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TRIPLE DIFFUSIVE CONVECTION OF A NON-NEWTONIAN FLUID UNDER THE COMBINED EFFECT OF COMPRESSIBILITY AND VARIABLE GRAVITY

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In this paper, triple diffusive convection in a Rivlin-Ericksen fluid layer, which is permeated with suspended particles in the porous medium under the effect of compressibility and variable gravity, is investigated. Linear stability theory and normal mode analysis have been used to study the problem under consideration. It is observed that, for stationary convection, suspended particles, compressibility and medium permeability have destabilizing/stabilizing effects under certain conditions. The variable gravity parameter destabilizes the system whereas stable solute gradients have a stabilizing effect.

Key words: triple diffusion, Rivlin-Ericksen fluid, suspended particles, porous medium, compressibility, variable gravity.

1. Introduction

Due to the growing importance of non Newtonian fluids such as the Walters fluid, Rivlin-Ericksen fluid and couple stress fluid in many fields such as in industries and in modern technologies, the study on these fluids is eminently required. In convective problems it is more recommendable to consider fluid flow in the presence of solute gradient (Because of wide applications in ionosphere, astrophysics, atmospheric physics etc.) with free boundaries. The principles and theory of thermosolutal convection are investigated by Bejan [1]. The convection problem when the fluid layer is heated and soluted from below is studied by Veronis [2] and Nield [3]. The instability of non-Newtonian fluids has been studied by several authors [4-15] and [16, 17, 18, 19, 20, 21]. There are so many problems in engineering, oceanography and limnology which can be studied by double diffusive convection. The problems of astrophysics, geophysics, hydrology, etc. can be investigated through double diffusive convection in a porous medium. The assumption of uniform gravity field is removed in case of large scale convective phenomenon such as the mantle of earth or ocean so condition of variable gravity is considered in the present problem. Study on compressible fluids was started by Landau [22] and Lees [23] earlier investigated by Dunn and Linn [24]. Further, convection in a multicomponent fluid was investigated by Straughan and Walker [25]. The problem of multi diffusive-convection (when more than one salt is present in the fluid) is of great importance because of its usefulness in describing so many natural phenomena such as acid rain effects, underground water flow, warming of stratosphere etc.

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The present paper is an effort to study triple diffusive convection which arises due to temperature and the presence of two salt components in a horizontal layer of the Rivlin-Ericksen fluid, under the effect of compressibility and variable gravity in a porous medium. Pearlstein *et al.* [26] obtained very interesting results regarding triple diffusive convection. Triple diffusion was investigated by Rionero [27], Kango and Rana [28] for Walter's fluid under varying gravity.

2. Mathematical model of the problem

Here we consider a compressible Rivlin-Ericksen fluid layer (horizontal) of infinite length and the bounds of the layer are $z = \theta_L$ (lower bound) & $z = d_U$ (upper bound), in the presence of suspended particles. This fluid layer is subjected to variable gravity g(0,0,-g) where $g = \lambda g_0$, $g_0(>0)$ is the value of g at z = 0 and λ is the variable gravity parameter (can be positive or negative) in a porous medium. This layer is heated and salted from below. The temperature T at $z = \theta_L$ & $z = d_U$ are T_0 and T_1 and solute concentrations C^1 and C^2 at the bottom and top surfaces are C_0^1 and C_1^1 ; C_0^2 and C_1^2 , respectively. The uniform solute concentration gradients $\left(\beta' = \frac{dC^1}{dZ} & \beta'' = \frac{dC^2}{dZ}\right)$ and a uniform temperature gradient $\left(\beta = \frac{dT}{dZ}\right)$ are maintained.

Under the above assumptions mathematical equations suitable for this model are

$$\frac{\rho}{\varepsilon} \left[\frac{\partial \boldsymbol{q}_F}{\partial t} + \frac{l}{\varepsilon} (\boldsymbol{q}_F \cdot \nabla) \boldsymbol{q}_F \right] = -\nabla p - \rho g \boldsymbol{\lambda} + \frac{KN}{\varepsilon} (\boldsymbol{q} - \boldsymbol{q}_F) - \frac{l}{k_I} \left(\boldsymbol{\mu} + \boldsymbol{\mu}' \frac{\partial}{\partial t} \right) \boldsymbol{q}_F , \qquad (2.1)$$

$$\nabla \boldsymbol{q}_F = \boldsymbol{0} \,, \tag{2.2}$$

$$mN\left[\frac{\partial \boldsymbol{q}}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q}.\nabla)\boldsymbol{q}\right] = KN(\boldsymbol{q}_F - \boldsymbol{q}), \qquad (2.3)$$

$$\varepsilon \frac{\partial N}{\partial t} + \nabla (N\boldsymbol{q}) = 0, \qquad (2.4)$$

$$\left[\rho C_V \varepsilon + \rho_S C_S \left(1 - \varepsilon\right)\right] \frac{\partial T}{\partial t} + \rho C_V \left(\boldsymbol{q}_F \cdot \nabla\right) T + m N C_{pt} \left(\varepsilon \frac{\partial}{\partial t} + \boldsymbol{q} \cdot \nabla\right) T = q_F \nabla^2 T, \qquad (2.5)$$

$$\left[\rho C_V \varepsilon + \rho_S C_S \left(1 - \varepsilon\right)\right] \frac{\partial C^I}{\partial t} + \rho C_V (\boldsymbol{q}_F \cdot \nabla) T + m N C_{pt} \left(\varepsilon \frac{\partial}{\partial t} + \boldsymbol{q} \cdot \nabla\right) T = \boldsymbol{q}' \nabla^2 C^I, \qquad (2.6)$$

$$\left[\rho C_V \varepsilon + \rho_S C_S \left(1 - \varepsilon\right)\right] \frac{\partial C^2}{\partial t} + \rho C_V (\boldsymbol{q}_F \cdot \nabla) T + m N C_{pt} \left(\varepsilon \frac{\partial}{\partial t} + \boldsymbol{q} \cdot \nabla\right) T = q'' \nabla^2 C^2 .$$
(2.7)

The density equation of the state is

$$\rho = \rho_0 \left[1 - \alpha \left(T - T_a \right) \right] + \alpha^l \left(C^l - C_a^l \right) + \alpha^2 \left(C^2 - C_a^2 \right),$$

$$\delta \rho = -\rho_0 \left[\alpha \theta - \alpha^l \gamma^l - \alpha^2 \gamma^2 \right]$$

where T_a (average temperature) = $\frac{T_0 + T_1}{2}$ and average concentrations are given by $C_a^l = \frac{C_0^l + C_1^l}{2}$ and $C_a^2 = \frac{C_0^2 + C_l^2}{2}$.

The equations obtained after linearization are as follows

$$\frac{l}{\varepsilon} \left[\frac{\partial \boldsymbol{q}_F}{\partial t} \right] = -\frac{l}{\rho_m} \nabla \delta p + \frac{KN_0}{\rho_m \varepsilon} (\boldsymbol{q} - \boldsymbol{q}_F) - \frac{l}{k_l} \left(\boldsymbol{\nu} + \boldsymbol{\nu}' \frac{\partial}{\partial t} \right) \boldsymbol{q}_F + g \left(\alpha \theta - \alpha^l \gamma^l - \alpha^2 \gamma^2 \right) \boldsymbol{\lambda} , \qquad (2.8)$$

$$\nabla \boldsymbol{.}\boldsymbol{q}_F = \boldsymbol{0} \,, \tag{2.9}$$

$$\left[\frac{m}{k}\frac{\partial}{\partial t}+I\right]\boldsymbol{q}=\boldsymbol{q}_F\,,\tag{2.10}$$

$$\varepsilon \frac{\partial N}{\partial t} + N_0 \left(\nabla . \boldsymbol{q} \right) = 0 , \qquad (2.11)$$

$$\left(e+h_{I}\varepsilon\right)\frac{\partial\theta}{\partial t}=\beta\left(\frac{G-I}{G}\right)\left(w+h_{I}s\right)+\kappa\nabla^{2}\theta,$$
(2.12)

$$\left(e'+h_{2}\varepsilon\right)\frac{\partial\gamma^{l}}{\partial t}=\beta'\left(w+h_{2}s\right)+\kappa'\nabla^{2}\gamma^{l},$$
(2.13)

$$\left(e'' + h_2\varepsilon\right)\frac{\partial\gamma^2}{\partial t} = \beta''\left(w + h_3s\right) + \kappa''\nabla^2\gamma^2$$
(2.14)

where

$$\mu, \mu', \nu = \frac{\mu}{\rho_m}, \quad \nu' = \frac{\mu'}{\rho_m}, \quad \kappa \left(= \frac{q}{\rho_m C_\nu} \right), \quad \kappa' \left(= \frac{q'}{\rho_m C_\nu'} \right) \quad \text{and} \quad \kappa'' \left(= \frac{q''}{\rho_m C_\nu''} \right).$$

Also,
$$h_1 = f \frac{C_{pt}}{C_v}, \quad h_2 = f \frac{C_{pt}'}{C_v'}, \quad h_3 = f \frac{C_{pt}''}{C_v''}, \quad f = \frac{mN_0}{\rho_m} \quad \text{and} \quad G = \frac{C_p\beta}{g}.$$

Now, we linearize the perturbation equations (Boussingue approximation) and obtain non dimensional equations as

$$p^{-l}\left[\frac{\partial \boldsymbol{q}}{\partial t}\right] = -\nabla \delta p + B(\boldsymbol{q}_F - \boldsymbol{q}) - P_l^{-l}\left(l + A\frac{\partial}{\partial t}\right)\boldsymbol{q} + \left(R\Theta - S_I\gamma^l - S_2\gamma^2\right)\boldsymbol{\lambda}, \qquad (2.15)$$

$$\nabla \boldsymbol{.} \boldsymbol{q} = \boldsymbol{0} \,, \tag{2.16}$$

$$\left[\tau \frac{\partial}{\partial t} + I\right] \boldsymbol{q}_F = \boldsymbol{q} \,, \tag{2.17}$$

$$\varepsilon \frac{\partial M}{\partial t} + (\nabla . \boldsymbol{q}_F) = 0 , \qquad (2.18)$$

$$\left(e+h_{I}\varepsilon\right)\frac{\partial\theta}{\partial t} = \left(\frac{G-I}{G}\right)\left(w+h_{I}s\right) + \nabla^{2}\theta, \qquad (2.19)$$

$$\left(e'+h_2\varepsilon\right)\frac{\partial\gamma^I}{\partial t} = \left(w+h_2s\right) + \xi\nabla^2\gamma^I, \qquad (2.20)$$

$$\left(e''+h_3\varepsilon\right)\frac{\partial\gamma^2}{\partial t} = \left(w+h_3s\right) + \xi'\nabla^2\gamma^2$$
(2.21)

where

$$R = \frac{g\alpha\beta d^4}{\nu\kappa}, \quad S_1 = \frac{g\alpha^1\beta' d^4}{\nu\kappa'}, \quad S_2 = \frac{g\alpha^2\beta'' d^4}{\nu\kappa''}, \quad e = \varepsilon + (I - \varepsilon)\frac{\rho_S C_S}{\rho C_V} \text{ is a constant};$$

$$\mathbf{e}' = \varepsilon + (1 - \varepsilon) \frac{\rho_S C_S}{\rho C_V'}, \quad \mathbf{e}'' = \varepsilon + (1 - \varepsilon) \frac{\rho_S C_S}{\rho C_V''} \text{ are analogous to } \mathbf{e}; \quad \boldsymbol{\xi} = \frac{\kappa'}{\kappa} \quad \& \quad \boldsymbol{\xi}' = \frac{\kappa''}{\kappa'}.$$

3. Exact solution

After maintaining uniform temperature and solute concentrations at the boundaries (stress free), the boundary conditions suitable for this mathematical model are

$$w = \frac{\partial^2 w}{\partial z^2} = 0, \quad \theta = \gamma^I = \gamma^2 = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = I.$$
(3.1)

Now solving Eqs (2.15)-(2.21), we get

$$\begin{bmatrix}
L_{I} + \frac{L_{2}}{P_{I}} \left(I + A \frac{\partial}{\partial t} \right) \left[\left(e + h_{I} \varepsilon \right) \frac{\partial}{\partial t} - \nabla^{2} \right] \left[\left(e' + h_{2} \varepsilon \right) \frac{\partial}{\partial t} - \xi \nabla^{2} \right] \left[\left(e'' + h_{3} \varepsilon \right) \frac{\partial}{\partial t} - \xi' \nabla^{2} \right] \nabla^{2} w = \\
= R\lambda \left(\frac{G - I}{G} \right) \left[\left\{ \left(e' + h_{2} \varepsilon \right) \frac{\partial}{\partial t} - \xi \nabla^{2} \right\} \left\{ \left(e'' + h_{3} \varepsilon \right) \frac{\partial}{\partial t} - \xi' \nabla^{2} \right\} \left\{ \left(\tau \frac{\partial}{\partial t} + H_{I} \right) \right\} \right] \nabla^{2} I w + \\
-S_{I} \lambda \left[\left\{ \left(e + h_{I} \varepsilon \right) \frac{\partial}{\partial t} - \nabla^{2} \right\} \left\{ \left(e'' + h_{3} \varepsilon \right) \frac{\partial}{\partial t} - \nabla^{2} \right\} \left\{ \left(\tau \frac{\partial}{\partial t} + H_{2} \right) \right] \nabla^{2} I w + \\
-S_{2} \lambda \left[\left\{ \left(e + h_{I} \varepsilon \right) \frac{\partial}{\partial t} - \nabla^{2} \right\} \left\{ \left(e' + h_{2} \varepsilon \right) \frac{\partial}{\partial t} - \xi \nabla^{2} \right\} \left\{ \left(\tau \frac{\partial}{\partial t} + H_{3} \right) \right] \nabla^{2} I w.
\end{cases}$$
(3.2)

Now we take the solution in the form of the following expression (using normal mode method for analyzing disturbances)

$$w = W(z) \exp\left(ik_x \cdot x + ik_y \cdot y + nt\right).$$
(3.3)

Now by Eqs. (3.2) and (3.3), we obtain

$$\begin{bmatrix} L_{l} + \frac{L_{2}}{P_{l}}(l+An) \end{bmatrix} [(e+h_{l}\varepsilon)n - (D^{2}-k^{2})] [(e'+h_{2}\varepsilon)n - \xi(D^{2}-k^{2})] \\ [(e''+h_{3}\varepsilon)n - \xi'(D^{2}-k^{2})] (D^{2}-k^{2})W = \\ = -R\lambda \Big(\frac{G-l}{G}\Big) [\{(e'+h_{2}\varepsilon)n - \xi(D^{2}-k^{2})\} \{(e''+h_{3}\varepsilon)n - \xi'(D^{2}-k^{2})\} \{(\tau n+H_{1})\}] k^{2}W + \\ +S_{l}\lambda \Big[\{(e+h_{l}\varepsilon)n - (D^{2}-k^{2})\} \{(e''+h_{3}\varepsilon)n - (D^{2}-k^{2})\} (\tau n+H_{2})] k^{2}W + \\ +S_{2}\lambda \Big[\{(e+h_{l}\varepsilon)n - (D^{2}-k^{2})\} \{(e'+h_{2}\varepsilon)n - \xi(D^{2}-k^{2})\} (\tau n+H_{3})] k^{2}W \Big]$$
(3.4)

where

$$L_1 = p_1^{-l} (\tau n^2 + Fn), \quad L_2 = (\tau n + l) \quad \& \quad D = \frac{d}{dz}.$$

4. The stationary convection

The neutral state, obtained by putting n = 0 in Eq.(3.4), is given by

$$\frac{-\xi\xi'}{P_l} \left(D^2 - k^2 \right)^2 W = -R\lambda \left(\frac{G-l}{G} \right) H_l k^2 W \xi\xi' + S_l \lambda\xi' H_2 k^2 W + S_2 \lambda\xi H_3 k^2 W , \qquad (4.1)$$

when the two boundaries are free, the solution of Eq.(3.4) for the lowest mode is

$$W = W_0 \sin \pi z$$
 where W_0 is a constant. (4.2)

From Eqs (4.1) and (4.2), we get

$$R = \left(\frac{G}{G-I}\right) \left[\frac{\left(\pi^{2} + k^{2}\right)^{2}}{k^{2}H_{I}P_{I}\lambda} + \frac{S_{I}H_{2}}{\xi H_{I}} + \frac{S_{2}H_{3}}{\xi' H_{I}}\right].$$
(4.3)

This relation is termed dispersion relation.

5. Instability analysis

Differentiating Eq.(4.3) with respect to P_1 , H_1 , G, S_1 , S_2 and λ respectively, we get

$$\frac{dR}{dP_l} = -\left(\frac{G}{G-l}\right) \frac{\left(\pi^2 + k^2\right)^2}{k^2 H_l P_l^2 \lambda},\tag{5.1}$$

$$\frac{dR}{dH_{I}} = -\left(\frac{G}{G-I}\right) \left[\frac{\left(\pi^{2}+k^{2}\right)^{2}}{k^{2}H^{2}{}_{I}P_{I}\lambda} + \frac{S_{I}H_{2}}{\xi H^{2}{}_{I}} + \frac{S_{2}H_{3}}{\xi' H^{2}{}_{I}}\right],$$
(5.2)

$$\frac{dR}{dG} = -\frac{1}{\left(G-I\right)^2} \left[\frac{\left(\pi^2 + k^2\right)^2}{k^2 H_1 P_l \lambda} + \frac{S_l H_2}{\xi H_l} + \frac{S_l H_3}{\xi' H_l} \right],\tag{5.3}$$

$$\frac{dR}{dS_1} = \left(\frac{G}{G-I}\right) \left[\frac{H_2}{\xi H_1}\right],\tag{5.4}$$

$$\frac{dR}{dS_2} = \left(\frac{G}{G-I}\right) \left[\frac{H_3}{\xi' H_I}\right],\tag{5.5}$$

$$\frac{dR}{d\lambda} = -\left(\frac{G}{G-I}\right) \frac{\left(\pi^2 + k^2\right)^2}{k^2 H_1 P_1 \lambda^2}.$$
(5.6)

It is evident from Eqs (5.1), (5.2) and (5.3) that medium permeability, suspended particles and compressibility show a destabilizing/stabilizing effect when λ is positive/negative and Eq.(5.6) is always negative and thereby implies that the effect of the variable gravity parameter is to destabilize the system. On the other hand, the positive sign of Eqs (5.4) and (5.5) implies stabilizing character of solute gradients.

6. Graphical representation and verification of analytical results

Now we analyze the dispersion relation graphically. It is clearly evident from Figs 1-3 that as the value of permeability, compressibility and suspended particles increases, the value of the Rayleigh number decreases which thereby proves the destabilizing effects and the flow is no longer stable. On the other hand, it is clear from Figs 5 and 6, that as the values of the solute gradient increase the values of the Rayleigh number increase. Figure 4 shows the destabilizing effect of varying gravity. As the value of the gravity parameter increases, the Rayleigh number decreases.

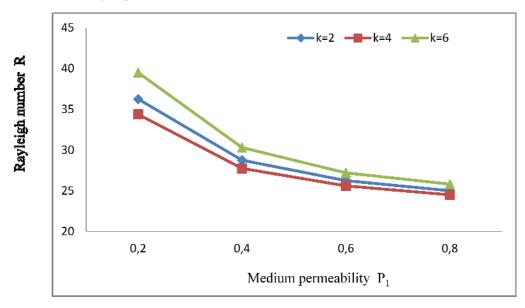


Fig.1. The variation of *R* with *k* (2, 4, 6...) for $H_1=10$, $H_2=20$, $H_3=30$, $S_1=20$, $S_2=30$, G=5, $\lambda = 2$, $\xi = 5$, $\xi' = 10$ and medium permeability P_1 (=0.2, 0.4, 0.6, 0.8).

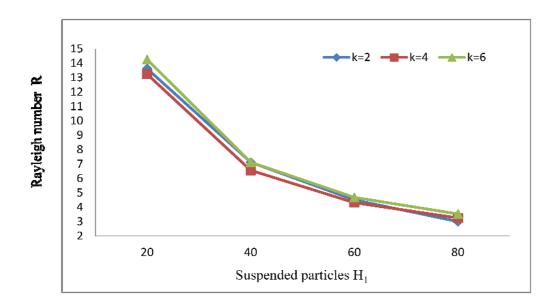


Fig.2. The variation of *R* with *k* (2, 4, 6...) for $P_1=0.5$, $H_2=20$, $H_3=30$, $S_1=20$, $S_2=30$, G=5, $\lambda = 2$, $\xi = 5$, $\xi' = 10$ and suspended particle parameter H_1 (=20, 40, 60, 80).

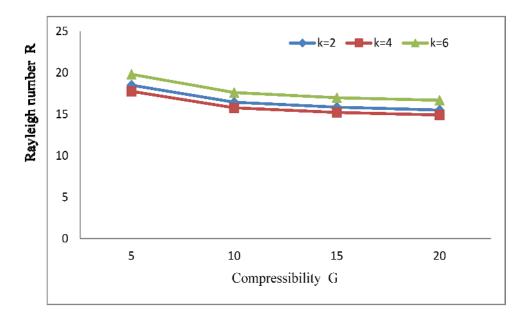


Fig.3. The variation of *R* with *k* (2, 4, 6...) for $P_1=0.5$, $H_1=10$, $H_2=20$, $H_3=30$, $S_1=20$, $S_2=30$, $\lambda = 2$, $\xi = 5$, $\xi' = 10$ and compressibility parameter *G* (=5, 10, 15, 20).

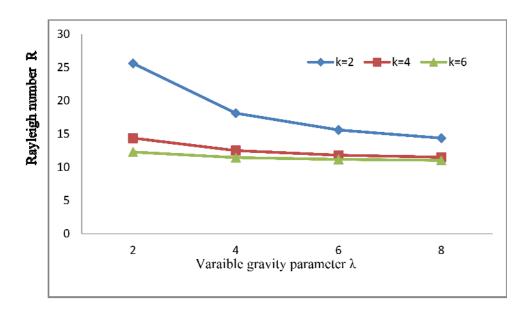


Fig.4. The variation of *R* with *k* (2, 4, 6...) for $P_1 = 0.2$, $H_1 = 10$, $H_2 = 20$, $H_3 = 30$, $S_1 = 20$, $S_2 = 30$, G = 5, $\xi = 5$, $\xi' = 10$ and variable gravity parameter λ (= 2, 4, 6, 8).

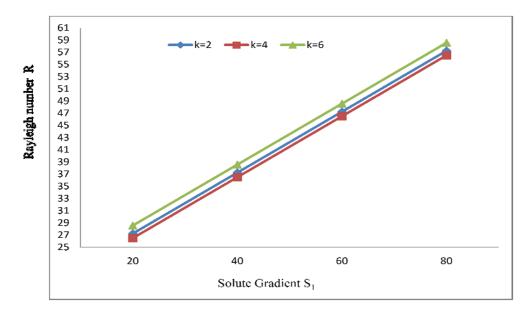


Fig.5. The variation of *R* with *k* (2, 4, 6...) for $H_1=10$, $H_2=20$, $H_3=30$, $S_2=30$, $P_1=0.5$, G=5, $\lambda = 2$, $\xi = 5$, $\xi' = 10$ and solute gradient parameter S_1 (=20, 40, 60, 80).

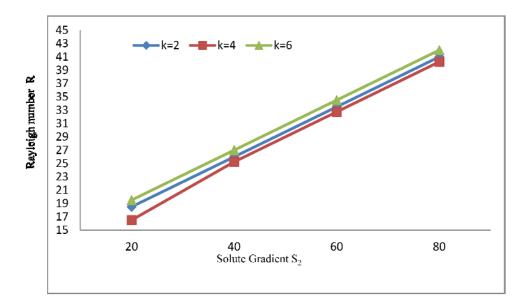


Fig.6. The variation of with k (2, 4, 6...) for $H_1=10$, $H_2=20$, $H_3=30$, $S_1=10$, $P_1=0.5$, G=5, $\lambda = 2$, $\xi = 5$, $\xi' = 10$ and solute gradient parameter $S_2(=20, 40, 60, 80)$.

7. Conclusions

We have examined the effect of medium permeability, suspended particles, compressibility, stable solute gradients and variable gravity parameter in triple diffusive convection and obtained the following results:

Medium permeability, compressibility and suspended particles exhibit a destabilizing effect when gravity increases and hence the flow is no longer stable.

Solute gradients exhibit a stabilizing effect, and the flow is more stable. This stabilizing effect is in agreement with the earlier work of Kango and Rana [28]. The variable gravity parameter also has a destabilizing effect on the system.

Nomenclature

 C_p – specific heat at constant pressure

 C_{pt} – specific heat of particles

- $C_{\rm s}$ specific heat of solid material
- C_v specific heat at constant volume

 C_a^l, C_a^2 – average concentrations

$$D - \frac{d}{dz}$$

- *G* compressibility parameter
- H_1 suspended particle parameter
- K- Stokes drag coefficient
- k wave number of disturbances
- N- perturbation in number density
- N_0 number density of suspended particles
- P_l medium permeability
- p- pressure of the fluid
- q effective thermal conductivity of pure fluid

q',q'' – analogous effective solute conductivities

 $q_F(u,v,w)$ – velocity of fluid

$$q(r,l,s)$$
 – velocity of suspended particles

R – Rayleigh number

- $S_1 \& S_2$ analogous solute Rayleigh number
 - s vertical particle velocity
 - T_a average temperature
 - w vertical fluid velocity
 - α coefficient of thermal expansion
- α^{l}, α^{2} analogous coefficient of solvent expansion
 - β temperature gradient

 $\beta' \& \beta''$ – analogous solute gradients

- $\gamma^{I} \& \gamma^{2}$ perturbation in solute concentrations
 - δp perturbation in pressure
 - $\delta \rho$ perturbation in density
 - ϵ medium porosity
 - θ perturbation in temperature
 - κ thermal diffusitivity
- $\kappa' \& \kappa''$ analogous solute diffusivities
 - λ variable gravity parameter
 - μ viscosity of the fluid
 - μ' viscoelasticity of the fluid
 - v kinematic viscosity
 - v' kinematic viscoelasticity
 - ρ density of the fluid
 - ρ_m constant space average of density
 - ρ_s density of solid material

$$\nabla^2 - \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

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