Int. J. of Applied Mechanics and Engineering, 2021, vol.26, No.1, pp.122-134
DOI: 10.2478/ijame-2021-0008

HEAT TRANSFER EFFECTS ON FREE CONVECTION OF VISCOUS DISSIPATIVE FLUID FLOW OVER AN INCLINED PLATE WITH THERMAL RADIATION IN THE PRESENCE OF INDUCED MAGNETIC FIELD

P.PRAMOD KUMAR*
Department of Mathematics, B V Raju Institute of Technology
Narsapur, Telangana, 502313, INDIA
E-mail: pramodkumarpatlolla36@gmail.com

BALA SIDDULU MALGA
Department of Mathematics, GITAM University
Hyderabad, Telangana, 502329, INDIA

LAKSHMI APPIDI and SWETA MATTA
Department of Mathematics, CMR Technical Campus Kandlakoya
Telangana, 501401, INDIA

Abstract: The principal objective of the present paper is to know the reaction of thermal radiation and the effects of magnetic fields on a viscous dissipative free convection fluid flow past an inclined infinite plate in the presence of an induced magnetic field. The Galerkin finite element technique is applied to solve the nonlinear coupled partial differential equations and effects of thermal radiation and other physical and flow parameters on velocity, induced magnetic field, along with temperature profiles are explained through graphs. It is noticed that as the thermal radiation increases velocity and temperature profiles decrease and the induced magnetic field profiles increases.

Keywords: thermal radiation; induced magnetic field; magnetic Prandtl number; free convection; Galerkin finite element method; viscous dissipation.

1. Introduction

Free convection flow of heat and mass transfer problems has diverse applications in industrial processes. The Influence of a magnetic field on free convection flow problems has many applications in science and technology. In the study of MHD flow, the influence of free convection plays a vital role in the fields of petroleum, engineering and agriculture. The final result of the magnetic field and thermal radiation on the induced magnetic field has attracted many researchers. It has been adopted in various fields such as aerodynamics, science and technology, astrophysics and geophysics. In polymer processing industry radiation effects play a major role in getting the final product, by adjusting the heat and mass transfer. The impact of heat and mass transfer plays a major role in distribution of moisture and temperature in agriculture, modeling the chemical equipment, emergence of fog, harm of a crop due to freeze, pollution in the ecosystem, etc. Chen [1] investigated heat and mass transfer in MHD stream on a slant plate in free convection by considering viscous dissipation and Ohmic heat. Pop and Soundalgekar [2] studied the effect of free convection flow of a fluid through an accelerated infinite plate. Emery and England [3] observed the influence of radiation on a narrow gray gas surrounded by a non-moving plate. Soundalgekar *et al*. [4] studied an electrically conducting fluid flow through an infinite plate in the presence of heat and suction constant at the plate by taking into account an induced magnetic field. Raptis *et al*. [5] by applying a finite

* To whom correspondence should be addressed

difference scheme studied an unsteady convective stream of a fluid passing through a porous medium in a semi-infinite vertical plate. Rashidi *et al*. [6] investigated a free convective flow of a viscous fluid of a steady laminar two dimensional fluid through a porous medium over a stretchy surface. The effect of heat and mass transfer on a free convection flow through an inclined plate was reviewed by Shafie *et al*. [7]. Raju *et al*. [8] investigated flow of a fluid from a permeable medium with viscous dissipation and Joule heating with a horizontal impermeable bottom which is insulated. Prakash *et al*. [9] examined the effect of radiation and induced magnetic intensity over a vertical porous plate on the convection flow of a viscous magnetic dissipation. Raju *et al*. [10] investigated the influence of heat transfer in the presence of an induced magnetic field on a vertical plate of viscous dissipative fluid. Chamkha [11] analyzed the hydro- magnetic flow through an accelerating penetrable semi-infinite surface, accompanied by radiation and heat generation or inclusion. Ahmed [12] investigated the effect of induced magnetic domains with viscous dissipation bordered by a vertical permeable plate in the presence of thermal radiation. Ahmed *et al*. [13] explored transform of radiation and MHD on the chemical behavior of heat mass transfer fluid with periodic solutions. Raptis *et al*. [14] investigated the fluid flow through a thin gray gas and its effects in the presence of radiation. Bhattacharya *et al*. [15] has studied the flow of a fluid in a vertical plate and established a solution for the similarity of convective mixed boundary layer. Chamkha *et al*. [16] analyzed the solution of similarity of the free convection of the inclined plate in the presence of a transverse magnetic field with internal heat and absorption. Chamkha and Issa [17] studied the flow of MHD with thermal inclusions or heat absorption on flat surfaces. Siddiqal and Hossian [18] investigated the flow of a fluid over a vertical flat plate in a mixed convection boundary layer in the presence of radioactive heat transfer. Sami and Steinruck [19] studied the flow of a mixed convection through a horizontal plate. Bala Siddulu Malga *et al*. [20] studied the heat transfer effect of a viscous dissipative radiating fluid past a vertical porous plate in the presence of an induced magnetic field. Recently, Ram Mohan *et al.* [21] studied the effect of an induced magnetic field of a viscous dissipative fluid flow through an inclined plate in the presence of a heat source. In the present paper, we study the effect of radiation on the flow of a free convection viscous dissipative fluid through an inclined plate in the presence of a heat source. The non-linear coupled partial differential equations are intended by the Galerkin finite element method, and the effect of various physical parameters on velocity, induced magnetic field and temperature is discussed graphically.

2. Mathematical Formulation

Let $(\bar{u},\bar{v},0)$ be the velocity of the fluid and $\left(\bar{H}_x,\bar{H}_y,0\right)$ be the magnetic induction vector of a point $(\bar{x},\bar{y},\bar{z})$ in the fluid. The ambient temperature $\bar{T}_\infty$ is less than the external wall temperature $\bar{T}_W$. Conservation of electric charge is $\nabla.J=0,\ \ J=\left(J_x,J_y,J_z\right)$ is the propagation direction along the an $\bar{y}-axis$ and it does not include changes along the $\bar{y}-axis$,. then $\dfrac{\partial J_y}{\partial y}=0$, which gives $J_y=const$, under the above assumption, the fluid is governed by the x- momentum equation.

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-g+v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\mu_0 H_0}{\rho}\frac{\partial \bar{H}_x}{\partial \bar{y}}\ . \tag{2.1}$$

The first term in RHS equation (1) gives the mixed convection term. Since the velocity gradient is very small, the viscous term in the above equation will disappear. Then without the induced magnetic field

$$\frac{\partial p}{\partial x}=-\rho_\infty g\ \ . \tag{2.2}$$

Neglecting pressure from Eqs (2.1)-(2.2), using the Boussinesq estimate:

$$\rho_\infty - \rho = \rho_\infty \beta\left(\bar{T} - \bar{T}_\infty\right).$$

Then Eq. (2.1) becomes:

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta\cos\alpha\left(\bar{T} - \bar{T}_\infty\right) + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\mu_0 H_0}{\rho}\frac{\partial \bar{H}_x}{\partial \bar{y}} \quad . \tag{2.3}$$

The energy and magnetic- induction equations are as follows:

$$\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{v}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \frac{1}{\sigma\rho C_p}\left(\frac{\partial \bar{H}_x}{\partial \bar{y}}\right)^2 + \frac{1}{\rho C_p}\frac{\partial q_r}{\partial \bar{y}} + \bar{Q}\frac{\partial}{\partial y}\left(\bar{T} - \bar{T}_\infty\right), \tag{2.4}$$

$$\bar{v}\frac{\partial \bar{H}_x}{\partial \bar{y}} = \frac{1}{\rho\mu_0}\frac{\partial^2 \bar{H}_x}{\partial \bar{y}^2} + H_0\frac{\partial \bar{u}}{\partial \bar{y}}. \tag{2.5}$$

The boundary conditions are

$$\left.\begin{array}{lllll} at\ \bar{y} = 0: & \bar{u} = 0, & \bar{v} = -v_0, & \bar{T} = T_w, & \bar{H}_x = 0, \\ as\ \bar{y} \to \infty: & \bar{u} \to U_0, & & \bar{T} \to \bar{T}_\infty, & \bar{H}_x \to 0. \end{array}\right\} \tag{2.6}$$

The non-dimensional quantities are:

$$\left.\begin{array}{llll} y = \dfrac{v_0 \bar{y}}{v}, & u = \dfrac{\bar{u}}{U_0}, & \theta = \dfrac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}\ , & \mathrm{P_r} = \dfrac{\rho v C_p}{k} \\ \mathrm{G_r} = vg\beta\left(\dfrac{\bar{T}_w - \bar{T}_\infty}{U_0 v_0^2}\right), & \mathrm{H} = \sqrt{\dfrac{\mu_0 H_x}{\rho U_0}}\ , & \mathrm{E_c} = \dfrac{U_0^2}{C_p\left(\bar{T}_w - \bar{T}_\infty\right)} & \\ \mathrm{P_{rm}} = \sigma v \mu_0, & M = \sqrt{\dfrac{\mu_0 H_x}{\rho v_0}}\ , & Q = \dfrac{\bar{Q}^*}{v_0}\ , & R = \dfrac{64\alpha v\bar{\sigma}\bar{T}_\infty^3}{\rho v_0^2 C_p} \end{array}\right] \tag{2.7}$$

Radiant absorption in case of narrow gray gas is expressed by Raptis *et al*. [22]

$$\frac{\partial q_r}{\partial \bar{y}} = -4a\bar{\sigma}\left(\bar{\mathrm{T}}_\infty^4 - \bar{\mathrm{T}}^4\right); \tag{2.8}$$

a is the mean absorption, $\bar{\sigma}$ is the Stefan-Boltzmann constant.

Here assuming that the temperature differences of the flow are small, so $\bar{T}^4$ can be expressed as a linear function temperature $\bar{T}$. It is obtained by expanding $\bar{T}^4$ in Taylor series about $\bar{T}_\infty$ and omitting higher order terms as [22,23] and then

$$\overline{T}_\infty^4 \cong 4\overline{T}_\infty^3\overline{T} - 3\overline{T}_\infty^4 . \tag{2.9}$$

From Eq.(2.7) and with the help of Eqs (2.8) and (2.9) the set of governing equations (2.1)-(2.5) are transformed into the following non-dimensional form

$$\frac{d^2u}{dy^2} + \frac{du}{dy} + M\frac{dH}{dy} = -G_r\theta cos\alpha , \tag{2.10}$$

$$\frac{d^2\theta}{dy^2} + P_r\frac{d\theta}{dy} = -\left(E_cP_r\left(\frac{du}{dy}\right)^2 + \frac{E_cP_r}{P_{rm}}\left(\frac{dH}{dy}\right)^2 \right) + Q\frac{d\theta}{dy} + \frac{P_rR}{4}\theta , \tag{2.11}$$

$$\frac{d^2H}{dy^2} + MP_{rm}\frac{du}{dy} + P_{rm}\frac{dH}{dy} = 0 . \tag{2.12}$$

The boundary conditions in the non-dimensional form of equation (6) are as follows;

$$\left.\begin{aligned} &u = 0;\ \theta = 1;\ H = 0; \quad at \quad y = 0, \\ &u \to 1, \quad \theta \to 0, \quad H \to 0 \quad as\ y \to \infty. \end{aligned}\right\} \tag{2.13}$$

3. Method of Solution

Let us consider the steady mixed MHD free convection heat transfer flow of a conductive Newtonian incompressible viscous dissipative fluid perpendicular to a semi-infinite inclined plate at an angle α. Let us consider that the $\overline{x}$. -axis is along the direction of the plate and the $\overline{y}$. -axis along the direction of the fluid and perpendicular to the plate. A uniform magnetic field of strength (H_0) is applied perpendicular to the plate. If the plate is non -conductive, the flow is taken into account by considering the induced magnetic field. Here we assume that all the physical parameters are independent of $\overline{x}$ except the pressure, since it is in the $\overline{x}$ -direction with an infinite length.

The steps involved in finite element inspection are:

- domain disconnected into elements;
- formulation of equations of components;
- assembly of element equations;
- establishment of constraints conditions;
- result of assembled equations.

The Galerkin finite element method is applied to solve Eqs (2.10)-(2.13) over a two nodded linear element (e) $(y_j \le y \le y_k)$:

$$\int_{y_j}^{y_k} N^T\left[\frac{d^2u}{dy^2} + \frac{du}{dy} + M\frac{dH}{dy} + G_r\theta cos\alpha\right]dy , \tag{3.1}$$

$$\int_{y_j}^{y_k}\left[\frac{\partial N}{\partial y}.\frac{\partial u^{(e)}}{\partial y}-N^T\left(\frac{du}{dy}+R\right)\right]dy=0 \tag{3.2}$$

where

$$R=\left(M\frac{dH}{dy}+G_r\theta cos\alpha\right), N=\left[N_j, N_k\right],\ \phi^{(e)}=\begin{bmatrix}u_j\\ u_k\end{bmatrix},$$

$$u^{(e)}=N.\phi^{(e)},\quad N_j=\frac{y_k-y}{l^{(e)}},\quad N_k=\frac{y-y_j}{l^{(e)}},\quad l^{(e)}=y_k-y_j=h,$$

the (e) element equation is given by

$$\int_{y_j}^{y_k}\begin{bmatrix}N_j'N_j' & N_j'N_k'\\ N_k'N_j' & N_k'N_k'\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}dy-\begin{bmatrix}N_jN_j' & N_jN_k'\\ N_kN_j' & N_kN_k'\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}dy-R\begin{bmatrix}N_j\\ N_k\end{bmatrix}dy=0, \tag{3.3}$$

$$\frac{1}{l^{(e)}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}-\frac{1}{2}\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}-R\frac{l^{(e)}}{2}\begin{bmatrix}1\\ 1\end{bmatrix}=0 \tag{3.4}$$

where the prime denotes differentiation with respect to y. The element equations of two consecutive elements are assembled together $\left(y_{i-1}\le y\le y_i\right)and\left(y_i\le y\le y_{i+1}\right)$.
The following results are obtained:

$$\frac{1}{l^{(e)}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}+\frac{1}{2}\begin{bmatrix}1 & 1 & 0\\ -1 & 0 & 1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}=R\frac{l^{(e)}}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}. \tag{3.5}$$

Applying the trapezoidal rule, following system equations in Crank –Nicolson to the Eq. (3.5), we obtain:

$$A_1u_{i-1}^{n+1}+A_2u_i^{n+1}+A_3u_{i+1}^{n+1}=A_4u_{i-1}^n+A_5u_i^n+A_6u_{i+1}^n+4hR \tag{3.6}$$

where

$$R=M\frac{dH}{dy}+GrT_i^j cos\alpha.$$

here $l^{(e)}=h$.
Similarly, for Eqs (2.11), (2.12) the following equations can be obtained:

$$B_1u_{i-1}^{n+1}+B_2u_i^{n+1}+B_3u_{i+1}^{n+1}=B_4u_{i-1}^n+B_5u_i^n+B_6u_{i+1}^n+48hR^{**} \tag{3.7}$$

where

$$R^{**}=\mathrm{P_r}\,\mathrm{Ec}\left(\frac{\partial u}{\partial y}\right)^{2}+\frac{\mathrm{P_r E_c}}{\mathrm{P_{rm}}}\left(\frac{\partial H}{\partial y}\right)^{2},\qquad R^{***}=M\,\mathrm{P_r}\,m\frac{du}{dy},$$

$$D_1u_{i-1}^{n+1}+D_2u_i^{n+1}+D_3u_{i+1}^{n+1}=D_4u_{i-1}^{n}+D_5u_i^{n}+D_6u_{i+1}^{n}+4hR^{***},$$

$$A_4=2-h,\ A_5=-4,\quad A_6=2+h,\qquad B_1=-24+12h\left(\mathrm{P_r}-Q\right)+R\,\mathrm{P_r}\,h^2,$$

$$B_2=48+4R\,\mathrm{P_r}\,h^2,\quad B_3=-24-12h\left(\mathrm{P_r}-Q\right)+R\,\mathrm{P_r}\,h^2, \tag{3.8}$$

$$B_4=24-12h\left(\mathrm{P_r}-Q\right)-R\,\mathrm{P_r}\,h^2,\qquad B_5=-48-4R\,\mathrm{P_r}\,h^2,$$

$$B_6=24+12h\left(\mathrm{P_r}-Q\right)-R\,\mathrm{P_r}\,h^2,\qquad D_1=-2+\mathrm{P_{rm}}\,h,\qquad D_2=4,$$

$$D_3=-2-\mathrm{P_{rm}}\,h,\qquad D_4=2-\mathrm{P_r}\,mh,\quad D_5=-4,\qquad D_6=2+\mathrm{P_{rm}}\,h$$

Here h is the grid size, and the lattice system consists of $h=0.1$. Velocity, temperature and induced magnetic field profiles are used in the calculation of Eqs (2.8)-(2.10), $i = 1(1)n$ and the initial boundary conditions (2.6)-(2.11) are applied to obtain the following equation $A_iX_i=B_i$, $i=1,2,3\ldots$. Let A_i's be a matrix of order n, also X_i's, B_i's are column matrices with n-components. The Thomas algorithm method is applied to get the solution of the above system equations. The numerical solutions of these equations are obtained by the C-programme with small changes in the values of h, and no obvious changes have been noticed in the profiles of u, θ and H. Here we noticed that the Galerkin finite element is anchored and convergent.

4. Results and discussions

The non-dimensional fluid flow, velocity, induced-magnetic field and temperature profiles for discrete estimations of flow parameters on an inclined plate at an angle $\alpha=0$, $\frac{\pi}{6}$, $\frac{\pi}{4}$, are studied from Figures 1(a)-3(e). The effect of radiation on convection is discussed. Figure 1(a) shows the influence of the magnetic parameter (M). It is noticed that the induced-magnetic field profile increases as the magnetic number becomes larger. Figures 1(b), 2(c) and 3(b) show the influence of the Prandtl number on air $\left(\mathrm{P_r}=0.71\right)$ and water $\left(7.0\right)$. Here it is noticed that as the Prandtl number rises the induced-magnetic field also increases but the velocity and temperature profiles reduce. Figure 1(d) explains the effect of the Grashof number $\left(\mathrm{G_r}\right)$ on the induced magnetic field. It is observed that if it increases the profile of the induced magnetic field decreases. Figure 1(c) shows the induced magnetic field for different observations of the heat source $\left(Q\right)$. It is noticed that if the heat source parameter increases, then there is a drop in the induced magnetic field profile. Figures 1(e), 2(e) and 3(c) show the effect of the Eckert number $\left(\mathrm{E_c}\right)$. The induced-magnetic field profile increases, whereas velocity and temperature profiles reduce as the Eckert number rises. The effect of radiation $\left(R\right)$ on velocity, temperature and induced magnetic field profiles is shown in Figs 1(f), 2(f) and 3(e). Here the temperature and velocity profiles decrease within the increment of the radiation parameter in the fluid region, though a reverse effect is seen on the induced-magnetic field. The effect of the magnetic parameter $\left(M\right)$ on the velocity and temperature is shown in Figs 2(a)-3(a). Here it is observed that as the magnetic number increases, the velocity and temperature profiles will decrease. The influence of the Grashof number (G_r) on the velocity and temperature is illustrated in Figs 2(b)-3(d). Here it is observed that velocity profile increases, whereas the temperature profile decreases as the values of the Grashof number increase.

The skin friction (τ) at the plate is $\tau = \left(\dfrac{\partial u}{\partial y}\right)$ at $y = 0$.

Table 1. Skin friction coefficient for different values for M, Q, P_{rm}, P_r, R, and G_r.

M	Q	P_{rm}	P_r	G_r	R	E_c	α	τ
0.5	0.1	0.1	0.71	5.0	0.1	0.001	0.0	-1.37417
0.5	0.1	0.1	7	5.0	0.1	0.001	0.0	-0.75477
2	0.1	0.1	0.71	5.0	0.1	0.001	0.0	-1.42249
4	0.1	0.1	0.71	5.0	0.1	0.001	0.0	-1.57835
0.5	0.2	0.1	0.71	5.0	0.1	0.001	0.0	-1.38699
0.5	0.3	0.1	0.71	5.0	0.1	0.001	0.0	-1.4003
0.5	0.1	0.1	0.71	6	0.1	0.001	0.0	-1.23207
0.5	0.1	0.1	0.71	7	0.1	0.001	0.0	-1.27902
0.5	0.1	0.2	0.71	5.0	0.1	0.001	0.0	-1.37729
0.5	0.1	0.3	0.71	5.0	0.1	0.001	0.0	-1.38066
0.5	0.1	0.4	0.71	5.0	0.1	0.001	0.0	-1.38407
0.5	0.1	0.1	0.71	5.0	0.2	0.001	0.0	-1.37425
0.5	0.1	0.1	0.71	5.0	0.4	0.001	0.0	-1.37460

It is observed that as these values increase, the skin- friction coefficient decreases, but it is reverse in the case of the Grashof number.

The heat transfer coefficient in terms of the Nusselt number (N_u) is $N_u = -\left(\dfrac{\partial \theta}{\partial y}\right)$ at $y = 0$.

Table 2. Heat transfer coefficient (N_u.) for different values of M, Q, P_{rm}, P_r, R, and G_r.

M	Q	P_{rm}	P_r	G_r	R	E_c	α	Nu
0.5	0.1	0.1	0.71	5.0	0.1	0.001	0.0	0.280112
0.5	0.1	0.1	7	5.0	0.1	0.001	0.0	0.614554
2	0.1	0.1	0.71	5.0	0.1	0.001	0.0	0.280097
4	0.1	0.1	0.71	5.0	0.1	0.001	0.0	0.280112
0.5	0.2	0.1	0.71	5.0	0.1	0.001	0.0	0.236899
0.5	0.3	0.1	0.71	5.0	0.1	0.001	0.0	0.192538
0.5	0.1	0.1	0.71	6	0.1	0.001	0.0	0.280007
0.5	0.1	0.1	0.71	7	0.1	0.001	0.0	0.280112
0.5	0.1	3.0	0.71	5.0	0.1	0.001	0.0	0.280002
0.5	0.1	5.0	0.71	5.0	0.1	0.001	0.0	0.279218
0.5	0.1	6.0	0.71	5.0	0.1	0.001	0.0	0.276252
0.5	0.1	0.1	0.71	5.0	0.2	0.001	0.0	0.277698
0.5	0.1	0.1	0.71	5.0	0.4	0.001	0.0	0.272855

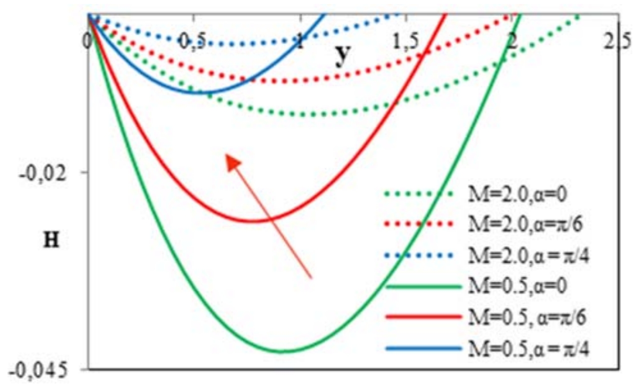

Fig.1(a). Induced magnetic field for discrete values of M, $P_r = 0.71$, $G_r = 5.0$, $Q = 0.1$, $P_{rm} = 0.5$, $E_c = 0.001$.

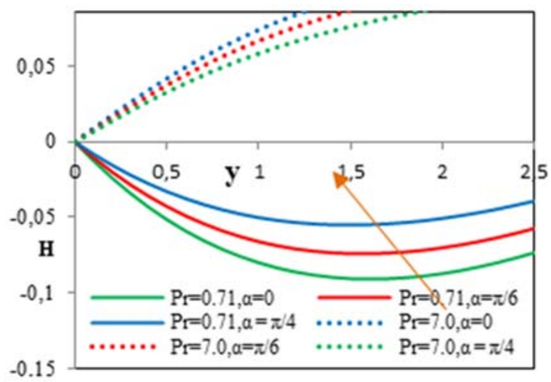

Fig.1(b). Induced magnetic field for discrete values of P_r, $M = 0.5$, $G_r = 5.0$, $Q = 0.1$, $P_{rm} = 0.5$, $E_c = 0.001$.

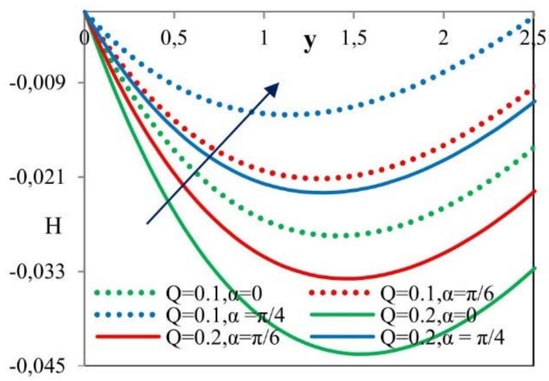

Fig.1(c). Induced magnetic field for discrete values of Q, $M = 0.5$, $G_r = 5.0$, $P_{rm} = 0.5$, $E_c = 0.001$.

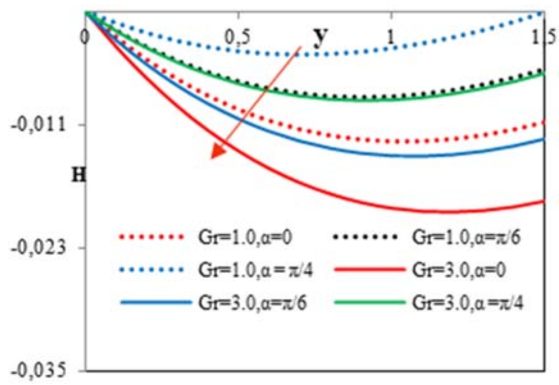

Fig.1(d). Induced magnetic field for discrete values of G_r, $M = 0.5$, $P_{rm} = 0.5$, $P_r = 0.71$, $P_{rm} = 0.5$, $E_c = 0.001$.

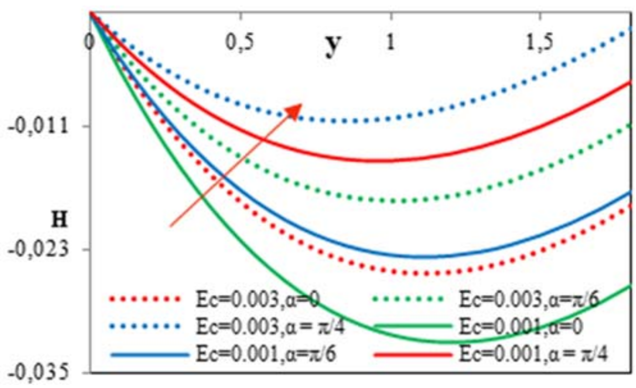

Fig.1(e). Induced magnetic field for discrete values of E_c, $M = 0.5$, $P_{rm} = 0.5$, $P_r = 0.71$, $P_{rm} = 0.5$, $G_r = 5.0$.

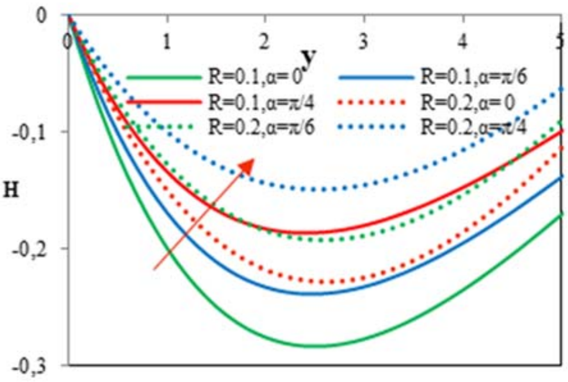

Fig.1(f). Induced magnetic field for discrete values R, $M = 0.5$, $P_r = 0.71$, $Q = 0.1$, $P_{rm} = 0.5$, $G_r = 0.5$, $E_c = 0.001$.

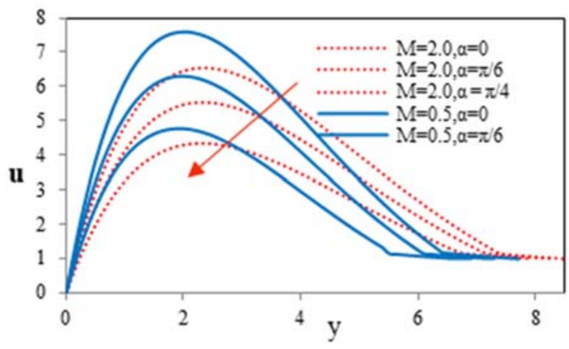

Fig.2(a). Velocity for discrete values of M, $P_r = 0.71$, $Q = 0.1$, $P_{rm} = 0.5$, $G_r = 0.5$, $E_c = 0.001$.

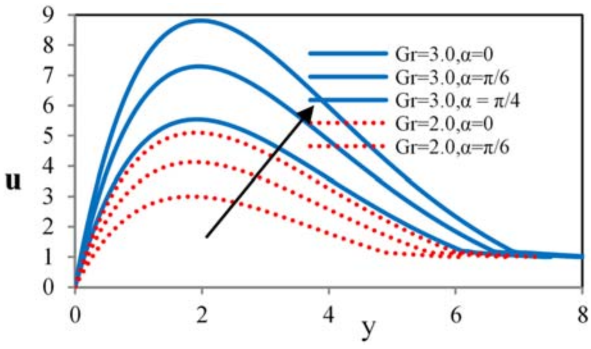

Fig.2(b). Velocity for discrete values of G_r, $P_r = 0.71$, $Q = 0.1$, $P_{rm} = 0.5$, $M = 0.5$, $E_c = 0.001$.

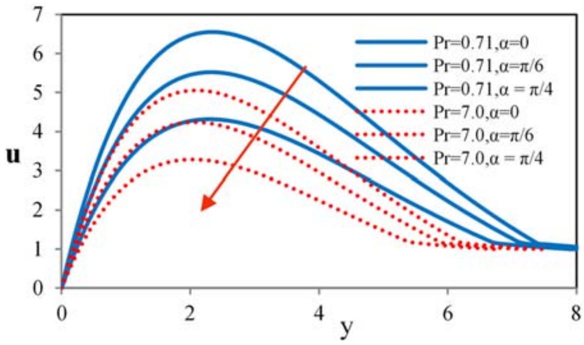

Fig.2(c). Velocity for discrete values of P_r, $M = 0.5$, $Q = 0.1$, $P_{rm} = 0.5$, $G_r = 0.5$, $E_c = 0.001$.

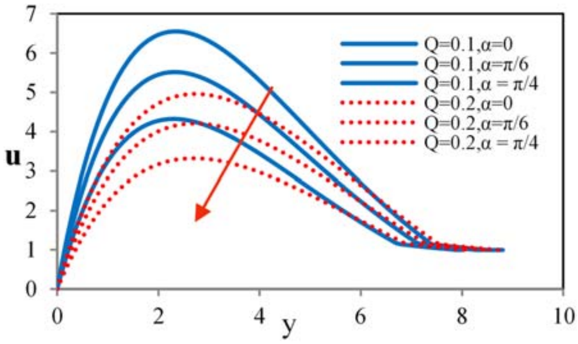

Fig.2(d). Velocity for discrete values of Q, $P_r = 0.71$, $M = 0.5$, $P_{rm} = 0.5$, $G_r = 0.5$, $E_c = 0.001$.

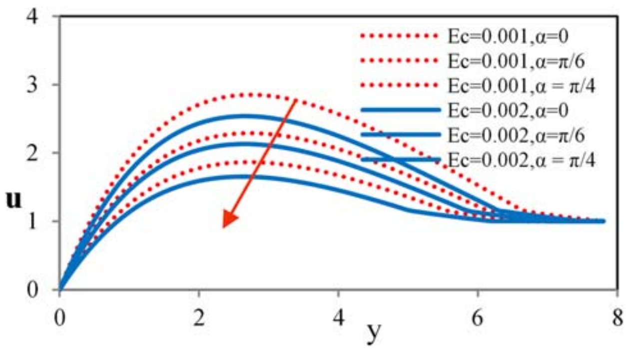

Fig.2(e). Velocity for discrete values of E_c, $P_r = 0.71$, $Q = 0.1$, $P_{rm} = 0.5$, $G_r = 0.5$, $M = 0.5$.

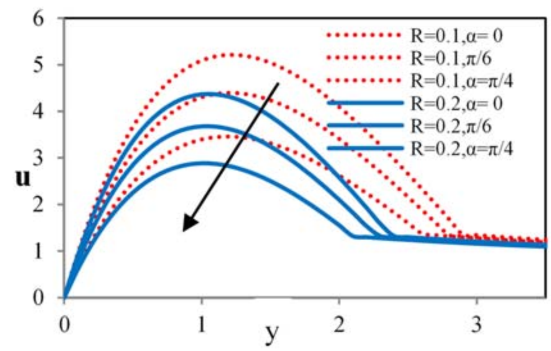

Fig.2(f). Velocity for discrete values of *R*, $M = 0.5$, $P_r = 0.71$, $Q = 0.1$, $P_{rm} = 0.5$, $G_r = 0.5$, $E_c = 0.001$.

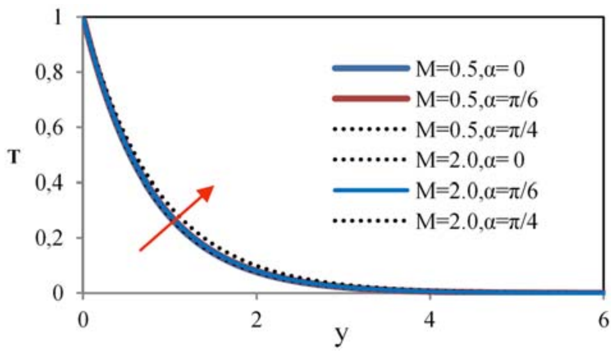

Fig.3(a): Temperature for discrete values of M, $\mathrm{P_r} = 0.71$, $Q = 0.1$, $\mathrm{P_{rm}} = 0.5$, $\mathrm{G_r} = 0.5$, $\mathrm{E_c} = 0.001$.

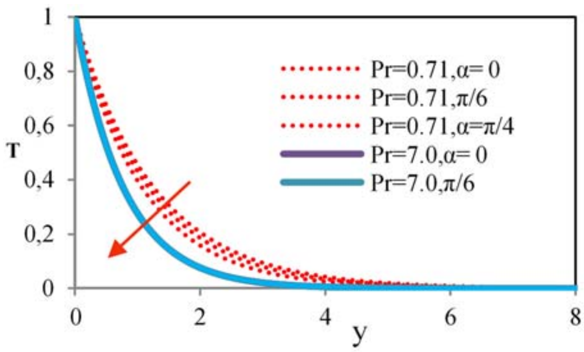

Fig.3(b), Temperature for discrete value of $\mathrm{P_r}$, $M = 0.5$, $Q = 0.1$, $\mathrm{P_{rm}} = 0.5$, $\mathrm{G_r} = 0.5$, $\mathrm{E_c} = 0.001$.

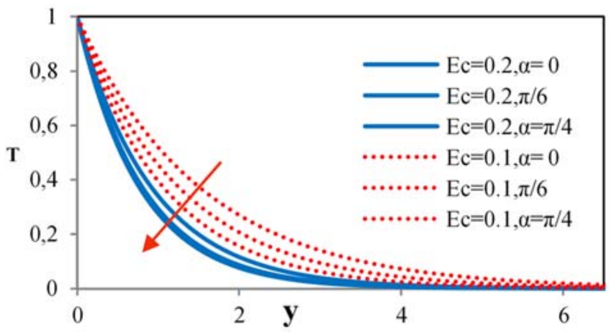

Fig.3(c). Temperature for discrete values of $\mathrm{E_c}$, $\mathrm{P_r} = 0.71$, $Q = 0.1$, $\mathrm{P_{rm}} = 0.5$, $\mathrm{G_r} = 0.5$, $M = 0.5$.

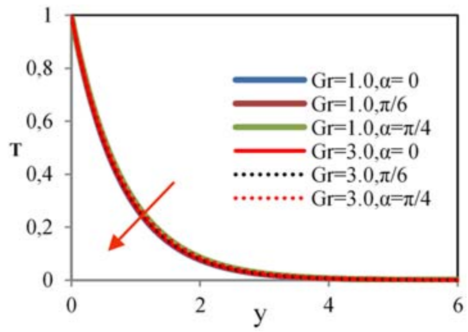

Fig.3(d). Temperature for discrete values of $\mathrm{G_r}$, $\mathrm{P_r} = 0.71$, $Q = 0.1$, $\mathrm{P_{rm}} = 0.5$, $M = 0.5$, $\mathrm{E_c} = 0.001$.

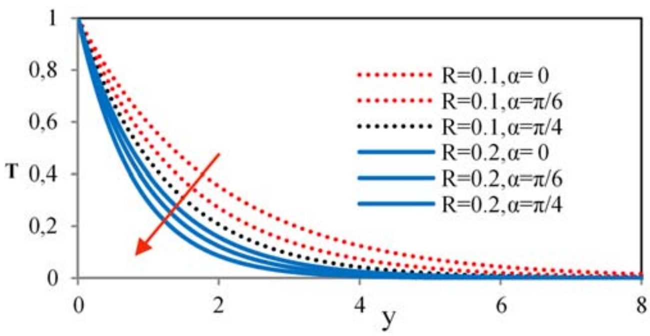

Fig.3(e). Temperature for discrete values of R, $M = 0.5$, $\mathrm{P_r} = 0.71$, $Q = 0.1$, $\mathrm{P_{rm}} = 0.5$, $\mathrm{G_r} = 0.5$, $\mathrm{E_c} = 0.001$.

5. Conclusion

The study was carried out to analyze the influence of the induced magnetic field on the free convection flow of a viscous dissipative fluid passing through the vertical inclined plate at an angle $\alpha = 0, \frac{\pi}{6}, \frac{\pi}{4}$. The effects of flow parameters on profiles of velocity, temperature and induced magnetic field are as follows:

- the flow velocity reduces as there is an increment in the angle of inclination;
- as the Lorentz force acts on the flow of the fluid the magnetic number (M) will reduce the velocity, whereas the induced magnetic profile increases;
- as the radiation parameter (R) rises, the temperature and velocity profiles decrease, but a reverse effect is noticed in the induced magnetic field;
- as the heat source (Q) becomes larger, the velocity and induced-magnetic profiles decrease;
- as the Prandtl number becomes larger, the induced magnetic field will increase, and the velocity and temperature profiles will decrease;
- the increase of the Eckert number (E_c) causes a rise in the induced magnetic field, and a decrease in the velocity and temperature profiles.

Nomenclature

C_p – specific heat at constant pressure
E_c – Eckert number
g – acceleration due to gravity
G_m – mass Grashof number
G_r – thermal Grashof number
H – induced magnetic field
H_0 – uniform induced magnetic field
H_x – induced magnetic field along the x-axis
J – current density
H – Hartmann number
Q – heat source
P_r – Prandtl number
u – velocity component in the x-direction
U_0 – non-dimensional free stream velocity
β – coefficient of volume expansion due to temperature
ρ – density
μ_0 – magnetic diffusivity
ν – kinematic viscosity
σ – electrical conductivity
w – condition at the wall
∞ – free stream conditions

References

[1] Chen CH. (2004): *Transfer of mass and heat of natural convection through a porous in a MHD flow in an inclined surface with variable concentration and temperature*.– Acta Mechanica, vol.172, pp.219-235.

[2] Pop I. and Soundalgekar V.M. (1980): *Free convection flow of a fluid through an accelerated infinite plate*.– Z. Angew. Math. Mech., vol.60, pp.167-168.

[3] England W.G. And Emery A.F. (1969): *Thermal radiation effects on laminar convection free boundary layer of an heat absorbing gas*.– Journal of Heat Transfer, vol.91, pp.37-44.

[4] Raptis A.A. and Soundalgekar V.M. (1982): *Hydro magnetic fluid flow passing through a constant infinite vertical porous plate with heat flux*.– Nuclear Engineering and Design, vol.72, pp.373-379, DOI: 10.1016/0029-5493(82)90050-4.

[5] Raptis A.A., Singh A.K. and Rai K.D. (1987): *Unsteady flow of a free convective fluid past through a semi infinite plate using finite difference scheme*.– Mer. Res. Comm., vol.14, pp.9-16.

[6] Rashidi M.M, Momoniat E. and Rostami. B (2012): *Approximate solutions for hydro magnetic boundary layer viscoelastic flow over moving stretching surface with two auxiliary parameters*.– Journal Applied Math., vol.2012, Article ID 780415, https://doi.org/10.1155/2012/780415.

[7] Shafie S., Ali F., Khan I. and Samiulhaq (2013): *Conjugate effects of mass transfer and heat on hydro magnetic free convection flow of a fluid over an inclined plate in a porous medium*.– PLOS One, vol.8, No.6, DOI:10.1371/journal.pone.0065223.39.

[8] Raju K.V.S, Sudhakar Reddy T., Raju M.C., Satyanarayana P.V. and Venkataramana S. (2014): *MHD free convective flow of a fluid through a porous medium in a horizontal channel with insulated impermeable bottom wall in presence of viscous dissipation and joule heating*.– Aim Shams Eng. J., vol.5, No.2, pp.543-551.

[9] Prakash J., Vijay Kumar A.G., Kumar B.R. and S. V. K. Varma (2014): *The effect of induced magnetic field and radiation on MHD free mixed convection fluid flow past through a porous vertical plate of viscous dissipation*.– International Conference on Advances in Marine, Industrial and Mechanical Engineering, April 15-16, Johannesburg, South Africa.

[10] M.C. Raju, Varma S.V.K. and Seshaiah B. (2015): *Heat transfer effects on a dissipative fluid flow past through a plate in presence of induced magnetic field*.– Aims Shams Eng. Journal, vol.6, pp.333-339.

[11] Chamkha, A. J. (2000): *Thermal radiation and buoyancy effects on MHD flow through an accelerated surface in the presence of heat*.– International Journal of Engineering Science, vol.38, No.15, pp.1699-1712.

[12] Ahmed S. (2012): *Mathematical modeling of induced magnetic field with viscous dissipation surrounded by a vertical porous plate in the presence of radiation*.– International Journal of Mathematics and Mechanics, vol.8, No.1, pp.86-104.

[13] Ahmed S. and Tridip (2010): *Thermal radiation and MHD effects on mass transfer and heat chemically reacting fluid with suction*.– International Journal of Applied Mathematics, vol.23, No.5, pp.778-789.

[14] Raptis A. A. and Perdikis C. (1999): *Radiation and free convection flow of a fluid past through a moving plate*.– International Journal of Applied Mechanics and Engineering. vol.4, No.4, pp.817-821.

[15] Bhattacharya K., Mukhopadhya S. and Layek G.C. (2013): *Similarity solutions of a mixed convective boundary layer slip flow over a flat vertical plate*.– Aim Shams Eng. Journal, vol.4, No.2, pp.299-305.

[16] Chamkha A.J. and Khaled A. (2001): *Solutions of similarity for MHD heat and mass transfer through an inclined plate with heat generation or absorption*.– Heat and Mass Transfer, vol.37, pp.117-123.

[17] Chamkha A.J. and Issa C. (2000): *Effects of heat absorption and thermophoresis on hydro magnetic flow with heat and mass transfer through a flat surface*.– International Journal of Numerical Methods Fluid Heat Flow, vol.10, pp.432-448.

[18] Siddiqal S. and Hossian M.A. (2012): *Mixed convection boundary flow over flat plate with radiative heat transfer*.– Applied Mathematics, vol.3, No.7, p.12.

[19] Savic L.J. and Steinruck H. (2005): *Mixed convection fluid flow passed through a horizontal plate*.– Theory App. Mech., vol.32, No.1, pp.1-19.

[20] Malga B.S., Kumar P.P., Matta S. and Appidi L. (2020): *The consequence of heat transfer on viscous dissipative radiating stream over a permeable medium in presence of induced magnetic field*.– Int. J. Innovative Tech. and Exploring Eng. (IJITEE).

[21] Ram Mohan S., Bhaskarudu P., Vishwanath Reddy G. and Thajoddin S. (2017): *Effect of induced magnetic field on a free viscous dissipative fluid over vertical porous plate in presence of heat source*.– Int. J. of Trend and Research and Development. vol.3, ISSN: 2394-9333.

[22] Raptis A.A., Perdikis C. and Leontitsis A. (2003): *Effect of radiation optically narrow gas over a vertical plate in presence of magnetic field*.– Heat and Mass Transfer, vol.39, pp.771-773.

[23] Hossian M.A., Wilson M. and Husain S. (2000): *Effect of difference surface temperature on natural convection flow over a flat vertical plate*.– Engineering Computations vol.17, pp.789-812.

[24] Kumar P.P., MalgaB.S., Matta S. and Appidi L. (2020): *Effect of viscous dissipation on unsteady MHD free convection mass transfer flow of polar fluid over a vertical porous plate in presence of thermal diffusion and heat source*.– International Journal of Advanced Science and Technology vol.29, No.5, pp.10507-10519

Received: March 30, 2020
Revised: January 3, 2021