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ELECTRO-MAGNETOHYDRODYNAMIC TWO FLUID FLOW OF IONIZED-GASES WITH HALL AND ROTATION EFFECTS

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An electro-magnetohydrodynamic (EMHD) two fluid flow and heat transfer of ionized gases through a horizontal channel surrounded by non-conducting plates in a rotating framework with Hall currents is investigated. The Hall effect is considered with an assumption that the gases are completely ionized and the strength of the applied transverse magnetic field is strong. The governing equations are solved analytically for the temperature and velocity distributions in two fluid flow regions. The numerical solutions are demonstrated graphically for various physical parameters such the Hartmann number, Hall parameter, rotation parameter, and so on. It was noticed that an increment is either due to the Hall parameter or the rotation parameter reduces the temperature in the two regions.

Keywords: magnetic and electric fields, immiscible flow, plasma, Hall effect, rotating frame, heat transfer, insulating plates.

1. Introduction

The principle of electro-magneto hydrodynamics describes all phenomena associated with the interaction of magnetic and electric fields with magnetic or electrically conducting fluids, and it has been adopted for various technological as well as in industrial applications. This subject has been known for over a century and there are few admirable research papers presented by many scientists [1-13] who highlighted the diverse aspects of this expeditiously growing field.

Modeling of plasma flows in a rotating frame of reference can also provide significant insights into electro-magneto hydrodynamic flows for a single fluid. It has also been reported in the literature that in magnetized plasma, the impacts of Hall currents become significant when the vitality of the magnetic field is very large. Numerous investigations have been made accessible in the literature on the subject of MHD plasma flows with or without rotation effect under the impact of a strong magnetic field in varied geometrical situations [14-23].This type of flow turned out to be a significant domain of research for both academic and scientific community in view of the enormous applications in aero-space science, engineering, technology and in several industrial contexts such as energy conversion systems, extraction of the products of oil in the geothermal regions, in designing the thermonuclear fusion reactor, MHD power generation model and so forth [24-32]. Likewise, Lohrasbi and Sahai [33] investigated the magnetohydrodynamic heat transfer in a two-phase flow between parallel plates. Umavathi *et al.* [34] studied an oscillatory Hartmann two fluid flow and heat transfer in a horizontal channel. The problem of a hydromagnetic two phase flow in a channel was discussed by Chamkha [35]. Li e*t al*. [36] investigated the linear stability of two fluid interfaces for electro-hydro-dynamic mixing in a channel. Hasnain Qaisrani *et al.* [37] studied the statistical properties of three-dimensional two fluid plasma models. Abd Elmaboud *et al*. [38] studied an electromagnetic flow for two-layer immiscible fluids. Raju [39] investigated the magnetohydrodynamic heat transfer in two ionized fluids flow between two parallel plates with Hall currents was investigated. Recently, Raju and Gowri [40] studied the effects of Hall currents on unsteady magnetohydrodynamic two-ionized fluid flow and heat transfer in a horizontal channel.

It follows from the aforesaid studies that a significance piece of investigation work has been reported in the literature with single liquid flow phenomena and very limited theoretical studies exist for channel flow system of two - fluid flows of electrically conducting fluids such as, gases and liquids without or with Hall and rotation effects. The electro-magnetohydrodynamic two fluid plasma flows in a rotating frame of reference is presumed to be the novel model study and has applications in the development of the conceptual design of fusion reactors and liquid metal magnetohydrodynamic rotating power generators, etc. So, in this article the impacts of Hall currents and Coriolis forces on an EMHD two-fluid flow in a horizontal channel bounded by two parallel non-conducting plates is contemplated. The combined effects of the proposed model may have applications in the design of the energy extraction framework from the blanket of the thermonuclear fusion reactor and in liquid metal MHD rotating power generation models, Hall accelerators and the conceptual design of fusion reactors, which may be interested in Engineers.

2. Mathematical model

The schematic diagram for a steady two-dimensional EMHD two-fluid flow of ionized gases driven by a constant pressure gradient $-\dfrac{\partial p}{\partial x}$ in a horizontal channel bounded by two parallel rigid plates with Hall currents is shown in Fig.1. The two plates are kept at $y = h_1$ and $y = h_2$ and are infinitely long in the *x*-and z-directions choosing the origin halfway between the two plates. The *x*-axis is taken toward the hydrodynamic pressure gradient in the plane parallel to the channel plates however not towards the flow.

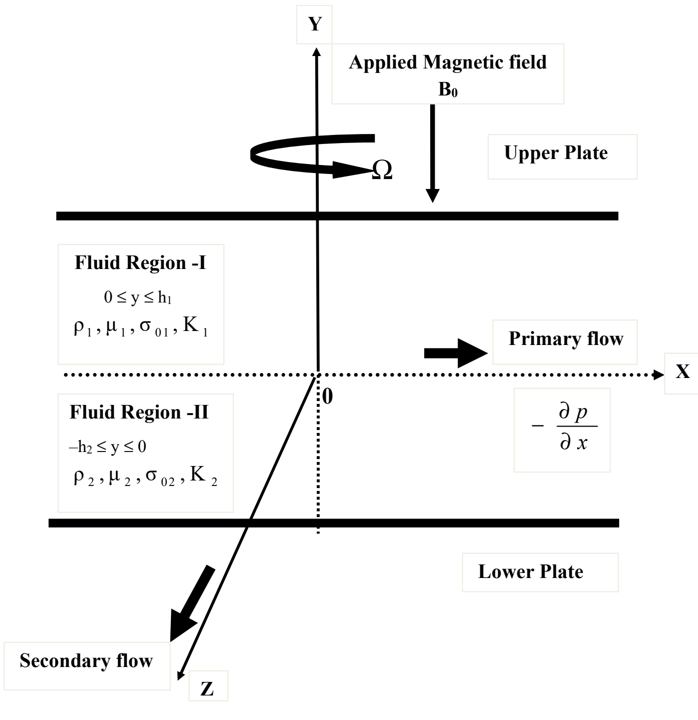

Fig.1. Schematic flow diagram.

A constant magnetic field B_0 is imposed in the *y*-direction, which is transverse to the flow field. The whole system is rotated with an angular velocity Ω about an axis perpendicular to the plates. The plates are considered to be electrically non-conducting. The Hall Effect is considered with the assumption that the gases are totally ionized and the strength of the magnetic field is strong. In order to disregard the induced magnetic field, the magnetic Reynolds number is taken as small. The upper fluid region and lower fluid regions are considered as $0 \le y \le h_1$ and $-h_2 \le y \le 0$, also these fluid regions are labeled as Regions I, II. These two regions are involved by two immiscible electrically conducting incompressible fluids with different densities ρ_1, ρ_2, viscosities μ_1, μ_2, electrical conductivities σ_{01}, σ_{02} and thermal conductivities K_1 and K_2. The channel plates are kept up at uniform temperature T_w, so that $T_{w1} = T_{w2}$. It is supposed that the thermal boundary conditions affect the infinite channel plates everywhere. The thermal conduction in the flow direction and electron heating is also ignored. The interface between the two immiscible fluids is preferred as flat, stress-free and undisturbed.

To define the governing equations, we assume in the basic equations of EMHD flow that the velocity field as $\overline{V_i} = (u_i, 0, w_i)$, magnetic field intensity $\overline{B} = (0, B_0, 0)$, angular velocity $\overline{\Omega} = (0, \Omega, 0)$, current density $\overline{J_i} = (J_{ix}, 0, J_{iz})$ since all over the flow field $J_{iy} = 0$, and the electric field $\overline{E_i} = (E_{ix}, 0, E_{iz})$, where $(i = 1, 2)$ in both the upper and lower liquid stream regions. Likewise, the equations designed for *x*- and *z*- components of the momentum and current as well as the equation of energy in the two fluid areas (namely, Region-I and Region-II) are obtained:

Region-I

$$2\rho_1 \Omega\, w_1 = \mu_1 \frac{d^2 u_1}{dy^2} - \left\{ 1 - s\left(1 - \frac{\sigma_{11}}{\sigma_{01}} \right) \right\} \frac{\partial p}{\partial x} + B_0 \left\{ -\sigma_{11} \left(E_{1z} + u_1 B_0 \right) + \sigma_{21} \left(E_{1x} - w_1 B_0 \right) \right\}, \tag{2.1}$$

$$-2\rho_1 \Omega\, u_1 = \mu_1 \frac{d^2 w_1}{dy^2} + s \frac{\sigma_{11}}{\sigma_{01}} \frac{\partial p}{\partial x} + B_0 \left\{ \sigma_{11} \left(E_{1x} - w_1 B_0 \right) + \sigma_{21} \left(E_{1z} + u_1 B_0 \right) \right\}, \tag{2.2}$$

$$K_1 \frac{d^2 T_1}{dy^2} = -\left[\mu_1 \left\{ \left(\frac{du_1}{dy} \right)^2 + \left(\frac{dw_1}{dy} \right)^2 \right\} + \frac{J_1^2}{\sigma_{01}} \right], \tag{2.3}$$

$$J_{1x} = \sigma_{11} E_{1x} - B_0 \sigma_{11} w_1 + \sigma_{21} E_{1z} + B_0 \sigma_{21} u_1 + \frac{s \sigma_{21}}{B_0 \sigma_{01}} \frac{\partial p}{\partial x}, \tag{2.4}$$

$$J_{1z} = \sigma_{11} \left(\frac{E_{1z}}{B_0} + u_1 \right) - \sigma_{21} \left(\frac{E_{1x}}{B_0} - w_1 \right) - \left(1 - \frac{\sigma_{11}}{\sigma_{01}} \right) \frac{s}{B_0} \frac{\partial p}{\partial x}. \tag{2.5}$$

Region-II

$$2\rho_2 \Omega\, w_2 = \mu_2 \frac{d^2 u_2}{dy^2} - \left\{ 1 - \left(1 - \frac{\sigma_{12}}{\sigma_{02}} \right) s \right\} \frac{\partial p}{\partial x} + B_0 \left[-\sigma_{12} \left(E_{2z} + u_2 B_0 \right) + \sigma_{22} \left(E_{2x} - w_2 B_0 \right) \right], \tag{2.6}$$

$$-2\rho_2 \Omega\, u_2 = \mu_2 \frac{d^2 w_2}{dy^2} + s \frac{\sigma_{22}}{\sigma_{02}} \frac{\partial p}{\partial x} + B_0 \left\{ \sigma_{12} \left(E_{2x} - w_2 B_0 \right) + \sigma_{22} \left(E_{2z} + u_2 B \right)_0 \right\}, \tag{2.7}$$

$$K_2 \frac{d^2 T_2}{dy^2} = -\left[\mu_2 \left\{ \left(\frac{du_2}{dy} \right)^2 + \left(\frac{dw_2}{dy} \right)^2 \right\} + \frac{J_2^2}{\sigma_{02}} \right], \tag{2.8}$$

$$J_{2x} = \sigma_{12}E_{2x} - B_0\sigma_{12}w_2 + \sigma_{22}E_{2z} + B_0\sigma_{22}u_2 + \frac{s\sigma_{22}}{B_0\sigma_{02}}\frac{\partial p}{\partial x}, \tag{2.9}$$

$$J_{2z} = \sigma_{12}(\frac{E_{2z}}{B_0} + u_1) - \sigma_{22}(\frac{E_{2x}}{B_0} - w_2) - (1 - \frac{\sigma_{12}}{\sigma_{02}})\frac{s}{B_0}\frac{\partial p}{\partial x}. \tag{2.10}$$

The boundary conditions for Eqs (2.1)-(2.10) are assumed in the form:

$$u_1(h_1) = 0, \quad w_1(h_1) = 0, \quad u_2(-h_2) = 0, \quad w_2(-h_2) = 0, \tag{2.11}$$

$$u_1(0) = u_2(0), \quad w_1(0) = w_2(0) \quad \text{for} \quad h_1 = h_2, \tag{2.12}$$

$$\mu_1\frac{du_1}{dy} = \mu_2\frac{du_2}{dy} \quad \text{and} \quad \mu_1\frac{dw_1}{dy} = \mu_2\frac{dw_2}{dy} \quad \text{at} \quad y = 0, \tag{2.13}$$

$$\begin{gathered} T_1(h_1) = T_{w1}, \quad T_2(h_2) = T_{w2}, \quad T_1(0) = T_2(0) \quad \text{for} \quad h_1 = h_2, \\ K_1\frac{dT_1}{dy} = K_2\frac{dT_2}{dy} \quad \text{at} \quad y = 0. \end{gathered} \tag{2.14}$$

In the above equations (2.1)-(2.10) and conditions (2.11)-(2.14), the subscripts 1 and 2 refer to the quantities for Region-I and II separately. The quantities u_1, u_2 and w_1, w_2 are the x-and z- components of the velocity in the two fluid regions, likewise these are known as the primary and secondary velocity distributions of the flow field. The quantities T_1 and T_2 are the temperatures in the two regions, and $Cp_i\,(i = 1, 2)$ is the specific heat at constant pressure. The quantities E_{ix} and E_{iz}, J_{ix} and J_{iz} $(i = 1, 2)$ are the x- and z-components of the electric field, and current densities separately. The term $s = p_e / p$ is called as the proportion of electron pressure to the total pressure. The symbol Ω denotes the angular velocity, likewise the symbols σ_{11}, σ_{12} and σ_{21}, σ_{22} are the modified conductivities parallel and normal to the direction of the electric field separately.

The under stated non-dimensional quantities are utilized to make the flow equations (2.1)-(2.10) and conditions dimensionless:

$$\begin{gathered} u^\bullet_1 = \frac{u_1}{u_p}, \quad u^\bullet_2 = \frac{u_2}{u_p}, \quad w^\bullet_1 = \frac{w_1}{u_p}, \quad w^\bullet_2 = \frac{w_2}{u_p}, \quad y^\bullet_i = \frac{y_i}{h_i}, \\ u_p = -\frac{h_1^2}{\mu_1}\frac{\partial p}{\partial x}, \quad m_{ix} = \frac{E_{ix}}{B_0 u_p}, \quad m_{iz} = \frac{E_{iz}}{B_0 u_p}, \quad I_{ix} = \frac{J_{ix}}{\sigma_{0i}B_0 u_p}, \\ I_{iz} = \frac{J_{iz}}{\sigma_{0i}B_0 u_p}, \quad J_i^2 = J_{ix}^2 + J_{iz}^2 \quad (i = 1, 2) \end{gathered} \tag{2.15}$$

and the Hartmann number is $H_a = \sqrt{\sigma_{01} B_0^2 h_1^2 / \mu_1}$, the Taylor number or rotation parameter $K = \sqrt{h_1^2 \rho_1 \Omega / \mu_1}$ (i.e., the reciprocal of the Ekman number), the density ratio is $\rho = \dfrac{\rho_2}{\rho_1}$, the viscosity ratio is $\alpha = \dfrac{\mu_1}{\mu_2}$, the height ratio is $h = \dfrac{h_2}{h_1}$, the electrical conductivity ratio is $\sigma_0 = \dfrac{\sigma_{01}}{\sigma_{02}}$, $\sigma_{01} = \dfrac{\sigma_{12}}{\sigma_{11}}$, $\sigma_{02} = \dfrac{\sigma_{22}}{\sigma_{21}}$, $\dfrac{1}{1+m^2} = \dfrac{\sigma_{11}}{\sigma_{01}}$, $\dfrac{m}{1+m^2} = \dfrac{\sigma_{21}}{\sigma_{01}}$, the Hall parameter is $m = \dfrac{\omega_e}{\left(\dfrac{1}{\tau} + \dfrac{1}{\tau_e}\right)}$, the Prandtl number is $P_{ri} = \dfrac{\mu_i C_{pi}}{K_i}$, the thermal conductivity ratio is $\beta = \dfrac{K_1}{K_2}$ and the temperature distribution is $\theta_i = \dfrac{T_i - T_{wi}}{(u_p^2 \mu_1 / K_i)}$, and ω_e is the gyration frequency of electrons, τ, τ_e are the mean collision time among electron and ion, electron and neutral particles respectively. Likewise, the expression for Hall parameter m is valid in the case of a partially-ionized gas agrees with that of completely ionized gas when τ_e approaches infinity. Thus, with the assistance of the previously mentioned non-dimensional factors (2.15) and ignoring the asterisks for simplicity, the non-dimensional type of principal equations (2.1)-(2.10) and conditions in Region-I and Region-II become:

Region-I

$$2K^2 w_1 = \frac{d^2 u_1}{dy^2} - \frac{(m_{1z} + u_1) H_a^2}{1+m^2} + \frac{(m_{1x} - w_1) m H_a^2}{1+m^2} + k_1 , \tag{2.16}$$

$$-2K^2 u_1 = \frac{d^2 w_1}{dy^2} + \frac{(m_{1x} - w_1) H_a^2}{1+m^2} + \frac{(m_{1z} + u_1) m H_a^2}{1+m^2} + k_2 , \tag{2.17}$$

$$\frac{1}{P_{r1}} \frac{d^2 \theta_1}{dy^2} = -\left[\left\{ \left(\frac{du_1}{dy} \right)^2 + \left(\frac{dw_1}{dy} \right)^2 \right\} + H_a^2 I_1^2 \right], \tag{2.18}$$

$$I_{1x} = \frac{(m_{1x} - w_1)}{1+m^2} + \frac{(m_{1z} + u_1) m}{1+m^2} - \left(\frac{m}{1+m^2} \right) \frac{s}{H_a^2} , \tag{2.19}$$

$$I_{1z} = \frac{(m_{1z} + u_1)}{1+m^2} - \frac{m(m_{1x} - w_1)}{1+m^2} + \left(1 - \frac{m}{1+m^2} \right) \frac{s}{H_a^2} \quad and \quad I_1^2 = I_{1x}^2 + I_{1z}^2 . \tag{2.20}$$

Region-II

$$2\rho\alpha h^2 K^2 w_2 = \frac{d^2 u_2}{dy^2} - \frac{(m_{2z} + u_2) \alpha \sigma_1 h^2 H_a^2}{1+m^2} + \frac{(m_{2x} - w_2) m \alpha \sigma_2 h^2 H_a^2}{1+m^2} + k_3 \alpha h^2 , \tag{2.21}$$

$$-2\rho\alpha h^2 K^2 u_2 = \frac{d^2 w_2}{dy^2} - \frac{(m_{2x} - w_2) \alpha \sigma_1 h^2 H_a^2}{1+m^2} + \frac{(m_{2z} + u_2) m \alpha \sigma_2 h^2 H_a^2}{1+m^2} + k_4 \alpha h^2 , \tag{2.22}$$

$$\frac{1}{P_{r2}}\frac{d^2\theta_2}{dy^2} = -\left[\frac{\beta}{\alpha}\left\{\left(\frac{du_2}{dy}\right)^2 + \left(\frac{dw_2}{dy}\right)^2\right\} + \sigma_0\beta h^2 H_a^2 I_2^2\right], \tag{2.23}$$

$$I_{2x} = \frac{(m_{2x} - w_2)\sigma_0\sigma_1}{1+m^2} + \frac{(m_{2z} + u_2)m\sigma_0\sigma_2}{1+m^2} - \left(\frac{m}{1+m^2}\right)\frac{\sigma_0^2\sigma_2 s}{H_a^2}, \tag{2.24}$$

$$I_{2z} = \frac{(m_{2z} + u_2)\sigma_0\sigma_1}{1+m^2} - \frac{(m_{2x} - w_2)m\sigma_0\sigma_2}{1+m^2} + \left(1 - \frac{\sigma_0\sigma_1}{1+m^2}\right)\frac{s\sigma_0}{H_a^2},$$

$$I_2^2 = I_{2x}^2 + I_{2z}^2 \tag{2.25}$$

where

$$k_1 = 1 - \frac{sm^2}{1+m^2}, \qquad k_2 = -\frac{sm}{1+m^2}, \qquad k_3 = 1 - \left(1 - \frac{\sigma_0\sigma_1}{1+m^2}\right)s, \qquad k_4 = -\frac{\sigma_0\sigma_2 sm}{1+m^2}.$$

The boundary conditions are

$$u_1(+1) = 0, \quad w_1(+1) = 0, \quad u_2(-1) = 0, \quad w_2(-1) = 0, \quad u_1(0) = u_2(0), \quad w_1(0) = w_2(0);$$

$$\frac{du_1}{dy} = \frac{1}{\alpha h}\frac{du_2}{dy} \quad \text{and} \quad \frac{dw_1}{dy} = \frac{1}{\alpha h}\frac{dw_2}{dy} \quad \text{at} \quad y = 0, \tag{2.26}$$

$$\theta_1(1) = 0, \quad \theta_2(-1) = 0, \quad \theta_1(0) = \theta_2(0) \quad \text{and} \quad \frac{d\theta_1}{dy} = \frac{1}{\beta h}\frac{d\theta_2}{dy} \quad \text{at} \quad y = 0. \tag{2.27}$$

3. Solution to the problem

Equations (2.16)-(2.25) as well as the conditions (2.26) and (2.27) represent a system of linear ordinary differential equations and conditions, and these are solved analytically. Firstly, Eqs (2.16), (2.17) and (2.21), (2.22) are solved for the velocity employing the prescribed conditions (2.26). Consequently, the closed form solutions for temperature in the two-fluid regions and heat transfer rate at the side plates are achieved when the plates are made of a non-conducting material. As the side plates are non-conducting, the total current should vanish and hence the following conditions are utilized [15, 39].

The conditions for the currents are

$$\int_0^1 I_{1x}dy = 0 \quad \text{and} \quad \int_0^1 I_{2x}dy = 0, \tag{3.1}$$

if the plates are kept at a huge distance in the direction of the x-axis.

Similarly,

$$\int_0^1 I_{1z} dy = 0 \quad \text{and} \quad \int_0^1 I_{2z} dy = 0, \tag{3.2}$$

when the plates are kept at a huge distance in the direction of the z-axis.

Using the above conditions (3.1)-(3.2), the constants involved in the solution of velocity distributions are determined and then the analytical solutions for u_1, u_2, w_1, w_2, I_1 and I_2 in the two regions are attained. Substituting the expressions of u_1, u_2, w_1, w_2, I_1 and I_2 in the energy equations (2.18) and (2.23), the following resultant ordinary differential equations are attained for the two-fluid regions.

Region-I:

$$\frac{1}{P_{r1}}\frac{d^2\theta_1}{dy^2} = -\left[\left(\frac{dQ_1}{dy}\right)\left(\frac{d\bar{Q}_1}{dy}\right) + \frac{H_a^2}{1+m^2}(Q_1 - 1)(\bar{Q}_1 - 1)\right]. \tag{3.3}$$

Region-II:

$$\frac{1}{P_{r2}}\frac{d^2\theta_2}{dy^2} = -\left[\frac{\beta}{\alpha}\left(\frac{dQ_2}{dy}\right)\left(\frac{d\bar{Q}_2}{dy}\right) + \left(\frac{h^2 H_a^2 \beta\sigma_0}{1+m^2}\right)(Q_2 - 1)(\bar{Q}_2 - 1)\right] \tag{3.4}$$

where

$$Q_1 = \frac{q_1}{q_{1m}}, \qquad Q_2 = \frac{q_2}{q_{2m}}, \qquad q_1 = u_1 + iw_1, \qquad q_2 = u_2 + iw_2,$$

$$u_1 = \frac{q_1 + \bar{q}_1}{2}, \qquad w_1 = \frac{q_1 - \bar{q}_1}{2i}, \qquad u_2 = \frac{q_2 + \bar{q}_2}{2}, \qquad w_2 = \frac{q_2 - \bar{q}_2}{2i},$$

$$q_{1m} = u_{1m} + iw_{1m}, \qquad q_{2m} = u_{2m} + iw_{2m}.$$

$\bar{Q}_1$ and $\bar{Q}_2$ are the complex conjugates of Q_1 and Q_2, respectively. The quantities q_1 and q_2 are the solutions of the velocities; q_{1m} , q_{2m} are the corresponding mean velocities. The solutions for q_1 and q_2 are obtained by solving Eqs (2.16), (2.17), (2.21) and (2.22) utilizing the conditions (2.26). By using these solutions, the above stated Eqs (3.3) and (3.4) are solved under the boundary and interface conditions (2.27) to obtain the temperature distributions θ_1, θ_2 in the two regions. The rates of heat transfer at the upper and lower plate of the channel are given by $Nu_1 = -\dfrac{d\theta_1}{dy}$ at $y = 1$ and $Nu_2 = \dfrac{1}{\beta h}\dfrac{d\theta_2}{dy}$ at $y = -1$ are also obtained. The details of calculation are omitted here as they are lengthy expressions.

4. Results and discussion

To acquire the physical insight into the problem and to discuss the results, numerical calculations are made for both the velocity and temperature fields in the two-ionized fluid regions. The resulted estimates for various values of the governing parameters are represented graphically in Figs 2-19. The specific parameters $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\rho = 1$ and $P_{r1} = 1 = P_{r2}$ are fixed for all the computations and the impact of auxiliary imperative flow parameters on both the velocity and temperature is studied. As expected, it is also observed that the solutions are independent of the ionization parameter s (that is, the ratio of electron pressure to the total pressure).

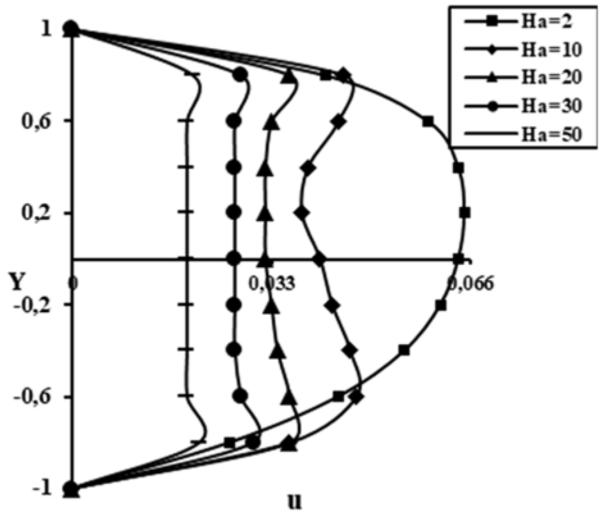

Fig.2. Primary velocity for different H_a and $\alpha = 0.333$, $\sigma_0 = 2$, $h = 0.8$, $\sigma_1 = 1.2$, $m = 2$, $\sigma_2 = 1.5$, $K = 2$, $\rho = 1$.

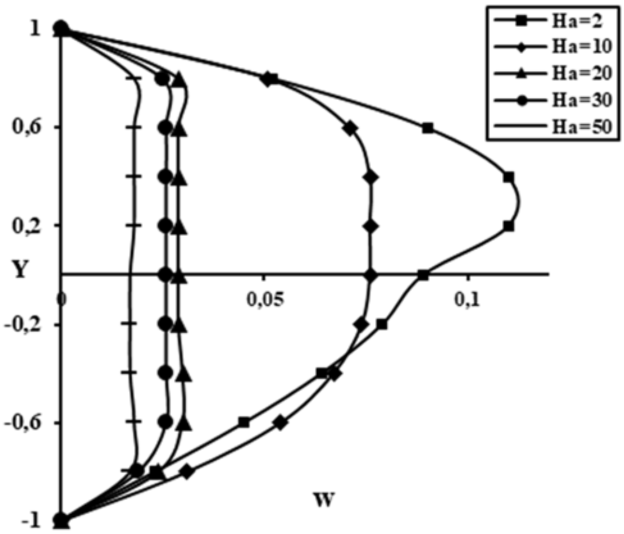

Fig.3 Secondary velocity for different H_a and $\alpha = 0.333$, $\sigma_0 = 2$, $h = 0.8$, $\sigma_1 = 1.2$, $m = 2$, $\sigma_2 = 1.5$, $K = 2$, $\rho = 1$.

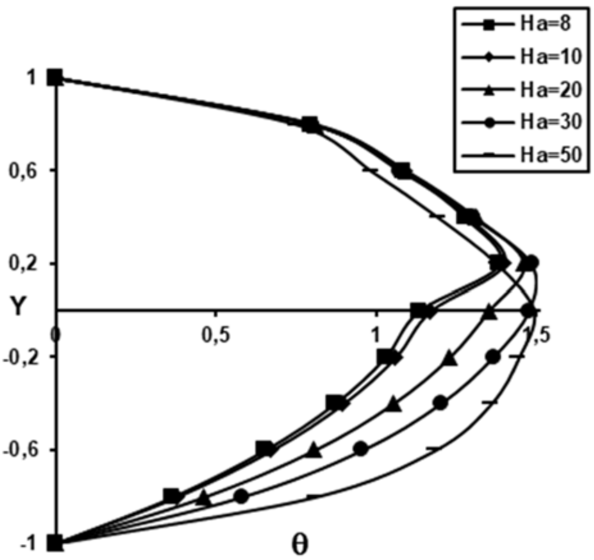

Fig.4. Temperature for different H_a and $\alpha = 0.333, s_0 = 2$, $s_1 = 1.2$, $s_2 = 1.5$, $h = 0.1$, $m = 2$, $r = 1$, $b = 1$, $K = 2$.

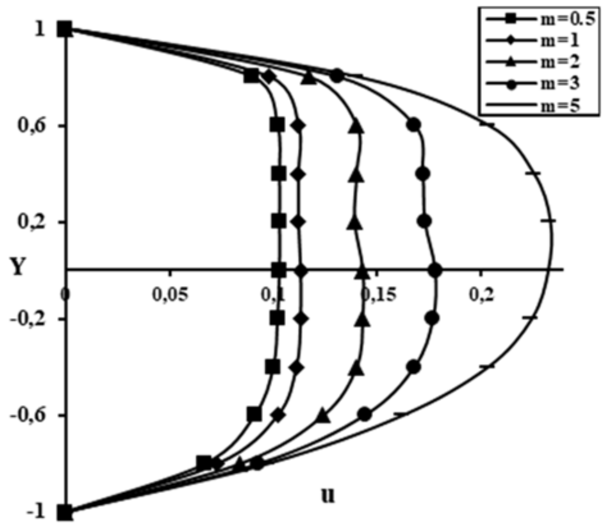

Fig.5. Primary velocity for different m and $H_a = 10, \sigma_0 = 2, h = 0.8, \sigma_1 = 1.2, \alpha = 0.333$, $\sigma_2 = 1.5$, $K = 2$, $\rho = 1$

The impact of varying the Hartmann number $\mathrm{H_a}$ on velocity and temperature fields in the two fluid regions is demonstrated in Figs 2-4. It is seen from Figs 2 and 3 that a rise in the Hartmann's number diminishes the primary and secondary velocities in the two regions. This reduction in the velocity components may be due to the presence of a magnetic field in an electrically conducting fluids, introduction of a Lorentz force which acts against the flow under the applied magnetic field in the normal direction (that is, along the *y*-direction).

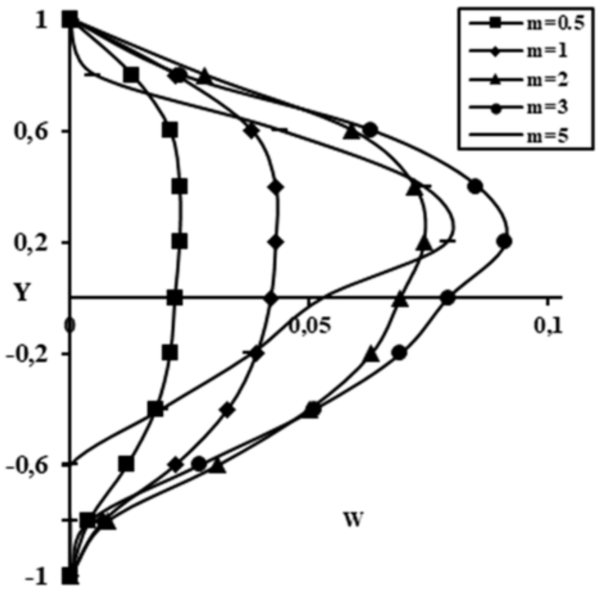

Fig.6. Secondary velocity for different m and $\alpha = 0.333, H_a = 10, \sigma_0 = 2, K = 2, \sigma_1 = 1.2, h = 0.3, \sigma_2 = 1.5, \rho = 1.$

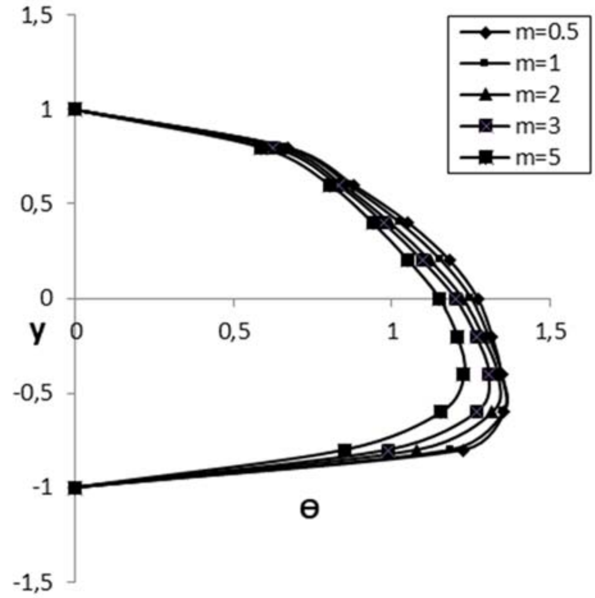

Fig.7. Temperature for different m and $H_a = 10, h = 0.8, \sigma_0 = 2, K = 2, \sigma_1 = 1.2, \sigma_2 = 1.5, \rho = 1, \beta = 1, \alpha = 0.333.$

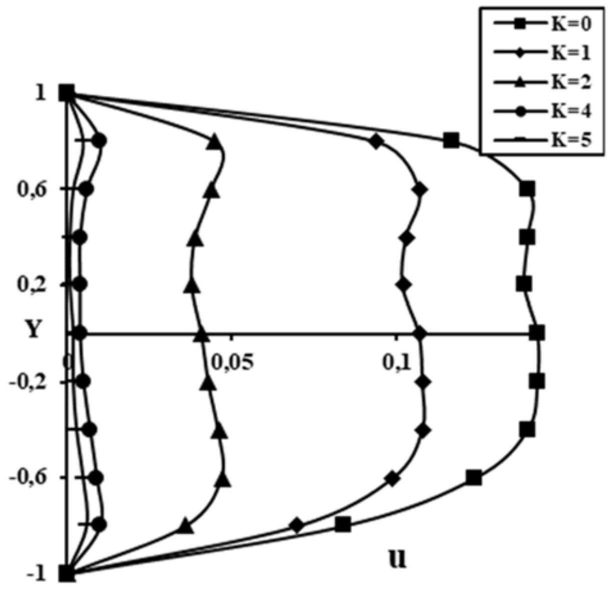

Fig.8 Primary velocity for different K and $H_a = 10, \alpha = 0.333, \sigma_0 = 2, m = 2, \sigma_1 = 1.2, \rho = 1, \sigma_2 = 1.5, h = 0.8.$

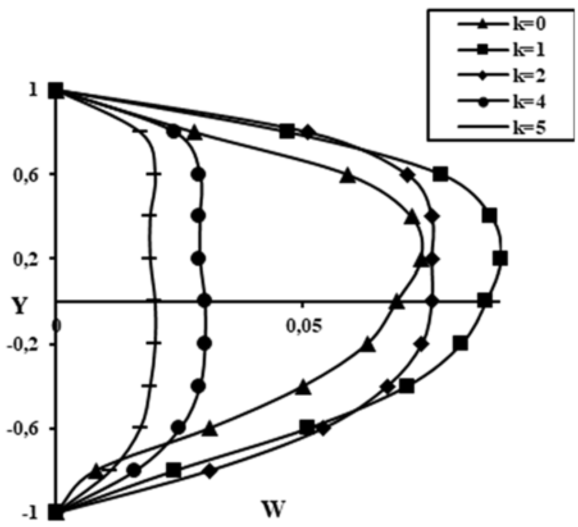

Fig.9 Secondary velocity for different K and $H_a = 10, \alpha = 0.333, \sigma_0 = 2, m = 2, \sigma_1 = 1.2, \rho = 1, \sigma_2 = 1.5, h = 0.8.$

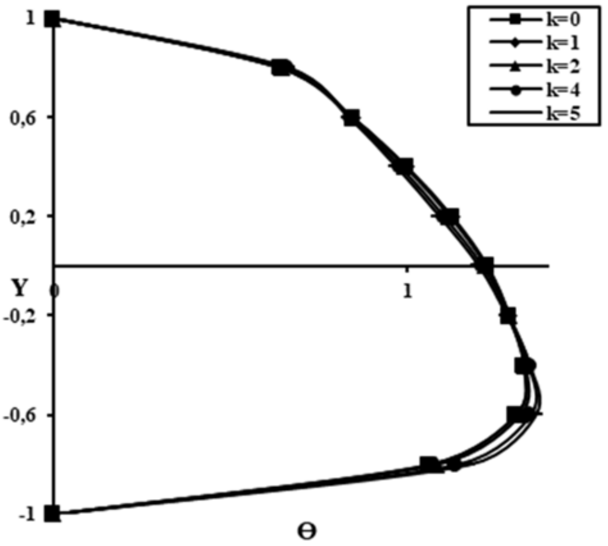

Fig.10. Temperature for different K and $\alpha = 0.333$, $\sigma_0 = 2$, $\beta = 1, \sigma_1 = 1.2$, $m = 2$, $\sigma_2 = 1.5, \rho = 1$, $H_a = 10$, $h = 0.8$.

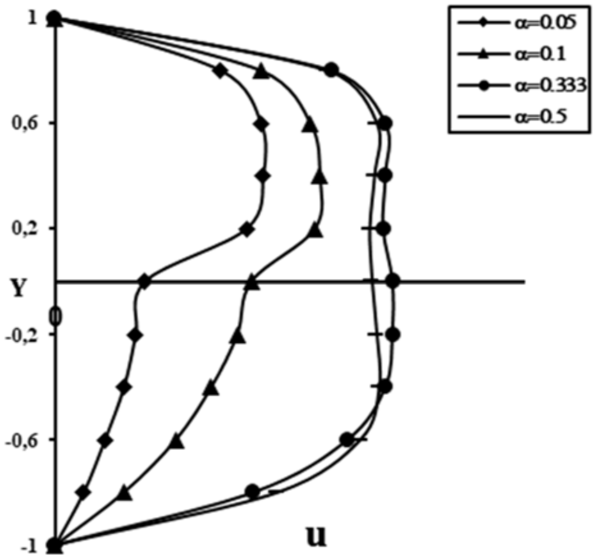

Fig.11. Primary velocity for different α and $H_a = 10, \sigma_0 = 2$, $h = 0.8, \sigma_1 = 1.2$, $m = 2$, $\sigma_2 = 1.5$, $K = 2, \rho = 1$.

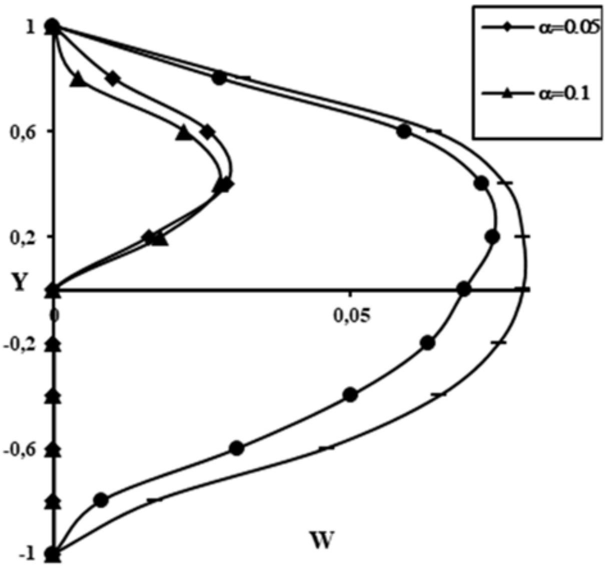

Fig.12. Secondary velocity for different α and $H_a = 10, \sigma_0 = 2$, $K = 2, \sigma_1 = 1.2, \sigma_2 = 1.5$, $h = 0.8$, $m = 2$, $\rho = 1$.

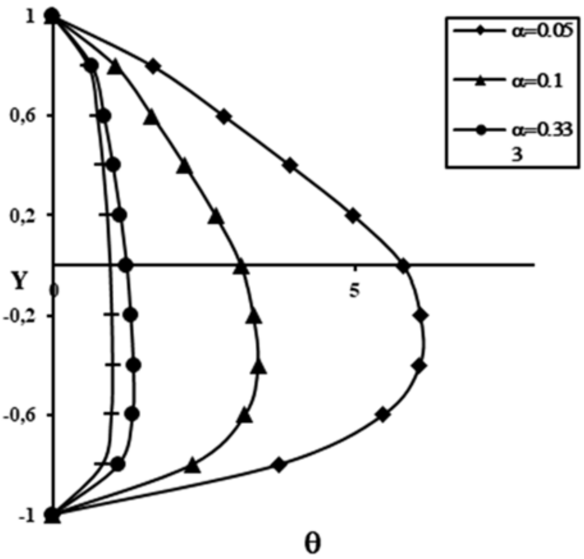

Fig.13. Temperature for different α and $H_a = 10, \sigma_0 = 2, \beta = 1, \sigma_1 = 1.2, \sigma_2 = 1.5$, $h = 0.8, \rho = 1$, $m = 2$, $K = 2$.

Hence, the resulting resistive force reduces the fluid velocity. Additionally, the maximum velocity in the channel falls below the channel center line close to the region-II for an estimate of the Hartmann number $H_a = 10$ in the case of the primary velocity component. While in the secondary velocity component, it is noticed that the magnitude of fluid velocity is greater in the upper region as compared to that of the lower fluid region for an estimate of the Hartmann number $H_a = 2$. It is noticed from Fig.4. that the temperature distribution in the upper fluid region increases up to a specific estimate of the Hartmann number, say $H_a = 30$ and then it falls beyond this estimate while it improves in the entire lower fluid region.

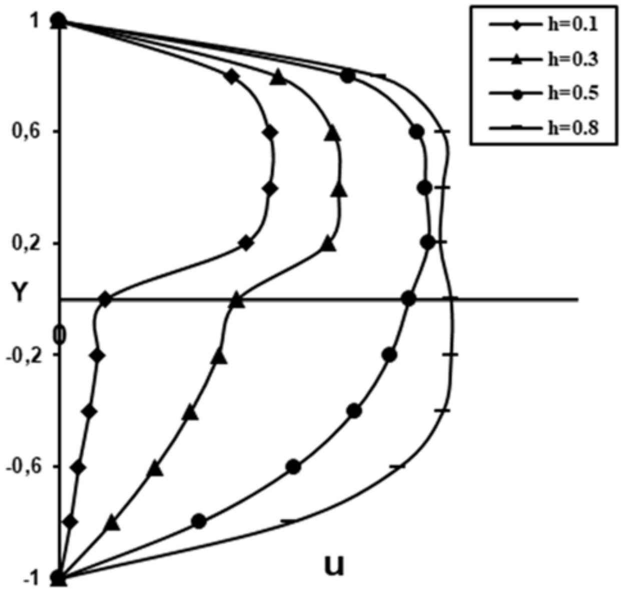

Fig.14. Primary velocity for different h *and* $H_a = 10$, $a = 0.333$, $s_0 = 2$, $m = 2$, $s_1 = 1.2$, $s_2 = 1.5$, $\rho = 1$, $K = 2$.

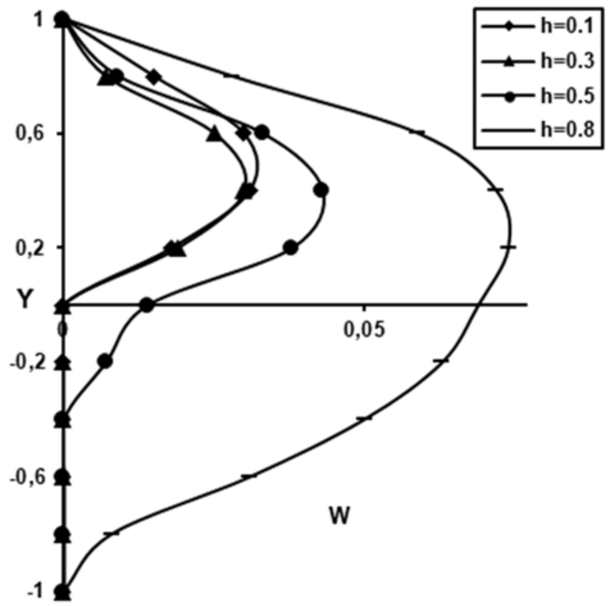

Fig.15. Secondary velocity for different h and $H_a = 10, s_0 = 2$, $K = 2$, $s_1 = 1.2$, $s_2 = 1.5$, $m = 2$, $\rho = 1$, $\alpha = 0.333$.

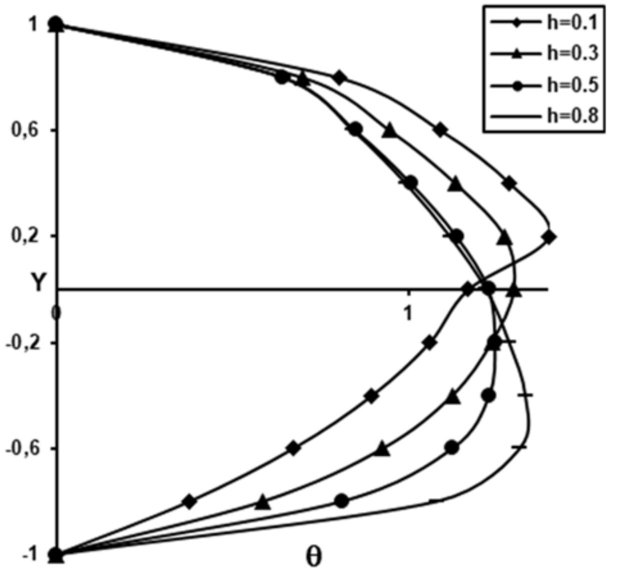

Fig.16. Temperature for different h and $a = 0.333$, $H_a = 10$, $s_0 = 2$, $K = 2$, $s_1 = 1.2$, $s_2 = 1.5$, $m = 2$, $r = 1$, $b = 1$.

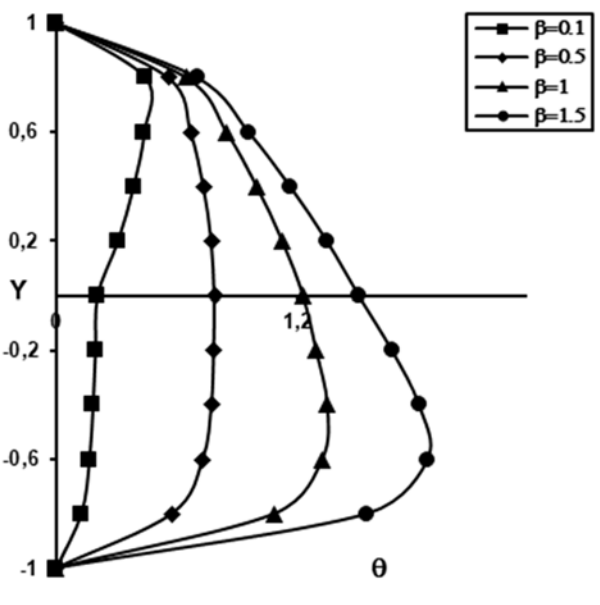

Fig.17. Temperature for different β and $H_a = 10$, $h = 0.8$, $\alpha = 0.333$, $s_0 = 2$, $m = 2$, $s_1 = 1.2$, $s_2 = 1.5$, $K = 2$, $\rho = 1$.

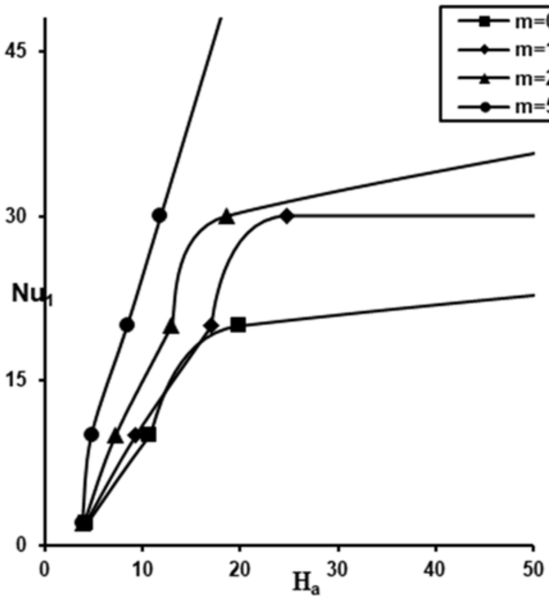

Fig.18. Nusselt number Nu_1 for different H_a and $a = 0.333$, $h = 0.8$, $s_0 = 2$, $K = 2$, $s_1 = 1.2$, $s_2 = 1.5$, $\rho = 1$, $b = 1$.

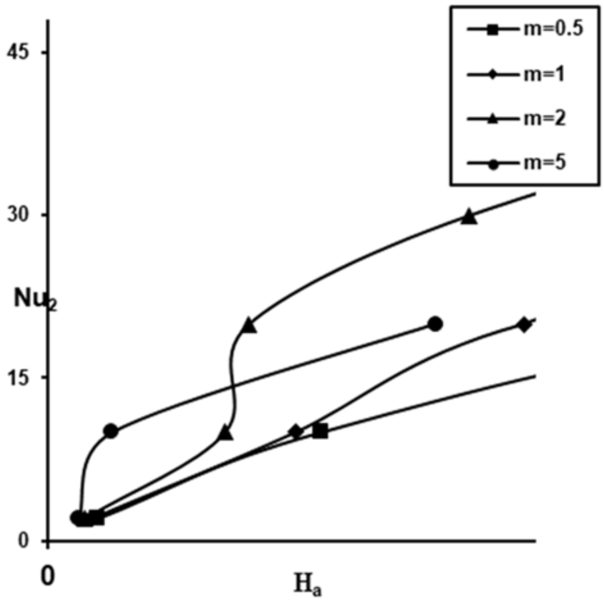

Fig.19. Nusselt number Nu_2 for different H_a and $a = 0.333$, $h = 0.8$, $s_0 = 2$, $s_1 = 1.2$, $K = 2$, $s_2 = 1.5$, $\rho = 1$, $\beta = 1$.

Figures 5-7 present the impact of the Hall parameter m on the primary flow velocities u_1, u_2, secondary flow velocities, w_1, w_2 and temperature distributions θ_1, θ_2 in the two fluid regions. It is observed from Fig. 5 that increasing Hall parameter values consistently boosts the primary flow in the two regions. It indicates that the Hall parameter accelerates velocity of the two fluids in the primary flow direction, and this is because of the fact that a rise in the Hall parameter m reduces the effective conductivity and hence the magnetic damping force is increased in the flow fields. But, it is noticed from Fig.6. that the secondary velocity component increases for minute values of the Hall parameter up to $m = 3$ and it decreases for $m > 3$. It is additionally seen that the magnitude of the fluid velocity is smaller in the lower region compared with that of the upper fluid region as the Hall parameter increases. This kind of tendency may be due to the fact that the minimum retarding force acting along the y-direction that is created by virtue of the interaction of the applied magnetic field and the Hall current in electrically conducting fluids. Figure 7 indicates that a rise in the Hall parameter reduces the temperature field in the two regions. It is further noticed that the magnitude of temperature is higher in the lower region in comparison to that in the upper region with a rise in the Hall parameter.

The effect of the Taylor number K on the velocity and temperature distributions is represented in the Figs 8-10. It is seen that an increment in the Taylor number decreases the velocity and temperatures in the two regions. The maximal temperature dispersal in the channel tends to move below the channel centerline towards the lower region as the Taylor number enhances. We have the larger temperature field in the upper fluid region in comparison with the lower region for a rise in the Taylor number.

Figures 11-13 display the effect of the viscosity ratio α on the velocity and temperature fields. It seen from Fig.11. that the primary velocity distribution increases up to a specific value of the parameter, say $\alpha = 0.333$, and beyond this estimate it decreases. While the secondary velocity distribution diminishes as the viscosity ratio α increases up to a specific estimate of the parameter, say $\alpha = 0.1$ and beyond the value of this parameter, it increases (as is seen in Fig.12.). Likewise, it is seen that the velocity profile is high in the upper region when compared to the lower region for smaller values of the viscosity ratio (for $\alpha = 0.05$ and 0.1). That is for smaller values of the viscosity ratio, the viscosity of the lower region becomes thick and hence the velocity in the lower region decreases. From Fig.13., it is noticed that an increment in the viscosity ratio α is to decrease the temperature distributions in the two regions. It is additionally seen that the magnitude of temperature is high in the lower region in comparison with the upper region as α increases. This means that the magnitude of contraction of temperature is lesser in the lower fluid region when compared to the upper region as the viscosity ratio increases.

From Fig.14., it is seen that an enhancement in the height ratio h increases the primary velocity in the two regions. But from Fig.15., it is seen that an increase in the height ratio h reduces the secondary velocity distribution for smaller values of the height ratio up to the value, say, $h = 0.3$ and beyond this value it increases in both the regions. Also, it is observed that the flow is high in the upper region as compared to the lower region for smaller values of the height ratio (for $h = 0.1$ and 0.3). It is noticed that the maximum secondary velocity distribution in the channel tends to move above the channel centerline towards the region-I as the height ratio augments. It is observed from Fig.16. that the temperature in the upper region reduces while it enhances in the lower region with an increase in the height ratio. Also, the extreme temperature in the channel tends to move beneath the channel centerline towards the region-II for the height ratio $h = 0.8$ and the maximum temperature in the channel moves above the channel centerline towards the region-I for the height ratio $h = 0.1$. That is, smaller the height of the upper fluid region compared to the lower region, larger in the magnitude of the temperature field.

Figure 17 portrays the impact of the thermal conductivity ratio β on the temperature distribution. It is seen that the temperature enhances as the thermal conductivity ratio β increases. This shows that the ratio of thermal conductivity accelerates the fluid temperature in the two fluid regions. In like manner, the extreme temperature in the channel moves below the channel centerline towards the region-II for the specific values of the thermal conductivity ratio (say, $\beta = 1.5$ and 2), and the extreme temperature in the channel tends to move above the channel centerline towards the region-I for smaller values of the thermal conductivity ratio.

From Figs 18 and 19 it is seen that an increment either in the Hartmann number or in the Hall parameter increases the rate of the heat transfer coefficient at both the plates when all other governing parameters are fixed.

Further, it is seen that there is no much significant variation in the velocity and temperature as electrical conductivity σ_0 increases.

The above mentioned results reveal that there is a significant influence on both the velocity and temperature fields, as well as the heat transfer rates at the plates. Hence, the present theoretical model may be useful in dealing with real engineering problems.

5. Conclusions

The impact of flow parameters, such as the Hartmann number, the Hall parameter, the Taylor number, the ratios of the viscosity, density, height, electrical conductivity and thermal conductivity on the velocity and temperature fields in two-fluid regions is studied. The major outcomes of this investigation are abridged as follows:

1. Fluid velocity decreases in the upper and lower fluid regions due to an in increase either in the Hartmann number or Taylor number.
2. An increase in the Hall parameter develops the primary velocity distribution in two regions. However, the secondary velocity distribution increases for small values of the Hall parameter for a specific value of this parameter and from that point diminishes.
3. The smaller the value of the viscosity of the fluid in the lower region compared to the upper fluid region, the larger the flow field.
4. A growth in the height ratio increases the primary velocity component in the two regions but decreases the secondary velocity component for smaller values of the height ratio and beyond that value it increases in the two regions.
5. Temperature in the upper fluid region increases up to a specific estimate of the Hartmann number and then it decreases beyond this estimate while it increases in the entire lower fluid region.

6. An increase either in the Hall parameter or the Taylor number or the viscosity ratio reduces the temperature distribution.
7. An increase in the thermal conductivity ratio enhances the temperature distribution.
8. The rate of heat transfer coefficient increases with an increase either in the Hartmann number or in the Hall parameter.

Nomenclature

B_0	– applied magnetic field
$\overline{B}$	– magnetic flux density
c_{p_i} $(i=1,2)$	– the specific heat at constant pressure in the two-fluid regions
E_{ix}, E_{iz}	– applied electric fields in x- and z- directions
H_a	– Hartmann number
h	– ratio of the heights of the two regions
h_1	– height of the channel in the upper region (Region-I)
h_2	– height of the channel in the lower region (Region-II)
$I_{ix}, I_{iz} (i=1,2)$	– dimensionless current densities along x- and z- directions in Region-I and Region-II
I_1, I_2	– symbols for currents in two-fluids
$\overline{J}_i$ $(i=1,2)$	– current density
J_{ix}, J_{iz}	– current densities along x- and z- directions in two fluid regions
K	– rotation parameter (or Taylor number)
K_1, K_2	– thermal conductivities of the two fluids in region-I and region-II
m	– Hall parameter
m_{ix}, m_{iz}	– dimensionless electric fields
$m_{1x}, m_{1z}; m_{2x}, m_{2z}$	– dimensionless electric field in Region-I and region-II
p	– pressure
p_e	– electron pressure
$P_{ri}(i=1,2)$	– Prandtl number of the two fluids
Q_1, Q_2	– symbols used for simplicity as $Q_1 = q_1/q_{1m}, Q_2 = q_2/q_{2m}$
$\overline{Q_1}, \overline{Q_2}$	– complex conjugates of Q_1, Q_2
q_1, q_2	– solutions of the velocity distributions for both fluid regions in complex form as $q_1 = u_1 + iw_1, q_2 = u_2 + iw_2$
q_{1m}, q_{2m}	– mean velocities in complex notations as $q_{1m} = u_{1m} + iw_{1m}$ and $q_{2m} = u_{2m} + iw_{2m}$
s	– ratio of electron pressure to the total pressure
$T_i(i=1,2)$: T_1, T_2	– temperatures of the fluids in Region-I and Region-II
$T_{w_i}(i=1,2): T_{w_1}, T_{w_2}$	– constant plate temperatures in Region-I and Region-II
$u_i, (i=1,2): u_1, u_2$	– Primary velocity distributions (velocity components along x-axis) in Region-I and Region-II
u_{1m}, u_{2m}	– primary mean velocity distributions in the two fluid regions
u_p	– the characteristic velocity

$\overline{V_i}, (i = 1,2)$ – fluid velocity

$w_i, (i = 1,2): w_1,\ w_2$ – secondary velocity distributions (component of velocity field along the Z-direction) in the two fluid regions

w_{1m}, w_{2m} – secondary mean velocity distributions in the two fluid regions

(x, y, z) – space co-ordinates in rectangular Cartesian co-ordinate system

$-\partial p / \partial x$ – common constant pressure gradient

Greek symbols

α – ratio of the viscosities

β – ratio of thermal conductivities

$\mu_i, (i = 1,2): \mu_1, \mu_2$ – viscosities of the two fluids

$\sigma_{0i}, (i = 1,2): \sigma_{01}, \sigma_{02}$ – electrical conductivities of the two fluids

σ_0 – ratio of the electrical conductivities

$\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}$ – modified conductivities parallel and normal to the direction of electric field

σ_1, σ_2 – symbols for the ratios $\sigma_1 = \dfrac{\sigma_{12}}{\sigma_{11}}, \sigma_2 = \dfrac{\sigma_{22}}{\sigma_{21}}$

ρ_1, ρ_2 – densities of the two fluids

ρ – ratio of the densities

$\theta_1,\ \theta_2$ – dimensionless temperature distributions for two-fluid regions

$\tau,\ \tau_e$ – mean collision time between electron and ion, electron and neutral particles

ω – frequency of oscillation

ω_e – gyration frequency of electron

Ω – angular velocity

Subscripts

1, 2 – refers to the quantities in the upper and lower fluid regions (Region-I and Region-II)

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