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HEAT AND MASS TRANSFER EFFECT ON AN INFINITE VERTICAL PLATE IN THE PRESENCE OF HALL CURRENT AND THERMAL RADIATION WITH VARIABLE TEMPERATURE

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MHD and radiated heat flow on a rotating system of an electrically conducting fluid in the presence of Hall current under the influence of variable temperature is studied analytically. An exact solution of a non-dimensional form of coupled partial differential equations is obtained by the technique of Laplace transform. The effect of temperature, velocity and concentration is analyzed for various parameters like the Hall parameter (m), thermal radiation (R),

rotation parameter (Ω) , Hartmann number (M) and results are discussed in detail with the help of graphs. A mixed analysis of a rotating fluid with Hall current and thermal radiation plays a very essential role in the research area such as plasma physics, MHD generator, fluid drift sensor, cosmological and geophysical level, etc.

Keywords: MHD, Hall effects, radiation, vertical plate, variable temperature.

1. Introduction

The effect of a magnetic field on a fluid with natural convection plays a major role in electrolytes, liquid metal and ionized gases. The problem of an unsteady free convection flow is encountered in many natural phenomena and engineering applications with MHD analysis. An MHD flow with Hall current is encountered in engineering applications in MHD pumps, MHD bearings, etc. A free convection unsteady flow is of great interest in industrial fields such as fibre insulation, nuclear reactor, spacecraft design, geothermal system, etc. It also plays a very vital role in plasma physics and astrophysics. The Hall effect principle used in Hall effect joysticks, is used to control hypersonic flows, hydraulic valves, nuclear power reactors, brushless DC motors, calculate magnetic fields, build plasma actuator, and Hall current accelerators.

In a rotating system, Hall current on an accelerated plate or sheet was investigated by Pop. [1]. Muthucumaraswamy *et al.* [2] analyzed a magnetic flow with variable temperature on a vertical accelerated plate of a rotation system without mass diffusion. Deka [3] analyzed Hall current with MHD flow on an accelerated plate. Hall current and thermal characteristics effect in a rotating system in MHD flow was studied by Ghosh *et al.* [4]. Muthucumaraswamy and Jeyanthi [5] studied a rotating system effect of a first order chemical reaction and variable temperature with Hall current on an MHD flow. An exact solution of the radiation effect in the presence of variable temperature on a vertical moving plate with mass flux was studied by Muthucumaraswamy [6]. Thamizhsudar *et al.* [7] investigated an MHD free convection flow with Hall effect with combined MHT (Mass and Heat Transfer)

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in a rotating system. A hydro- magnetic natural convection flow with Hall effect in a spinning fluid was studied by Ram [8]. A hydromagnetic flow with Hall effect in a porous medium was studied by Gupta [9]. The inverse Laplace algorithm was studied by Hetnarski [10].

The objective of this paper is to study the thermal radiation on a magneto hydrodynamic fluid in the presence of a rotating system with Hall current on an isothermal vertical plate relative to uniform mass diffusion and variable temperature. The analytic solutions of dimensionless governing equations are expressed as a function of exponential and complement error function. Numerical results for concentration, temperature and fluid velocity profiles are represented by graphs which are executed by MATLAB.

2. Mathematical formulation and solution

Consider a viscous, incompressible transient flow of an electrically conducting fluid on an infinite vertical plate or sheet (z'=0). The plate motion is in upward direction along the x'-axis and the y'-axis is perpendicular to both the x'-axis and the z'-axis. Initially, (t=0). The system and fluid rotate about the z'-axis with uniform angular velocity $-\Omega'$, which is normal to the sheet or plate. The fluid temperature - T'_{∞} remains same at $t' \le 0$ and fluid concentration- C'_{∞} also. At present work, the variable temperature effect on a transient flow of a viscous incompressible fluid on an infinite vertical plate with constant mass diffusion is considered. A uniform magnetic field B_0 acting parallel to the z'-axis and y'-axis respectively. At the same time, the system rotates along the z'-axis in a constant angular velocity in anticlockwise direction. The motion of the plate started with velocity $u'=u_0$ about the x'-axis and the fluid temperature linearly raised with time t' and levels of concentration (C'_w) are also raised and kept constant. Since the transient flow depends upon z' and t', the fluid flow is maintained at same level apart from the plate. Let us assume that the transient flow is represented by a couple of partial differential equations. All physical parameters exempting pressure depend on z' and t' and also the flow is undisturbed far away from the plate. The governing equations of the present model with the above assumption are as follows:

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial z'^2} x + g\beta \left(T' - T'_{\infty}\right) - \left(\frac{u' + mv'}{1 + m^2}\right) \frac{\sigma B_0^2 \mu_e^2}{\rho} + 2\Omega'_z v' + g\beta^* \left(C' - C'_{\infty}\right),\tag{2.1}$$

$$\frac{\partial v'}{\partial t'} = \upsilon \frac{\partial^2 v'}{\partial z'^2} - \left(\frac{v' - mu'}{1 + m^2}\right) \frac{\sigma B_0^2 \mu_e^2}{\rho} - 2\Omega_z u', \qquad (2.2)$$

$$\frac{\partial T'}{\partial t'} + \frac{1}{\rho C_p} \frac{\partial q_r}{\partial z'} = A \frac{\partial^2 T'}{\partial z'^2}, \quad \text{where} \quad A = \frac{k}{\rho C_p},$$
(2.3)

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2} - K_t \left(C' - C'_{\infty} \right).$$
(2.4)

The axial velocity component is denoted by u' along the x'-axis and the transverse velocity component is denoted by v' along the y'-axis of the flow. When the angular velocity Ω is equal to $\frac{mM^2}{l+m^2}$, the velocity reaches its peak value and the transverse velocity decreases to zero.

The initial value and boundary value are given as below:

for all
$$z'$$
, and $t' \le 0$, $T' = T'_{\infty}$, $u' = v' = 0$, and $C' = C'_{\infty}$, (2.5)

for all
$$t' > 0$$
, $T' = T'_w$, $u' = u_0$, $v' = 0$, $C' = C'_w$ at $z' = 0$,
 $T' \to T'_\infty$, $u \to 0$, $v \to 0$, $C' \to C'_\infty$ as $z \to \infty$.
$$(2.6)$$

Thin gray gas' local radiant expression is as noted below:

$$\frac{\partial q_r}{\partial z'} = -4a^* \left(T_{\infty}^{\prime 4} - T^{\prime 4}\right) \sigma^* \tag{2.7}$$

where σ^* denotes the Stefan-Boltzmann constant and a^* represents the coefficient of heat absorption. In the boundary layer, assume that the fluid difference in temperature is sufficiently small. Using Taylor's expansion at T'_{∞} , we get:

$$T'^{4} + 3T'_{\infty}^{4} = 4T'_{\infty}^{3}T' .$$
(2.8)

Using (2.7) and (2.8) in Equation (2.3), we obtain

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} + 16a^* \sigma^* T_{\infty}'^4 \left(T_{\infty}' - T_{\infty}' \right) + k \frac{\partial^2 T'}{\partial z'^2}.$$
(2.9)

Non-dimensional quantities are introduced by Eq.(2.10) and used in Eqs (2.1), (2.2), (2.4) and (2.9)

$$u = \frac{u'}{u_0}, \quad v = \frac{v'}{v_0}, \quad t = \frac{t'u_0^2}{\upsilon}, \quad z = z'\frac{u_0^2}{\upsilon^2}, \quad \Omega = \Omega'_z \frac{\upsilon}{u_0^2}, \quad \Pr = \frac{\mu C_p}{k},$$

$$Gr = \frac{\upsilon g\beta(T'_w - T'_w)}{u_0^3}, \quad Gc = \frac{\upsilon g\beta^*(C'_w - C'_w)}{u_0^3}, \quad R = \frac{16\,\sigma^{\bullet}a^*T'_w^{3}\upsilon^2}{k\,\Pr u_0^{2}},$$

$$M^2 = \frac{\sigma\mu_e^2 B_0^2 \upsilon}{2\rho u_0^2}, \quad \theta = \frac{T' - T'_w}{T'_w - T'_w}, \quad S_C = \frac{\upsilon}{D}, \quad K = K_I \frac{\upsilon}{u_0^2}, \quad C = \frac{C' - C'_w}{C'_w - C'_w}.$$
(2.10)

The governing equations are reduced to a non-dimension form

$$\frac{\partial u}{\partial t} = 2\Omega v + Gr\theta + \frac{\partial^2 u}{\partial Z^2} + GcC - M^2 \left(\frac{u + mv}{1 + m^2}\right),$$
(2.11)

$$\frac{\partial v}{\partial t} + 2\Omega \ u = \frac{\partial^2 v}{\partial z} + M^2 \left(\frac{mu - v}{l + m^2}\right),\tag{2.12}$$

$$\frac{\partial \theta}{\partial t} + R\theta = \frac{l}{\Pr} \left(\frac{\partial^2 \theta}{\partial z^2} \right), \tag{2.13}$$

$$\frac{\partial C}{\partial t} + KC = \frac{1}{S_C} \left(\frac{\partial^2 C}{\partial z^2} \right), \tag{2.14}$$

with initial and boundary conditions

at $t \leq 0$

$$u = 0, \quad v = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } z,$$
 (2.15a)

when t > 0,

$$u = l, \quad v = 0, \quad \Theta = t, \quad C = l \quad \text{at} \quad z = 0,$$

 $u \to 0, \quad \Theta \to 0, \quad v \to 0, \quad C \to 0 \quad \text{as} \quad z \to \infty.$

$$(2.15b)$$

Combining the governing Eq.(2.11) and (2.12), we get

$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial z^2} - aF + Gr\theta + GcC$$
(2.16)

where

$$a = \frac{2M^2}{l+m^2} + 2i\left(\Omega - \frac{M^2m}{l+m^2}\right) \quad \text{and} \quad F = u + iv,$$

$$\frac{\partial\theta}{\partial t} + R\theta = \frac{1}{\Pr}\left(\frac{\partial^2\theta}{\partial z^2}\right),$$

$$\frac{\partial C}{\partial t} + KC = \frac{1}{Sc}\left(\frac{\partial^2 C}{\partial z^2}\right).$$

The transverse velocity vanishes when the rotation parameter Ω is $\frac{mM^2}{1+m^2}$. Conditions (2.15) transform into

at $t \leq 0$

$$F = 0$$
, $\theta = 0$, $C = 0$ for all z ,

when t > 0

$$F = 1$$
, $\theta = t$, $C = 1$ at $z = 0$,
 $\theta \to 0$, $F \to 0$, $C \to 0$ as $z \to \infty$

3. Discussion of exact solution

Exact solutions of the fluid velocity, temperature and concentration are obtained by the inverse Laplace transform:

$$\begin{split} & \theta(z,t) = \frac{l}{2} \Big[t \Big(E_3 \operatorname{erfc} \left(\eta \sqrt{\Pr} - \sqrt{Rt} \right) + E_4 \operatorname{erfc} \left(\eta \sqrt{\Pr} + \sqrt{Rt} \right) \Big) + \\ & + \frac{\eta \sqrt{P_r t}}{\sqrt{R}} \Big(E_3 \operatorname{erfc} \left(\eta \sqrt{\Pr} - \sqrt{Rt} \right) + E_4 \operatorname{erfc} \left(\eta \sqrt{\Pr} + \sqrt{Rt} \right) \Big) \Big], \\ & C(z,t) = \frac{l}{2} \Big[E_1 \operatorname{erfc} \left(\eta \sqrt{S_c} - \sqrt{Kt} \right) + E_2 \operatorname{erfc} \left(\eta \sqrt{S_c} + \sqrt{Kt} \right) \Big], \\ & F(z,t) = \Big(I - \frac{A}{\gamma} - B \Big) \Big[E_5 \operatorname{erfc} \left(\eta - \sqrt{at} \right) + E_6 \operatorname{erfc} \left(\eta + \sqrt{at} \right) \Big] + \\ & + \frac{A}{2\gamma} \Big\{ \exp(\gamma t) \Big[E_7 \operatorname{erfc} \left(\eta - \sqrt{(a+\gamma)t} \right) + E_8 \operatorname{erfc} \left(\eta + \sqrt{(a+\gamma)t} \right) + \\ & - \Big(E_9 \operatorname{erfc} \left(\eta \sqrt{\Pr} - \sqrt{(R+\gamma)t} \right) + E_{10} \operatorname{erfc} \left(\eta \sqrt{\Pr} + \sqrt{(R+\gamma)t} \right) \Big) \Big] + \\ & + E_3 \operatorname{erfc} \left(\eta \sqrt{\Pr} - \sqrt{Rt} \right) + E_4 \operatorname{erfc} \left(\eta \sqrt{\Pr} + \sqrt{Rt} \right) + \\ & -\gamma t \left(E_5 \operatorname{erfc} \left(\eta - \sqrt{at} \right) + E_6 \operatorname{erfc} \left(\eta + \sqrt{at} \right) \right) + \\ & -\gamma t \Big[E_3 \operatorname{erfc} \left(\eta \sqrt{\Pr} - \sqrt{Rt} \right) + E_4 \operatorname{erfc} \left(\eta \sqrt{\Pr} + \sqrt{Rt} \right) \Big] + \\ & + \frac{\eta \sqrt{T}}{\sqrt{R}} \Big[E_3 \operatorname{erfc} \left(\eta \sqrt{\Pr} - \sqrt{Rt} \right) - E_4 \operatorname{erfc} \left(\eta \sqrt{\Pr} + \sqrt{Rt} \right) \Big] \Big\} + \\ & + \frac{B}{2} \Big\{ \exp(\alpha t) \Big[E_{I3} \operatorname{erfc} \left(\eta - \sqrt{(a+\alpha)t} \right) + E_{I2} \operatorname{erfc} \left(\eta \sqrt{Sc} + \sqrt{(K+\alpha)t} \right) + \\ & - \Big(E_{I1} \operatorname{erfc} \left(\eta \sqrt{Sc} - \sqrt{(K+\alpha)t} \right) + E_{I2} \operatorname{erfc} \left(\eta \sqrt{Sc} + \sqrt{Kt} \right) \Big] \Big\} \end{split}$$

where

$$\eta = \frac{Z}{2\sqrt{t}}, \quad \alpha = \frac{a - KSc}{(Sc - I)}, \quad \gamma = \frac{a - R\Pr}{(\Pr - I)}, \quad A = \frac{G_r}{\gamma(\Pr - I)}, \quad B = \frac{G_c}{\alpha(S_c - I)},$$
$$E_I = \exp\left(-2\eta\sqrt{S_cKt}\right), \quad E_2 = \exp\left(2\eta\sqrt{S_cKt}\right), \quad E_3 = \exp\left(-2\eta\sqrt{R\Pr t}\right),$$
$$E_4 = \exp\left(2\eta\sqrt{R\Pr t}\right), \quad E_5 = \exp\left(-2\eta\sqrt{at}\right), \quad E_6 = \exp\left(2\eta\sqrt{at}\right),$$

$$E_{7} = \exp\left(-2\eta\sqrt{(a+\gamma) t}\right) , \qquad E_{8} = \exp\left(2\eta\sqrt{(a+\gamma) t}\right) , \qquad E_{9} = \exp\left(-2\eta\sqrt{\Pr(R+\gamma) t}\right) ,$$
$$E_{10} = \exp\left(2\eta\sqrt{\Pr(R+\gamma) t}\right) , \qquad E_{11} = \exp\left(-2\eta\sqrt{Sc(K+\alpha) t}\right) ,$$
$$E_{12} = \exp\left(2\eta\sqrt{Sc(K+\alpha) t}\right) , \qquad E_{13} = \exp\left(-2\eta\sqrt{(a+\alpha) t}\right) ,$$
$$E_{14} = \exp\left(2\eta\sqrt{(a+\alpha) t}\right) .$$

4. Results and discussion

The impact of physical parameters such as the magnetic parameter (M), Hall parameter (m), radiation parameter (R), chemical reaction parameter (k), rotation parameter (Ω) and the non-dimensional number such as the Prandtl number - Pr, Schmidt number - S_c and thermal Grashof number - G_r on temperature (θ) , concentration (C) and axial velocity field (F) is studied for different numerical values.

The temperature profiles for different values of the parameters are described in Figs 1-4. For t = 2 and different values of R and Pr, the distribution of temperature is shown in Fig.1. In Fig.2., the temperature field is described for different values of Pr (air and water) with different values of radiation and time. It is noted that the temperature increases as the value of time increases. This trend reverses with respect to radiation and the Prandtl number. Figure 3 shows the temperature distribution for t = 1, 2, 3, corresponding to R = 0.2, 0.5, 1 and for constant value of Pr = 0.71, the temperature increases. In Fig.4, for t and Pr constant (2 and 0.71, respectively), for increasing values of R, the temperature decreases.

Figures 5 and 6 show that the concentration boundary rises with the fall of *Sc* (Schmidt number) and *k* (reaction parameter). Concentration in the presence of Sc = 2.01 (ethyl benzene) is less when compared to Sc = 0.16, (hydrogen) and Sc = 0.6 (water vapour) (see Fig.5) with various values of k = 0.2, 2, 5 and t = 2. Figure 6 shows that the concentration boundary falls with an increase in the chemical reaction parameter (*k*) at Sc = 2.01 and t = 2.



Fig.1. Temperature profile for different values of Pr and R and t = 2.



Fig.2. Temperature profile for different values of R, Pr and t.



Fig.3. Temperature profile for different values of R and t for Pr = 0.71



Fig.5. Concentration profiles for different values of Sc and k.



Fig.7. Velocity profiles for different values of the Hall parameter.



Fig.4. Temperature profile for different values of R.



Fig.6. Concentration profiles for different values of k for Sc = 2.01



Fig.8. Velocity profiles for different values of radiation.

The profiles of auxiliary velocity F with various parameters m, R, M, Gr, Gc, K, Sc, are illustrated in Figs 7-12. From Fig.7., it is clear that velocity increases with the increase in the Hall parameter (m); where Sc = 2.01, M = 2, $\Pr = 7$, k = 0.2, Gr = 5, Gc = 10, R = 2, $\Omega = 2$ and t = 2. For different values of radiation (R) when m = 0.5, Sc = 2.01, M = 2, $\Pr = 7$, k = 0.2, $\Omega = 2$, Gr = 5, Gc = 10 and t = 2, the velocity distribution is shown in Fig.8. It is evident that the velocity F falls when R is increased. Figure 9 shows that as the magnetic parameter (M) increases, the velocity decreases when $\Omega = 2$, $\Pr = 7$, k = 0.2, Gr = 5, Gc = 10, R = 2, t = 2 and m = 0.5. We can observe that in Fig.10., the velocity F increases with an increase in the thermal Grashof number (Gr) and mass Grashof number (Gc).



Fig.9. Velocity profiles for different values of the Hartmann number.



Fig.11. Velocity profiles for different values of the rotation parameter.

Fig.10. Velocity profiles for different values of Gr and Gc.



Fig.12. Velocity profiles for different values of chemical reaction.

Figure 11 shows that for different values of the rotation parameter Ω when Pr = 7, k = 0.2, Gr = 5, Gc = 10, R = 2, t = 2, m = 0.5 and Sc = 2.01, it is noticed that the velocity field goes up as the rotation parameter values drop down. One can also see in Fig.12. that velocity falls when the chemical reaction value goes up.

5. Conclusion

Hall effect and radiation effect have been studied under theoretical solution of heat and mass transfer in the presence of a rotating fluid with variable temperature. The effect of different parameters on velocity, concentration and temperature field is presented graphically.

The conclusions of the study are follows:

- The temperature distribution increases with an increase of time and decrease of Pr Prandtl number and radiation parameter (R).
- The velocity profiles decreas with a decrease in Gr, Gc, and Hall parameter (m).
- Velocity increases with a decrease in the rotation parameter, chemical reaction parameter, magnetic parameter (M) and radiation parameter (R).
- The concentration field decreases with a decrease of Sc and k.

Nomenclature

- B_0 imposed magnetic field
- C dimensionless concentration
- C' species concentration in the fluid
- c_w wall concentration
- G_c Grashof's number
- Gr thermal Grashof number
- J_z current density along the z-axis
- K chemical reaction parameter Pr l Prandtl number
- *M* Hartmann number
- m Hall parameter
- S_c Schmidt number
- T temperature of the fluid near the plate
- T_w temperature of the plate
- T_{∞} temperature of the fluid far away from the plate
- t non-dimensional time
- (u', v', w') velocity components of velocity
- (u, v, w) dimensionless velocity
 - v kinematic viscosity
- (x', y', z') Cartesian triplets
 - β volumetric coefficient of thermal expansion
 - θ dimensionless temperature
 - $\mu \quad \, coefficient \, of \, viscosity$
 - ρ fluid density
 - σ electrical conductivity
 - Ω non-dimensional angular velocity
 - $\mathbf{\Omega}_{z}$ angular velocity

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