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APPLICATION OF TWO CONDITIONS OF LOSS OF STABILITY IN ANALYSIS OF THE TUBE BENDING PROCESS

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In this paper, the derivation of expressions for admissible values of strains and stresses for vertex points of layers subjected to tension during tube bending at bending machines is presented. The conditions of the dispersed and located loss of stability of the bent tube were assumed as criteria of instability. The original element of this paper is the extension of the criterion of strain location in a form of possible initiation of a neck or furrow (introduced by Marciniak for thin plates [1]) to bending thin- and thick-walled metal tubes at bending machines. The conditions of the dispersed and localized loss of stability together with formation of the plane state of deformation (PSD) in the plane stress state (PSS) were assumed as the criteria of instability. The calculation results were presented as graphs being useful nomograms. We present also simple examples of calculations of permissible and critical strains and values of bending angles including and not including displacement of the neutral axis y_0, during cold bending metal thin-walled tubes at bending machines for bending angles $<0^o; 180^o>$.

Key words: allowable strains and stresses, bending angles, neutral layer, wall thickness of elbows.

1. Introduction

Tube bending, see e.g. (Śloderbach [2]; Śloderbach and Strauchold [3]; Beskin [4]; Boyle [5]; Dzidowski [6]; Dzidowski and Strauchold [7]; Franz [8]; Gruner [9]; Grunow [10]; Korzemski [11], [12]; Pesak [13]; Śloderbach and Rechul [14]; Śloderbach and Pajak [15]; Śloderbach [16], [17], [18]; Tang [19]; UDT [20]; Wick *et al*. [21]; Zhang *et al*. [22]; Zhiqiang *et al*. [23]; Zdankiewicz [24], [25]; Dobosiewicz and Wojczyk [26]; Śloderbach and Pajak [27]; Śloderbach [28], [29]) as a technological problem appeared in the end of the 19th century when production of tubes started on industrial scale. Tubes were delivered mainly to industry of steam engines and boilers, gas engineering, power engineering, civil engineering. At present, tubes and elbows are purchased by almost all branches of industry and tube bending is a typical activity in many technological processes in metal industry. Production of tubes and elbows is increasing more rapidly than production of steel because tubes and elbows are made also of other materials, for example plastics. Higher requirements concerning the quality of produced tubes and elbows must be met. The choice of a tube bending method is dependent on a kind of material, thickness of the tube, bending radius, the required accuracy and quality of bending, work conditions, bend angle, serial production and others.

In Śloderbach [18] a generalized model of strain during metal tubes bending on bending machines was derived. In the considered case, the tubes were bent with the wrapping method at a rotating template with the use of a lubricated steel mandrel. The model contains three strain components in an analytical form, including displacement of the neutral axis. The derived strain scheme satisfies initial and boundary kinematic conditions of the bending process, conditions of continuity and compatibility of strains. The obtained analytical expressions can be classified as kinematically admissible. The present paper is a further development of Śloderbach [18].

Tube bending on a bender is usually made by the method of wrapping at a rotating template using a suitable mandrel, see Fig.1.

Pipelines and tube installations can be operated during a definite time of life and safe work. The pipelines and tube installations contain straight parts, elbows, pipe fittings (tees, four-way pieces, reducing pipes, nozzles, etc.) and connecting elements, for example welds, screw joints and others. Their lifetimes are different. Lifetime of straight parts is the longest, next there are elbows and pipe fittings and lifetime of connecting elements such as welds is the shortest (Śloderbach [2]; Śloderbach and Strauchold [3]; Dzidowski [6]; Dzidowski and Strauchold [7]; Śloderbach and Rechul [14]; Śloderbach and Pajak [15]; Śloderbach [16], [17]; Dobosiewicz and Wojczyk [26]; Śloderbach and Pajak [27]; Życzkowski and Skrzypek [30]).

Premature damages of the elbows occurring during operation of the elbows can also be caused by applying an unsatisfactory method of strength calculations, see e.g. (Dzidowski [6]; Dzidowski and Strauchold [7]; Śloderbach and Rechul [14]; Śloderbach [16]; Tang [19]; UDT [20]; Dobosiewicz and Wojczyk [26]; Życzkowski and Skrzypek [30]). Such a situation can be caused by the lack of a precise method of determination of permissible distribution of the wall thickness at the points of maximum strains during the elbow bending. It concerns especially top parts of the elbow bending zone where this thickness is minimal. If the strain components and intensity in the bending zone, especially in the top part are well known, better calculations of strength of elbows and their better manufacturing will render it possible to improve reliability of machines and devices.

At present, tube bending with bending machines is usually performed applying the method of wrapping at a rotating template with the use of a suitable mandrel, see Fig.1.

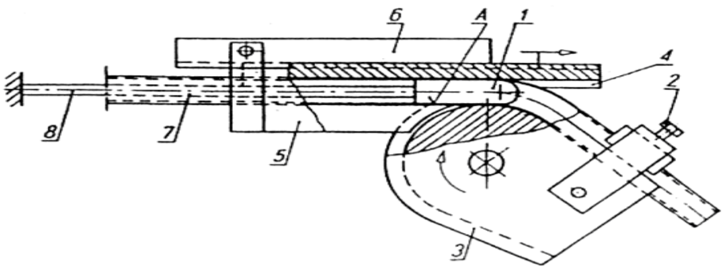

Fig.1. Scheme of a tube bending machine with a rotating template and a mandrel, 1-mandrel, 2-clamp bolt, 3-rotating template, 4-sliding slat, 5-planisher, 6-bed of sliding slat, 7-bending pipe, 8-mandrels rod.

Tube bending with the considered method always causes a reduction of the wall thickness in the layers subjected to tension, increase of wrinkling and the wall thickness in the layers subjected to compression, and deformation (ovalization) of the cross section. These unfavourable phenomena should be included into the tolerance limits given in the European and Polish Standards (UDT [20]; Zdankiewicz [25]; EN [31]). The reduction of the wall thickness and negative influence of large strains at the top points of the elbow are the most important factors influencing the operation life of the elbow (Dzidowski [6]; Dzidowski and Strauchold [7]; Korzemski [11], [12]; Śloderbach and Rechul [14]; Śloderbach [16], [17]; Tang [19]; Dobosiewicz and Wojczyk [26]; Śloderbach and Pajak [27]).

In this paper, previous ideas and concepts of (Marciniak [1]) are developed. The development consists in an extension of the stability loss condition in the localized form (possible initiation of the furrow) derived by (Marciniak [1]) for uniaxial and biaxial tension of sheets to the cases of bending thin- and thick-walled metal tubes at the bending machines, in particular with the use of the rotational template and the mandrel, Fig.1.

In papers (Śloderbach [2]; Śloderbach and Strauchold [3]; Śloderbach and Rechul [14]; Śloderbach and Pajak [15]; Śloderbach [16], [17], [18]) it was assumed that the bending of thin-walled metal tubes at the bending machines in the layers subjected to tension (tube bending with the method of wrapping at the rotational template using the mandrel) is a complex process of heterogeneous curvilinear tension (curvilinear wrap forming) under the biaxial stress state. In the case of the layers subjected to compression, it is something like a combination of heterogeneous curvilinear tension and bounded upsetting, respectively. In this paper, the authors analyse also the influence of parameters of hardening and normal anisotropy on admissible values of the bending angle, strains and stresses occurring in the layers subjected to tension during cold bending of thin- and thick walled metal tubes at the bending machines in the range of the bending angle $\alpha_b \in \langle 0^\circ, 180^\circ \rangle$ and for the case when $y_0 \geq 0$, where y_0 is the displacement of the neutral axis of plastic bending (Śloderbach [16], [17], [18], [19]; Wick [21]) and Fig.2. Moments of possible occurrence of the stability loss in the dispersed form for the case of uniaxial tension and occurrence of the localized stability loss (for example in a form of local initiation of the external furrow) under biaxial stress state were assumed as the criteria (Marciniak [1]; Śloderbach and Rechul [14]; Śloderbach [16], [17]; El-Sebaie and Mellor [32]; Gabryszewski and Gronostajski [33]; Hill [34]; Moore and Wallace [35]; Śloderbach [36]; Erbel *et al*. [37]; Marciniak and Kołodziejski [38]; Olszak *et al*. [39]; Swift [40]; Szczepiński [41]). For the case of thin-walled tubes, the problem of occurrence of the plane state of deformation (PSD) under the plane stress state (PSS) was also considered (Marciniak [1]; Śloderbach and Rechul [14]; Śloderbach [16], [17]; El-Sebaie and Mellor [32]; Gabryszewski and Gronostajski [33]; Moore and Wallace [35]; Śloderbach [36]; Erbel *et al.* [37]; Marciniak and Kołodziejski [38]; Olszak *et al*. [39]; Swift [40]; Szczepiński [41]. Analytical calculations were made for two extreme cases, i.e. for a generalized scheme of strain and simplification of the 3rd type (Śloderbach [2]; Śloderbach and Strauchold [3]; Śloderbach and Rechul [14]; Śloderbach [16], [18], [28]), because the calculation results are received from simplifications of the 1st and 2nd types (Śloderbach and Strauchold [3]; Śloderbach [16], [18], [28]) will be included in those two extreme cases.

The authors concentrate only on the analysis of the top points of the layers subjected to tension because from the previous experiments and operating tests it appears that the process of elbow damage in most pipelines (especially those used in energy engineering) usually starts and develops at external top points of the layers subjected to tension in the bending zone. From the collected statistical data (Dzidowski [6]; Dzidowski and Strauchold [7]; Śloderbach [16], [17]; Zdankiewicz [24], [25]; Dobosiewicz and Wojczyk [26]; Śloderbach and Pajak [27]) it appears that an average lifetime of elbows of the pipelines loaded by internal pressure and operating at elevated temperatures is shorter than that of straight intervals of the pipelines by *30*% and more. Thus, occurrence of stability loss states (especially those localized) during tube bending causes a further drop of the operating life. It is recommended in this paper to prevent such states in technology of metal tube bending for pipeline elbows (depending on their application). In the paper you can find an expression for displacement and a new position of the neutral axis of plastic bending y_0 for the considered elements of the stability loss and for the bending angles included in the range $\left(k\alpha_b \leq 180^\circ\right)$.

Then computational values of substitute deformations depending on the rate of the hardening coefficient n and depending on the rate of the normal anisotropy coefficient r are presented. The calculations were performed for the case of the formation of the plane state of the deformation PSD in conditions of the plane state of stress PSS. The formation of PSD in PSS conditions results from initiation of located loss of the stability state (e.g. in the form of the appearance of the local groove). The material of the pipe is accepted as rigid-plastic with the isotropic hardening.

The paper presents also exemplary calculations of some selected states of loss of stability during cold bending of thin-walled metal tubes at bending machines. The conditions of the dissipated and localized loss of stability together with formation of the plane state of deformation PSD in the plane stress state PSS and the cracking criterion based on the technological index A_5 were assumed as the criteria of unstability. The calculations were performed for a generalized model of strain and simplification of type 3 (Śloderbach and Strauchold [3]; Śloderbach and Rechul [14]; Śloderbach [16], [18], [28], [29]). It appears that there are two extreme cases: the one (generalized model) providing the greatest strains and bending angles and the simplification of type 3 providing the lowest strains and bending angles. For simplifications of types 1 and 2 (Śloderbach and Strauchold [3]; Śloderbach [16], [28], [29]) the values of permissible strains and bending angles are included between those extreme cases. The calculations were performed for the top points of the elbow where strains are the greatest and the wall thickness is the smallest. The calculation results are presented as graphs being useful nomograms.

2. Fundamental assumptions and relationships

The analytical-geometrical description and analysis of the process concerning tube bending with the use of the wrapping method at the rotating template with or without the mandrel, assuming that $d_{\text{int}} \cong \text{const}$

(admissible ovalization is below *6%*, according to (Zdankiewicz [25]; EN [31]), is presented. The analytic description of deformation is limited to determination of the plastic strain state because elastic strains are very small and they can be neglected.

Let us take into account the experimental data presented in paper (Franz [8]) and the data derived in (Śloderbach [16], [18]), i.e. the generalized logarithmic components of the strain state (including displacement of the neutral axis of plastic bending y_0) in the layers subjected to tension and compression; then we can write the following expressions

$$\varphi_1 \cong \lambda_j \ln \frac{2\left(R-y_0\right) \pm \left(d_j \cos\beta_j \pm 2\, y_0\right)\left(\cos(k\alpha) - \cos\left(k\frac{\alpha_b}{2}\right)\right)}{2\left(R-y_0\right)},$$

$$\varphi_2 \cong \ln\frac{d_j}{d_{ext}}, \qquad \varphi_3 \cong \ln\frac{g_j}{g_0} \tag{2.1}$$

where: d_j – local "actual diameter" of the bent elbow, $d_j = 2\, r_j$ ($j = 1$ and sign (+)for the elongated layers, $j = 2$ and sign (-) for the compressed layers),

g_0 and g_j - initial thickness of the tube and local actual thickness of the elbow wall in the bending and bent zone,

k – technological-material coefficient determined from experimental results, which defines a range of the bending zone in the bend zone, so that $k\alpha_b = 180^\circ$. From the theoretical point of view $k \in \langle 1; \infty \rangle$. For practical purposes we can assume that $k \in \langle 1; 6 \rangle$. Based on the known test results we can approximately assume that $k \in \langle 1 \div 3 \rangle$ (Śloderbach [2]; Franz [8]; Korzemski [11], [12]; Śloderbach and Rechul [14]; Śloderbach [16], [17], [18]). For the elbows bent at the bending angle $\alpha_0 = 180^\circ$, the coefficient k means the ratio of the bending angle α_0 to the real bend angle α_b, i.e. $k \cong \alpha_0/\alpha_b$. When $\alpha_0 = 180^\circ$, then $\alpha_0 = k\alpha_b = 180^\circ$. When, for example, $\alpha_b = 90^\circ$, then $k = 2$; when $\alpha_b = 60^\circ$, then $k = 3$ etc.

R – nominal radius of tube bending,

R_j and R_0 - "big active actual radius of bending" connected with longitudinal strain and the radius determining the actual position of the neutral layer, respectively,

r_j - "small active actual radius" of the elbow in the bending zone, $r_j = r_{int} + g_j$ and $d_j = 2r_j$,

r_{ext} and d_{ext}- external radius and diameter of the tube subjected to bending, respectively, $d_{ext} = 2\, r_{ext}$,

r_{int} and d_{int}- internal radius and diameter of the tube, respectively, $d_{int} = 2\, r_{int}$,

y_0–displacement of the neutral layer of plastic bending,

index $j = 1$ and sign (+) in Eqs (2.1) relate to the layers subjected to tension, $y_0 \in \langle 0; 1 \rangle$,

index $j = 2$ and sign (-) in Eqs (2.1) relate to the layers subjected to compression.

Greek symbols

$\alpha_b \equiv \alpha_g$ - bending angle measured in the bending zone. In the bending zone, the angles of bending and bend are equal, so $\alpha_b = \alpha_0$, where α_0 – bend angle (angle of rotation of the template of the bending machine),

α, β - angles of the point position in the bending zone,

β - angle of circulation of the layers subjected to tension and compression of the elbow, $\beta_j \in \left\langle 0; 90^\circ \pm \beta_0 \right\rangle$ and $\sin\beta_0 = y_0/r_{ext}$, where β_0 – angular range of displacement of the neutral axis of bending,

λ_i - correction coefficient (technological-material) of strain distribution in the layers subjected to tension (i=1) and compression ($i = 2$) of the bending and bend zone, defined from experimental results so that $\lambda_1 \cong 1$

and $\lambda_2 \in <0; 1>$. In the case of most known tests we can approximately assume that $\lambda_2 \approx 0.5$ (Śloderbach [2]; Franz [8]; Śloderbach and Rechul [14]; Śloderbach [16]).

In the strain model (1) we can introduce three simplifications of physical sense. Simplification 1 is obtained by introduction of ($d_j = d_{\text{ext}}$) into Eq.(2.1)$_1$. In simplification 2, we introduce ($d_j = d_{ext}$) into Eq.(2.1)$_2$ (then $\varphi_2'' = 0$) and in the case of simplification 3 we introduce ($d_j = d_{\text{ext}}$) into Eq.(2.1)$_1$ and Eq.(2.1)$_2$ (then $\varphi_2''' = 0$), see (Śloderbach [16], [28], [29]). The simplifications are denoted adequately by one, two or three signs: ('), ('') or (''') respectively (Śloderbach and Strauchold [3], Śloderbach [16], [28], [29]). The expression for strain intensity and plastic incompressibility of the material takes the following form (Marciniak [1], Śloderbach [2], [16]; Erbel *et al.* [37]; Marciniak and Kołodziejski [38])

$$\varphi_{(i)} = \sqrt{\frac{2}{3}\left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2\right)}, \qquad \varphi_1 + \varphi_2 + \varphi_3 = 0. \tag{2.2}$$

Expressions (2.1) and (2.2) are used for description of the strain state of the tube subjected to bending in the top, $\cos(k\alpha) = 1$, and external, $\cos\beta = 1$ points of the layers subjected to tension or compression where $\varphi_1, \varphi_2, \varphi_3$ - logarithmic components of plastic strains, $\varphi_{(i)} \equiv \varphi_i$ – intensity of logarithmic plastic strains. For simplifications of the 1, 2 or 3 order we have that $\varphi_i' \equiv \varphi_{(i)}', \varphi_i'' \equiv \varphi_{(i)}'', \varphi_i''' \equiv \varphi_{(i)}'''$.

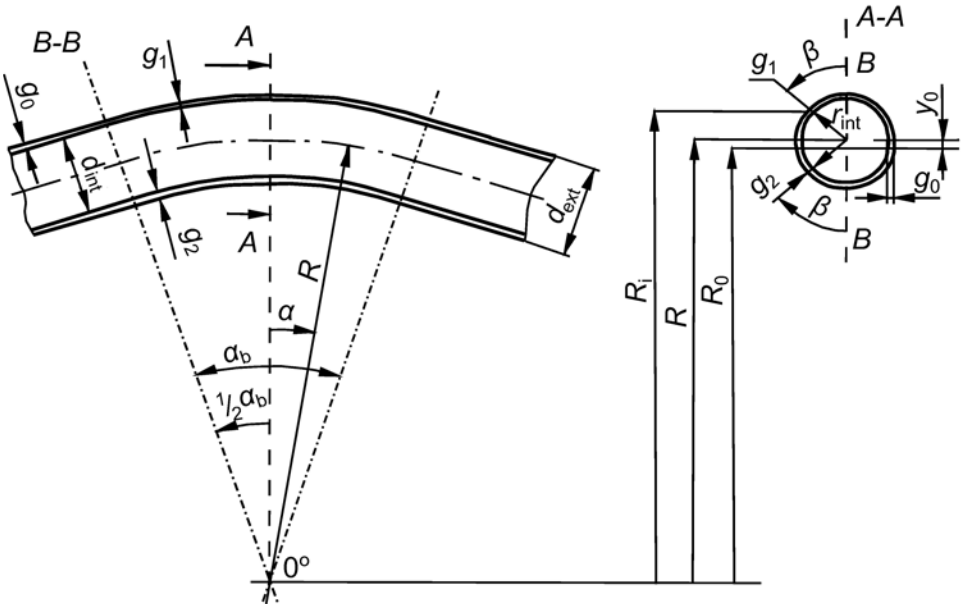

Fig.2. Geometrical quantities pertaining to pipe-bending processes.

Permissible values of the bending angle and the substitute strain are determined for suitable moments of stability loss occurrence (Śloderbach and Rechul [14]; Śloderbach [16]; El-Sebaie and Mellor [32]; Gabryszewski and Gronostajski [33]; Hill [34]; Moore and Wallace [35]; Śloderbach [36]; Swift [40]; Szczepiński [41]). It is obvious that the range of plastic strains seen from the point of view of application in plastic work processes is limited because of the possibility of material stability loss or coherence loss, i.e. cracking. Stability loss usually occurs as the first one, however, we are able to select suitable material properties, strain conditions and production technology where the fracture does not occur before stability loss

(Marciniak [1]; Dzidowski and Strauchold [7]; Śloderbach [16], [17]; Marciniak and Kołodziejski [38]). In this paper, the author concentrated only on the analysis of moments of possible occurrence of stability loss in the dispersed form (Śloderbach [16], [17]; Gabryszewski and Gronostajski [33]; Śloderbach [36]) and in the localized form at a top point of elbow (e.g. a beginning of local initiation of the neck or furrow (Marciniak [1]; Śloderbach and Rechul [14]; Śloderbach [16], [17]; El-Sebaie and Mellor [32]; Gabryszewski and Gronostajski [33]; Hill [34]; Moore and Wallace [35]; Olszak *et al*. [39]; Swift [40]; Szczepiński [41]) under biaxial stress state.

The case of initiation of the plane state of deformation (PSD) in the plane stress state (PSS) was also considered. Let as assume that the tube is made of a rigid-plastic material with isothropic hardening which satisfies the condition of the Huber-Mises-Hencky (H-M-H) condition of plasticity and the plastic flow laws formulated by Levy-Mises. Let us also assume that displacement of the neutral axis $y_0 \geq 0$, and that cold tube bending is performed at the bending machine at ambient temperature. It is a quasi-static and quasi-isothermic process. Thus, dynamic and thermal effects accompanying small and great plastic deformations are not taken into account (Raniecki and Sawczuk [42], [43]; Marciniak [44]; Nowacki [45]).

In the present considerations the following form of the hardening curve was assumed (Marciniak [1]; Swift [40])

$$\sigma_p = D\left(\varphi_0 + \varphi_{(i)}\right)^n \tag{2.3}$$

where σ_p – yield stress in [MPa], n – coefficient of hardening,
D – material constant in [MPa], φ_0 - logarithmic initial strain,
$\varphi_{(i)} \equiv \varphi_i$ - logarithmic substitute strain (strain intensity).

For most metals and alloys applied in engineering practice the value of the coefficient n is in the range $\langle 0.0 \div 0.6 \rangle$.

3. The cases of stability loss under consideration

Let us consider three special cases of stability loss in the tube bending process.

a. Uniaxial tension

Loss of stability in the dispersed form (maximum drawing force) (Gabryszewski and Gronostajski [33])

From above mentioned papers, it follows that

$$\varphi_{(i)a} = n - \varphi_0 \tag{3.1}$$

where $\varphi_{(i)a} \equiv \varphi_{ia}$ – substitute strain corresponding to this stability loss.

b. Biaxial stress state (Marciniak [1]; Śloderbach and Rechul [14])

Stability loss in the form of localized deformation when locally $d\left(\sigma_p g_1\right) = 0$, see (Marciniak [1])

Following (Marciniak [1]; Śloderbach [16], [17]; El-Sebaie and Mellor [32]; Moore and Wallace [35]; Swift [40]) and taking into account expressions for principal components of the strain state during tube bending (1), expressions for equivalent strain $(2)_1$ and plastic incompressibility $(2)_2$ and assuming that d_{int} = const. (d_{int} – internal diameter of the bent tube) and $\left(d\varphi_3 = dg_1/g_1\right)$, after transformations we obtain

$$\varphi_{(i)b1} \cong \sqrt{\frac{(1+r)\left[8\left(\frac{g_1}{d_1}\right)^2 + 4\left(\frac{g_1}{d_1}\right) + (1+r)\right]}{(1+2r)}}\, n - \varphi_0 , \qquad \varphi'''_{(i)b2} = \frac{1+r}{\sqrt{1+2r}} n - \varphi_0 . \tag{3.2}$$

Assuming some simplifying expressions, such as $\left(g_1/d_1 \approx g_0/d_{ext} = s^*\right)$, we get the influence of the pipe geometry parameter s^* on values of appropriate (admissible) strains and stresses and we obtain

$$\varphi_{(i)b1} \approx \sqrt{\frac{(1+r)\left[8(s^*)^2 + 4s^* + (1+r)\right]}{(1+2r)}}\, n - \varphi_0 \quad \text{and} \quad \varphi'''_{(i)b2} = \frac{1+r}{\sqrt{1+2r}} n - \varphi_0 \tag{3.3}$$

where $\varphi_{(i)b1}$ – values of the substitute strain corresponding to the generalized model of strain during metal tubes bending at bending machines (Śloderbach [16], [17], [18]) for this form of stability loss,

$\varphi'''_{(i)b2}$ for simplification 3 during the considered stability loss (Śloderbach and Strauchold [3]; Śloderbach [16], [17]),

g_1 and d_1- real thickness and diameter of the elbow in the layers subjected to tension,

s^* - thin-walled parameter of the bent tube defined as ($s^* = g_0/d_{ext}$), see (Korzemski [12]; Śloderbach [16], [17]).

s^*_w - thin-walled parameter of the bent tube defined as $\left(s^*_w = g_0/d_{\text{int}}\right)$, see (Dzidowski [6]; Śloderbach [18]; UDT, [20]), then $s^*_w = s^*/\left(1-2s^*\right)$, g_0 and d_{ext} – initial thickness and external diameter of the bent tube.

The parameter r is the coefficient of Lankford normal anisotropy, see (Marciniak [1]; Gabryszewski and Gronostajski [33]) which can be written as

$$r = \frac{\varphi_2}{\varphi_3} = \frac{\ln\frac{b}{b_0}}{\ln\frac{g}{g_0}}, \quad \text{and for tubes} \quad r = \frac{\varphi_2}{\varphi_3} = \frac{\ln\frac{d}{d_{ext}}}{\ln\frac{g}{g_0}} \tag{3.4}$$

where: b_0 and b – specimen width before and after deformation,

d_{ext} and d – external diameter of the tube before and after deformation,

g_0 and g – specimen thickness before and after deformation.

From the above relationship it appears that when the coefficient r increases, then the reduction of the specimen thickness is lower, i.e. resistance to reduction of thickness of the tube wall increases. The coefficient r is in the range ($1.0 \div 2.5$) for most steels used for tube manufacturing.

The cases of stability loss (**a** and **b**) are applied here for estimation of the instability state of tubes of thin-walled parameter ($0.00 < s^*_w < 0.15$). In (Śloderbach [16], [17]) thin-walled parameter ($0.0 < s^* \le 0.1$) was assumed as a more suitable from a technological point of view and admissible one, determined as a certain geometric mean of the thin-walled parameter ($0.0 < s^* < 0.2$) according to (Korzemski [12]) and

($0.00 < s_w^* < 0.05$) according to papers (Dzidowski [6]; UDT [20]). The case of bending of thin-walled tubes ($s^* \approx 0.101$) is considered in (Franz [8]). In most cases (or even in all the cases) technologically thick-walled tubes ($s_w^* > 0.15$) of big diameters ($d_{ext} > 160\ mm$) are subjected to hot or semi-hot bending (Marciniak [44]). During thick-walled tube bending, there is the triaxial stress state in the tubes, so $(\sigma_1 \geq \sigma_2 \geq \sigma_3)$, $\sigma_3(r_{ext}) \cong \sigma_3(r_1) \cong 0$. Let us note that during hot bending the coefficient of normal anisotropy ($r \approx 1$), and there is also no material hardening (Szczepiński [41]; Marciniak [44]).

The Huber-Mises-Hencky (H-M-H) condition of plasticity and Levy-Mises equations of plastic flow (Marciniak [1]) for an isotropic body expressed in principal stresses take the following forms

$$\sigma_p = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}\ , \tag{3.5}$$

$$\frac{d\varphi_1}{\sigma_1 - \sigma_m} = \frac{d\varphi_2}{\sigma_2 - \sigma_m} = \frac{d\varphi_3}{\sigma_3 - \sigma_m} = \frac{d\varphi_{(i)}}{\frac{2}{3}\sigma_p} \tag{3.6}$$

where: $\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ - mean stress, σ_1, σ_2 and σ_3 – principal stresses, and φ_1, φ_2 and φ_3 - logarithmic principal plastic strains.

When the deformation process is proportional, then suitable strain increments can be replaced by overall strains (Marciniak [1]; Gabryszewski and Gronostajski [33]; Hill [34]; Olszak *et al.* [39]; Szczepiński [41]).

The stress state components corresponding to state (**b1**) and expressions $(3.2)_1$ or $(3.3)_1$ (assuming for simplification that, $g_1/d_1 \approx g_0/d_{ext} = s^*$ and for thick-walled tube bending by hot or semi-hot method [44], then $r \approx 1$) are obtained from the equations of plastic flow (3.6), taking into account the H-M-H condition of plasticity (3.5) with no effect of hardening ($\sigma_p \cong \text{const}$). Taking then into account the low value of radial stress $(\sigma_3(r_{ext}) \cong \sigma_3(r_1) \cong 0)$ on the external surfaces of the layers subjected to tension, after transformations we obtain

$$\sigma_1 \cong \frac{2(1+s)}{\sqrt{3\left[4(s)^2 + 2s + 1\right]}}\sigma_p, \quad \sigma_2 \cong \frac{1-2s}{\sqrt{3\left[4(s)^2 + 2s + 1\right]}}\sigma_p, \quad \sigma_3(r_{ext}) \cong \sigma_3(r_1) \cong 0. \tag{3.7}$$

The stress state components corresponding to state (**b2**), expressed by Eqs $(3.2)_2$ or $(3.3)_2$, for thick-walled tube bending (made also of the isotropic material, then $r \cong 1$), can be obtained in a similar way, including an additional condition resulting from determination of almost zero stress value σ_3''' on the external (unloaded) surfaces of the layers subjected to tension. Thus

$$\sigma_1''' \cong \frac{2}{\sqrt{3}}\sigma_p\ , \quad \sigma_2''' \cong \frac{\sigma_p}{\sqrt{3}}\ , \quad \sigma_3'''(r_{ext}) \cong \sigma_3(r_1) \cong 0\ . \tag{3.8}$$

As it can be seen, the cases (**b1**) determined for the generalized scheme of strain and simplifications of the 1st type, see expressions $(3.2)_1$, $(3.3)_1$) and (3.7), depend not only on material parameters (n, r, φ_0), but also on the geometric parameters (g_1 and d_1) of the bent tube (they approximately depend on a value of the parameter s^*, $s^* \approx g_1/d_1$). It appears that when thin-walled parameter s^* of the tube increases, then are right expressions $(3.2)_1$, $(3.3)_1$) on $\varphi_{(i)b1}$. Other cases (**b2**), see expressions $(3.2)_2$, $(3.3)_2$ and (3.8) derived for the scheme of simplifications 2 and 3 respectively (Śloderbach and Strauchold [3]; Śloderbach and Rechul [14]; Śloderbach [16], [17], [28]), do not depend on the geometric parameters of the bent tube (g_1 and d_1) or s^*. In the case when $s^* = 0.5$ (full rod), from the Eqs (3.6) and (3.7) it appears that $\sigma_1 = \sigma_p, \sigma_2 = \sigma_3 = 0$ and ($\varphi_2 = \varphi_3 = -0.5\varphi_1$), also that $\varphi_{(i)b1p} \cong (2n - \varphi_0)$.

General distribution of principal stresses at all the points of the bending zone can be obtained from expressions (2.3), (3.5) and (3.6), suitable equations of equilibrium including friction forces for a case of asymmetric (of variable thickness) thin- and thick-walled closed shell (or asymmetric interval of toroidal closed shell) (Szczepiński [41]).

The H-M-H condition of plasticity for PSS and a material with hardening and properties of normal anisotropy has the following form (Marciniak [1])

$$(1+r)\sigma_p^2 = (1+r)\sigma_1^2 - 2r\sigma_1\sigma_2 + (1+r)\sigma_2^2. \tag{3.9}$$

As for thin-walled tubes (where there is the biaxial stress state in the layers subjected to tension during the bending process), the Levy-Mises equations of plastic flow expressed in logarithmic measures of strain have the following form (Marciniak [1])

$$\frac{d\varphi_1}{(1+r)\sigma_1 - r\,\sigma_2} = \frac{d\varphi_2}{(1+r)\sigma_2 - r\,\sigma_1} = \frac{d\varphi_3}{-(\sigma_1+\sigma_2)} = \frac{d\varphi_{(i)}}{(1+r)\,\sigma_p}, \tag{3.10}$$

where $$d\varphi_{(i)} = \sqrt{\frac{(1+r)\left(d\varphi_1^2 + d\varphi_2^2 + d\varphi_3^2\right)}{1+2\,r}}.$$

The stress state components corresponding to states **b1** and **b2**, (obtained from the equations of plastic flow (3.10), including the hardening curve (2.3) and the condition of plasticity (3.9) and respectively from expressions $(3.3)_1$ and $(3.3)_2$) for a case of the material showing properties of normal anisotropy, have the following form

$$\sigma_1 \cong \frac{(1+r)(1+r+2s)}{\sqrt{(1+r)(1+2r)\left[8(s)^2+4s+(1+r)\right]}}\sigma_p \quad \text{and} \quad \sigma_1''' \cong \frac{1+r}{\sqrt{1+2\,r}}\sigma_p''',$$

$$\sigma_2 \cong \frac{(1+r)(r-2\,s)}{\sqrt{(1+r)(1+2r)\left[8(s)^2+4s+(1+r)\right]}}\sigma_p \quad \text{and} \quad \sigma_2''' \cong \frac{r}{\sqrt{1+2\,r}}\sigma_p'''. \tag{3.11}$$

When $s^* = 0.5$ (bent bar) then from Eqs (3.10), (3.11) and (3.3) it appears, as previously, that $\sigma_1 = \sigma_p, \sigma_2 = \sigma_3 = 0$ and ($\varphi_2 = \varphi_3 = -0.5\varphi_1$) and $\varphi_{(i)b1p} \cong (2n - \varphi_0)$.

Thus, expressions $(3.2)_1$, $(3.3)_1$ and $(3.11)_1$ are formal [for thin ($s^* < 0.05$) and thick $\left(0.05 \le s^* < 0.10\right)$ walled tubes] extension of the expressions obtained by (Marciniak [1]) for isotropic sheets and the stress state when ($0 \le \sigma_2/\sigma_1 \le 1/2$). Assuming that for plane sheets (the diameters (d_1 and d_{ext}) $\to \infty$, then $s^* \to 0$), from Eqs (3.2), (3.3) and $(3.11)_2$ we obtain Eqs $(3.2)_2$, $(3.3)_2$ and $(3.11)_2$, respectively.

The obtained expressions $(5)_2$, $(6)_2$ and $(14)_2$ have already been cited in many papers (Marciniak [1]; Śloderbach and Rechul [14]; El-Sebaie and Mellor [32]; Moore and Wallace [35]).

In the case of metal tube bending at bending machines, from the condition $d\left(\sigma_p \cdot g\right)=0$ we have obtained the effects not recognized so far.

- in the case of the generalized model of strains and the simplification of the 1st kind, for the external top point of the bent elbow where there is PSS we can state that
  - when $s^* \neq 0$, then the condition $d\left(\sigma_p \cdot g\right)=0$ corresponds to the stress states occurring in hyperbolic range on the H-M-H ellipse of plasticity, when $\left(\sigma_1 \text{ and } \sigma_2\right)>0$ and $d\varphi_2<0$. At these points of stresses a set of static of quasi-linear partial differential equations have a hyperbolic character (Olszak *et al.* [34]; Szczepinski [41]),
  - when $s^* = 0$ (internal surface of the bent elbow $g = 0$ in thickness) or the plane specimen when $d_{ext} \to \infty$. Then, the condition $d\left(\sigma_p \cdot g\right)=0$ refers to the parabolic point on the ellipse of plasticity where $\left(d\varphi_2=0\right)$ – there is initiation of PSD,
- in the case of simplifications of the 2nd and 3rd kind (Śloderbach and Strauchold [3]; Śloderbach [16], [28], [29]), for the top point of the bent elbow we obtain that the condition $d\left(\sigma_p \cdot g\right)=0$ refers to the point ***s*** on the ellipse of plasticity where $\left(d\varphi_2=0\right)$, it physically signifies the local initiation of PSD.

c. Formation of (PSD) under (PSS), see (Marciniak [1]; El-Sebaie and Mellor [32]; Moore and Wallace [35]; Śloderbach [36])

In such a case

$$\varphi_{(i)c}=nz-\varphi_0 . \tag{3.12}$$

We can write that $\left(\varphi_{(i)c} \equiv \varphi'''_{(i)c}\right)$, and

$\varphi_{(i)c}$ – value of the substitute strain corresponding to the loss of stability,

z – subtangent including influence of the stress σ_p on the moment of stability loss under conditions of the plane stress state and in the moment of formation of the plane state of deformation, so

$$z=\frac{1+r}{\sqrt{1+2r}} . \tag{3.13}$$

This state of stability loss refers to thin-walled tubes because of the assumed conditions resulting from the plane stress state.

4. Evaluations of the allowable bending angles and critical strains and stresses

Let us consider a case of formation of plane stress state under plane state of deformation, see Eqs (3.12) and (3.13).

Using expressions obtained in papers (Śloderbach and Rechul [14]; Śloderbach [16], [17]) for allowable values of bending angels α_b, depending on admissible values of deformations intensity in the stretched layers $\varphi_{(i)}$, we obtain

$$\cos\left(k\frac{\alpha_{ball}}{2}\right)=\frac{2R+d_1-2R\exp\sqrt{1.5\varphi_{(i)}^2-\left(\varphi_2^2+\varphi_3^2\right)}}{d_1}=\frac{2R+d_1-2R\exp\varphi_1}{d_1}, \tag{4.1}$$

$$\cos\left(k\frac{\alpha'''_{ball}}{2}\right)=\frac{2R+d'''_1-2R\exp\sqrt{1.5\varphi'''^2_{(i)}-\varphi'''^2_3}}{d_{ext}}=\frac{2R+d'''_1-2R\exp\varphi'''_1}{d_{ext}} \tag{4.2}$$

where α_{ball} - allowable values of the bending angle calculated from expressions obtained for the generalized model of deformations, cf. (Śloderbach [2], [16], [17]), when the value of intensity of deformation $\varphi_{(i)}$ reaches permissible value $\varphi_{(i)all}$, Eqs $(3.2)_1$ or $(3.3)_1$,

α'''_{ball} - allowable values of the bending angle calculated from the model of deformation of 3rd-order (Śloderbach and Strauchold [3]; Śloderbach and Rechul [14]; Śloderbach [16], [17], [28], [29]), when intensity value of the deformation $\varphi'''_{(i)}$ reaches the allowable value $\varphi'''_{(i)all}$, obtained from expressions $(3.2)_2$ or $(3.3)_2$ and (3.12),

d_1 and d'''_1 - current diameters at points of top stretched layers of the knee for the generalized model of deformations and simplification of the 3rd order respectively (Śloderbach and Strauchold [3]; Śloderbach and Rechul [14]; Śloderbach [16], [17], [28], [29]).

The expressions presented in papers describing the generalized scheme of deformations and the simplifications of type 3 can be used for the analysis of allowable strain quantity. These relationships allow the description of initiation of the plane state of deformation in the plane stress state. It has been shown that the method resulting from the simplification of type 3 determines the safest values for admissible strains, stresses, bending angles or wall thickness. It means that for a given admissible value of the strain intensity $\varphi_{(i)all}$ in the bending zone, admissible values of the bending angle are lower than these resulting from the generalized model of simplification of type 2 and 3 (Śloderbach [2]; Śloderbach and Strauchold [3]; Śloderbach and Rechul [14]; Śloderbach [16], [17], [18], [28], [29]). Thus, a description resulting from simplification of type 3 (Śloderbach and Rechul [14]) will be used as the safest and corresponding to the formation of PSD.

Analysis of the problem for uniaxial tension of the layers of the bent tube is widely known and here it is limited to a solution of one simple example. Substituting (3.13) to (3.12), we obtain

$$\varphi'''_{(i)cr}=\frac{1+r}{\sqrt{1+2r}}\cdot n-\varphi_0\,. \tag{4.3}$$

In the case of initiation of PSD during PSS, condition (3.5) is equal to condition $(3.3)_2$.

From Eq.(4.3) we can calculate the value of $\varphi'''_{(i)cr}$ and then from Eq.(4.2) we can determine the critical value of the bending angle α'''_{bcr}, which corresponds to the beginning of stability loss (local initiation of PSD in PSS). It is widely known that the top (centre) of the layers subjected to tension is the most deformed in the bending zone ($\alpha=\beta=0^\circ$, see Fig.2.), so the beginning of the loss of stability should be expected there. Thus, the derived relationships applied to the top point of the layers subjected to tension for $(\lambda_1=1)$ can be applied in the following way: the values of $\varphi'''_{(i)cr}$ are determined from expression (4.3) for given material parameters (r,n,φ_0) and for given parameters of the bending process; the values of angle α'''_{cr} are determined from Eq.(4.2).

A value of the equivalent (reduced) critical plastic stress σ_{ps}, corresponding to the strain (4.3), is determined from the constitutive Eq.(2.3) after substituting a strain value $\varphi_{(i)} = \varphi'''_{(i)cr}$. Suitable stress components can be determined (see Fig.3) for the point ***s*** on the ellipse of plasticity. According to the theory of associated laws of plastic flow, this point determines initiation of PSD in PSS. The basic set of quasi-linear partial differential equations of statics for the characteristics under H-M-H yield condition of plasticity is parabolic type at that point (Marciniak [1], Śloderbach and Rechul [14]; Śloderbach [16]; Moore and Wallace [35]; Śloderbach [36]). Thus

$$\sigma'''_{1s} = \frac{1+r}{\sqrt{1+2r}}\sigma'''_p, \qquad \sigma'''_{2s} = \frac{r}{\sqrt{1+2r}}\sigma'''_p \,. \tag{4.4}$$

From considerations concerning the stress state during the process of tube bending it appears that at the top points of the cross section of the layers subjected to tension (the points of intense contact of the bent tube with the mandrel in the bending zone) there is the stress state included between the axis σ_1, and the point ***s*** on the ellipse of plasticity (see Fig.3).

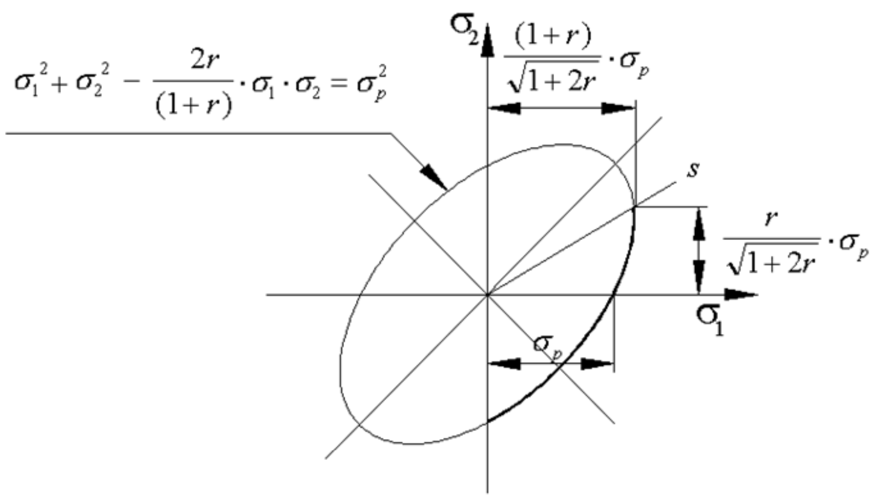

Fig.3. Stress condition arising at the H-M-H yield ellipsis at the onset of instability.

For this range of the stress states, the set of differential partial equations of statics is of hyperbolic type for the stress field characteristics (Śloderbach and Rechul [14]; Śloderbach [16]; Olszak *et al.* [39]; Szczepiński [41]). Occurrence of stability loss in dispersed and localized forms (especially as localized strains in the form of slip lines) during tube bending for pipeline elbows causes high acceleration of degradation processes of creep, and next failure during their operation. It results from the fact that at the points of the elbow where the localized strain states occur during the manufacturing stage, the strain localization processes are intensified during operation. Farther continuation and development of strain localization state can cause crack of the elbows and their failure.

When the stress states are above the point ***s*** (biaxial tension states, see Fig.3), then increments of the circumferential strain in the elongated layers will be positive $\left(d\varphi_2 > 0 \text{ or } d\varepsilon_2 > 0\right)$, see (Marciniak [1]). Such stress states and strain states occur during plastic expanding of tubes.

The calculations and analysis were performed for a generalized model of strain and for simplification of the 3rd order, see (Śloderbach [2]; Śloderbach and Strauchold [3]; Śloderbach and Rechul [14]; Śloderbach [16], [17], [18], [28], [29]). There are two extreme cases (the "gentlest" one, where the greatest strains and bending angles are admissible, and the "sharpest", where the lowest strains and bending angles are possible, respectively).

5. Results and discussion

Figure 4 presents a change of the wall thickness g_1''' of the elbow of the pipeline at its top (central) point of the elongated layers of the bending zones depending on a value of the bending angle $k\alpha_b$, for the assumed technological-material coefficient of correction of the strain distribution $(\lambda_1 = 1)$ (Śloderbach [2]; Śloderbach and Rechul [14]; Śloderbach [16], [17]). The graphs were drawn for different bending radii R, included in the period $R \in \langle (1 \div 5) \times d_{ext} \rangle$ and without taking into account displacement of the neutral axis of plastic bending ($y_0 = 0$). For calculations, a standard tube $\phi 44.5 \times 4.5 mm$, used for many calculations and tests, was applied (Śloderbach [2]; Śloderbach and Strauchold [3]; Franz [8]; Śloderbach and Rechul [14]; Śloderbach [16], [17], [18], [28], [36]). It follows from the graphs that wall thickness decreases as the bending angle $k\alpha_b$ increases and the bending radius R decreases.

Figure 5 shows a change of the plastic strain intensity $\varphi_{(i)}'''$ calculated at the central (top) point of elongated layers of the bending zone, depending on the bending angle $k\alpha_b$. The calculations were done for different bending radii R included, as previously, in the interval $R \in \langle (1 \div 5) \times d_{ext} \rangle$ for a diameter tube $\phi 44.5 \times 4.5 mm$, see (Śloderbach [2]; Śloderbach and Strauchold [3]; Franz [8]; Śloderbach and Rechul [14]; Śloderbach [16], [17], [18], [28], [36]). From Fig.5 it appears that the value $\varphi_{(i)}'''$ increases together with an increase of the bending angle $k\alpha_b$ and decreases as the bending radius R increases. Thus, our previous expectations seem to be right. When the bending radius R tends to infinity (straight tube), then $\varphi_{(i)}'''$ tends to zero and in consequence g_1 tends to g_0, so there is no bending process. Then bending angle $\alpha_b = 0^\circ$, because $k > 0$.

Figure 6 presents results of calculations of logarithmic strain components $\varphi_1, \varphi_2, \varphi_3$ and the substitute strain $(\varphi_i \equiv \varphi_{(i)})$, depending on the bending angle ($k\alpha_b$) for the top point of the elbow $(\alpha = \beta = 0^\circ)$. Let us put the experimental strain values of metal samples on the Y axis to determine a value of the uniform strain A_u (for example, determined under uniaxial tension). Then we are able to determine a value of the admissible bending angle after exceeding of which a technological values of permissible strains. For example A_u or the strains corresponding to the localized form of stability loss are exceeded. From the experimental data presented (Franz [8]) we have $A_u \approx 0.173$ for steel St. 35.8 according to DIN 17175. According to the graphs shown in Fig.6, it corresponds to the bending angle ($k\alpha_b \approx 145^\circ$). It means that after exceeding the bending angle 145°, for example for ($k\alpha_b = 180^\circ$) in the external elongated layers limited by the angle $\beta(0^\circ \le \beta \le 45^\circ)$, see Fig.2, and (Śloderbach [16], [17]) there are strains exceeding a value of the uniform strain A_u. After exceeding of uniform strain for given material A_u, a phenomenon of dispersed stability loss can occur and next localization of plastic strains can be observed, as in tests of uniaxial and biaxial tension (Marciniak [1]; El-Sebaie and Mellor [32]; Moore and Wallace [35], Marciniak *et al.* [37]; Marciniak and Kołodziejski [38]; Swift [40]; Szczepiński [41]).

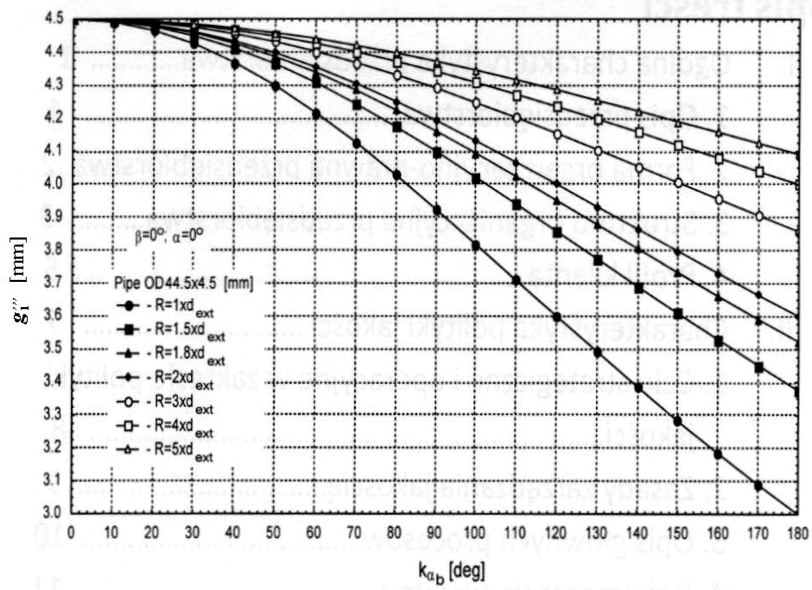

Fig.4. Wall thickness at the apex point as a function of the bending angle for selected values of the bending radius.

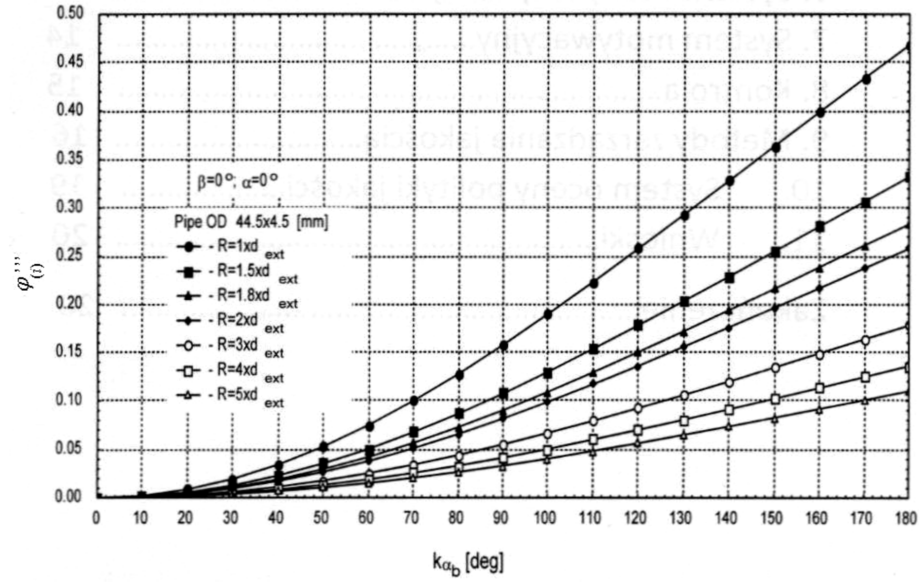

Fig.5. The equivalent strain value at the bend apex point as a function of the bending angle for selected values of the bending radius.

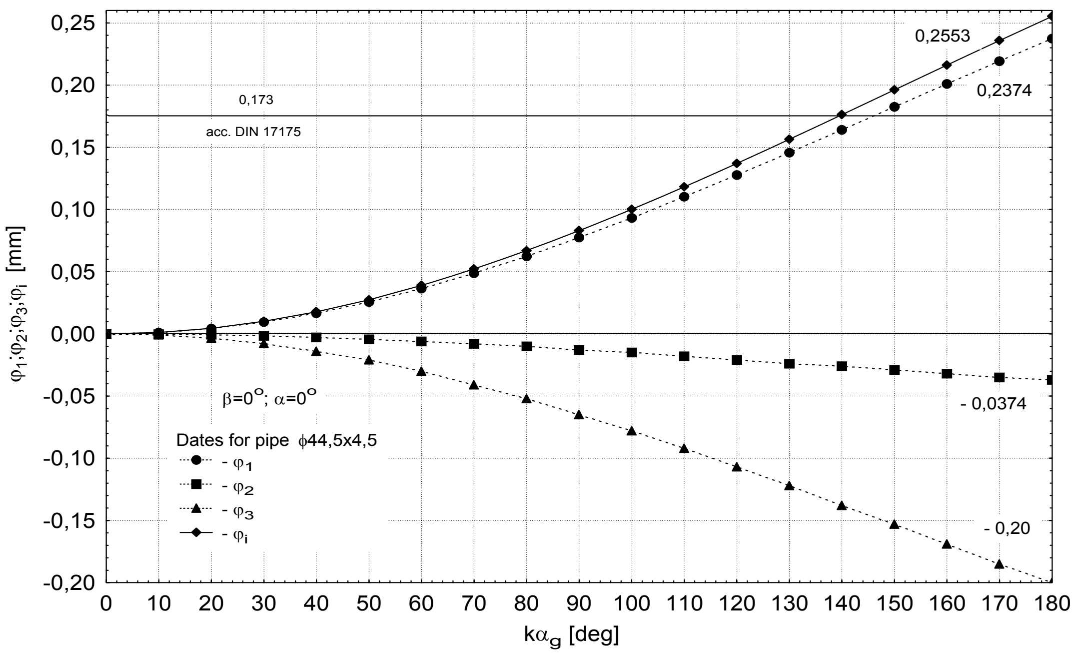

Fig.6. Strain and strain intensity components as functions of the bending angle, where ($\alpha_g = \alpha_b$).

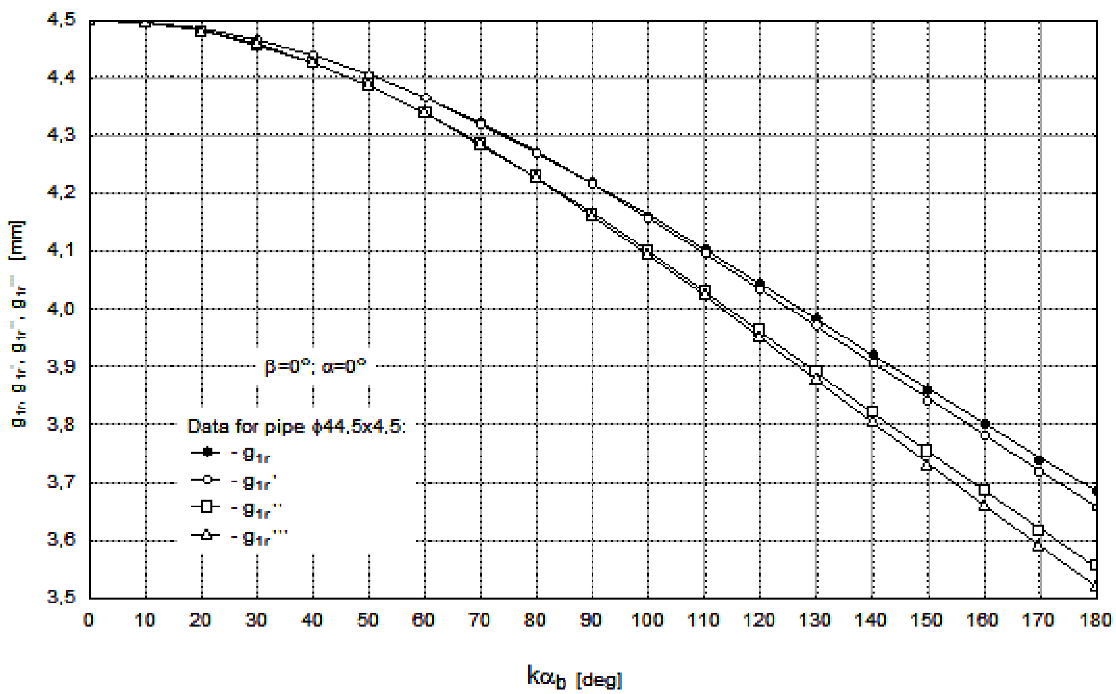

Fig.7. Variation of the wall thickness value at the bend apex point versus the bending angle according to four computing methods.

Figure 6 shows $\varphi_{(i)}$ equal to *0.173*; that value was determined for steel St 35.8 (Franz [8]) during a test of simple tension (according to the standard DIN 17175). Values of the bending angle α_g from the *X*-

axis for suitable strain intensities $\left(\varphi_i, \varphi_i^{'} \text{ and } \varphi_i^{'''}\right)$ determined for three calculation methods (the exact method and two simplified methods) oscillate around the following values of angles ($\alpha_{bcr} \approx 140^\circ$ for $k = 1$), ($\alpha_{bcr} \approx 57^\circ$ for $k = 2.5$) and ($\alpha_{bcr} \approx 47^\circ$ for $k = 3$) (Śloderbach and Rechul [14]; Śloderbach [16], [17]).

Figure 7 presents the calculation results for changes of the wall thickness $\left(g_{1r}, g_{1r}^{'}, g_{1r}^{''}, g_{1r}^{'''}\right)$ depending on the value of the bending angle $k\alpha_b$. The subscript r means calculations of real strains (logarithmic strains). The thickness is calculated at the top point of the elbow ($\alpha = \beta = 0^\circ$) of the elongated layers ($\lambda_1 = 1$) for the bent tube $\phi 44.5 \times 4.5mm$ and the radius $R = 80\ mm$ such that ($R = 1.8 \times d_{ext}$). The tube was made of steel St 35.8 according to DIN 17175, see (Franz [8]). The wall thickness g_1 at the top central points of the elbow in the elongated layers, corresponding to the bending angle $k\alpha_b \approx 145^\circ$ can be graphically determined from Fig.7 or analytically defined from Eqs.(2.1), using the condition of plastic incompressibility of the material ($\varphi_1 + \varphi_2 + \varphi_3 = 0$). Wall thickness calculated and read out from Fig.7 is $\left(g_1 \cong 3.88mm\right)$.

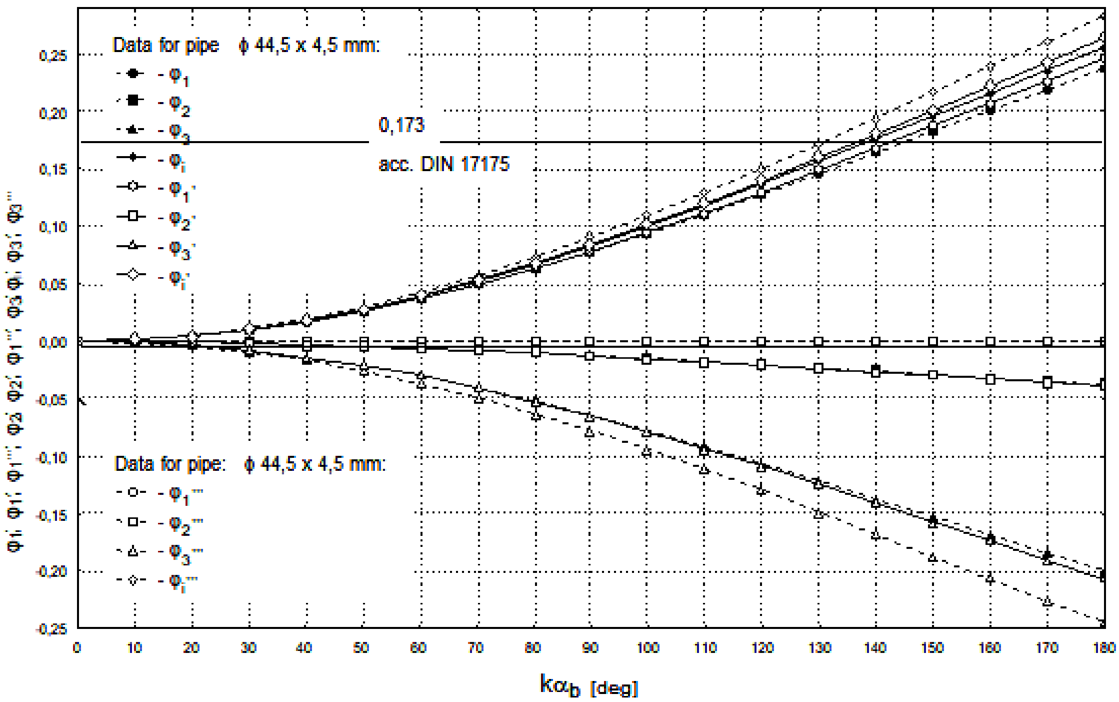

Fig.8. Strain components and strain intensity versus the bending angle according to three computing methods.

Figure 8 shows the obtained results of calculations of logarithmic measures of strain components and substitute strains $\left(\varphi_{(i)}, \varphi_{(i)}^{'}, \varphi_{(i)}^{'''}\right)$ depending on the bending angle ($k\alpha_b$) calculated according to the expressions for the general model and the simplified models of the 1st and 3rd order, describeded in papers (Śloderbach and Strauchold [3]; Śloderbach [16], [17], [28]). The calculations were made for the top point of the bent elbow in the bending zone, being also the central point of the elbow ($\alpha = \beta = 0^\circ$) in the elongated layers. Only two extreme simplifications (1st and 3rd order) are considered because additional graphs for simplification of the 2nd order could make the figures low-readable. Let us put admissible values of the experimental substitute strain on the Y-axis, as in Fig.5. Now we are able to determine an approximate value of the admissible critical bending angle α_{bcr} - (at first, the coefficient k should be determined). Exceeding that coefficient causes exceeding the permissible strains A_u (obtained under uniaxial tension) for tube steels.

From Figs 7 and 8 it also results that application of simplified models of the 1st, 2nd and 3rd order, respectively, causes a greater decrease of the tube thickness in the bending zone, greater strain components and substitute strain values as compared with the results obtained for the generalized model. Thus, the descriptions will determine lower (safer) values and safer limitations for the allowable bending angle α_{ball}. When the angle α_{ball} is exceeded, we can observe effects connected with localization of plastic strains or another form of stability loss and next cracking. Calculated values received from the simplified models derived in papers (Śloderbach [2]; Śloderbach and Strauchold [3]; Śloderbach [16], [17], [28], [29]) used in the analysis of the bending process determine a greater decrease of the wall thickness in the elongated layers, greater components of the strains and substitute strain in the bending zone. Because of a simplified form of the expressions they can be calculated with a calculator under real conditions (production, repair in situ).

6. Simple examples of calculations of critical states

Let us assume that the material of the bent tube has the dimensions $\phi 44.5 \times 4.5mm$ and is described by the following material parameters: $n \approx 0.2$; $\varphi_0 \approx 0.016$; $D \approx 550MPa$, $r \approx 1.5$, see expression (2.3), (3.4). These values can be related to the boiler steel K10 or St 35.8 according to DIN 17175. All the examples are calculated according to the scheme shown in Fig.9.

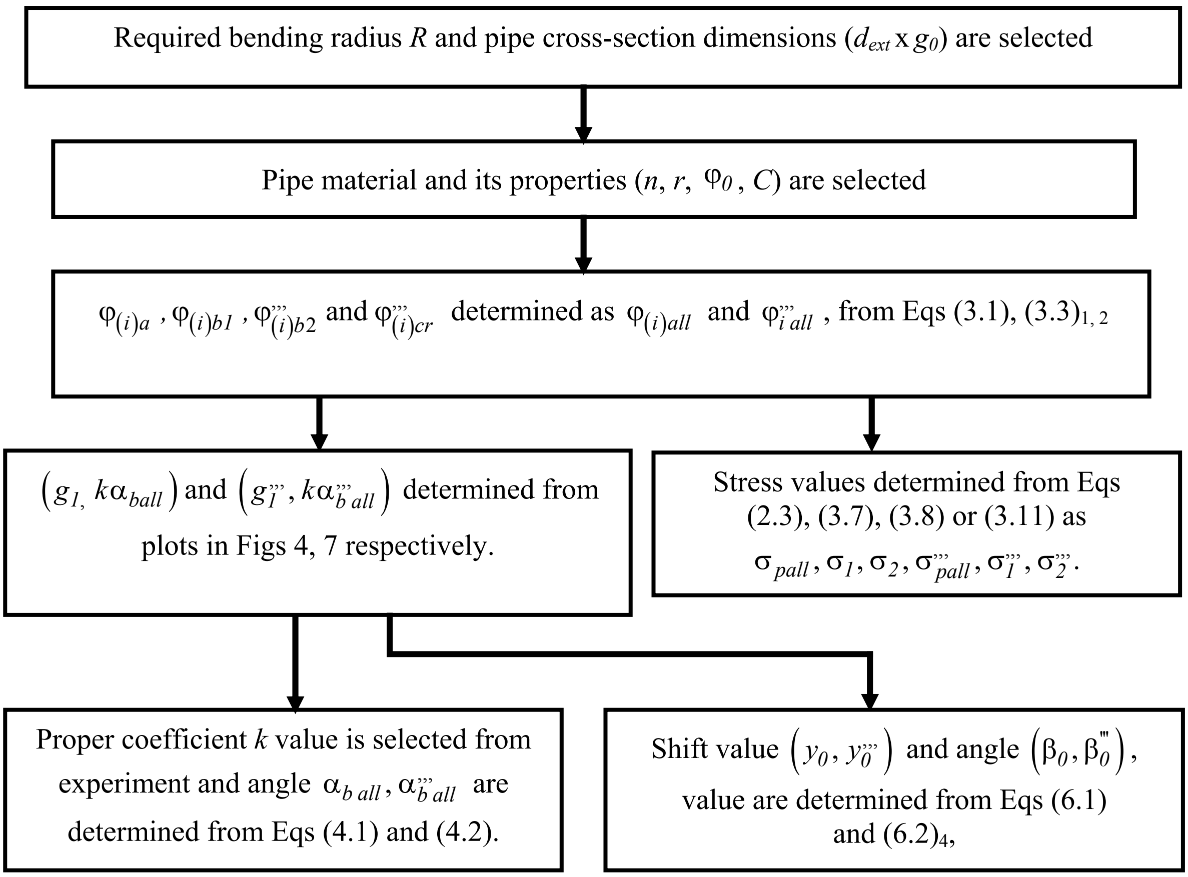

Fig.9. Flow chart showing the computation stages.

Example 1

When the strain state expressed by Eq.(3.1) is obtained, then we obtain $\varphi_{(i)a} = 0.184$.

The allowable value of the bending angle corresponding to the above value, read out from Fig.6 is $k\alpha_{ball} \cong 145^\circ$. Thus, $\alpha_{ball} \cong 145^\circ$ for ($k = 1$), $\alpha_{ball} \cong 72.5^\circ$ for ($k = 2$), $\alpha_{ball} \cong 58^\circ$ and for ($k = 2.5$) and $\alpha_{ball} \cong 48.3^\circ$ for ($k = 3$). The calculated values of wall thickness g_1 and g_2 for $(\lambda_1 = \lambda_2 = 1)$ are $g_1 \cong 3.88mm$, see Fig.10, and $g_2 \cong 5.66mm$. During a test of simple tension, (Franz [8]) obtained the strain value equal to ($A_u = 0.180$). The value φ_{1all} calculated from Eq.(2.1), equals $\varphi_{1all} \cong 0.173$ and it is the same as that given in (Franz [8]) for a case of uniform strains. From the calculations and Fig.8 it also follows that $k\alpha_b^{'''} \approx 135^\circ$. Thus, $\left(k\alpha_b > k\alpha_b^{'''}\right)$. The yield stress calculated according to Eq.(2.3), reaches the value $\sigma_{pall} \cong 398.6MPa$.

An approximate position of the neutral layer of plastic bending (see Fig.10) corresponding to the considered case can be determined from Eq.(6.1), see (Śloderbach [16], [17], [18]). Expression (6.1) is a generalization of the expression presented in (Tang [19]) for zones of active bending. Thus

$$y_0 \cong \lambda_0 \frac{0.42}{\tilde{r}}\left(r_{\text{int}} + \frac{g_0}{2}\right)\left[\cos(k\alpha) - \cos\left(k\frac{\alpha_b}{2}\right)\right] \tag{6.1}$$

where: λ_0 - technological-material correction coefficient of displacement of the neutral layer of plastic bending (Śloderbach and Pajak [15]; Śloderbach [16]-[18], [28], [29]).

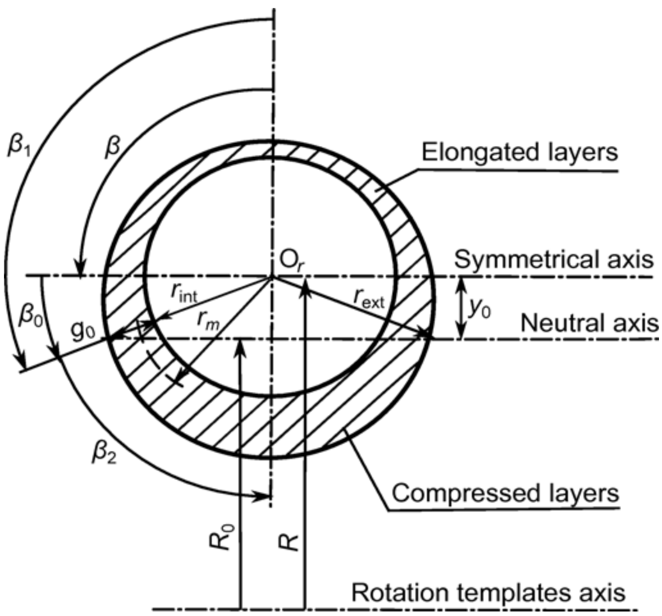

Fig.10. Schematic picture of cross-section of the elbow and its characteristic parameters.

According to tests, it is possible to assume that $\lambda_0 \in \langle 0; 1 \rangle$ (Śloderbach and Pajak [15]; Śloderbach [16], [17], [18]), $\tilde{r}$ - relative bending radius $\left(R = \tilde{r} \times d_{ext}\right)$. Thus

$$\tilde{r} = \frac{R}{d_{ext}}, \qquad r_m = r_{int} + \frac{g_0}{2}, \qquad R_0 = R - y_0 \qquad \text{and} \qquad \frac{y_0}{r_{ext}} \cong \sin\beta_0 . \tag{6.2}$$

The maximum displacement of the neutral axis for free bending can be determined by Eq.(6.1) for $\alpha = 0^\circ$ and $k\alpha_b = 180^\circ$ and $\lambda_0 = 1$. Assuming that in the considered case (bending on the bender machine) $\lambda_0 \approx 0.5$, after calculations we obtain $y_0 \approx 1.63mm$ and $y_{0\max} \approx 2.33mm$.

The radius R_0 defines a new (instantaneous for a given bending angle) position of the neutral layer of plastic bending for the angle $\alpha = 0^\circ$. Thus, $R_0 = R - y_0 \cong 78.37mm$. When $k\alpha_b = 180^\circ$ and $\lambda_0 = 0.5$ in the considered case bending on the bender, then $y_0 = y_{0max}$. Thus

$$\frac{y_0}{d_{ext}} \cong 0.037 \text{ or } \frac{y_0}{d_{ext}} \cong 3.7\% \text{. When } (y_0 = y_{0max})\text{, then } \frac{y_{0\max}}{d_{ext}} \cong 0.052 \text{ or } \frac{y_{0\max}}{d_{ext}} \cong 5.2\% .$$

Such a low value of y_0 does not strongly influence the plastic strain distribution. Moreover, the value of y_0, assumed in this paper ($\lambda_0 \approx 0.5$) for bending on the bender and removed clearances between devices of the bending machine and the bent tube, especially in the layers subjected to compression, can be even lower because of kinematic permissible displacement of the material particles upward along the perimeter. In the considered case of bending, it is displacement downward in the direction of the centre of the template rotation.

Example 2

When the strain state expressed by Eqs $(3.3)_{1,2}$ is obtained, then we have

$$\varphi_{(i)b1} \approx 0.257 \qquad \text{or} \qquad \varphi'''_{(i)b1} \cong 0.234 .$$

When the calculated allowable bending angle corresponding to a defined value of $\varphi_{(i)b1}$ is equal to $k\alpha_{ball} > 180^\circ$, see Figs 6 and 8, it means that for the generalized scheme of strain that form of stability loss does not occur. Thus, from Figs 4 and 7 for $k\alpha_b = 180^\circ$ we obtain $g_{1\min} \cong 3.68mm$ g_{1min}.

The numerically calculated g_{2max} for the layers subjected to compression for $(\lambda_2 = 1)$ is $g_{2\max} \cong 6.0mm$, and for $(\lambda_2 = 0.5)$, $g_2 \cong 5.0mm$. From Fig.7 we obtain $k\alpha'''_b \approx 158^\circ$. Thus, it appears that $\left(k\alpha_b > k\alpha'''_b\right)$. It means that for the strain scheme of the 3rd type such stability loss occurs, and this estimation is safer in comparison with the generalized scheme of strain. The value $\varphi'''_{1all} \approx 0.203$ calculated from Eq.$(3.3)_2$ was obtained.

The following values of the plasticizing stress and the principal stress components were obtained from Eqs.(2.3) and (3.7), (3.8): $\sigma_p \cong 424.2MPa$ and $\sigma'''_{ps} \cong 417MPa$ and $\sigma_1 \approx 524.4MPa$, $\sigma_2 \approx 251{,}8MPa$ and $\sigma'''_1 \cong 521MPa$, $\sigma'''_2 \cong 313MPa$.

When the bending state is reached for the strain value resulting from the generalized scheme of strain equal to $\varphi_{(i)b1} \approx 0.257$, then $y_0 = y_{0\max} \approx 2.33mm$ and $R_0 = R_{0\min}$. As in Example 1, the following values were obtained: $\left(y_{0\max}/d_{ext} \approx 0.052\right)$ and $\left(y_{0\max}/d_{ext} \approx 5.2\%\right.$ and $\left.R_{0\min} \approx 77.67mm\right)$.

7. Final remarks and conclusions

1. The condition of possible localized stability loss for the case of initiation of the plane (biaxial) state of deformation in the plane stress state determines higher permissible strain intensities than in the case of stability loss in the dispersed form (maximum drawing force) and lower ones for the localized stability loss $d\left(\sigma_p \cdot g\right) = 0$ during biaxial tension. In the case of stability loss in the dispersed form under uniaxial uniform tension, see (Marciniak [1], Śloderbach and Rechul [14]; Śloderbach [16], [17]), admissible strain intensity is comparable to a value of the coefficient of plastic strain hardening of a metal. The important contribution of the present paper is the formal extension of the criterion of strain localization formulated for sheets by (Marciniak [1]) for the case of tube bending. In the case of the generalized strain scheme and simplification of the 1st type, such extended criterion (with and without including displacement of the neutral axis of plastic bending y_0) depends additionally on geometric dimensions of the bent tube (approximately on its thin-walled parameter s^*).

2. In the case of metal tube bending at the bending machines and considering the condition that $d\left(\sigma_p \cdot g\right) = 0$, new effects (unknown in literature) have been obtained:

a). for the generalized and of the 1st order strain model and for the external top point of the bent elbow where plane stress state (PSS) is appearing, we obtain (Fig.3):

- when $s^* \neq 0$, then the condition $d\left(\sigma_p \cdot g\right) = 0$ concerns the stress states included in the hyperbolic range of a set of quasi-linear partial differential equations of statics on the H-M-H ellipse of plasticity, i.e. for the case when $\left(\sigma_1 \text{ and } \sigma_2\right) > 0$ and $d\varphi_2 < 0$,

- when $s^* = 0$ (internal surface of the bent elbow $g = 0$ in thickness) or plane specimen when $d_{ext} \to \infty$ then the condition $d\left(\sigma_p \cdot g\right) = 0$ refers to the parabolic point on the H-M-H ellipse of plasticity, in which $\left(d\varphi_2 = 0\right)$ – it means of the initiation of plane state of deformation (PSD). At this point of stresses a set of static of quasi-linear partial differential equations have a parabolic character (Olszak *et al*. [39]; Szczepinski [41]),

- when $s^* = 0.5$ (bent bar), where the condition $d\left(\sigma_p \cdot g\right) = 0$ corresponds to the point $\sigma_1 > 0$ and $\sigma_2 = 0$, and $\left(d\varphi_2 = d\varphi_3 = -0.5 d\varphi_1\right)$ on the ellipse of plasticity. Thus, if the coefficient $s^* \in \left(0.5; 0\right)$ then $\sigma_2/\sigma_1 \in \left(0; 0.5\right)$. When at the external points or the point of the layers subjected to tension the strain scheme represented by the strain components described by the averaged equations (when the reference area is the central layer in the tube wall) is acting, then for $s^* \in \left(0.5; 0\right)$ we obtain $\sigma_2/\sigma_1 \in \left(0.2; 0.5\right)$. It means that for the bar (when $s^* = 0.5$) in the central layer the stress is $\left(\sigma_2 \neq 0\right)$, so $\sigma_2/\sigma_1 = 0.2$.

b). for the strain model resulting from the simplification of the 3rd type we can state that for the external top point of the bent elbow where PSS occurs, the condition $d\left(\sigma_p \cdot g\right) = 0$ refers to the parabolic point ***s*** on the ellipsis of plasticity where $\left(d\varphi_2 = 0\right)$, which means local initiation of PSD.

3. The condition of possible initiation of the localized stability loss for the biaxial (plane) strain state in the plane stress state determines greater admissible strain intensities than those for the case of stability loss in the dispersed form (maximum of the drawing force) and lower ones for the localized stability loss $d\left(\sigma_p \cdot g\right) = 0$ during biaxial tension [Eqs $(3.2)_1$ and $(3.3)_1$]. In the case of dispersed stability loss during uniaxial uniform tension, see (Marciniak [1]; Śloderbach [16], [17], Gabryszewski and Gronostajski [33]; Marciniak and Kołodziejski [38]), a permissible value of strain intensity is comparable with the value of the coefficient of metal plastic hardening. A new element of this paper is the extension of the criterion of strain localization formulated by (Marciniak [1]) for sheets for the case of tube bending. In the case of the generalized scheme of strain and simplification of the 1st order, such extended criterion (with and without including the neutral axis displacement of plastic bending y_0) depends on geometrical dimensions of the bent tube (approximately on its wall thickness parameter s^*).

4. In the case of the strain model resulting from the simplifications of 2nd and 3rd order we can state that at the external top point of the bent elbow where there is the plane stress state, the condition $d\left(\sigma_p \cdot g\right) = 0$ corresponds to the parabolic point ***s*** at the H-M-H ellipse of plasticity, where $\left(d\varphi_2 = 0\right)$. Physically it means a local initiation of PSD.

5. In tube bending processes described by the generalized strain model and by suitable simplified methods presented in papers (Śloderbach and Pajak [15]; Śloderbach [16], [17], [18]), the influence of displacement of the neutral axis of plastic bending y_0 on the plastic strain state is included. The displacement of the neutral layer of the plastic bending y_0 is moving downward (in the direction of the layers subjected to compression and in the direction from of the rotation axis of the template, see Figs 2 and 10) and increases as the bending angle increases. In the considered examples, a value of this displacement for the top point of the elongated layers (see examples 1, 2) is ~ $\left(1 \div 2mm\right)$. This value is proportionally comparable to the value of decrease of the wall thickness for these points of elongated layers, see Figs 4 and 7.
6. We can formulate practical recommendations resulting from the calculations. The elbows of the pipelines in pressure devices working at elevated or high temperatures should be bent in such a way as not to exceed the allowable values (of the bending angles and also admissible strains) obtained in Examples 1, 2. Exceeding the allowable values of the bending angles (and also admissible strains) will cause a reduction of the time of their operation (lifetime). It particularly applies to elbows working at elevated and high temperatures. Occurrence of localized strains during tube bending for elbows of pipelines causes accelerated degradation processes of creep (because in such places these processes could concentrate), and leads to occurrence of cracks and dangerous failures, see (Dzidowski [6]; Dzidowski and Strauchold [7]; Śloderbach and Pajak [14]; Śloderbach [16], [17]; Zdankiewicz [24], [25]; Dobosiewicz and Wojczyk [26]; Śloderbach and Pajak [27]). Let us note that the analytical expressions for principal components of the strain state derived in (Śloderbach [2], [16], [17], [18]) can help in future analysis and evaluation of tube usability for bending by means of the methods of defining the curves of limit strains, as in the case of evaluation of sheet drawability (Marciniak [1]; El-Sebaie [32]; Gabryszewski and Gronostajski [33]; Hill [34]; Moore and Wallace [35]; Swift [40]).
7. During hot, semi-hot or with preheating tube bending, it is necessary to consider another form of the constitutive equation than (2.3), in which material parameters will be dependent on temperature. For example, for the majority of metallic materials the coefficient of metal plastic hardening n decreases as the temperature increases (Marciniak [1]; Marciniak and Kołodziejski [38]; Szczepiński [41]; Marciniak [44]). Other terms of loss of stability should also be considered. At suitably high temperatures it is assumed that the strength limit is almost the same as the yield point (without hardening effect), (Szczepinski [41]; Marciniak [44]).

Nomenclature

- b_0 and b – specimen width before and after deformation
- D – material constant in [MPa]
- d_{ext} and d – external diameter of the tube before and after deformation
- d_1 and d_1''' – current diameters at points of top stretched layers of the knee for the generalized model of deformations and simplification of 3rd order, respectively
- g_0 and g – specimen thickness before and after deformation
- g_1 and d_1 – real thickness and diameter of the elbow in the layers subjected to tension
- n – coefficient of hardening
- r – coefficient of Lankford normal anisotropy
- $\tilde{r}$ – relative bending radius $\left(R = \tilde{r} \times d_{ext}\right)$
- R – nominal radius of tube bending
- R_j and R_0 – "big active actual radius of bending" connected with longitudinal strain, and radius determining actual position of the neutral layer, respectively
- r_j – "small active actual radius" of the elbow in the bending zone, $r_j = r_{int} + g_j$ and $d_j = 2r_j$
- r_{ext} and d_{ext} – external radius and diameter of the tube subjected to bending, respectively and $d_{ext} = 2\, r_{ext}$
- r_{int} and d_{int} – internal radius and diameter of the tube, respectively, $d_{int} = 2\, r_{int}$
- r_m – mean radius of bending pipe, $r_m = r_{int} + \frac{g_0}{2}$
- s^* – thin-walled parameter of the bent tube defined as ($s^* = g_0/d_{ext}$)
- s_w^* – thin-walled parameter of the bent tube defined as $\left(s_w^* = g_0/d_{\text{int}}\right)$, then $s_w^* = s^* \big/ \left(1 - 2s^*\right)$

y_0 – displacement of the neutral layer of plastic bending, $y_0 \in \langle 0;1 \rangle$,
index $j = 1$ and sign (+) in Eqs (2.1) refers to the layers subjected to tension,
index $j = 2$ and sign (-) in Eqs (2.1) refers to the layers subjected to compression

s_w^* – thin-walled parameter of the bent tube defined as $\left(s_w^* = g_0/d_{\text{int}}\right)$

z – subtangent including the influence of the stress σ_p on the moment of stability loss under conditions of the plane stress state and in the moment of formation of the plane state of deformation

$\alpha_b \equiv \alpha_g$ – bending angle measured in the bending zone. In the bending zone, the angles of bending and bend are equal, so $\alpha_b = \alpha_0$, where α_0 – bend angle (angle of rotation of the template of the bending machine)

α_{ball} – allowable values of the bending angle calculated from expressions obtained for the generalized model of deformations, when the value of intensity of deformation $\varphi_{(i)}$ reaches permissible value $\varphi_{(i)all}$

α'''_{ball} – allowable values of the bending angle calculated with use the model of deformation of 3rd-order when intensity value of the deformation $\varphi'''_{(i)}$ reaches the allowable value $\varphi'''_{(i)\,all}$

d_1 and d_1''' – current diameters at points of top stretched layers of the knee for the generalized model of deformations and simplification of 3rd order appriopriately

α, β – angles of the point position in the bending zone

β – angle of circulation of the layers subjected to tension and compression of the elbow, $\beta_j \in \left\langle 0; 90^\circ \pm \beta_0 \right\rangle$ and $\sin\beta_0 = y_0/r_{ext}$, where

β_0 – angular range of displacement of the neutral axis of bending

λ_i – technological-material corrective coefficients of strain distribution in the layers subjected to tension $(i = 1)$ and compression ($i = 2$) of the bending and bend zone, defined from the experimental results so, that $\lambda_1 \cong 1$ and $\lambda_2 \in < 0; 1 >$. In the case of most known experiments we can approximately assume that $\lambda_2 \approx 0.5$

λ_0 – technological-material correction coefficient of displacement of the neutral layer of plastic bending for components, intensity and three simplifications

φ_1, φ_2 and φ_3 – logarithmic principal plastic strains

φ_0 – logarithmic initial strain

$\varphi_{(i)} \equiv \varphi_i$ – logarithmic substitute strain (strain intensity)

$\varphi_{(i)c}$ – value of the substitute strain corresponding to the loss of stability

$\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ – mean stress

σ_1, σ_2 and σ_3 – principal stresses

σ_p – yield stress in [MPa]

References

[1] Maciniak Z. (1971): *Limit Deformations in Sheet Metal Stamping* [in Polish]. – Warsaw: WNT.

[2] Śloderbach Z. (1999): *A model of deformation geometry in pipe bending processes*. – Engineering Transactions, vol.47, No.1, Warsaw, pp.3-20.

[3] Śloderbach Z. and Strauchold Sz. (1999): *Approximate methods for evaluating strains in pipe bending processes* [in Polish]. – Technical Inspection, No 1/99, Warsaw, pp.1-6.

[4] Beskin L. (1945): *Bending of thin curved tubes*. – Journal of Applied Mechanics, Transactions of the ASME, vol.12.

[5] Boyle M. (1971): *Bending thin wall stainless tubing*. – Machinery, No77/71, London.

[6] Dzidowski E.S. (2001): *Reliability and solving problems in thin-walled pipe bending processes for the case of great manufacturing tolerance* [in Polish]. – Fourth Conference PIRE 2001, ISBN 83-909539-3-5, Lądek Zdrój, pp.91-101.

[7] Dzidowski E.S. and Strauchold Sz. (1998): *Effect of technological factors in pipe bending on damage characteristics and reliability of power pipelines* [in Polish]. – Scientific Books of the Opole University of Technology, Series Electricity, vol.6, Opole, pp.119-125.

[8] Franz W.D. (1961): *Das Kalt-Biegen von Rohren*. – Berlin: Springer-Verlag.

[9] Gruner P. (1960): *Über rohrbiegeverfahren*. – Maschinenmarkt, vol.30/31, pp.120-129.

[10] Grunow O. (1985): *Praktisches Rohrbiegen.* – Berlin: Springer-Verlag.

[11] Korzemski J.W. (1968): *Thin-walled pipe bending with use of the mandrels* [in Polish]. – Mechanic, 4/68, Warsaw, pp.207-210.

[12] Korzemski J.W. (1963): *Thin-Walled Pipe Bending* [in Polish]. – Warsaw: WNT.

[13] Pesak F. (1963): *Bending thin wall tubing.* – Machinery, vol.60, pp.147-151.

[14] Śloderbach Z. and Rechul Z. (2000): *Effect of strain hardening and normal anisotropy on allowable values of strain and stress in pipe-bending processes.* – Journal of Theoretical and Applied Mechanics, vol.38, No.4, Warsaw, pp.843-859.

[15] Śloderbach Z. and Pajak J. (2015): *Determination of the thicknesses of pipe elbows on the basis of expressions resulting from EU-Directive*. – Archive of Applied Mechanics, vol.85, No.5, pp.629-640.

[16] Śloderbach Z. (2002): *Some problems of mechanics in pipeline bending processes* [in Polish]. – Publishing House of the Wrocław University of Technology, ISBN 83-7085-665-9, Wrocław.

[17] Śloderbach Z. (2010): *The conditions of instabilitiy in the plastic deformations analysis* [in Polish]. – Publishing House of the Opole University of Technology ISBN 978-83-60691-85-4, Opole.

[18] Śloderbach Z. (2014): *A derivation of the generalized model of strains during bending of metal tubes at bending machines.* – International Journal of Applied Mechanics and Engineering, vol.19, No.1, University of Zielona Góra, pp.145-163.

[19] Tang N.C. (2000): *Plastic-deformation analysis in tube bending, International Journal of Pressure Vessels and Piping.* – Publisher Elsevier UK, vol.77, No.12, pp.751-759.

[20] UDT CONDITIONS (WUDT-UC-WO-O/02:10). (2003): *Pressure Installations. General Requirements. Bending Elbows. Strength Calculations* [in Polish]. – No.1, Warsaw.

[21] Wick CH., Benedict J.T. and Veilleux R.F. (2001): *Tool and manufacturing engineers handbook. A reference book for manufacturing engineers, managers and technicans*. Volume II, *Forming, Fourth Edition*. – Society of Manufacturing Engineers, One ASME Drive, Dearbon, Michigan, USA.

[22] Zhang Z., Yang H., Li H., Ren N. and Tian Y. (2011): *Bending behaviors of large diameter thinwalled CP-Ti tube in rotary draw bending* – Progress in Natural Science: Materials International, vol.21, pp.401-412.

[23] Zhiqiang J., Mei Z., He Y., Xudong X. and Guangjun L. (2011): *Deformation behavior of medium strength TA18 high-pressure tubes during nc bending with different bending radii.* – Chinese Journal of Aeronautics, vol.24, pp.657-664.

[24] Zdankiewicz M. (1970): *Investigations of the cold tubes bending processes* [in Polish]. – Mechanic, No 7/70, Warsaw, pp.609-519.

[25] Zdankiewicz M. (1998): *European directive concern a pressure installations. design recommendations for pressure vessels – manufacturing* [in Polish]. – Dozór Techniczny, No 2/98, Warsaw, 25-33 and 48.

[26] Dobosiewicz J. and Wojczyk K. (1998): *Life prediction for steam pipeline bends* [in Polish]. – Energetics, nr 3/88, Katowice, pp.88-90.

[27] Śloderbach Z. and Pająk J. (2013): *Stored energy of plastic deformations in tube bending processes* [in Polish]. – International Journal of Applied Mechanics and Engineering, vol.18, No.1, pp.235-248.

[28] Śloderbach Z. (2015): *Derivation of relations and analysis of tube bending processes using discontinous fields of plastic strains. Part I. Derivations of geometric-analytic relationships*. Part II. *Discussion and analysis of the obtained results*. – International Journal of Applied Mechanics and Engineering, vol.20, No.2, pp.417-435.

[29] Śloderbach Z. (2017): *Application of the continuous and discontinuous fields of plastic deformations to the evaluations of the initial thickness of bent tubes*. – Journal of Engineering Materials and Technology, vol.139, No.1, pp.011009-1 011009-10.

[30] Życzkowski M. and Skrzypek J. (1971): *Influence of bending, torsion and tension on the stationary creep of a pipe line, archives of mechanics*. – Vol.23, No.4, pp.465-494.

[31] EN 13445-4. (2009): „*Unfired Pressure Vessels" – Part 4: Fabrication*, (point 9.2.4 Tube Bents, expression 9.2-4), or earlier: Draft Standart EN UFPV. (May 1996)"Unfired Pressure Vessels" – Part 4. "Manufacture", CEN/TC54/267 JWGB N277, rev.5.

[32] El-Sebaie M.G. and Mellor P.B. (1972): *Plastic instability conditions in the deep-drawing of a circular blank of sheet metal*. – International Journal of Mechanical Science, vol.14, pp.535-556.

[33] Gabryszewski Z. and Gronostajski J. (1991): *Fundamentals of Metal-Working Processes* [in Polish]. – Warsaw: PWN.

[34] Hill R. (1985): *Mathematical Theory of Plasticity*. – London: Oxford, Clarendon Press.

[35] Moore G.G. and Wallace J.F. (!964/1965): *The effect of anisotropy on instability in sheet- metal forming* [in Polish]. – Journal of the Institute of Metals, vol.93, No.2, pp.33-38.

[36] Śloderbach Z. (2015): *Conditions of stability loss during the test of hydraulic forming of drawpieces. Part I. Derivation of basic equations and relationship*. Part II. *Discussion and analysis of the obtained results.* – International Journal of Applied Mechanics and Engineering, vol.20, No.4, pp.917-938.

[37] Erbel S., Kuczyński K. and Marciniak Z. (1986): *Plastic Metal-Working Processes* [in Polish]. – Warsaw: PWN.

[38] Marciniak Z. and Kołodziejski J. (1983): *Fundamentals of Metal-Working Processes* [in Polish]. – Warsaw: Warsaw University of Technology.

[39] Olszak W., Perzyna P. and Sawczuk A. (1985): *Theory of Plasticity* [in Polish]. – Warsaw: PWN.

[40] Swift H.W. (1962): *Plastic instability under plane stress*. – Journal of the Mechanics and Physics of Solids, vol.1, No.1, pp.1-76.

[41] Szczepiński W. (1993): *Theory of Plastic Working of Metals* [in Polish]. – Warsaw: PWN.

[42] Raniecki B. and Sawczuk A. (1975): *Thermal effects in plasticity*. Part I. *Coupled theory*. – Zeitschrift fur Angewandte Mathematik und Mechanik, ZAMM, vol.55, pp.333-341.

[43] Raniecki B. and Sawczuk A. (1975): *Thermal effects in plasticity*. Part II. *Uniqueness and aplications.* – Zeitschrift fur Angewandte Mathematik und Mechanik, ZAMM, vol.55, pp.363-371.

[44] Marciniak Z. (editor). (1987): *Plastic Metal-Working Processes. Which Methods of Forming? On Cold, Hot or Half-Hot?* [in Polish]. – Mechanics, SIMP, No2, Warsaw.

[45] Nowacki W.K. (1985): *Thermal effects in dynamic plasticity, numerical solution and experimental investigations* (*thermographic-infrared detection*). – Journal de Physique, vol.48, No.8, pp.113-119.

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