn. Mater., vol.331, pp.122-128.

[21] Sekar R., Murugan D. and Raju K. (2013a): *Stability analysis of thermohaline convection in ferromagnetic fluid in densely packed porous medium with Soret effect.* – World J. of Engng., vol.10, pp.439-447.

[22] Sekar R. and Raju K. (2013): *Effect of magnetic field dependent viscosity on Soret-driven thermoconvective instability of ferromagnetic fluid in the presence of rotating anisotropic porous medium of sparse particle suspension.* – Int. J. of Math. Sci., vol.12, pp.13–31.

[23] Sekar R. and Raju K. (2014): *Effect of magnetic field dependent viscosity on Soret-driven ferrothermohaline convection saturating an anisotropic porous medium of sparse particle suspension.* – World J. of Engng., vol.11, pp.213-228.

[24] Sekar R. and Raju K. (2014a): *Effect of Soret and temperature dependent viscosity on thermohaline convection in a ferrofluid saturating a porous medium.* – Int. J. of Appl. Mech. and Engng., vol.19, pp.321-336.

[25] Sekar R., Raju K. and Vasanthakumari R. (2013b): *Linear stability analysis of coriolis force on ferrothermohaline convection saturating an anisotropic porous medium with Soret effect.* – Global J. of Math. Analy., vol.1, pp.37-47.

[26] Sekar R., Murugan D. and Raju K. (2016): *Densely distribution pores and Coriolis force on thermohaline convection in a ferrofluid with Soret and anisotropy effects.* – Int. J. Advanced Sci. and Tech., vol.87, pp.15-30.

[27] Sekar R., Murugan D. and Raju K. (2016a): *Ferrothermoconvective instability in Soret driven convection saturating a densely packed anisotropic porous medium.* – Int. J. Appl. Math. Electronics and Computers vol.4, No.2, pp.58-64.