

Int. J. of Applied Mechanics and Engineering, 2022, vol.27, No.3, pp.103-114 DOI: 10.2478/ijame-2022-0038

DEVELOPMENT OF DYNAMIC LOAD FACTORS FOR HUMAN WALKING EXCITATION FOR FLOOR VIBRATION DESIGN

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This paper discusses the derivation of a set of dynamic load factors for calculation of walking response on the basis of measurements made during a biomechanics research carried out with young adults. Firstly, a quite large number of experimental data on single footstep force were collected. The single footstep forces were then superimposed to generate the force time history for a continuous walk. This was followed by the transformation of the resultant force to the frequency domain from which the dynamic load factors for the first ten harmonics of a pacing rate can be extracted. A statistical analysis was employed on the dynamic load factors to acquire their design values in terms of the *90-th* or *95-th* percentile. The waking force function recommended by various design guides and that developed in the paper were then used in a comprehensive finite element model to predict the vibration level of a building floor. Current design guides on floor vibration normally suggest using four harmonics in the walking force whereas load factors for ten harmonics were developed in this paper. The acceleration response of the floor was found to increase by *5-33%* when walking harmonics beyond the fourth harmonic were considered. The inclusion of higher harmonics would therefore lead to a more conservative estimation of the floor response.

Key words: walking excitation, step frequency, dynamic load factor, floor vibration, design guide.

1. Introduction

Vibration serviceability has become a vital requirement in the design and construction of long span floor systems featuring slender structural sections and open-plan layout [1-3]. Building floors with fundamental frequency below a threshold of approximately *10 Hz* are usually considered as low-frequency floors which might exhibit a resonant build-up response due to various human activities such as walking, running and dancing [4, 5]. During the design stage, the potential vibration level of a floor should be comprehensively calculated and checked against acceptable limits because rectification of the floor after completion of construction could be troublesome and expensive [6-10]. The National Building Code of Canada requires a dynamic analysis of a floor structure in case its fundamental frequency is less than *6 Hz* [11]. Contemporary guidelines on vibration design including the North American design guide AISC DG11 [5], the ISO 10137 [12] and the UK document SCI P354 [13] all specify acceptance criteria for human comfort or sensitive equipment attached to the floor.

Walking excitation is a common load case scenario to be considered in the design of long span floors [14, 15]. The serviceability design of floor systems subjected to human walking requires mathematical models representing footfall excitation. In accordance with current design guides, the vertical force induced by walking is normally assumed to be periodic and can be represented by a Fourier series including some harmonic

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components of the footstep frequency. The general form of the time dependent loading function F(t) for continuous walking can be expressed as [5, 12, 13, 16]:

$$F(t) = P\left[I + \sum_{i=1}^{n} \alpha_i \sin\left(2\pi i f_p t + \varphi_i\right)\right]$$
(1.1)

where *P* is the walker's weight, f_p is the step frequency, α_i and φ_i are the dynamic load factor (Fourier coefficient) and phase lag for the *i*-th harmonic component of the walking excitation, respectively; *n* is the total number of harmonics considered. A harmonic frequency if_p is an integer multiple of the pacing frequency. The design static weight of a walker *P* is usually assumed to be in the range of 700 N to 800 N. The recommended number of harmonics considered and the associated load factors are different among researchers and guidelines. The forcing function in the form of (1.1) adopted in current guidelines is a deterministic design approach which has not considered the inter- and intra-subject diversity in gait parameter. Although the problem of inter- and intra-subject gait variability is familiar to biomechanics researchers [17, 18], it has only recently been discussed by civil engineering researchers [19-22].

Popular guidelines suggest the step frequency being in the range of 1.6 to 2.2 Hz and include up to four harmonics in the walking-induced force of formula (1.1). Commonly used design values of the dynamic load factors for walking are presented in Tab.1. The phase angles φ_i have been found to scatter significantly by relevant literature and are therefore normally assigned with quite arbitrary values. For instance, phase angles can be taken as 0 for the first harmonic and $-\pi/2$ for the others as proposed by Bachmann [16]. On the other hand, the SCI P354 suggests using the phase angle values of θ , $-\pi/2,\pi$ and $\pi/2$ for the first, second, third and fourth harmonics, respectively. Alternatively, all phase angles can be taken as zero to simplify the walking force function [23].

Harmonic number, <i>i</i>	α_i suggested by			
	Bachmann [16]	AISC DG11 [5]	SCI P365 [13]	
1	0.4 - 0.5	0.5	$0.436 \left(if_p - 0.95 \right)$	
2	0.1	0.2	$0.006 \left(if_p + 12.3 \right)$	
3	0.1	0.1	$0.007(if_p + 5.2)$	
4		0.05	$0.007 \left(if_p + 2.0 \right)$	

Table 1. Dynamic load factors for walking excitation.

The present paper discusses the development of a set of vertical dynamic load factors α_i for walking excitation on the basis of statistical analysis of experimental gait data collected from an Australian biomechanics research. To examine the influence of higher harmonics and to compare with current floor vibration guidelines, the dynamic coefficients for the first ten harmonics will be derived from the measured footfall data. The newly proposed load factors and those suggested by current guidelines are then used in a finite element model of a building floor to estimate the vibration level of the floor under expected walking excitations.

2. Methods

2.1. Measurement of single footfall force

The Vicon motion capture system developed by Oxford Metrics Group, UK has seen application to life sciences, biomechanics and sport science [24]. A co-author of this paper conducted a biomechanics research relating to human gait using the Vicon motion visual-tracking system in which the motion history of reflective markers attached to a test subject can be recorded by high resolution cameras. The system also allows force plate measurements of ground reaction force to be captured. The footfall data used to develop the new dynamic load factors for walking in this paper were based on a sample of 158 single-footfall forces performed by 23 young adults. The test subjects had an average height of 170 cm and body weight of 72 kg.

The test subjects were asked to walk normally at self-selected speeds during which their motion was captured by cameras and the footfall-induced force traces were recorded by force plates. Footfall measures including step time, step frequency, step length and ground reaction force can then be extracted. Figure 1(a) shows a walking activity reconstructed by the Vicon Nexus software. The step frequency of the test subjects was found to be in the range of 1.55 to 2.38 Hz with the average of 2.02 Hz. Figure 1b shows a typical butterfly-shaped force versus time graph resulted from a single footfall with a step frequency f_p of 2 Hz, where the

force values F were normalized against the walker's body weight P.



Fig.1. Measurement of footfall-induced force.

2.2. Derivation of dynamic load factors

In order to generate the force time history for a continuous walk, firstly a large number of consecutive similar single footstep forces were superimposed as illustrated by Fig.2a. Because the step period $1/f_p$ was shorter than the loading duration of a single footfall T_p , there were instances when both feet were in contact with the ground during which the individual footstep forces were overlapped. The fast Fourier Transform (FFT) was then performed on the generated force time history for continuous walk of Fig.2a to acquire the forcing spectrum in the frequency domain. Figure 2b depicts an example of the resultant forcing spectrum where the FFT magnitudes on the vertical axis were scaled by the walker's static weight to obtain the dynamic load factors. A MATLAB code was written to perform the above analysis procedure.

The harmonic frequencies if_p and dynamic load factors α_i found for the first ten harmonics associated with the force spectrum shown in Fig.2b are presented in Tab.2. The lower the harmonic, the larger the dynamic load factor.



Fig.2. Generation of continuous walking force.

Harmonic i	Frequency $if_p(Hz)$	α_i
1	2	0.426
2	4	0.085
3	6	0.076
4	8	0.070
5	10	0.059
6	12	0.043
7	14	0.031
8	16	0.029
9	18	0.026
10	20	0.023

Table 2. Example of measured dynamic load factors.

2.3. Statistical analysis for design values of dynamic load factors

The analysis procedure discussed in Section 2.2 was repeated for 158 footfall force time histories of the investigated sample. Descriptive statistics were then conducted to acquire the mean and standard deviation values of the dynamic load factors α_i for the first ten harmonics as presented in Tab.3.

Table 3. Statistical measures of dynamic load factors α_i .

Harmonic <i>i</i>	Mean	Standard deviation	90-th percentile	95-th percentile
1	0.324	0.088	0.439	0.493
2	0.090	0.043	0.145	0.173
3	0.059	0.020	0.081	0.086
4	0.054	0.016	0.079	0.085
5	0.048	0.016	0.066	0.070
6	0.037	0.014	0.052	0.058
7	0.029	0.012	0.043	0.047
8	0.023	0.010	0.036	0.041
9	0.019	0.009	0.031	0.034
10	0.017	0.007	0.025	0.030

The cumulative distribution functions of the first ten dynamic load factors (DLFs) calculated for the sample are plotted in Fig.3. The 90-th and 95-th percentile values of α_i determined from these cumulative distribution curves are given in Tab.3. It is now proposed to consider the 90-th or 95-th percentile as the characteristic value of α_i with an exceedance probability of 10% or 5% which can conservatively be used in floor vibration design. The proposed 95-th percentile dynamic load factors for the first three harmonics were a bit lower than the corresponding values suggested by the AISC DG11. In relation to the fourth walking harmonic, the proposed dynamic load factor was about 1.6 times larger the ASIC DG11 value. Moreover, no information about the walking harmonics beyond the fourth harmonic is given by the AISC DG11 whereas the design values for up to the tenth harmonic can be developed in this paper.



Fig.3. Cumulative distribution functions of dynamic load factors.

3. Numerical results and discussion

3.1. Case study floor description

The dynamic load factors recommended by current guidelines and those found by this research were used to numerically examine the footfall-induced vibration of a floor bay in an office building. The floor was

of prestressed concrete construction with a 180 mm thick slab spanning 10 m between 2400x350 mm band beams which were in turn supported by concrete columns. Figure 4 shows a three-dimensional finite element (FE) model including the floor and the columns and shear walls attached to the floor. The investigated bay was located close to the upper-right corner of the floor, which was intended for office use. The FE model was created using SAP2000 software package.



Fig.4. FE model of case study floor.

The dynamic properties of the floor including the natural frequencies and mode shapes were predicted using eigenvalue analysis. Figure 5 shows a natural mode shape which would be most critical to the investigated bay since antinodes with maximum modal displacements were found to concentrate around the center of the bay. The natural frequency associated with this mode shape was 6.04 Hz which was about three times the average human pacing rate of 2 Hz. As the natural frequency of the floor bay was less than 10 Hz, the floor could be classified as a low-frequency floor with a possibility of resonant built-up response due to human walking excitation [4, 5, 13].



Fig.5. Mode shape critical to investigated floor bay.

3.2. Walking force simulation

The response level of the case study floor was determined via a series of time history analysis with modal superposition method in SAP2000. Various dynamic load factors suggested by current design guides as well as those developed in Section 2.3 were used to form the walking force functions for the time history analysis of the floor model. It should be noted that the forcing function in the form of Eq.(1.1), recommended by contemporary guidelines, is for a continuous stationary walk, i.e. walking on the same spot. Manipulation of formula (1.1) was hence needed to take account of the translation of the footfall along the walking path from

one end of the floor bay to the other end, passing the center of the floor bay. This can be done by incorporating the modal displacement z corresponding to various footstep locations along the walking path into the stationary-walk force. Accordingly, the forcing function can be written as:

$$F(t) = P \sum_{i=1}^{n} \alpha_i \sin\left(2\pi i f_p t + \varphi_i\right) z.$$
(3.1)

The static weight component was already subtracted from the above equation so that only the dynamic variation in forces was used for time history analysis. It is essential to first examine the step frequency of 2 Hz because its third harmonic closely matched the natural frequency of the floor (6.04 Hz) and could lead to resonance. Figure 6 depicts the forcing function calculated from Eq.(3.1) for a step frequency of 2 Hz using the proposed 90-th percentile dynamic load factors (Tab.3.) for the first four and ten harmonics. The phase angles were taken as 0 for the first harmonic and -p/2 for the others.



Fig.6. Walking force functions including four and ten harmonics.

3.3. Comparison of floor response due to four walking harmonics

For comparative purposes, the dynamic load factors suggested by Bachmann, the AISC DG11 and SCI P354 (Tab.1.) were also used with Eq.(3.1) in the time history analysis of the floor model. In addition, the floor response to walking at off-resonant step frequencies of 1.8 Hz and 2.2 Hz considered as well. The response level of the case study floor was determined via a series of time history analyses in which the weight of the walker P was taken as 750 N and the modal damping ratio was assumed to be 1.5% with regards to current guidelines for a paperless office of prestressed concrete construction [12, 25].



Fig.7. Response to walking at $f_p = 2 Hz$ with Bachmann dynamic load factors

Figures 7-10 show the floor acceleration response due to a walking activity at a pacing rate of 2 Hz across the floor bay span with the force functions including four walking harmonics. In these figures, the response in the frequency domain was obtained from the acceleration time history by means of the FFT technique. The response spectra revealed peaks with different magnitudes at different harmonic frequencies. It can be seen that the response magnitude was greatest at the 6 Hz frequency, i.e. three times the step frequency, although the maximum dynamic load factor was associated with the first harmonic frequency of 2 Hz. Furthermore, there was a relationship between the number of spectrum peaks observed and the number of harmonics included in the walking force function.



Fig.8. Response to walking at $f_p = 2 Hz$ with AISC dynamic load factors.



Fig.9. Response to walking at $f_p = 2 Hz$ with SCI dynamic load factors.



Fig.10. Response to walking at $f_p = 2 Hz$ with proposed 90-th percentile dynamic load factors for four harmonics.

The maximum floor acceleration computed using the dynamic load factors suggested by Bachmann, the AISC and the SCI guidelines was 0.318%g, 0.362%g and 0.290%g, respectively with, g being the gravitational acceleration (Figs 7-9). Using the 90-th percentile load factors proposed in the present paper for the first four harmonics resulted in a maximum response of 0.312%g which compared well with these guidelines (Fig.10.). The good agreement between the paper's proposal and the design guides [5, 13, 16] was not only for the resonant pacing rate of 2 Hz but also for the off-resonant pacing rates of 1.8 Hz and 2.2 Hz as can be seen from Fig.11.



Fig.11. Maximum response due to four harmonics of walking.

3.4. Effect of higher walking harmonics

Further time history analysis was carried out in which the floor acceleration was determined in the event that the number of harmonics included in the walking force function, n, increased from four to ten with the dynamic load factors being obtained from Tab.3. As can be seen in Fig.12, the inclusion of ten walking harmonics strongly excited ten frequency components of the floor response spectrum, resulting in a response level which was 33% higher than that coming from the first four harmonics.



Fig.12. Response to walking at $f_p = 2 Hz$ with proposed 90-th percentile dynamic load factors for ten harmonics.

The observed contribution of the walking harmonics beyond the fourth harmonic to the floor response level is summarized in Tab.4. for the *1.8-2.2 Hz* pacing rate. Compared with the resonant step frequency (2 Hz), either a slower (1.8 Hz) or faster (2.2 Hz) pacing rate resulted in a lower response level of the floor. In

addition, the peak floor acceleration due to the 95-th percentile load factors was found to be 7-10% higher than that acquired from the corresponding 90-th percentile. For instance, the peak acceleration caused by ten harmonics of the 2-Hz step frequency with the 95-th percentile load factors was 0.448%g whilst that coming from the 90-th percentile load factors was 0.414%g, i.e. about 8% lower.

			Peak floor acc	eleration (%g)		
n	<i>90-th</i> percentile, $f_p =$		95-th percentile, $f_p =$			
	1.8 Hz	2.0 Hz	2.2 Hz	1.8 Hz	2.0 Hz	2.2 Hz
4	0.184	0.312	0.193	0.200	0.338	0.213
6	0.215	0.328	0.211	0.233	0.351	0.226
8	0.247	0.396	0.235	0.271	0.429	0.258
10	0.279	0.414	0.249	0.308	0.448	0.275

Table 4. Floor acceleration response due to 4-10 walking harmonics.

4. Conclusions

In this paper, attempts have been made to determine the dynamic load factors that constitute a Fourier series to represent walking excitations. A large number of experimental data on single footstep force were obtained from a biomechanics research. The forces from consecutive footsteps were superimposed to generate a continuous walking force time history on which the Fourier transform can be performed to extract the dynamic load factors. The characteristic values in terms of 90-th percentile and 95-th percentile dynamic load factors were proposed for the first ten walking harmonics. A dynamic load factor at the 95-th percentile was found to be 6%-19% larger than that at the 90-th percentile. It is suggested using either the 90-th or 95-th percentile dynamic load factors in the prediction of floor vibration with 10% or 5% probability of exceedance. Moreover, the paper clearly showed that the dynamic load factor tends to decrease with increasing harmonic. The 10-th dynamic load factor was seen to be about 6% of the first one. The proposed dynamic load factors would be useful for footfall-induced vibration analysis of floors in general, not just for the case study of the floor presented here.

A comprehensive dynamic analysis to predict the footfall-induced vibration of a building floor was conducted using various forcing functions recommended by relevant literature as well as this paper. Whilst most currently used floor vibration guidelines suggest combining four harmonics in the walking force, the consideration of six, eight and ten harmonics was found to increase the predicted response level by 5%, 27% and 33%, respectively. The inclusion of forcing harmonics beyond the fourth harmonic would hence lead to a more conservative design.

Acknowledgements

This work was supported by the Faculty of Civil Engineering, University of Architecture Ho Chi Minh City (research project no.XD-NCKH22), supervised by the Government of Vietnam.

Nomenclature

- f_p step frequency
 - *i* harmonic number
- α_i dynamic load factor
- φ_i phase angle

- F(t) walking force
 - P walker's weight
 - z modal displacement
 - g gravitational acceleration

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Received: February 3, 2022 Revised: June 8, 2022