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FLOW FEATURES OF THERMOPHORETIC MHD VISCOUS FLUID FLOW PAST A CONVERGING CHANNEL WITH HEAT SOURCE AND CHEMICAL REACTION

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A boundary layer flow of an electrically conducting viscous fluid past a converging channel in the presence of thermophoresis, heat source, chemical reaction, viscous dissipation and simultaneous heat and mass transfer characteristics is studied in the paper. An external magnetic field of uniform strength is applied transversely to the channel. The similarity solution has been used to transform the partial differential equations that represent the problem into a boundary value problem of coupled ordinary differential equations, which in turn are solved numerically using MATLAB's built in solver bvp4c. Numerical computations are carried out to solve the problem and graphical illustrations are made to get the physical insight of the same. The convergent channel flow problem of an incompressible electrically conducting viscous fluid in the presence of a magnetic field has a wide range of applicability in different areas of engineering, specially in industrial metal casting and control of molten metal flow.

Key words: MHD, viscous fluid, heat source, thermophoresis, Nusselt number, skin friction.

1. Introduction

The application of incompressible viscous fluid flow through a convergent or divergent channel is most relevant in various areas such as aerospace, biomechanical, chemical, civil, environmental and mechanical engineering as well as in understanding rivers and canals. Jeffery [1] and Hamel [2] have worked out the mathematical formulations of the problem of convergent channel flow in 1915 and 1916, respectively. Navier-Stokes equations, when simplified, in the particular case of two-dimensional flow through a channel with inclined walls modifies into Jeffery-Hamel problem [3-5]. Jeffery-Hamel flows have been comprehensively studied by a number of authors and discussed in various textbooks[6-9]. Magnetohydrodynamic flow of a viscous fluid through a convergent or divergent channel has been scrutinized by authors, viz. Asadullah *et al.* [10] and Hosseini *et al.*[11].

The study of an electrically conducting viscous fluid that flows through convergent or divergent channels when an external magnetic field is applied is not only captivating theoretically but also appealing in mathematical modeling of numerous industrial and biological systems. An application of the theory is observed in the field of industrial metal casting and the control of molten metal flows. The motion of liquid metals or alloys in the cooling systems of advanced nuclear reactors is another area in which the theoretical study of this topic can be applied. Different applications and aspects of magnetohydrodynamic flow have been discussed by Nijsing and Eifler [12], Bansal [13], Moreau [14], Abel *et al.* [15], etc. The effect of a transverse magnetic field on an electrically conductiong fluid in channels has been studied by Linga Raju *et al.* [16]. The influence of chemical reaction on a magnetohydrodynamic flow in different flow configurations has been examined by a few authors, viz. Chandrakala [17], Muthucumaraswamy and Geetha [18], Zigta [19], Fenuga *et al.* [20] and Vijayalakshmi and Selvajayanthi [21].

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The phenomenon in which minuscule particles suspended in a non-isothermal fluid acquire a velocity relative to the fluid in the direction of diminishing temperature is termed thermophoresis. Such type of phenomenon is applicable in drug discovery process, aerosol reactors, heat exchange fouling etc. The effect of thermophoresis on a flow over a stretching sheet has been investigated by Srinivasacharya and Jagadeeshwar [22]. Numeruos authors, viz. Selim *et al.* [23], Postelnicu [24], Bakier and Gorla [25], Noor *et al.* [26], Kundu *et al.* [27], Zueco *et al.* [28] etc. have explored the impact of thermophoresis on fluid flows of different nature and under various physical configurations.

The present study is devoted to examine significant features of a magnetohydrodynamic viscous incompressible fluid flow through a convergent channel in the presence of thermophoresis, heat source and chemical reaction.

2. Mathematical formulation

We consider a steady two-dimensional incompressible viscous fluid flow past a converging channel under the influence of a magnetic field of uniform strength B(x) in the presence of thermophoresis, heat source and first order chemical reaction. The fluid flow is assumed to be in the direction of the x- axis along the wall of the converging channel and the y- axis is normal to it. The magnetic field acts in the transverse direction of the channel and with the assumption of a small magnetic Reynolds number we ensure that the induced magnetic field is of negligible order in comparison with the applied magnetic field.

Under the above assumptions, the governing equations of motion are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (2.1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + v\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B^2}{\rho}(U-u) + g\beta(T-T_{\infty}) + g\beta^*(C-C_{\infty}), \qquad (2.2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{v}{C_p} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q}{\rho C_p} (T - T_{\infty}), \qquad (2.3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - K_C(C - C_\infty) - \frac{\partial}{\partial y} \{V_t(C - C_\infty)\}.$$
(2.4)

The boundary conditions are given by

$$y = 0; \quad u = 0, \quad v = 0, \quad T = T_w, \quad C = C_w,$$
(2.5)

$$y \to \infty; \quad u \to U(x), \quad T \to T_{\infty}, \quad C \to C_{\infty}$$
 (2.6)

where u, v are velocity components along the x and y axes respectively, U(x) is the velocity of potential flow, ρ is the fluid density, σ is electric conductivity, C_p is the specific heat, κ is the thermal conductivity, T and C are dimensional species temperature and concentration respectively, D is the coefficient of chemical molecular diffusivity, K_C is the rate of chemical reaction, Q is the heat generation coefficient, ν is the kinematic viscosity, $V_t \left(= -\frac{K_t \nu}{T_r} \frac{\partial T}{\partial y} \right)$ is the thermophoretic velocity with K_t and T_r being the thermophoretic coefficient and reference temperature respectively, T_w , T_∞ are temperatures and C_w , C_∞ are concentrations at the channel wall and the free stream, respectively.

Following Schlichting (1968), we consider $U(x) = -\frac{a}{x}$ where a > 0 is the strength of the sink placed at the origin, $B(x) = \frac{B_0}{x}$ where B_0 is constant, dimensionless variable $\eta = y \sqrt{-\frac{U(x)}{xv}} = \frac{y}{x} \sqrt{\frac{a}{v}}$ and stream function $\psi(x, y) = -\sqrt{va} f(\eta)$ such that $u = \frac{\partial \psi}{\partial v}$ and $v = -\frac{\partial \psi}{\partial x}$ which satisfies Eq.(2.1).

We bring forth the following non-dimensional variables and parameters

$$\theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \qquad \varphi = \frac{C - C_{\infty}}{C_w - C_{\infty}},$$

$$G_r = \frac{g\beta(T_w - T_{\infty})x^3}{a^2}, \qquad G_m = \frac{g\beta^*(C_w - C_{\infty})x^3}{a^2},$$

$$M = \frac{\sigma B_0^2}{\rho a}, \qquad P_r = \frac{\mu C_p}{\kappa}, \qquad S = \frac{Qx}{\rho C_p U}, \qquad \gamma = \frac{K_C x}{U},$$

$$E = \frac{U^2}{C_p (T_w - T_{\infty})}, \qquad \tau = -\frac{K_t (T_w - T_{\infty})}{T_r}, \qquad S_C = \frac{v}{D}.$$
(2.7)

The modified forms of Eqs (2.2)-(2.4) are:

$$f''' + (f')^{2} + Mf' = (M+1) - G_{r}\theta - G_{m}\phi, \qquad (2.8)$$

$$\boldsymbol{\theta}'' + P_r E \left(f'' \right)^2 - P_r S \boldsymbol{\theta} = 0, \qquad (2.9)$$

$$\varphi'' + S_C \gamma \varphi - S_C \tau(\varphi \theta'' + \varphi' \theta') = 0.$$
(2.10)

The corresponding boundary conditions are:

$$\eta = 0; f' = 0, \theta = 1, \phi = 1,$$
 (2.11)

$$\eta \to \infty; f' \to l, \quad \theta \to \theta, \quad \phi \to \theta.$$
 (2.12)

3. Method of solution

Equations (2.8)-(2.10) under the boundary conditions (2.11) and (2.12) are solved numerically using MATLAB's built in solver byp4c. The non-dimensional velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are obtained in graphical forms.

4. Results and discussion

The features of boundary layer flow of an electrically conducting magnetohydrodynamic viscous fluid past a converging channel in the presence of thermophoresis, heat source, viscous dissipation, chemical reaction and simultaneous heat and mass transfer have been analyzed. The relevant flow characteristics are illustrated graphically in Figs 1-32. In this problem, we examine the effect of salient non-dimensional parameters involved in the study. The solution is obtained for parameters with values $P_r = 2.5$, M = 2, $G_r = 5$, $G_m = 5$, S = 3, E = 0.05, $S_c = 0.5$, $\gamma = 1$, $\tau = 1.5$ unless otherwise stated.

The variations of the fluid velocity u against the distance y are exhibited in the Figs 1-7 to illustrate the effects of different dimensionless constants on the velocity field in the presence of other flow parameters involved in the problem. It is noticed that the fluid velocity steps up near the plate and converges to the stream velocity as one moves away from the plate. Figure 1 illustrates that the increase in the Prandtl number causes a fall in the fluid velocity. The ratio of the buoyancy force to the viscous force characterizes the Grashof number for heat transfer (G_r) which is very useful in many technological applications. It is noticed from Fig.2 that the rising value of G_r enhances the speed of the fluid velocity. Figure 3 displays the fluid velocity for various values of the magnetic parameter M. It is observed that with the increase of M, the fluid velocity decreases. This is because the transverse magnetic field in action gives rise to a resistive force called the Lorentz force which has a tendency to retard the velocity of the flow. Figure 4 shows the effect of G_m on the fluid flow. With the increase of G_m the flow velocity increases. The combined effect of momentum and mass diffusion characterizes the Schmidt number. Figure 5 reveals that with the rise of the Schmidt number S_c the fluid velocity exhibits a decelerating trend. Figure 6 demonstrates the effect of the chemical reaction parameter γ on the fluid velocity. It is noticed that as the chemical reaction parameter goes up, the velocity increases but there is not much of significant difference between two adjacent velocity graphs for most part along the y-axis. This can be summed up by stating that chemical reaction of order one has a comparatively meager effect on the fluid velocity than other flow parameters. Figure 7 brings out the effect of the thermophoretic parameter on the fluid velocity. It is observed that the fluid velocity gets diminished with the increase of the thermophoretic parameter.

The variations of the temperature θ against distance y are displayed in Figs 8-10 to explain the effect of the Prandtl number, Eckert number and heat source parameter on the flow temperature in the presence of different fixed flow parameters. Figure 8 illustrates that with the rise of the Prandtl number the temperature of the flow field decreases. It is also observed that for very small values of the Prandtl number, temperature decreases almost linearly as one moves away from the plate. However, for comparatively larger values of the Prandtl number the temperature profile diminishes steeply near the plate and then decreases a symptotically as one moves to infinity. Figure 9 exhibits that a growth of the Eckert number causes a rise in the flow temperature, but the difference in temperature is not much significant. Figure 10 demonstrates that with the rise of the heat source parameter the flow temperature gets reduced.

Figures 11-15 reveal that the concentration profile diminishes sharply near the plate and then proceeds uniformly away from the plate. The concentration profile accelerates with the enhancement of numerical values of the chemical reaction parameter but decelerates with the rise of the heat source parameter, Prandtl number, Schmidt number and thermophoretic parameter.

Figures 16-21 illustrate the effects of different flow parameters on the skin friction of the fluid under consideration. The skin friction declines with the increasing value of the Prandtl number, magnetic parameter, heat source parameter, Schmidt number and thermophoretic parameter but increases with the increase of the chemical reaction parameter. Figures 22-27 exhibit the influence of flow parameters on the Nusselt number. From these figures, it is observed that the increase in the Prandtl number, magnetic parameter, Schmidt number, heat source parameter and thermophoretic parameter enforces a declination in the heat flux of the flow but the chemical reaction parameter increases the rate of heat flux. The effect of different flow parameters on the Sherwood number is shown in Figs 28-32. It is observed that with the increase in the Prandtl number, heat source parameter, Schmidt number and thermophoretic parameter, the

Sherwood number gets diminished but a rise in the chemical reaction parameter exhibits an opposite scenario.



Fig.1. Velocity for different values of $P_r = 1.7$, 2.5, 3.5, 4.5.



Fig.3. Velocity for different values of M = 1, 2, 3, 4.



Fig.5. Velocity for different values of $S_C = 0.25, 0.5, 1.0, 1.5$.



Fig.2. Velocity for different values of $G_r = 2, 3, 4, 5$.



Fig.4. Velocity for different values of $G_m = 2, 3, 4, 5$.



Fig.6. Velocity for different values of $\gamma = 1, 2, 3, 4$.



Fig.7. Velocity for different values of $\tau = 1, 1.5, 2.0, 3.0$.



Fig.9. Temperature for different values of E = 0.05, 0.15, 0.30, 0.40.







Fig.8. Temperature for different values of $P_r = 1.7, 2.5, 3.5, 4.5$.



Fig.10. Temperature for different values of S = 1, 2, 3, 4.



Fig.12. Concentration for different values of $P_r = 1.7, 2.5, 3.5, 4.5$.



Fig.13. Concentration for different values of $S_C = 0.25, 0.5, 1.0, 1.5$.



Fig.15. Concentration for different values of $\tau = 1, 1.5, 2.0, 3.0$.







Fig.14. Concentration for different values of $\gamma = 1, 2, 3, 4$.



Fig.16. Skin friction for different values of $P_r = 1.7, 2.5, 3.5, 4.5$.



Fig.18. Skin friction for different values of S = 1, 2, 2.5, 3.5.



Fig.19. Skin friction for different values of $\tau = 0.5, 1.0, 1.5, 2.0$.



Fig.21. Skin friction for different values of $S_C = 0.5, 0.7, 1.0, 1.5$.



Fig.23. Nusselt number for different values of $\gamma = 0.5, 1.0, 1.5, 2.0$.



Fig.20. Skin friction for different values of $\gamma = 0.5, 1.0, 1.5, 2.0$.



Fig.22. Nusselt number for different values of M = 1, 2, 3, 4.



Fig.24. Nusselt number for different values of $\tau = 0.5$, 1.0, 1.5, 2.0.



Fig.25. Nusselt number for different values of $S_C = 0.5, 0.7, 1.0, 1.5$.



Fig.27. Nusselt number for different values of $P_r = 1.7, 2.5, 3.0, 3.5$.



Fig.29. Sherwood number for different values of S = 1, 2, 2.5, 3.5.



Fig.26. Nusselt number for different values of S = 1, 2, 2.5, 3.5.



Fig.28. Sherwood number for different values of $P_r = 1.7, 2.5, 3.5, 4.5$.



Fig.30. Sherwood number for different values of $S_C = 0.5, 0.7, 1.0, 1.2$.



Fig.31. Sherwood number for different values of $\gamma = 0.5, 1.0, 1.5, 2.0$.



Fig.32. Sherwood number for different values of $\tau = 0.5, 1.0, 1.5, 2.0$.

5. Conclusion

The salient conclusions are cited below:

- the velocity and temperature fields are affected significantly by the variations of the pertinent flow parameters in the fluid flow region;
- the species concentration exhibits the diminishing trend with the growth of the Schmidt number, heat source parameter and thermophoretic parameter while a reverse nature is noticed in case of the chemical reaction parameter;
- with the acceleration of the chemical reaction parameter the skin friction gets expedited but an
 opposite pattern is noticed with the enhancement of other flow parameters, viz. the Prandtl number,
 magnetic parameter, heat source parameter and thermophoretic parameter;
- the Nusselt number and the Sherwood number get elevated with the increase of the chemical reaction parameter but show an opposite behavior in case of the magnetic parameter, Prandtl number, heat source parameter and thermophoretic parameter.

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Nomenclature

- B strength of the applied magnetic field
- C species concentration
- C_p specific heat at constant pressure
- C_W species concentration at the wall
- C_{∞} free stream species concentration
- D coefficient of chemical molecular diffusivity
- E Eckert number
- g acceleration due to gravity
- G_m solutal Grashof number
- G_r thermal Grashof number

- K_C rate of chemical reaction
- K_t coefficient of thermophoresis
- M magnetic parameter
- P_r Prandtl number
- *O* heat generation coefficient
- S heat source parameter
- S_C Schmidt number
- T species temperature
- T_r reference temperature
- T_W temperature at the channel wall
- T_{∞} temperature of the stream
- component of fluid velocity along the x-axis и
- stream velocity U
- v –component of fluid velocity along the y- axis
- V_t thermophoretic velocity
- coefficient of thermal expansion β
- coefficient of spatial expansion β*
- γ chemical reaction parameter
- nondimensional temperature θ
- κ thermal conductivity
- kinematic viscosity ν
- fluid density ρ
- electric conductivity σ
- thermophoretic parameter τ
- nondimensional species concentration

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