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C o n t e n t s

4.3

Synthesis of Quasi-Optimal Minimum Time Control by means of Approximating Signum-Functions

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1. Intrnduction

The problem of synthesis of time-optimal trajectory is reduced to the determination of the optimal control function whose geometrical interpretation in the phase space is an optimal switching hyper-plane. The optimal function found as a solution to the problem of synthesis is, of course, a complex nonlinear function of phase coordinates. It is difficult to realize an optimal function in a controller. Those difficulties are caused, first of all, by the fact that it is not easy to construct nonlinear functional converters of a few independent variables as well as to use numerous converters and multipliers that are necessary to realize precisely an optimal switching hyper-surface.

The basic approach to the solution of the problem of constructing a quasi-optimal controller consists in obtaining, in the phase space, an approximating switching hyper-surface close to that strictly optimal. With this approach, the approximating functions should be related with a class of those suitable for technical realization. It is possible, of course, to seek a solution to this problem in the class of all nonlinear suitable for practical construction functions of phase coordinates [1,2]. These are, for example, second-power functions, some parabolic functions, nonlinear functions of one independent variable, etc.

The theory of optimal processes was developed at the beginning of 1950s but its practical application to construct control systems with an invariable part of the third and higher orders proved to be extremely difficult.

Works [3] intended to answer the question of how to approximate a complex nonlinear control function began to be published also in 1950s . To achieve the required exactness of approximation to an optimal switching hyper-surface it was necessary to widen the class of approximating functions and to complicate them. However, in these cases the practical construction of a controller is associated with great troubles and sometimes it fails to be feasible in practice.

It is an inquestionable interest to impose strong restrictions on the function adopted for technical construction and to seek a solution to the problem of approximation of time-optimal control in a class of linear functions and signum functions of linear combinations of phase coordinates. They should be possibly most expendient for technical realization.

2. Equivalent Signum-Functions

An optimal function $\mathcal{U}_o(x_1,\dots,x_m)$, obtained as a solution to the problem of synthesis corresponds to a defined class of equivalent synthesizing functions $\mathcal{U}_{cэ}(x_1,\dots,x_m)$ that are signum-equivalent to the optimal function, that is,

$$\operatorname{sign}\left[\mathcal{U}_{cэ}(x_1,\dots,x_m)\right] = \operatorname{sign}\left[\mathcal{U}_o(x_1,\dots,x_m)\right], \qquad /1/$$

where $x_1, \ldots, x_m$ are phase coordinates of the system.

The forming of the time-optimal control $V_0(t)$, applied to the input of the system can be based on an arbitrary synthesizing function because

$$V_0(t) = \mu \, \mathrm{sign}\left[\mathcal{U}_{\text{сэ}}(x_1, \ldots, x_m)\right] = \mu \, \mathrm{sign}\left[\mathcal{U}_0(x_1, \ldots, x_m)\right], \qquad /2/$$

where μ is a modulus of the input control.

The procedure of approximating the optimal control can be performed by means of equivalent approximating functions $\mathcal{U}_{\text{аэ}}(x_1, \ldots x_m)$ that are signum-equivalent to functions chosen for realization $\mathcal{U}_{\text{рэ}}(x_1, \ldots, x_m)$, that is,

$$\mathrm{sign}\left[\mathcal{U}_{\text{аэ}}(x_1, \ldots, x_m)\right] = \mathrm{sign}\left[\mathcal{U}_{\text{рэ}}(x_1, \ldots, x_m)\right]. \qquad /3/$$

If the approximating function is chosen in the form of the equivalent function $\mathcal{U}_{\text{аэ}}(x_1, \ldots, x_m)$, it is sufficient to construct, in the controller, a signum-equivalent function so that the indentity of the input control is ensured because

$$V_a(t) = \mu \, \mathrm{sign}\left[\mathcal{U}_{\text{рэ}}(x_1, \ldots, x_m)\right] = \mu \, \mathrm{sign}\left[\mathcal{U}_{\text{аэ}}(x_1, \ldots, x_m)\right] \qquad /4/$$

The equivalent approximating functions are necessary for the approximation process but they are not feasible. A controller characteristic of the quasi-optimal system is realized with the help of simple functions suitable for technical construction. These functions are employed to synthesize the equivalent realizing function $\mathcal{U}_{\text{рэ}}(x_1, \ldots, x_m)$. As the functions adopted for technical realization, we shall consider the following linear functions of phase coordinates

$$W_i(x_1,\dots,x_m)=C_{i1}x_1+C_{i2}x_2+\dots+C_{im}x_m+C_{i0} \qquad /5/$$
$$(i=1,2,\dots,n)$$

as well as signum-functions of linear combinations of the coordinates. During the process of synthesis of the realizing function, we can perform operations that also correspond to the requirement of simplicity of technical realization. These can be, for example, sum of signum-functions, product of signum-functions, sum of a linear function and a signum-function, product of a linear function and a signum-function.

The control determined by a signum-function of a sum of signum-function of linear combination of coordinates W_1 and linear function W_2, that is

$$U^2(x_1,\dots,x_m)=\mu\,\mathrm{sign}\left[U^2_{p\ni}(x_1,\dots,x_m)\right]=\mu\,\mathrm{sign}\left[\mathrm{sign}\,W_1(x_1,\dots,x_m)+W_2(x_1,\dots,x_m)\right] \qquad /6/$$

appears to be a "quadratic" equation, namely

$$U^2_\ni(x_1,\dots x_m)=\mu\,\mathrm{sign}\left[W_1(x_1,\dots,x_m)+\left|W_1(x_1,\dots,x_m)\right|W_2(x_1,\dots,x_m)\right]= \qquad /7/$$
$$=\mu\,\mathrm{sign}\left[U^2_{a\ni}(x_1,\dots,x_m)\right].$$

Indeed, it is easy to show

$$\mathrm{sign}\,U^2_{p\ni}(x_1,\dots,x_m)=\mathrm{sign}\left[\mathrm{sign}\,W_1(x_1,\dots x_m)+W_2(x_1,\dots,x_m)\right]= \qquad /8/$$
$$=\mathrm{sign}\left[W_1(x_1,\dots x_m)+\left|W_1(x_1,\dots,x_m)\right|W_2(x_1,\dots,x_m)\right]=$$
$$=\mathrm{sign}\,U^2_{a\ni}(x_1,\dots,x_m)$$

that is, the equivalent approximating function $U^2_{a\ni}(x_1,\dots,x_m)$, involves the product of linear function and, thereby, also the quadratic terms of phase coordinates. It is possible, moreover, to determine other forms of the equivalent "quadratic" control with the aid of a sum of two signum-functions or the product of linear function and signum-functions or the product of linear function and signum-function.

3. Approximating Equivalent Signum-Functions

The approximating function $\mathcal{U}^{2}_{a\mathfrak{z}}(x_1,\dots,x_m)$ in the form /8/ is one of possible forms of approximating equivalent signum-functions of the second order. Making use of a sequence of three signum-functions of linear combinations of coordinates, it is possible to obtain an equivalent approximating function $\mathcal{U}^{3}_{a\mathfrak{z}}(x_1,\dots,x_m)$ in the form of a polynomial of the third order. It is also possible to determine a control in the form of signum-function of a sum of three signum-functions and so forth. The approximating equivalent function $\mathcal{U}^{3}_{a\mathfrak{z}}(x_1,\dots x_m)$ involves the products of three linear functions and thus, the control defined by this function is an equivalent "cubic" control.

The results obtained with the help of a few signum-functions of linear functions of coordinates can be generalized to achieve the expressions for the approximating functions in the class of linear functions and of signum-functions of linear combinations of coordinates.

If the approximating function $\mathcal{U}^{n}_{a\mathfrak{z}}(x_1,\dots,x_m)$ is defined in the form of the approximating signum-polynomial of the n-th order

$$G^{n}(x_1,\dots x_m)=\sum_{j=1}^{n} g_j(x_1,\dots,x_m), \qquad /9/$$

where

$$g_j(x_1,\dots,x_m) = W_j(x_1,\dots,x_m)\left|\sum_{i=1}^{j-1} g_i(x_1,\dots,x_m)\right|, \qquad /10/$$

that is, $g_j(x_1,\dots,x_m)$ is an order of the signum-functions of linear combinations of coordinates $W_j(x_1,\dots,x_m)$, namely

$$g_1(x_1,\dots,x_m) = W_1(x_1,\dots,x_m);$$
$$g_2(x_1,\dots,x_m) = W_2(x_1,\dots,x_m)\,|g_1(x_1,\dots,x_m)|;$$
$$g_3(x_1,\dots,x_m) = W_3(x_1,\dots,x_m)\,|g_1(x_1,\dots,x_m) + g_2(x_1,\dots,x_m)$$

and so on, there exists, in the class of linear functions and signum-functions of linear combinations of coordinates, an equivalent realizing function

$$\begin{aligned} U_{p\vartheta}(x_1,\dots,x_m) = \mathrm{sign}\Big\{\mathrm{sign}\Big[\mathrm{sign}\dots\mathrm{sign}\big(\mathrm{sign}\,W_1(x_1,\dots,x_m) + \\ + W_2(x_1,\dots,x_m)\big) + \dots + W_{n-2}(x_1,\dots,x_m)\Big] + W_{n-1}(x_1,\dots,x_m)\Big\} + W_n(x_1,\dots,x_m), \end{aligned} \qquad /11/$$

that satisfies the relation

$$\mathrm{sign}\,U_{a\vartheta}(x_1,\dots,x_m) = \mathrm{sign}\,G^n(x_1,\dots,x_m) = \mathrm{sign}\,U_{p\vartheta}(x_1,\dots,x_m). \qquad /12/$$

The validity of the above theorem on the equivalent control can be proved if the polynomial $G^n(x_1,\dots,x_m)$ of the form /9/ is written in the expanded form

$$\begin{aligned} G^n(x_1,\dots,x_m) = G^1(x_1,\dots,x_m) + W_2(x_1,\dots,x_m)\left|G^1(x_1,\dots,x_m)\right| + \dots + \\ + W_i(x_1,\dots,x_m)\left|G^{i-1}(x_1,\dots,x_m)\right| + \dots + W_n(x_1,\dots,x_m)\left|G^{n-1}(x_1,\dots,x_m)\right|. \end{aligned} \qquad /13/$$

A polynomial of the i-th order is obtained from a polynomial of the i-th order by means of multiplication by a piece-wisely linear function, namely

$$G^{i}_{(x_1,\dots,x_m)} = G^{i-1}_{(x_1,\dots,x_m)}\left[\operatorname{sign} G^{i-1}_{(x_1,\dots,x_m)}\right]\left[W_i(x_1,\dots,x_m) + \operatorname{sign} G^{i-1}_{(x_1,\dots,x_m)}\right] \qquad /14/$$

According to this property of the G -polynomials, it is possible to write the relation for signum of the i-th polynomial

$$\operatorname{sign} G^{i}(x_1,\dots,x_m) = \operatorname{sign}\left[\operatorname{sign} G^{i-1}_{(x_1,\dots x_m)} + W_i(x_1,\dots,x_m)\right]. \qquad /15/$$

We shall apply a successively recurrent relation for some values of the order of the polynomial, namely

$$\begin{aligned} i=3,\ \operatorname{sign} G^{3}(x_1,\dots,x_m) &= \operatorname{sign}\left[\operatorname{sign} G^{2}(x_1,\dots,x_m) + W_3(x_1,\dots,x_m)\right] = \qquad /16/ \\ &= \operatorname{sign}\left[\operatorname{sign}\left(\operatorname{sign} W_1(x_1,\dots x_m) + W_2(x_1,\dots,x_m)\right) + W_3(x_1,\dots,x_m)\right]; \\ i=4,\ \operatorname{sign} G^{4}(x_1,\dots,x_m) &= \operatorname{sign}\left[\operatorname{sign} G^{3}(x_1,\dots,x_m) + W_4(x_1,\dots x_m)\right] \end{aligned}$$

or

$$\operatorname{sign} G^{4}_{(x_1,\dots,x_m)} = \operatorname{sign}\left\{\operatorname{sign}\left[\operatorname{sign}\left(\operatorname{sign} W_1(x_1,\dots,x_m) + W_2(x_1,\dots,x_m)\right) + W_3(x_1,\dots,x_m)\right] + W_4(x_1,\dots,x_m)\right\}$$

and so on.

Thus, by making use of indication for $i = n$, we get the expression /11/ that is used to determine a signum of the approximating polynomial $G^{n}(x_1,\dots,x_m)$.

If the approximating function $U_{a3}(x_1,\dots,x_m)$ is determined in the form of the signum-polynomial

$$G_{\Sigma}^{n}(x_1,\dots,x_m)=\sum_{i=1}^{n}\dot{G}^{i}(x_1,\dots,x_m)\left|\prod_{\substack{j=1\\ j\neq i}}^{n}\dot{G}^{j}(x_1,\dots,x_m)\right|, \qquad /17/$$

where

$$\dot{G}^{j}(x_1,\dots,x_m)=\sum_{i=1}^{j} g_i(x_1,\dots,x_m),$$

there exists, in the class of linear functions and signum-functions of linear combinations of coordinates, an equivalent realizing function

$$\begin{aligned} U_{p\vartheta}(x_1,\dots,x_m) &= \operatorname{sign} W_1(x_1,\dots,x_m)+\operatorname{sign}\big(\operatorname{sign} W_1(x_1,\dots,x_m)+ \qquad /18/\\ &+ W_2(x_1,\dots,x_m)\big)+\operatorname{sign}\big[\operatorname{sign}\big(\operatorname{sign} W_1(x_1,\dots,x_m)+W_2(x_1,\dots,x_m)\big)+\\ &+ W_3(x_1,\dots,x_m)\big]+\operatorname{sign}\big\{\operatorname{sign}\big[\operatorname{sign}\big(\operatorname{sign} W_1(x_1,\dots,x_m)+W_2(x_1,\dots,x_m)\big)+\\ &+ W_3(x_1,\dots,x_m)\big]+W_4(x_1,\dots,x_m)\big\}+\dots+\\ &+\operatorname{sign}\big\{\operatorname{sign}\big\{\operatorname{sign}\big[\operatorname{sign}\dots\operatorname{sign}\big(\operatorname{sign} W_1(x_1,\dots,x_m)+W_2(x_1,\dots,x_m)\big)+\\ &+\dots+W_{n-2}(x_1,\dots,x_m)\big]+W_{n-1}(x_1,\dots,x_m)\big\}+W_n(x_1,\dots,x_m)\big\}. \end{aligned}$$

A signum of the polynomials $G_{\Sigma}^{n}(x_1,\dots,x_m)$ remains unchanged after G_{Σ}^{n} is multiplied by a defined positive expression and it is then justified to write

$$\operatorname{sign} G_{\Sigma}^{n}(x_1,\dots,x_m)=\operatorname{sign}\left\{\frac{1}{\prod_{i=1}^{n}\left|\dot{G}^{i}(x_1,\dots,x_m)\right|}\cdot \qquad /19/\right.$$

$$\left.\cdot\left[\sum_{i=1}^{n}\dot{G}^{i}(x_1,\dots,x_m)\left|\prod_{\substack{j=1\\ j\neq i}}^{n}\dot{G}^{j}(x_1,\dots,x_m)\right|\right]\right\}=\sum_{i=1}^{n}\frac{\dot{G}^{i}(x_1,\dots,x_m)}{\left|\dot{G}^{i}(x_1,\dots,x_m)\right|}$$

We obtain thus an expression for a signum of the approximating polynomial $G_{\Sigma}^{n}(x_1,\dots,x_m)$:

$$\operatorname{sign} G_E^n(x_1,\dots x_m) = \sum_{i=1}^{n} \operatorname{sign} G^i(x_1,\dots,x_m) \qquad /20/$$

On the foundation of the recurrent relation /15/, we get the realizing Signum-functions

$$\operatorname{sign} G^i(x_1,\dots,x_m) = \operatorname{sign}\Big\{\operatorname{sign}\Big[\operatorname{sign}\dots\operatorname{sign}\Big(\operatorname{sign} W_1(x_1,\dots,x_m) + W_2(x_1,\dots,x_m)\Big) + \dots + W_{i-1}(x_1,\dots,x_m)\Big] + W_i(x_1,\dots,x_m)\Big\} \qquad /21/$$

$$(i=1,2,\dots,n)$$

After the signum function /21/ is inserted, for various values of i , into the expression /20/, we obtain the expression for the equivalent realizing function /18/. In this way we have proved the generalized theorem on the equivalent control.

4. Construction of some Quasi-Optimal Controller by means of Equivalent Signum-Functions

After some classes of approximating functions signum-equivalent to those chosen for realization are determined, we are able to solve the problem of the structure of a quasi-optimal controller. The essence of the operations carried-out consists in determination of the forms of approximating functions which are signum-equivalent to the realizing functions constructed by means of linear functions and signum-functions of coordinates. It is easy to select proper apparatus for a controller on basis of the expressions of the type of linear functions of coordinates; also the signum-functions of linear combinations of coordinates can be achieved without a difficulty by employing relay elements.

The approximating polynomial $G^{n}(x_1,\dots,x_m)$ in the form /9/ determines the equivalent realizing function in the form /11/ that can be used to obtain an equivalent control by means of a sequence of n signum-functions composed of sums of signum-functions and linear combinations of the phase coordinates of the system.

The structure of a controller built-up on the basis of the equivalent realizing function /11/ is shown in Fig.1.

The possibility of transformations of realizing structures is infinite and, in this way, it is possible to widen the class of approximating functions. Signum-equivalent transformations can be used to modify equivalent approximating functions and to approximate them to the form of equivalent synthesizing functions. The last functions belong to the class of optimal functions and are, at the same time, optimal switching functions. This is so because having known those functions, it is possible to realize the strictly optimal control. It is obvious that the optimal control is unique but the functions synthesising it constitute a set. This set is infinite since it is possible to transform infinitely one function to another being signum-equivalent to the first.

The substance of these operations lies in the fact that the obtained equivalent approximating functions can be utilized to construct equivalent controllers of a system that have a simpler structure for technical implementation. To realize the optimal control by means of functions adopted for technical construction, it is sufficient to present an arbitrary synthesizing function in the form of the approximating polynomials given in the present work.

The structure of a controller of a system is determined by the choice of the type of an equivalent approximating polynomial. The form of approximating function is determined in dependence on complexity and the type of the model of an invariable part of control system. The approximating function in the form /17/ is used to obtain the equivalent realizing structure in the form /18/ that consists of relay elements interconnected in series-parallel. This only slightly complicates the structure of controller as compared to that shown in Fig.1.

The required accuracy of approximation specifies an order of the equivalent approximating polynomial. According to the statements of the theorem discussed, a number of components of controller is equal to the order of this polynomial. Arbitrary approximations are possible by increasing the quantity of terms of the approximating polynomial and by adding correspondingly new members in the block diagram of control system. In this way, the problem of synthesis of a quasi-optimal controller is reduced to the problem of choosing the expression form which is best for the approximation procedure in the class of approximating functions and of determining the form of this expression in dependence on the required accuracy of approximation to the precisely optimal control. The considered approximating -polynomials determined in the forms /9/ and /17/ are expressed by means of linear combinations of coordinates $W_i(x_1,\dots,x_m)$. The equivalent realizing functions determined correspondingly in the form /11/ and /18/ are expressed also with the aid of linear combinations of coordinates. It is very important that the equivalent functions contain the same linear combinations $w_i(x_1,...,x_m)$, which are involved in the approximating polynomials corresponding to these functions. The last feature of the considered equivalent control functions enable to solve the problem of determining parameters of controller of a quasi-optimal system by using the approximation method. Indeed, to determine linear functions and signum-functions that are used to construct equivalent realizing functions it is required only to find coefficients of the linear functions of phase coordinates. Thus, the problem of synthesis of parameters of a quasi-optimal control member is led to finding, according to the approximation criterion adopted for the approximation process, unknown coefficients of linear functions of coordinates.

While solving the problem of determination of parameters of a quasi-optimal controller, the analytical expression for the optimal control function is expanded into a series. After the last term of the series is estimated, it is possible, with an arbitrary predetermined accuracy, to express the optimal function by a finite number of terms of the series, for example, by n terms:

$$\mathcal{U}(x_1,...x_m) = \mathcal{U}_{(0)} + \frac{\partial \mathcal{U}_{(0)}}{\partial x_1} \overset{\circ}{x}_1 + \frac{\partial \mathcal{U}_{(0)}}{\partial x_2} \overset{\circ}{x}_2 + \frac{\partial \mathcal{U}_{(0)}}{\partial x_3} \overset{\circ}{x}_3 + \cdots \qquad /22/$$
$$+ \sum_{i=1}^{n} \frac{1}{i!} \left\{ \frac{\partial \mathcal{U}_{(0)}}{\partial x_1} \overset{\circ}{x}_1 + \frac{\partial \mathcal{U}_{(0)}}{\partial x_2} \overset{\circ}{x}_2 + \frac{\partial \mathcal{U}_{(0)}}{\partial x_3} \overset{\circ}{x}_3 + \cdots \right]^{(i)},$$

where

$$\overset{o}{x}_i = x_i - x_{i0}, \quad (i = 1,2,...,m);$$

$$\mathcal{U}_{(0)} = \mathcal{U}_{(x_1,x_2,x_3,\dots)}\Big|_{\substack{x_1=x_{10},\\ x_2=x_{20},\\ x_3=x_{30},}} ; \frac{\partial \mathcal{U}_{(0)}}{\partial x_1} = \frac{\partial \mathcal{U}_{(x_1,x_2,x_3\dots)}}{\partial x_1}\Big|_{\substack{x_1=x_{10}\\ x_2=x_{20}\\ x_3=x_{30}}}$$

and so on,

and the bracketed exponents of the power to which the expressions are risen have a symbolic meaning to indicate the order of derivatives. Moreover, the form of the approximating polynomial of n-th order can be described by means of an arbitrary set of auxiliary functions which is determined by the theorems discussed above. For example, the approximating signum-polynomial of the n-th order can be, by means of the function $g_j(x_1,\dots,x_m)$ in the form /10/, written as such a polynomial. This description is as follows

$$G^n(x_1,\dots,x_m) = W_1(x_1,\dots,x_m) + W_2(x_1,\dots,x_m)\left|W_1(x_1,\dots,x_m)\right| + \dots + W_n(x_1,\dots,x_m)\left|\sum_{j=1}^{n-1} g_j(x_1,\dots,x_m)\right|. \qquad /23/$$

The unknown coefficients c_{ij} of linear combinations $W_i(x_1,\dots,x_m)$ involved in /23/ can be determined by employing any familiar methods. In particular, this can be done by first writing expression /23/ into an expanded form in terms of increasing powers of phase coordinates and then by equating the coefficients standing at the terms of the some powers of expressions /22/ and /23/.

To illustrate application of this method of synthesis of a quasi-optimal controller, we shall consider an example of the third order whose invariable part consists of two aperiodic components connected in series /time constants: $T_1 = 0{,}0625$, $T_2 = 0{,}134$, transfer coefficients $k_1 = k_2 = I$ / and one integrating component / $T_3 = 0.007$ /. As an approximating polynomial we assume the signum-function

$$G^2(x_1,x_2,x_3) = \mu_1 \left[C_{11}x_1 + C_{12}x_2 + C_{13}x_3\right] + \qquad /24/$$
$$+ \sigma_1\left[C_{11}x_1 + C_{12}x_2 + C_{13}x_3\right]\left[C_{21}x_1 + C_{22}x_2 + C_{23}x_3\right],$$

where

$$\sigma_1 = \operatorname{sign} U^1(x_1,x_2,x_3) = \operatorname{sign}\left[C_{11}x_1 + C_{12}x_2 + C_{13}x_3\right]$$

A simple structure of a quasi-optimal control device in the form of two relay components connected in series in shown in Fig.2. This structure has been determined by using the method of equivalent signum-functions.

An analytical approach has been used to calculate the time of the strictly optimal control with applying a unit function at the system input. After the shown structure of a quasi-optimal controller has been realized a model has been utilized to measure the duration of a transient response and this time deviates by 20% from that of the strictly optimal process. Of course, the controller can be built-up of n relay components and, thus, the order of approximating G-polynomial as well the accuracy of approximation to optimal process can be more higher.

We shall consider an example of synthesis of another third order system whose invariable part consists of three series-connected integrating components /$T_1 = T_2 = T_3 = \ = I$/.

The structure of controller of this system is the same as that depicted in Fig.2 where are marked the coefficients C_{ij} of linear combination of coordinates $W_i/x_1, x_2, x_3/$. We determine the coefficients C_{ij} of linear combination of coordinates, namely:

$$C_{11} = \frac{\partial U_{(0)}}{\partial x_1}; \quad C_{21} = C'_{21}\sigma_1, \quad C'_{21} = \frac{1}{2}\,\frac{\dfrac{\partial^2 U_{(0)}}{\partial x_1^2}}{\dfrac{\partial U_{(0)}}{\partial x_1}};$$

$$C_{12} = \frac{\partial U_{(0)}}{\partial x_2}; \quad C_{22} = C'_{22}\sigma_1, \quad C'_{22} = \frac{1}{2}\,\frac{\dfrac{\partial^2 U_{(0)}}{\partial x_2^2}}{\dfrac{\partial U_{(0)}}{\partial x_2}}; \qquad /25/$$

$$C_{13} = \frac{\partial U_{(0)}}{\partial x_3}; \quad C_{23} = C'_{23}\sigma_1, \quad C'_{23} = \frac{1}{2}\,\frac{\dfrac{\partial^2 U_{(0)}}{\partial x_3^2}}{\dfrac{\partial U_{(0)}}{\partial x_3}}.$$

Fig.3a illustrates an optimal transient response with applying a unit function at the system input. Very satisfactory results have been obtained from tests performed on the model set-up according to the controller of Fig.2 and to coefficients C_{ij}, determined in the first approximation by formulae /25/. To improve the behaviour of transient response the coefficient C'_{22} has been slightly corrected. The quasi-optimal process obtained by approximation with using the polynomial

$G^2(x_1, x_2, x_3)$ and the realization of equivalent signum-function on the model is shown in Fig.3 b. It is seen that the minimum time quasi-optimal process does not differ practically from the optimal process. An analytical method has been employed to determine the time of optimal process with various deviating errors. With the change of initial condition from 100% to 200%, the deviation from the optimum time is not greater than 60%. The system performance is satisfactorily also in case when initial conditions change by 10 times, namely, within the range from 25% to 250% of a rated step signal, the time deviation of quasi-optimal process from that optimal does not exceed 80% with proper initial conditions. Transient response of quasi-optimal system with a linear input signal is presented in Fig.3c. It can be seen that the quasi-optimal process does not, almost at all, differ from that optimal one. With an input signal in the form of a parabola of the second order, the transient response in the quasi-optimal system also satisfactorily approaches that optimal one. All the above experimental results have been obtained for various classes of input signals and for initial conditions varying in a wide range but with unchanged values of coefficients C_{ij} determined by using the methods described above.

In a general case with a high order of the system invariable part and a complex character of restrictions, the mathematical description of strictly optimal switching hyper-surface may be difficult. In this case it is not allowed to use an analytical method to determine parameters of a control device. The possibility exists here to employ any known methods of searching for values of the coefficients of linear combinations of coordinates $W_i(x_1,\dots,x_m)$, which are capable of being involved in equivalent realizing functions. For instance, if a model of the controlled plant is known, the structure of controller is built-up on this model and unknown coefficients are found experimentaly.

We shall consider an example of constructing a controller of a system of a high order.

For illustration, we shall cite the results of testing an approximative optimal follow system of automatic control used in tracing machine tools. The proceeding works [4, 5] are devoted to determination of the controller structure, especially easy for realization, with making use of the method of equivalent functions presented by the author of the present report. The invariable part of the system of Fig.4 was in the form of two series connected dynamic blocks

a/ the first block C 1 consists of the second cascade of an electronic and electromechanical amplifier /aperiodic link of the first order/

b/ the second block C 2 is composed of slave motor and reductor /link of the second order/.

An error of the control system has been assumed to be an output of the block C 2 , whereas current δ of armature circuit of the slave motor is taken as an output from the block C 1 . The controller is fed with a signal composed of these variables and their derivatives in the form of linear combinations, namely

$$W_2(\delta,\dot{\delta}) = k_\delta \delta + k_{\dot{\delta}} \dot{\delta} = k_{x2} x_2 + k_{\dot{x}2} \dot{x}_2 ; \qquad /26/$$
$$W_1(I,\dot{I}) = k_I I + k_{\dot{I}} \dot{I} = k_{x1} x_1 + k_{\dot{x}1} \dot{x}_1 .$$

As an approximating function we assume the following polynomial of the second order

$$G^2(\delta,\dot{\delta},I,\dot{I}) = k_\delta \delta + k_{\dot{\delta}} \dot{\delta} - \sigma \left[k_\delta k_I \delta I + k_\delta k_{\dot{I}} \delta \dot{I} + k_{\dot{\delta}} k_I \dot{\delta} I + k_{\dot{\delta}} k_{\dot{I}} \dot{\delta} I \right], \qquad /27/$$

where

$$\sigma = \operatorname{sign} \left[k_\delta \delta + k_{\dot{\delta}} \dot{\delta} \right].$$

The oscillogram of the transient response of controlled variable /displacement of a cutter/ $S = f(t)$ in the linear system is shown in Fig. 5a. The transient was tested with a step input signal representing an instantaneous maximum detuning between tracing machine and cutter. After the values of unknown coefficients in the linear combinations of coordinates $W_1(I,\dot{I})$ and $W_2(\delta,\dot{\delta})$ have be choosen and applied in the system, we have obtained the minimum time $t_m = 0,13$ s of transient response. The oscillogram of the process with maximum detuning is given in Fig. 5 b. Tests performed on a model have exhibited that decreasing the amplitude of input signal by 40 % in relation to its maximum exerts very small effect on the transient response time. Further decrease of input signal representing detuning of cutter causes the transient response time to be shorter as compared to the time with the maximum detuning of cutter. Thus, the high-speed action of the system in markedly improved /for example, 7 times/ in comparison with transient response in a linear system. Also the over-control of the output variable is thus avoided.

5. Conclusion

1. It has shown in the present work that it is possible to solve the problem of synthesis of a quasi-optimal system by employing the method of equivalent control functions. This method is used to determine signum-equivalent control functions in this class of them which is most suitable for technical realization.

2. Making used of the theorem obtained in the present work it has been proved that a sign of approximating function

coincides with a sign of the function selected for technical realization. This function is synthesizable by means of linear functions and signum-functions of the system phase coordinates.

3. If the procedure of approximation of the optimal function is conducted with the help of equivalent functions, the realization of the obtained approximating expression is not necessary. When using the present method, it is sufficient, for approximation, to realize an equivalent function defined in the class of functions adopted for technical implementation.

4. The method presented can also be used to determine the structure of a quasi-optimal controller of system. This structure depends on the choice of the form of equivalent approximating functions. The required accuracy of approximation to the strictly optimal system predetermines a number of members of the structure.

5. In the case when the mathematical description of the optimal control function is known, the obtained expressions for equivalent functions enable to determine parameters of quasi-optimal controller by using an analytical manner. The method presented here is applicable also in the case when the mathematical description of an optimal control function is unknown.

6.1

SOME PROBLEMS OF CONTROL PLANTS IDENTIFICATION

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Identification of control plants has become a major field in both theory and practice of control; considerable attention has been paid in recent years to stochastic and deterministic procedures intended to solve problems of identification [1-7]. Because the plants and the control equipment are becoming increasingly complex the scope of such problems expends. Whereas initially identification was to yield chiefly the control equation, at present identification involves estimates of how strong are the links between input and output variables, finding the equation of links and its parameters, quantitative estimate of the model isomorphity to the actual plant, development of decomposition and composition techniques, estimates of non-linearity, etc.

This paper will essentially deal with identification of stochastic plants which make a large class of complex actual industrial processes. The results obtained can be considered as an extension of results cited in [8,9] where determined plants were identified whose input and output parameters are random functions or random quantities. We will first deal with complete characteristics of a stochastic and determined plant i.e. conditional (output input variables) or combined (input and output) multidimensional probability densities. Since in practice complete characteristics for non-Gaussian distributions are hard to find, their approximation by Gaussian densities and perturbational polynomials are discussed instead. Then instantaneous characteristics of a stochastic plant are discussed and the concept of mean linearity introduced. Since instantaneous characteristics used in description of stochastic plants by the data of their normal operation can lead to erroneous results in case where conditional dispersion of the output variable the input variable is heteroskedastic.(Results of research into skedastic functions are reported). In the last portion of the paper we propose to study the estimates of dispersion functions. The Appendix presents some results for instantaneous functions of Gaussian distributions.

1. Stochastic plants and their complete characteristics. A complete characteristic of a dynamic plant is the operator $\mathcal{A}$, which links the input x , and output, y , variable: $y=\mathcal{A}x$. Generally this link can be given to the plant equation $\mathcal{B}y=\mathcal{C}x$ ($\mathcal{B}$ and $\mathcal{C}$ are certain operators) which is equivalent to $y=\mathcal{A}x$, $\mathcal{A}=\mathcal{B}^{-1}\mathcal{C}$, provided that there is an operator $\mathcal{B}^{-1}$.

The operator $\mathcal{A}$ can be regarded as random or non-random; the plants can be stochastic or determined, respectively. In other words the internal parameters of the plant (e.g. linear differential equation factors for a linear system) can be either random or not. Besides, both types of plants can be studied at random and determined input signals x ; this means that each type can be approached in two different ways depending on whether external actions are random or not. In further discussion we will assume that the operator $\mathcal{A}$ (form and parameters) does not depend on the input signal x either in the probabilistic or functional or any other sense; a less rigit requirement that this condition be met at least for input signals belonging to a certain class e.g. constrained by $\ell_1 \leq x \leq \ell_2$. Besides (and this will be only for the sake of simplicity) we will discuss the case of one-dimensional inputs and outputs: x and y , where $x(t)\equiv x_t, y(t)\equiv y_t$ are some functions (processes) random or otherwise, of time t . The assumption that $\mathcal{A}$ is independent of x_t makes a linear plant such a plant whose operator $\mathcal{A}$ is linear and does not depend on input action. The superposition principle is thus fulfilled. Determined systems are identified completely when the form of operator A and its parameters, the most comprehensive characteristics of the plant, is found. When we know A we can determine the output y unambignously for any known input x .

A complete identification of stochastic systems consists in finding the form of the operator A and the distribution laws for its parameters (instead of the parameters proper). However, even if the operator A is known we cannot find the output y with the known x unambignously; we can just describe the distribution of y at the given x , or the conditional density of the probability y against x: $\Psi(y/x)$, which will depend on probabilistic characteristics of the plant internal parameters. Identification by the data of normal operation and the use of results thus obtained will reduce to analysis of the output signal y characteristics, provided that the input signal x was at the input. A complete characteristic is $\Psi(y/x)$. Therefore, identification of a stochastic system can be described as finding the conditional density $\Psi(y_t/x_s, s_0 \leq s \leq t)$ (s_0 is the origin of count), or the operator which enables to find the distribution of the output y_t with the known input realization $x_s, s_0 \leq s \leq t$. In the case of discontinuous processes $\Psi(y_n/x_1,...,x_n)$ will be a similar characteristic. It is thus necessary to find the functions $\Psi(y_n/x_1,...,x_n)$. The functions Ψ cannot be calculated directly from statistical data, therefore the approximating formulae. Further on we will cite the results of approximation by Gaussian densities and perfurbational polynomials. For static plants a two dimensional density $\Psi(y_t/x_s)$ will be the complete characteristic.

2. Approximation of statistical distributions

Ref.[10] described a technique whereby statistical distribution curves $\varphi(x)$ are approximated by the functions $f(x)=P_n(x)\Gamma(x)$ where $\Gamma(x)$ is the Gaussian distribution and $P_n(x)=\sum_{k=0}^{n} a_k x^k$ is the n -powered polinomial chosen appropriately.

Factors a_i of that polynomial are found from the condition

$$\mathcal{J}=\int_{-\infty}^{\infty}\left[\varphi(x)-P_n(x)\Gamma(x)\right]^2 e^{\frac{x^2}{2}}\,dx=min. \qquad (2.1)$$

which leads to equations of moments:

$$m[x^k]=\int_{-\infty}^{\infty}x^k P_n(x)\Gamma(x)dx=\sum_{i=0}^{n}a_i\int_{-\infty}^{\infty}x^{i+k}\Gamma(x)dx=\sum_{i=0}^{n}a_i M[x^{i+k}] \qquad (2.2)$$

Here and below m denotes the moments of statistic distribution and M the moments of Gaussian distribution. Let us assume that all random quantities x are normed and centered; otherwise let us make the substitutions $u=\frac{x-m_x}{\sigma_x}$ and consider u. To approximate the complete characteristics of a stochastic plant let us use the method of[10]. We will approximate the multidimensional densities $\varphi(x_1,...,x_k)$ by the functions $f(x_1,...,x_k)=P_n(x_1,...,x_k)\Gamma(x_1,...,x_k)$ where $\Gamma(x_1,...,x_k)$ is a multidimensional Gaussian distribution whose parameters (mathematical expectation dispersion in our case $m_i=0, \sigma_i=1$ and correlation factors) are chosed on the basis of the given statistic distribution while $P_n(x_1,...,x_k)=\sum_{(i_1,...,i_k)=0}^{i_1+...+i_k=n} a_{i_1...i_k}x_1^{i_1}...x_k^{i_k}$ is an appropriately chosen polynomial. $\Gamma(x_1,...,x_k)$ has the form $Ce^{-Q(x_1,...,x_k)}$ where $C=const., Q(x_1,...,x_k)>0$ is a quadratic form. The criterion for factors $a_{i_1...i_k}$ will be similar to (2.1):

$$\mathcal{J}=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left[\varphi(x_1,...,x_k)-P_n(x_1,...,x_k)\Gamma(x_1,...,x_k)\right]^2 e^{Q(x_1,...,x_k)}dx_1...dx_k=min. \qquad (2.3)$$

This leads to equations of moments similar to (2.2)

$$m[x_1^{i_1}...x_k^{i_k}]=\int_{-\infty}^{\infty}...\int_{-\infty}^{\infty}x_1^{i_1}...x_k^{i_k}P_n(x_1,...,x_k)\Gamma(x_1,...,x_k)dx_1...dx_k=\sum_{(j_1,...,j_k)=0}^{j_1+...+j_k=n}a_{j_1...j_k}M[x_1^{i_1+j_1}...x_k^{i_k+j_k}] \qquad (2.4)$$

If we assume $n\leq 2$, then the solution will be $P_n(x_1,...,x_k)=1$ i.e. for $n\leq 2$ the best approximation by criterion of Eq.(2.3) will be the Gaussian distribution $f(x_1,...,x_k)=\Gamma(x_1,...,x_k)$.

Let us take the case where $n=3$. But first we have to

orthogonalyze the quantities so as $r'_{ij} = M[u_i u_j] = 0$. This is know orthogonalization process $u_1 = x_1, u_2 = x_2 - r_{12} u_1, \ldots$
Then $\Gamma(u_1,\ldots,u_k) = \Gamma(u_1)\ldots\Gamma(u_k)$ and Eq.(2.4) will look like

$$m[u_1^{i_1}\ldots u_k^{i_k}] = \sum_{(j_1,\ldots,j_k\geq 0)}^{j_1+\ldots+j_k\leq 3} a_{j_1\ldots j_k} M[u_1^{i_1+j_1}]\ldots M[u_k^{i_k+j_k}] \quad (i_1+\ldots+i_k \leq 3)$$

Then the solution will be

$$P_3(u_1,\ldots,u_k) = 1 + \sum_{i=1}^{k} \frac{S_i}{3}(u_i^3 - 3u_i) + \sum_{i\neq j} m_{ij} u_i (u_j^2 - 1) + \sum_{i\neq j\neq k} m_{ijk} u_i u_j u_k \qquad (2.5)$$

where $S_i = \frac{m[x_i^3]}{2}$ is the assymetry factor $x_i, m_{ij} = \frac{1}{2} m[u_i u_j^2], m_{ijk} = m[u_i u_j u_k]$.
For n =4 we will have (e.g. for two dimensional density)

$$\begin{aligned}
&a_{00} = 1 + E_1 + E_2 + E, \ a_{10} = -(S_1 + m_{12}), \ a_{01} = -(S_2 + m_{21}), \\
&a_{20} = -(E + 2E_1), \ a_{11} = -(e_{12} + e_{21}), \ a_{02} = -(E + 2E_2), \\
&a_{30} = \frac{S_1}{3}, \ a_{21} = m_{21}, \ a_{12} = m_{12}, \ a_{03} = \frac{S_2}{3}, \\
&a_{40} = \frac{E_1}{3}, \ a_{31} = \frac{e_{21}}{3}, \ a_{22} = E, \ a_{13} = \frac{e_{12}}{3}, \ a_{04} = \frac{E_2}{3},
\end{aligned} \qquad (2.6)$$

where $E_i = \frac{m[x_i^4] - 3}{8}$ is in excess of $x_i, E = \frac{m[u_1^2 u_2^2] - 1}{4}, e_{ij} = \frac{1}{2} m[u_i u_j^3]$.
These formulas are applicable at small $S_i, m_{ij}, m_{ijk}, E_i, E, e_{ij}$.
A transition from the functions $f(u_1,\ldots,u_k)$ to the functions $f(x_1,\ldots,x_k)$ is cumbersome. Therefore for the case of two-dimensional density we will have formulas for the factors of the polynomial $P_3(x_1, x_2)$ i.e. where x_1 and x_2 are not orthogonal. From Eqs. ($\Pi.5$) to ($\Pi.8$) we will have

$$a_{00} = 1, \ a_{20} = a_{11} = a_{02} = 0,$$

$$a_{10} = \frac{r_{12} S_2 - S_1 + 3r_{12} m_{21} - m_{12}(1 + 2r_{12}^2)}{(1 - r_{12}^2)^2}, \quad a_{01} = \frac{r_{12} S_1 - S_2 + 3r_{12} m_{12} - m_{21}(1 + 2r_{12}^2)}{(1 - r_{12}^2)^2},$$

$$a_{30} = \frac{S_1 - r_{12}^3 S_2 + 3r_{12}^2 m_{12} - 3r_{12} m_{21}}{(1 - r_{12}^2)^3}, \quad a_{03} = \frac{S_2 - r_{12}^3 S_1 + 3r_{12}^2 m_{21} - 3r_{12} m_{12}}{(1 - r_{12}^2)^3},$$

$$a_{21} = \frac{r_{12}^2 S_2 - r_{12} S_1 + m_{21} + 2r_{12}^2 m_{21} - r_{12}^3 m_{12} - 2r_{12} m_{12}}{(1 - r_{12}^2)^3}, \quad a_{12} = \frac{r_{12}^2 S_1 - r_{12} S_2 + m_{12} + 2r_{12}^2 m_{12} - r_{12}^3 m_{21} - 2r_{12} m_{21}}{(1 - r_{12}^2)^3}$$

We can take the functions $\frac{f(y,x_1,\ldots,x_k)}{f(x_1,\ldots,x_k)}=\frac{P_{n+1}(y,x_1,\ldots,x_k)\Gamma(y,x_1,\ldots,x_k)}{P_n(x_1,\ldots,x_k)\Gamma(x_1,\ldots,x_k)}$

to approximate the conditional probabilities $\Psi(y/x_1,\ldots,x_k)$; in other words we can approximate separately the functions $\varphi(y,x_1,\ldots,x_k)$ and $\varphi(x_1,\ldots,x_k)$ by the above technique. Then we will have $f(x_1,\ldots,x_k)=\int_{-\infty}^{\infty} f(y,x_1,\ldots,x_k)\,dy$, i.e. when we approximate $\varphi(y,x_1,\ldots,x_k)$ we approximate simultaneously $\varphi(x_1,\ldots,x_k)$ by the same criterion, so that to find $f(y,x_1,\ldots,x_k)$ is sufficient.

3. Moment characteristics and mean linearity. In some practical cases we can deal with conditional instantaneous characteristics which are not so complete as conditional densities but are more convenient. In particular we can use conditional mathematical expectation of the output against the input $M(y_t/x_s, S_0 \le S \le t)$ in the continuous case and $M(y_n/x_1,\ldots,x_n)$ in the discontinuous case.

These conditional mathematical expectations are considered for any t or n and any $x(S)$ or x_i and are determined by a certain operator $\mathcal{B}$ so that

$$M(y_t/x_s, S_0 \le S \le t) = \mathcal{B}_t x_s \quad \text{in a continuous case}$$

$$M(y_n/x_1,\ldots,x_n)=\mathcal{B}\{x_1,\ldots,x_n\} \quad \text{in a discontinuous case} \tag{3.1}$$

Let us introduce the following definition: the system S will be termed linearity in the mean if the operator $\mathcal{B}$ is linear, or the conditional mathematical expectation is linearly dependant on the input. This definition was found to be a natural extension of the conventional definition of linearity. Indeed, for linear systems the operator $\mathcal{B}$ will look like

$$M(y_t/x_s, S_0 \le S \le t)=\int_{S_0}^{t} K(t,S)\,x(S)\,dS \tag{3.2}$$

or $M(y_n/x_1,\ldots,x_n)=\sum_{i=1}^{n} K_i x_i$.

For linear determined systems we will have

$$y(t)=\int_{s_0}^{t} W(t,s)x(s)ds \qquad \text{or} \qquad y_n=\sum_{i=1}^{n} W_i x_i \tag{3.3}$$

It is easy to see that if (3.3) is valid so is (3.2). Indeed in (3.2) $y(t)$ is described unambiguously by the values $x(s), s_0 \le s \le t$, and then $y(t)=M(y_t/x_s, s_0 \le s \le t)$; this means that we obtain (3.2) and $W(t,s)=K(t,s)$. This is also true for the discontinuous case. The reverse is not true, since $y(t)$ and $x(s)$ are generally related probabilistically. Thus formula (3.3) is a case of formula (3.2) where y_t and x_s are related unambiguously. Therefore the definition of linearity in (3.2) is broader than (3.3). The function $K(t,s)$ in (3.2) is a generalization of the weight function $W(t,s)$ for determined systems, therefore it could be termed an averaged weight function of the stochastic system. To find the meaning of the term "averaged" let us consider a generalized equation of a linear stochastic plant in the form

$$y_t = A x_s = \int_{s_0}^{t} \mathcal{K}(t,s)x(s)ds \qquad \text{or} \qquad y_n=\sum_{i=1}^{n} \mathcal{K}_i x_i \tag{3.4}$$

Here $\mathcal{K}(t,s)$ or $\mathcal{K}_1,\dots,\mathcal{K}_n$ are random functions since the operator A is random. Due to the above assumption of Eq. 3,4 in independence of A from x we will have

$$M(y_t/x_s, s_0 \le s \le t)=\int_{s_0}^{t} \overline{\mathcal{K}(t,s)}x(s)ds \quad \text{or} \tag{3.5}$$
$$M(y_n/x_1,\dots,x_n)=\sum_{i=1}^{n} \overline{\mathcal{K}_i}\, x_i .$$

Comparison of (3.5) and (3.2) yields

$$K(t,s)=\overline{\mathcal{K}(t,s)}$$
$$K_i=\overline{\mathcal{K}_i} \qquad (i=1,\dots,n) \tag{3.6}$$

i.e. $K(t,s)$ is the mean value of a random weight function $\mathcal{K}(t,s)$ of a stochastic system.

Let us note that every linear system is also linear in the mean (if A does not depend on x) whereas the reverse is not true.

To find the averaged weight function $K(t,s)$ we can use the well known Wiener-Hopf equation which follows from

$$R_{yx}(t,s)=\int_{s_0}^{t}\overline{\mathcal{K}(t,\tau)}R_{xx}(\tau,s)d\tau=\int_{s_0}^{t}K(t,\tau)R_{xx}(\tau,s)d\tau. \qquad (3.7)$$

The function $K(t,s)$ found from (3.7) yields a certain "average" model of the actual plant. How successfull this model is we can judge partially from the second conditional instantaneous characteristic, the conditional dispersion $\mathcal{D}(y_t/x_s, s_0\le s\le t)$ or $\mathcal{D}(y_n/x_1,\dots,x_n)$.

The conditional instantaneous characteristics underlie the dispersion methods of random functions [8,9].

Suppose that we have a plant $y(t)=\int_{s_0}^{t}K(t,s)x(s)ds+v(t)=Ax(s)$. This classical circuit with the noise $v(t)$ can be considered as a "noisy" stochastic plant whose operator A is random, linear, non-unform. The random parameter of the operator A , $v(t)$ is assumed normally independent from $x(s)$.

Then $M(y_t/x_s, s_0\le s\le t)=\int_{s_0}^{t}K(t,s)x(s)ds+m_v(t)=\mathcal{B}x(s)$

If $m_v(t)=0$ then ~~taking~~ we will obtain a conventional record of a determined linear model of the plant $z(t)=\int_{s_0}^{t}K(t,s)x(s)ds$, *where* $z(t)=M(y_t/x_s, s_0\le s\le t)$.

4. <u>A skedastic function and its properties</u>. Let us deal exclusively with the first conditional and inconditional functions only in that case of identification where the conditional dispersion $\mathcal{D}(y_t/x_s)$ is homoskedastic. If this requirement is not met the normed correlational and normed dispersional functions $r_{yx}(t,s)$ and $\eta_{yx}(t,s)$ describe the relation between the output variable $y(t)$ and input variable $x(s)$ with an error which increases with the "decrease" in the homoskedasity of $\mathcal{D}(y_t/x_s)$. We can show that $\eta_{yx}(t,s)$ of a cubatur with a Gaussian process at its input is characterized by the degree of the link between $y(t)$ and $x(s)$ for any instant of time t and s less accurately than by $\eta_{yx}(t,s)$ of a squarer with the same inputs. In a sense a boundary case is when $y(t)$ and $x(s)$ are pseudonormally correlated[2]. Then both $r_{yx}(t,s)$ and $\eta_{yx}(t,s)$

are identically zero though the processes studied are interpendent.

To find the magnitude of error when $\eta_{yx}(t,s)$ and $\eta_{yx}(t,s)$ and are used in the case of non-constant conditional dispersion we will introduce the function

$$\zeta_{yx}(t,s)=\left\{\frac{\int_{-\infty}^{\infty}[\mathcal{D}(y_t/x_s)-M\{\mathcal{D}(y_t/x_s)\}]^2 g_1(x_s)\,dx_s}{\int_{-\infty}^{\infty}[y_t^2-M^2(y_t)-M\{\mathcal{D}(y_t/x_s)\}]^2 g_2(y_t)\,dy_t}\right\}^{1/2} \qquad (4.1)$$

and call this a mutual skedastic function of random processes $y(t)$ and $x(s)$.

Let us look into the properties of the definition introduced. The mutual skedastic function lies within the range $0\le\zeta_{yx}(t,s)\le 1$. Indeed

a) it follows from (4.1) that $\zeta_{yx}(t,s)\ge 0$.

The notation $\Psi(y_t/x_s)$ will mean the conditional density of probability $y(t)$ against $x(s)$.

b) To prove that $\zeta_{yx}(t,s)\le 1$ let us use the inequality

$$\int_{-\infty}^{\infty}\Big[\int_{-\infty}^{\infty}(y_t^2-M^2(y_t))\Psi(y_t/x_s)\,dy_t\Big]g_1(x_s)\,dx_s\ge\int_{-\infty}^{\infty}\Big[\int_{-\infty}^{\infty}(y_t^2-M^2(y_t/x_s))\Psi(y_t/x_s)\,dy_t\Big]g_1(x_s)\,dx_s \qquad (4.2)$$

which becomes evident considering that the left-hand side of (4.2) contains the value of $\mathcal{D}(y_t)$ and the right-hand side only parts of the dispersion $y(t)$. Inequality (4.2) can be valid only in the case where

$$\int_{-\infty}^{\infty}(y_t^2-M^2(y_t))\Psi(y_t/x_s)\,dy_t\ge\int_{-\infty}^{\infty}(y_t^2-M^2(y_t/x_s))\Psi(y_t/x_s)\,dy_t \qquad (4.3)$$

which is used in the proof below

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\big(y_t^2-M^2(y_t)-M\{\mathcal{D}(y_t/x_s)\}\big)^2\Psi(y_t/x_s)g_1(x_s)\,dy_t\,dx_s\ge$$

$$\ge\int_{-\infty}^{\infty}\Big[\int_{-\infty}^{\infty}(y_t^2-M^2(y_t)-M\{\mathcal{D}(y_t/x_s)\})\Psi(y_t/x_s)\,dy_t\Big]^2 g_1(x_s)\,dx_s\ge$$

$$\geqslant \int_{-\infty}^{\infty}\Big[\int_{-\infty}^{\infty}\big(y_t^2 - M^2(y_t/x_s) - M\{\mathcal{D}(y_t/x_s)\}\big)\Psi(y_t/x_s)\,dy_t\Big]^2 g_1(x_s)\,dx_s =$$
$$= \int_{-\infty}^{\infty}\{\mathcal{D}(y_t/x_s) - M\{\mathcal{D}(y_t/x_s)\}\}^2 g_1(x_s)\,dx_s.$$

2. A mutual skedastic function is zero only in the case where $\tau_{yx}(t,s)$ or $\eta_{yx}(t,s)$ accurately describe the link between the random process $y(t)$ and $x(s)$. Indeed, it follows from (4.1) that $\xi_{yx}(t,s)=0$ at

a) $\mathcal{D}(y_t/x_s)=M\{\mathcal{D}(y_t/x_s)\}=const.$ - - the condition for homoskedasity;

b) $\mathcal{D}(y_t/x_s)=0$ is the condition for the functional link between the processes $y(t)$ and $x(s)$.

3. The mutual skedastic function reaches its maximal value when $\tau_{yx}(t,s)$ (in the case of a linear relation between the processes $y(t)$ and $x(s)$) or $\eta_{yx}(t,s)$ (in the case of a non-linear relation between them) are zero.

Indeed, it is known [8] that generally

$$\mathcal{D}(y_t)=\mathcal{D}\{M(y_t/x_s)\}+M\{\mathcal{D}(y_t/x_s)\}, \tag{4.5}$$

but in this specific case

$$\mathcal{D}(y_t)=M\{\mathcal{D}(y_t/x_s)\}. \tag{4.6}$$

It follows from (4.6) that the denominator in (4.1) reaches its maximal value, and since the function $\mathcal{D}(y_t/x_s)$ is positive at any $x(s)$ the numerator is easily shown to reach its maximal value.

4. The closer the relations between the random processes $y(t)$ and $x(s)$, the greater the value of $\xi_{yx}(t,s)$ and can be as close as wished, provided that $\eta_{yx}(t,s)=0$.

Therefore the mutual skedastic function is an essential characteristic for identification of stochasic plants.

A study of the random process $x(s)$ requires estimation of how accurately its self-correlating and self-dispersion functions are used as characteristic of the link. This characteristic is provided by the self-skedastic function of the random process $X(s)$:

$$\xi_{xx}(s_1,s_2)=\left\{\frac{\int_{-\infty}^{\infty}\left[\mathcal{D}(x_{s_2}/x_{s_1})-M\{\mathcal{D}(x_{s_2}/x_{s_1})\}\right]^2 g_1(x_{s_1})dx_{s_1}}{\int_{-\infty}^{\infty}\left[x_{s_2}^2-M^2(x_{s_2})-M\{\mathcal{D}(x_{s_2}/x_{s_1})\}\right]^2 g_2(x_{s_2})dx_{s_2}}\right\}^{1/2} \quad (4.7)$$

5. <u>On estimates of dispersional functions</u>. Mutual correlational R_{yx} and dispersional θ_{yx} functions [8,9] are used to find characteristics of links between the input, X and output, y signals. Therefore these functions have to be estimated from experimental data. The quantity θ_{yx} is known to characterize the link between random quantities better than R_{yx} does. However, generally the more sophisticated is the characteristic of the relations (i.e. the better the dependence between random quantities are described) the worse is the convergence of its estimate, that is the greater the samplicy must be to achieve the same accuracy of approximation.

Let us assume that observations of random quantities X and y will be result in pars $(x_1,y_1),\dots,(x_n,y_n)$ a consistent and unbiased estimate for R_{yx} is found by the formula

$$R^*_{yx}=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{n-1},$$

where $\bar{x}=\frac{\sum_{i=1}^{n}x_i}{n}$, $\bar{y}=\frac{\sum_{i=1}^{n}y_i}{n}$ are sampliny averages for X and y respectively.

Two techniques can yield an estimate of θ_{yx}. Normally the data are grouped into K intervals of the values of x and each interval is associated with the average value of x_i^* in that interval. Then y is also divided into K groups: y belongs to i-th the group if the corresponding x belongs to the i-th interval. Thus we group y into a table.

$y_{11},\dots,y_{1n_1}$

- - - - - - -

$y_{K1},\dots,y_{Kn_K}$ The i-th group can be associated with the group average $\bar{y}_i=\frac{\sum_{j=1}^{n_i}y_{ij}}{n_i}$. Which is known to be a consistent and unbiased estimate for the mathematical expectation of provided that this belongs to the i-th group, or for x_i^*. For the function θ_{yx} a consistent and unbiased estimate can be

$$\theta_{yx}^* = \sum_{i=1}^{k} \frac{n_i}{n}(\bar{y}_i - \bar{y})^2 + \frac{\mathcal{D}_y^*}{n} - \sum_{i=1}^{K} \frac{\mathcal{D}_i^*}{n},$$

where $\mathcal{D}_y^* = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij}-\bar{y})^2}{n-1}$ is the estimate of complete dispersion y and $\mathcal{D}_i^*$ is the estimate of dispersion of y in the i-th group.

However a dispersional function can also be found by another technique; this assumes monotonous regression of y over X. This assumption is valid for linear regression. The following theorem is proved by Hans (Prague, UTIA) if $X_1 \le X_2 \le \ldots \le X_n$ and $(x_1, y_1), \ldots, (x_n, y_n)$ are measured data, the sampling estimate for a normed correlation η_{yx}^2 in the case of a non-decreasing regression is $\eta_{yx}^{*2} = \frac{\sum_{i=1}^{n}(y_i - f(x_i))^2}{\sum_{i=1}^{n}(y_i - \frac{1}{n}\sum_{j=1}^{n} y_j)^2}$,

where for the function $f(x_i)$ the following relations are true

$$f(x_l) = \frac{1}{K_{i+1}^* - K_i^*} \sum_{j=K_i^*}^{K_{i+1}^*-1} y_j, \quad K_0^* = K_0 = 1, \quad K_{i+1} = \min_{x_K > x_{Ki}} k$$

$$K_{i+1}^* = \max_{j>i} \left\{ k_j : \frac{1}{K_j - K_i^*} \sum_{m=K_i^*}^{K_j-1} y_m \le \frac{1}{K_l - K_i^*} \sum_{m=K_i^*}^{K_l-1} y_m, \; l = i+1, i+2, \ldots \right\}.$$

Appendix

Moments of Gaussian distributions

Let X and y be linked by a Gaussian density. Mutual Gaussian density will be denoted as f, one-dimensional density, as g.

For this case the following relations hold:

I. Moments of a one-dimensional Gaussian distribution are described by the formula.

$$M(x^{n+1}) = n\sigma_x^2 M(x^{n-1}) + m_x M(x^n) \quad \text{(A.1)}$$

Indeed $M(x^{n-1})\cdot n\cdot \sigma_x^2 = n\sigma_x^2 \cdot \int_{-\infty}^{\infty} x^{n-1} g_x(x)dx = \sigma_x^2 \int_{-\infty}^{\infty} g_x(x)d(x^n)$

after integration in parts $n\sigma_x^2 M(x^{n-1}) = \sigma_x^2 \left[x^n g_x(x)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} x^n g_x'(x)dx\right] =$

$$= \sigma_x^2 \int_{-\infty}^{\infty} x^n \frac{x-m_x}{\sigma_x^2} g_x(x)dx = \int_{-\infty}^{\infty} x^{n+1} g_x(x)dx - m_x \int_{-\infty}^{\infty} x^n g_x(x)dx = M(x^{n+1}) - m_x M(x^n)$$

2. For co-variation we have

$$R_{y^n x} = n R_{yx} M(y^{n-1}). \qquad \text{(A.2)}$$

Indeed, $R_{y^n x} = M(y^n x) - m_x M(y^n) =$

$$= M[y^n M(x/y)] - m_x M(y^n) = M[y^n(m_x + r_{xy}\frac{\sigma_x}{\sigma_y}(y-m_y))] - m_x M(y^n) =$$

$$= \frac{R_{yx}}{\sigma_y^2}[M(y^{n+1}) - m_y M(y^n)],$$

Hence, by virtue of (A.1), follows Eq.(A.2)

In particular, if $m_y = 0$, then

$$R_{y^n x} = \begin{cases} 0 & n = 2k \\ (2k-1)!!\, R_{xy}\sigma_y^{2k-2} & n = 2k-1 \end{cases} \qquad \text{(A.3)}$$

3. If $m_x = m_y = 0$, then

$$M(x^{2k} y^{2l-1}) = M(x^{2k-1} y^{2l}) = 0 \qquad \text{(A.4)}$$

E.g. $M(x^{2k} y^{2l-1}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^{2k} y^{2l-1} f_{xy}(x,y)dx\,dy$

Replacement of x by $-x$ and of y by $-y$ yields .

$$M(x^{2k} y^{2l-1}) = -\int_{\infty}^{-\infty}\int_{\infty}^{-\infty} x^{2k} y^{2l-1} f_{xy}(-x,-y)dx\,dy =$$

$$= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^{2k} y^{2l-1} f_{xy}(x,y)dx\,dy = -M(x^{2k} y^{2l-1}),$$ and hence (A.4)

4. If $m_x = m_y = 0, \sigma_x = \sigma_y = 1$, then :

$$M(xy^3) = M(x^3 y) = 3\, r_{xy} \qquad \text{(A.5)}$$

$$M(x^2 y^2) = 1 + 2\, r_{xy}^2 \qquad \text{(A.6)}$$

$$M(x^2y^4) = M(x^4y^2) = 3 + 12\, r_{xy}^2 \qquad \text{(A.7)}$$

$$M(x^3y^3) = 9\, r_{xy} + 6\, r_{xy}^3 \qquad \text{(A.8)}$$

(A.5) follows from (A.3) at n =3 (K =2).

Since the conditional distribution $\psi_{yx}(y/x)$ will be a Gaussian one with the parameters $r_{xy}\frac{\sigma_y}{\sigma_x}$ and $\sigma_y\sqrt{1-r_{xy}^2}$ then (A.2) for conditional moments will be written as

$$M(y^{n+1}/x) = n\sigma_y^2(1-r_{xy}^2)M(y^{n-1}/x) + r_{xy}\frac{\sigma_y}{\sigma_x}\, x\, M(y^n/x) \quad (\sigma_y=\sigma_x=1) \qquad \text{(A.9)}$$

Hence $M(y^2/x) = r_{xy}^2 x^2 + (1 - r_{xy}^2)$

$$M(y^3/x) = 2(1-r_{xy}^2)M(y/x) + r_{xy}\, x\, M(y^2/x) = r_{xy}^3 x^3 + 3r_{xy}(1-r_{xy}^2)x$$

Then we shall have (A.6)

$$M(x^2y^2) = M[x^2 M(y^2/x)] = 3\, r_{xy}^2 + 1 - r_{xy}^2 = 1 + 2\, r_{xy}^2$$

Formulas (A.7) and (A.8) are evident

$$M(x^4y^2) = M[x^4 M(y^2/x)] = 15\, r_{xy}^2 + 3(1 - r_{xy}^2) = 3 + 12\, r_{xy}^2$$

$$M(x^3y^3) = M[x^3 M(y^3/x)] = 15\, r_{xy}^3 + 9(1 - r_{xy}^2)r_{xy} = 9\, r_{xy} + 6\, r_{xy}^3$$

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6.2

ON SOME CLASS OF ADAPTIVE (SELF-LEARNING) SYSTEMS

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At present one can see a rapid development of the trend in the theory of adaptive systems, connected with the usage of the mathematical apparatus of the stochastic approximation (Tsypkin[I], Fu[2] and others). A long list of the papers involved is supplied in the book[I].

This report gives another approach to the problem of cre - ating a theory of some particular kind of adaptive systems, based on the usage of the finitely convergent algorythms for the solution of the infinite systems of inequalities[3]. The general outline of the problem is as follows. The system is said to be adaptive if its functioning law changes with the experience acquired. Some information is supplied to the system on 'success' or 'failure' of its conduct to some aim condition. It is essential, that some characteristics of the surroundings and the system as well as, probably, some parameters of the aim condition are assumed to be unknown to the designer. They may be any of a certain class M . The adaptive system (AS) is said to be reasonable in class M , if for any aim condition and any characteristics of this class there exists an instant after which the aim condition is fulfilled every time. The problem consists in construction of a system, reasonable in the given class M .

The report gives a formalized stating of the simplest variant of this problem, and, along with some additional suppositions, the solution of this strictly formulated problem. The results are given of simulation on the computer of the self-learning process of one simple mathematically styled system "reason - able" in the given rather conventional sense.

I° Precise setting of the problem

Let's consider that the time t has the values $t = 0$, I,2... The values changing (generally speaking) in time will be called variables, and the values fixed for the given system(and, consequently, not changing in time) will be called parameters. Among the latter we shall distinguish the so called variable parameters ξ, which can take any values of some given set M. Here $\xi = \| \xi_j \|$ is a multidimentional vector.x)

The given set of some elements z will be denoted by $\{z\}$. The value of the variable z at the instant t will be denoted by z_t. We shall consider the sets $\{x\}, \{s\}, \{\sigma\}, \{u\}$ as given and the set $\{\tau\}$ as having to be defined (in accordance with the conditions stated below). Their elements will be called as follows: x - external coordinates of AS, s - surrondings, σ - sensor, u - control, τ - tactics. Let the function $\mu(x, s, \xi)$ be given with the values 0 or I, which is called the aim condition start signal, and also given is the real function $F(x, s, \xi)$. By aim condition (AC) we shall call: if $\mu_t = \mu(x_t, s_t, \xi) = 1$ then $F(x_{t+1}, s_{t+1}, \xi) > 0$. We shall say that AC is fulfilled at the moment $t+1$, if either $\mu_t = 0$ and $F(x_{t+1}, s_{t+1}, \xi) > 0$ or $\mu_t = 0$. (It should be noted that $\mu_t = 0$ means, in fact, that AC is not set).

We shall consider as given: the sensor equation

$$\sigma_t = \sigma(x_t, s_t, \xi) \tag{I}$$

(defining what AS "sees"), the motor equation

$$x_{t+1} = X(x_t, s_t, u_t, t, \xi) \tag{2}$$

(defining the motion of AS), and the equation of the change of the surroundings

$$s_{t+1} = S(x_t, s_t, t, \xi). \tag{3}$$

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x) The number of components of vector ξ may also be a variable parameter: $\xi = \| k, \xi_1, \dots, \xi_k \|$.

We must define the following "brain equations" x) of AS:

$$\mathcal{U}_t = \mathcal{U}(\sigma_t, \tau_t), \tag{4}$$

$$\tau_{t+1} = T(\sigma_t, \sigma_{t+1}, \tau_t). \tag{5}$$

Having x_0, s_0, τ_0 as given, the equations (I) - (5) allow to find subsequently the values of *all* variables mentioned at all instants, AC being either fulfilled or not for any t = I,2,... . It is to be underlined that the right parts of equations (I)-(3) (unlike equations (4), (5)) depend, generally speaking, on the variable parameters. The change of the variable parameters $\xi \in M$ means the change of the task of the fulfilment of AC, or the change of AC, under which the fixed problem is solved. (The change of motor and sensor equations means the change, in the process of exploitation, of the functional characteristics of AS, the change in the functions S, μ, F takes place in the change of the problems solved by AS). The initial values x_0, s_0, τ_0 may also depend on the variable parameters.

If all mentioned above is defined, we say, that AS is set. AS is said to be reasonable in class M of problems, if for any values of the variable parameters $\xi \in M$ there exists some instant t_0, such as for all $t \geqslant t_0$ AC will be fulfilled and τ_t = const for $t \geqslant t_0$ xx). The number τ_0 of instants t, for which AC is not fulfilled, will be said to be the number of errors of the system.

After all these formal definitions are made, we may give

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x) The author apologies for his having used the stale term"brain", and asks to consider this term only as the name of the automata (not finite), whose work is described by equations (4), (5).

xx) The requirement τ_t = const at $t \geqslant t_0$ may be cancelled. Yet, some considerations make it rather convenient and besides it is automatically fulfilled for some solution we get.

the exact setting of the problem of how to build "the brain" of a "reasonable" (in a rather conventional and limited sense) AS. This problem consists in constructing over the given class M and the given functions μ, F, σ, X, S of the brain equations (4),(5) such, that AS became reasonable (in the above mentioned sense) in the class M of problems.

2° General suppositions and the main result.

Let's denote the Euclidian space of dimention n by R_n. Suppose, that $\{\sigma\}$ is a closed bounded set of some R_n, $\sigma = \|\sigma_j\|_{j=1}^n$. Let's consider the following four conditions as fulfilled:

(I) We may introduce the new control v, where $\{v\}$ is a bounded set of some R_q $(v = \|v_j\|_{j=1}^q)$ so that $u = u(v)$ is some function, and that AC at the instant t is fulfilled, if the control $v = v_{t-1}$ satisfies k inequalities

$$|\eta_t^{(j)}| < \varepsilon_j \qquad j = 1, \dots, k, \tag{6}$$

where $\eta_t^{(j)} = (c_j, v_{t-1}) - \varphi_t^j$. Here ε_j are parameters, $c_1, \dots, c_k$ are linear independent known vectors and $\varphi_t^j = \varphi_j(x_{t-1}, x_t, s_{t-1}, s_t, \xi)$ are some functions of the mentioned arguments.

(II) There exists the function $v = V^u(\sigma, \xi)$ called ideal control such that for any x_{t-1}, s_{t-1} and $\xi \in M$ for $v_{t-1} = V^u(\sigma_{t-1}, \xi)$ (6) is fulfilled with ε_j, substituted by some $\varepsilon_j^* < \varepsilon_j$. Here in (6) σ_{t-1}, x_t, s_t are defined according to the natural succession of ~~(rations)~~ equations: $\sigma_{t-1} = \sigma(x_{t-1}, s_{t-1}, \xi)$, $u_{t-1} = u(v_{t-1})$, $x_t = X(x_{t-1}, s_{t-1}, u_{t-1}, t-1, \xi)$, $s_t = S(x_{t-1}, s_{t-1}, \xi)$.

(III) For any value of the control v_{t-1} the value $\eta_t^{(j)}$ may be expressed by $v_{t-1}, \sigma_{t-1}, \sigma_t$, i.e. $\eta_t^{(j)} = \Omega_j(v_{t-1}, \sigma_{t-1}, \sigma_t)$ where Ω_j are some functions.

(IV) For all $\xi \in M$, $\sigma \in \{\sigma\}$ there exist $\partial V^u / \partial \sigma_j$ and $|V^u(\sigma, \xi)| \le C_1$, $|\partial V^u / \partial \sigma_j| \le C_2$ where C_1, C_2 are the parameters.

We shall try to make these suppositions clear. Condition (I) means that, firstly, AC requires that "something was sufficiently little different from something", and secondly, that this "something" had a linear dependance on new control. Condition (II) roughly speaking, is equal to the possibily of solving this problem. (To use the control $v_t = V^u(\sigma_t, \xi)$ certainly is impossible, since the values of the variable parameters ξ are unknown). Condition (III) requires that the error at the instant $t-1$ might be measured by means of the evidence available at the instants $t-1$ and t. Condition (IV) is practically not restricting.

Theorem I. While fulfilling conditions (I)-(IV) the brain equations may be constructed so that AS obtained became reasonable in class M of problems and so that the number $\varkappa_0$ of errors of the system satisfied the inequality

$$\varkappa_0 \leq C \max_{\sigma} |v(\sigma,\xi) - V^u(\sigma,\xi)|^2$$

where the constant C do not depend on the variable parameters.

The brief proof of this theorem is given in the article[5]. It is constructive: in the process of proving the theorem we get the procedure of constructing the brain equations (4), (5) of reasonable (in class M) AS with the estimation of the number of errors given.

We shall describe the general idea[5] of construction of brain equations, constructing these equations on a number of simplifying assumptions.

(A) Ideal control $v = V^u(\sigma, \xi)$ should be rather strictly (in C - metric) approximated by the "degenerated nucleus"

$$V^u(\sigma,\xi) \approx \sum_{h=1}^{N} \vartheta_h(\sigma) \cdot \tau_h(\xi) \tag{7}$$

where $\tau_h(\xi)$ are scalars, $\vartheta_h(\sigma)$ are vectors functions (of the order of q)[x]. It should be noted that the functions $\tau_h(\xi)$ are not further needed. So it is not the particular kind of representation

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x) Sometimes it is more convenient to take τ_h as vector functions (of the order of q and $\vartheta_h(\sigma)$ as scalar ones. If the condition (IV) is fulfilled, the approximation (7) is always possible.

(7) that is needed, but the mere possibility of such approximation with functions $\vartheta_h(\sigma)$. Here it is more profitable (in the sense explained below) to have such a representation (7), where the number of linear independent functions among the set $\{\vartheta_h(\sigma)\}$ will be as small as possible. If we have an exact representation (7), then the "neuron" functions $\vartheta_j(\sigma)$ are defined by this representation. To make the system more reliable we should, however, increase the number N of functions, introducing additional, linear dependant functions $\vartheta_j(\sigma)$ (see below).

(B) As the tactics τ we take the vector $\tau = \| \tau_h \|_{h=1}^{N}$ (consequently, $\{\tau\} = R_N$), and the equation (4) is defined by the formulae

$$u_t = u(v_t), \quad v_t = \sum_{h=1}^{N} \vartheta_h(\sigma_t) \cdot \tau_{h,t} \tag{8}$$

where $\vartheta_h(\sigma)$ are the functions in condition (7).

(C) The equation (5) must be chosen so that the tactics τ_t as $t \to \infty$ be convergented to the unknown tactics $\tau(\xi) = \|\tau_h(\xi)\|$, or to some value close to it. Indeed, it is sufficient that the instant t_0 existed such that as $t \geqslant t_0$, $\mu_t = 1$ the inequatilies (6) were fulfilled. Then, according to condition (I), AC will be fulfilled as $t > t_0$. Inequalities (6) for $\mu_t = 1$ give the infinite system of inequalities, and the algorythm (5) must be a finitely convergent algorithm for the solution of these inequalities (see[3]). Such algorythm is given by theorem 3 or 5 in the paper[3] (the theorem 5 proof is given in the article[4], in the supplement). Here the algorythm of theorem 3 or 5 of the paper[3] should be applied subsequently k times to inequalities (6). Let's make a simplifying assumption holding that in condition (II) $\varepsilon_j^* < \varepsilon_j / 2$. Then we may use a more simple algorythm of theorem 4 of the paper[3]. Let k=I. Inequalities (6) have theform $|\eta_t| < \varepsilon_1$ where

$$\eta_t = \sum_{h=1}^{N} (c_1, \vartheta_h(\sigma_{t-1})) \cdot \tau_{h,t-1} - \varphi_t' . \tag{9}$$

According to theorem 4 of the paper [3], we have

$$\begin{cases} \tau_{h,t} = \tau_{h,t-1} & \text{if } |\eta_t| < \varepsilon_1, \\ \tau_{h,t} = \tau_{h,t-1} - \chi_t \cdot \eta_t \cdot (c_h, \vartheta_h(\sigma_t)) & \text{if } |\eta_t| \geqslant \varepsilon_1 \end{cases} \tag{10}$$

where

$$\varkappa_t = [\sum_{h=1}^{N} (c_1, \vartheta_h(\sigma_{t-1}))^2]^{-1}. \qquad (11)$$

The number τ_0 of errors of the system satisfies the inequality

$$\tau_0 \leq |\tau_0 - \tau(\xi)|^2 / \varkappa^2 \varepsilon(\varepsilon - 2\varepsilon_1^*)$$

where $\varkappa^2 = \max_{\sigma} \sum_{h=1}^{N} (c_1, \vartheta_h(\sigma))^2$ (see [3]).

Equations (8), (10), (11) for $\eta_t = \Omega(v_{t-1}, \sigma_{t-1}, \sigma_t)$ define τ_t through $\tau_{t-1}, \sigma_{t-1}, \sigma_t$ i.e. they are the brain equation (5). Equations (8)-(11) are brain equations of a reasonable(in the sense mentioned) AS.

It is essential to note, that these equations do not include (just as it should be) variable parameters, and this affords additional requirements to the choice of this or that finitely convergent algorythm. In a number of cases the numbers ε_j in (6) are variable parameters, but here the sygnal $\tilde{\mu}_t$ comes into the brain showing, whether AC ($\tilde{\mu}_t = 1$) is fulfilled at the instant t or not ($\tilde{\mu}_t = 0$). Without dwelling on a rather simple explanation, we shall note that in this case equations (10)may be substituted by the equations

$$\begin{cases} \tau_{h,t} = \tau_{h,t-1} & \text{if } \tilde{\mu}_t = 0, \\ \tau_{h,t} = \tau_{h,t-1} - \varkappa_t \eta_t (c_1, \vartheta_h(\sigma_t)) & \text{if } \tilde{\mu}_t = 1 \quad (h = 1, \ldots, N). \end{cases} \qquad (12)$$

Equations (8), (11), (12) are in this case the brain equations of reasonable AS. For $k > 1$, $\varepsilon_j^* < \varepsilon_j / 2$ the brain equations are constructed in the same way.

Consider the problem of ensuring the reliability of the brain work. The brain equations (4),(5) serve for the approximation of the unknown ideal control $u_t = u(v_t)$, $v_t = V^u(\sigma_t, \xi)$. The functions $\vartheta_h(\sigma)$ in equations (8) are not singularly defined. They may be any with sufficient approximation (7).Thus, in the functional space of functions $f(\sigma)$ the functions $V^u(\sigma, \xi)$ for all $\xi \in M$ should lie near the underspace $\mathcal{L}$, "stretched" on the functions $\vartheta_j(\sigma)$. For the fixed N functions $\vartheta_j(\sigma)$ may be chosen in different ways. If $\vartheta_j(\sigma)$ are linear independent, the brain work will hardly be reliable.In this case any drawback leading to the change of one of functions $\vartheta_j(\sigma)$ may result in vanishing of representation (7). The situation will change if we take a number of "extra" functions $\nu_j(\sigma)$. It means that we shall take the number N larger than it is necessary, furnishing the set of functions $\nu_j(\sigma)$ by additional functions which are linear dependent with the former ones. In this case the change of some functions $\nu_j(\sigma)$ will but stretch the space $\mathcal{L}$ (or leave it as it was). Representation (7) will, as before, take place, i.e. the infinite system of inequalities (6) ($t=0,I,2\ldots$, $\mu_t = I$) will, as before, have a solution (in the already mentioned sense). And this is just what is necessary for the algorythms to converge[3]. Thus, to increase the reliability of the brain the number of N should be increased, taking linear dependent functions as $\nu_j(\sigma)$. It goes without saying, that the system reasonable in class M will not be totally indifferent to the drawbacks due to some changes of some functions $\nu_j(\sigma)$. After the "damage" of its brain the system stops functioning regularly. For a certain period of time AC (generally speaking) will not be fulfilled. It is important, however, that algorythms (IO) and (I2) will cope with such a drawback by means of redesribution of weights τ_j. Indeed, after such kind of "damage", of its brain the system will learn itself again: in some time AC will again be fulfilled regularly. The discussed above is, certainly, valid if the drawbacks are not too bad to rule out the possibility of representation (7).

3°. Example; the system "grasshopper".

In this section a simple and mathematically styled example of the system of the class described above is given. Evidently, in the present state of development of the theory of adaptive systems consideration of such kind of examples is useful. These examples allow for simple simulation on the computors, and experiments on the computors may serve to test the efficiency of this as that theory and for comparative examination of the efficiency of different algorythms of selflearning.

Imagine the field F as a cirle of radius L, with some landmark (wood) in its center. Two "organisms" are moving (sumultaneously leaping) within the field: the target and the grasshopper (G). The target is not adaptive: it moves according to some definite, but unknown beforehand (defined by the values of the variable parameters) law. At fixed values of the variable parameters the movement of the target is fully defined by the relative location of the target, the landmark and the grasshopper. The grasshopper is an adaptive system. AC for it is the requirement to catch the target at the next instant. Since G and the target jump simultaneously, G should learn to foresee the leap of the target. Making it still more complicated, G (according to the condition set) "sees" the target every time from its moving system of coordinates. Moreover, according to our assumption, some parameters of its "eyes" are variable, i.e. some distortion unknown to the designer is possible. If the target is caught by the grasshopper at some instant, then at the next instant a new target appears accidentally in the field. This target moves on according to the same law of motion, and it again should be caught by the grasshopper. The grasshopper is said to be reasonable, if, beginning with some instant, it catchtes any target appearing before it within one measure of time, and it is valid for any law of motion (of some class) of the target and for any values of the variable parameters of its eyes. The task consists in constructing the brain (i.e. in defining equations (4), (5)) of a reasonable grasshopper[x].

We shall state the problem more strictly. The external coordinates of G are $x = \| z, \varphi \|$ where z is a complex number ($|z| \leq L$), defining cartesian coordinates G, a "course" angle φ defines the oriantation of G. The surroundings s is identificated with the complex number s (coordinates of the target), $|s| \leq L$. The number L is a variable parameter. The system of coordinates of G will be the system with the center in point z, turned at the angle φ. G sees the landmark at the beginning of the inmovable system of coordinates and the target. To be more precise, sensors $\sigma = \| \zeta, \psi, \zeta_0, \psi_0, \mu \|$ are the following values, connected with the coordinates of the target and the landmark in the system of coordinates Grasshopper:

$$\zeta = |z - s|^{-1}\delta, \quad \psi = \arg(s - z) - \varphi, \quad \zeta_0 = (|z| + \nu)^{-1}\delta_0, \quad \psi_0 = \arg z - \varphi.$$

Here $\delta_0 > 0$, $\nu > 0$ are variable parameters, δ parameters[xx]. The va-

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x) Instead of the infinite measure of time, during which the trained grasshopper catches the targets accidentally appearing before it within one measure of time, it is furthes considered that this measure of time is finite. After that a new problem is set before the grasshopper (the variable parameters are changed) and ect. (See for more detail below 4°).

xx) We may consider the case, when δ is also variable parameter. (see p.10)

riable μ is a sygnal of fulfillment of AC: $\mu_t = 1$ if $|z_t - s_t| \geq \varepsilon$ and $\mu_t = 0$, if $|z_t - s_t| < \varepsilon$. Here $\varepsilon > 0$ is a variable parameter, $\varepsilon < L$. AC consists in the requirement to catch the target at the next moment, if it is not yet caught; if $\mu_t = 1$, then $\mu_{t+1} = 0$. Thus, G must leap in the ε -circumference of this point, where the target will be at the next instant. The movement of the grasshopper is fulfilled in the following way. The grasshopper turns at the angle f_t , the leaps over the distance τ_t. So, the control is $u = \|f, \tau\|$. The motion equations have the form $\varphi_{t+1} = \varphi_t + f_t$, $z_t = z'_{t+1}$ where $z'_{t+1} = z_t + \tau_t \cdot \exp i\varphi_{t+1}$ if only $|z'_{t+1}| \leq L$. If $|z'_{t+1}| > L$ (which means, that the grasshopper "wants to leap" out of the circle F), then z_{t+1} is defined from the condition of "sticking" (according to some law) to the wall $|z| = L$. The target sees the landmark and the grasshopper, and its motion depends on where it sees them. Suppose, the target pays no attention to the orientation of the grasshopper: $s_{t+1} = S(s_t, s_t - z_t, \xi)$. Here ξ is a variable vector parameter. If the target is caught at a certain instant, then as is said above, at the next instant in some quasisudden way a new target appears in the circle F, with the same law of functioning. Since the target sees only the landmark, but not the system of coordinates connected with it , the function $S(s, w, \xi)$ must satisfy the following condition: $S(e^{i\chi}s, e^{i\chi}w, \xi) = e^{i\chi}S(s, w, \xi)$, where χ is any real number . Besides, let's consider that the function S and its derivatives over $\mathrm{Re}\, s, \mathrm{Im}\, s, \mathrm{Re}\, w, \mathrm{Im}\, w$ are limited as $|s| \leq L$, $\varepsilon \leq |w| \leq 2L$ uniformly over $\xi \in M$.

It is easy to prove that assumtions (I)-(IV) of section 2° are fulfilled. The new control $v_t = \|X_t, Y_t\|$ is connected over the old ones $u_t = \|\tau_t, f_t\|$ by $X_t + iY_t = \tau_t \exp(if_t)$. Inequalities (6) (for k=2) are the inequalities $|X_t - \mathrm{Re}\, \tilde{s}_{t+1}| < \varepsilon/\sqrt{2}$, $|Y_t - \mathrm{Im}\, \tilde{s}_{t+1}| < \varepsilon/\sqrt{2}$ where $\tilde{s}_t = (s_t - z_t) \times \exp(-i\varphi_t)$ are the coordinates of the target in the system G. The ideal control X_t^u, Y_t^u is defined by the requirement $z_{t+1} = s_{t+1}$ i.e. $X_t^u + iY_t^u = \tilde{s}_{t+1}$, with $\tilde{s}_{t+1}$ having to be expressed through σ_t and ξ . In accordance with the discussed above the brain equations of the reasonable G are

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Here, however, for the brain equation more complicated (compare to the mentioned above) finite convergent algorythms for solving the system of inequalities (6) must be used.

built.

Let's outline the structural scheme of the brain of G. For any AS of the class involved the brain has the same structure. The brain of G consists of identical (as to its functioning) elements, which we shall call as segments (S). The sensor information is supplied to the input S (not necessarily the whole of it), as well as the signal of disaccordance η_t and the norming signal $\varkappa_t$. Every S works in two regimes: the usual one and the adaptive one. In the usual regime S works as the functional transformator with the coefficient of the increase τ_h, which does not change in this regime ($\tau_{h,t} = \tau_{h,t-1}$). The outputs of S are the values $\vartheta^{(h)}(\sigma_t)$ and $\tau_{h,t}\, \nu^{(h)}(\sigma_t)$. (Here h is the number of S, and $\vartheta^{(h)}(\sigma) = (c_1, \vartheta_h(\sigma))$). At the instant t the regime of adaptation is set up, if AC has been given at the instant $t-1$ and turned out to be not fulfilled (which becomes clear at the instant t). Here the new value of the coefficient of increase $\tau_{h,t} = \tau_{h,t-1} - \varkappa_t \eta_t \nu^{(h)}(\sigma_t)$ is being worked out (see the second formula (12)), and the outputs of S are the former values. Thus various S differ only by their functional transformators $\vartheta_h(\sigma)$. The most important instant, defining the degree of "reasonability" of the system is contained in the algorythm of defining $\tau_{h,t}$. (Besides the mentioned above, a number of other algorythms were tried, of which some failed. These algorythms practically did not succeed in teaching G).

The following structural unit of the brain is "the field", connecting p segments with the similar sensor information. The field is outlined by the subspace (of the order of q) in the space $\{f(\sigma)\}$. In accordance with the discussed above, we should take $p > q$. For G we took $p=5$, $q=2$. The functions $\vartheta^{(h)}(\sigma)$ of the first field had the following form:

$$\vartheta^{(1)} = \cos\psi,\quad \vartheta^{(2)} = \cos(\psi+\alpha),\ \ldots,\ \vartheta^{(5)} = \cos(\psi+4\alpha). \quad (13)$$

For the second field the functions $\vartheta^{(h)}$ were obtained from (13) by adding the multiplier ζ, and for the third by the multiplier ζ^{-1}. Three other fields had the functions, obtained from the mentioned above by substituting ψ for ψ_o and ζ for ζ_o.

Different fields are combined into k sections, where k is the number of inequalities (6). (For the grasshopper $k=2$). In any section the outputs of the fields are combined to form the norming signal (II). From the outputs $\tau_{h,t}\, \nu^{(h)}(\sigma_t)$ the corres ponding component of the control is formed. For G they were X_t (first section) and y_t (second section). Also here the signals of disaccordance are formed: $\eta_t = \Delta X_t$ and $\eta_t = \Delta y_t$, where

$$\Delta X_t = \delta/\zeta_t \cdot \cos(\psi_t + f_t), \quad \Delta y_t = \delta/\zeta_t \sin(\psi_t + f_t).$$

For G every section consists of six fields with the mentioned $\nu^{(h)}(\sigma)$. Consequently, the brain G consists all in all of sixty S.

The material discussed shows, that in the construction of the brain of all AS discribed above a "neurodynamic principle" is valid. That is, the brain of AS consists of a large number of elements having the similar construction (S). At any instant general information is spread over the whole brain, estimating the work of the system as a whole (the signals of AC being fulfilled, of disaccordance, and of norming). In accordance with this information every element readjusts its work so that as a result of this readjustment, the whole system starts to work out an expedient (in accordance with AC) behaviour.

4°. Description of experiments on the computor on simulation of the process of self-learning of a grasshopper

There were a number of serii of experiments carried out, of which every one refers, so to say, to the "life" of one particular grasshopper. Below the results are given for one such seria, and in some cases comparison is given with the results of the similar experiments with "other" grasshoppers. We considered a three parameter law of the movement of the target

$$\mathfrak{z}_{t+1} = \varkappa \cdot e^{i\omega}\mathfrak{z}_t + \gamma \rho_t^{-1} \cdot \exp[i \arg(\mathfrak{z}_t - z_t)]$$

where $\varkappa, \omega, \gamma$ are real parameters. If $\gamma = 0$ the target moves along a circle ($\varkappa = 1$) or a spiral ($\varkappa < 1$). The last term describes the response of the target to the grasshopper. It adds the component, directed along the line grasshopper - target from the grasshopper ($\gamma > 0$) or to it ($\gamma < 0$). The grasshopper with the constructed brain is reasonable in a wider twenty four parameter class of the laws of the movement of the target, and, consequently, instead of the mentioned above, any of these laws might have been taken. In particular, we could have taken γ as complex number, which would have resulted in deviation of the target from the line target-grasshopper. (There were some experiments carried out with grasshoppers reasonable in a class of laws of the target depending on n=48, n=80 parameters).

An experiment was stopped after G caught τ = 10000 targets, then came the following experiment. The instant t_0, after which the grasshopper catches τ targets, one each measure of time, is thus, an "empirical" time of selflearning. (Theoretical time of selflearning corresponds to $\tau = \infty$).

Consider the series of experiments, describing the life of some "particular" grasshopper with all the drawbacks resulting from the surroundings and its own deseases.

The life of this grasshopper is going on in the "field" of diameter $2L$ = 30 m; the target is caught, if after the simultaneous leap of the target and the grasshopper, the distance between their "centers of weight" does not appear to be exceeding ε = 1 m. The accidentally appearing target begins to move around the landmark in leaps at angles $\omega = 10^{\circ}$ keeping at the same distance from the landmark. The grasshopper learned this law rather quickly (in t_0 = 22 leaps) in the process of pursuing $m=5$ targets all in all. After that it caught, one by one and in one leap, all τ the accidentally appearing before him targets. Then it turned out that the grasshopper failed to catch the target. It resulted from the change of the variable parameter ε : the target was caught if the distance between the target and the grasshopper did not exceed ε =0,5 m. The grasshopper managed to concieve this circumstance in t_0=8 leaps, pursuing only m=3 targets. After that it caught again all τ accidentally appearing before it targets in one leap (already for ε = 0,5 m).

Leaving our grasshopper alone, it should be noted that down the grasshopper bears very easily the decrease of ε down to 10 cm, the further decrease ε makes it more and more difficult for it. (The value of t_0 increases rapidly). The increase of L , for the grasshopper turns out to be acceptible in this sense.

Let's turn back to our grasshopper. After a quiet period of τ measures of time the targets again fail to be caught. This happens because the targets changed their law of motion, beginning to leap around the landmark in a circle not at 10°, like before, but at 20°. The grasshopper became familias with this circumstance in t_0=54 leaps in the process of pursuing m = 22 targets. After the following quiet period of τ measures of time the targets again changed their law of motion, beginning to leap again at 10° but in the opposite direction. This "trouble" made the grasshopper most "nervous": the numbers t_0=72, m=28 are the largest of all the previous ones. (Though in future it will be still worse for it).

After the following quiet period of τ measures of time the targets again fail to be caught, and this time it is because the targets began to react at the grasshopper. On the one hand, the target began to move not in a circle, but in a spiral, approaching a little in its leap to the landmark, on the other hand, it began to leap away from the grasshopper and the nearer it was to the grasshopper, the farther it did so.

The new law of motion the grasshopper learnt with almost the same parameters as the previous one: t_0=78, m=28. Then the targets began to come nearer to the grasshopper in their leaping, then again they changed their direction around the land - mark, they changed theirs reaction at the grasshopper differ - ently. The grasshopper, as it should be, "learned" all these changes of the law of motion of the target, displaying it by acquiring ability to catch the targets with a new law of motion τ times one after another after the period of self-learning t_0 . The value of t_0 changed within the limits t_0=5 up to t_0=59.

After the last quiet period the grasshopper had troubles of some other kind due to deformations of its own "organism". A kind of confusion begain: the target was seen by the grasshopper to be two times farther that it really was (δ_0 =0.2 was taken instead of δ_0=0,1). Naturally, the grasshopper again failed to catch the targets, though they did not change their law of motion. The brain of the grasshopper recovered from this "desease" in t_0=230 measures of time, and this number is the largest of all considered so far. Defomation of the " organs of seeing" of the same kind continued and the brain managed to cope with it every time. Then came nonlinear distortions of the distance to the landmark percieved (the change ϑ=

$=0,I, \div 0,00I)$, apparently, not to big because the grasshopper bore these deseases very easily ($t_a=I \div 8, m=I \div 3$). It should be stressed that according to the sense of the problem under consideration, every time (after "the quiet period") G "did not know" what had happened: it found it out only after the target could not be caught any longer. The brain G cleared out what had happened itself, to be more precise, worked out a new, "correct" law of control..

Now we shall do some digression. As it was shown above, every field of the brain of our grasshopper contains five S , while for the above two S would be sufficient. Five S in the field was taken to increase the reliability of the brain , which was said above. It was, however, not clear beforehand, whether this increase of S will lead to the worsening of the work of the brain as a whole. Theoretically it is difficult to answer this question, The practice of work with algorythms[3] shows that with the increase of space dimentions n (which coincides in this case with the number S in one section of the brain) the convergence of algorythms[3] usually becomes worse and sometimes much worse. (It is to be noted, that the finite convergence[3] remaines for $n = \infty$ as well). So the worsening of the work of the "reliable" brain with five S in the field is possible, compare to the "unreliable" brain with two S in the field. In this connection the experiments of the type discussed above with two S in the field were carried out. Instead of (I3) the neuron functions had the form $\vartheta^{(1)}=\cos\psi, \vartheta^{(2)}=\sin\psi$ and the same form for the rest fields.

The experiments showed that contrary to our expectations, "reliable" brain as a rule works better than "unreliable" one though some exceptions are possible. Besides, as other experiments showed, addition of S with linear independent neuron functions as a rule make the result much worse, and this the more adding of such S makes the result still worse.

Now let's return to our grasshopper. The aim of our further experiments with it is to check the principle described above of the insuring the reliability of the brain, making clear, in particular, whether the time of relearning of the grasshopper will be permittable after all other sections of its brain are damaged. In accordance with some other considerations, on which we shall not dwell, S of its brain were damaged in different ways. (Usually, the signal of the "damaged" was substituted by zero. Sometimes S "functioned" wrongly). Localization of the damage was various, but for any S of five fields there were at least two left undamaged. After every such damage of G's brain, it naturally failed to catch the target and had to learn again. The values of $\mathfrak{m}$, t_0 were varied within large limits. The maximum were the values $\mathfrak{m}=430$, $t_0=993$. Thus, G's brain can sufficiently well cipe with all the problems set.

For checking up a "lethal destruction" of G's brain was made with the damage of four fields of one S, after which G, as was expected, failed to learn (within 10000 measures of time).

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7.1

Controllability and Synthesis of Optimal Dynamical Systems

by

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A great amount of works have been devoted to the extension of Pontryagin's maximum principle [I] to systems described by equations which are others than ordinary differential ones. In the present time this problem still attracts scientists'attension. The theory of singular controls has been risen up too. This theory has been stimulated by the problems of the space navigation and the discovering of the sliding modes as well. The interest increased to the controllability, the existence of optimal controls, sufficient conditions of the optimality, the computational algorithms, The latter problem has evoked the theory of optimal processes in discrete systems. In conclusion we have to mention the new branch of the theory of the optimal processes, namely, the differential games .

The purpose of this paper is to present results of authors on some aspects of the optimal processes theory. This results have been obtained by two methods. The first method (the increments method [2] is concerned with the increments of vector--valued and scalar-valued functions, defined along of trajectories of dynamical systems. The second method (the functional analysis method [3] is based on some theorems and facts of the functional analysis

I. The Controllability

Consider the system described by the equation

$$\frac{dx}{dt} = f(x,u),\ x=(x_1,\dots x_n),\ u=(u_1,\dots,u_r),\ f=(f_1,\dots f_n),\ f(0,0)=0, \quad (I)$$

where x is a state vector, u is a control vector.

Definition I. The state x_c is called controlled if there exists a piecewise continuous control $u(t)$ such that for the correspond trajectory $x(t)=x(x_c,u,t)$ is carried out

$x(x_0,u,T)=0, \quad T<+\infty.$

Definition 2. The system (I) is called completely controlled if for any T, $T=t_1-t_0$ there exists a number $\alpha=\alpha(T)>0$ such that all states x_0, $\|x_0\|<\alpha$, are T-controlled.

Now consider the system

$$\frac{dx}{dt} = Ax + b(u) \quad (2)$$

Here $x=(x_1,\dots,x_n)$, $u=(u_1,\dots u_r)$, A - is a constant matrice, $b(u)$ is a continuous function.

Let $\Omega(\mathcal{I})$ be the convex hull of the set $\{b(u),\ \|u\|\le\mathcal{I}\}$ and $b^1,\dots,b^r$ be a basis of the minimal subset (the dimension is $r=r(\mathcal{I})$) which contains the set $\Omega(\mathcal{I})$.

It can be established with the increment method of vector-valued functions that holds

Theorem I. The systems (2) is completely controlled in the class of bounded controls ($\|u\|\le\mathcal{I}$), if carried out either

A. I) rank $\{B, AB,\dots, A^{n-1}B\}=n$, $B=\{b^1,\dots,b^r\}$;

2) the origin is an inner point of the set $\Omega(\mathcal{I})$;

or

B. Rank $\left\{C, AC, \ldots, A^{n-I}C\right\} = n$, $C = (c^I, \ldots, c^p)$;
here $c^I, \ldots, c^p$ are non zero vectors such that $c^I, \ldots, c^p \in \Omega(\mathcal{Z})$ and $\beta_1 c^I + \ldots + \beta_p c^p = 0$, $\beta_1 \neq 0, \ldots, \beta_p \neq 0$.

The second condition is necessary one for the system (2) to be completelly controlled with $\|u\| \leq \mathcal{Z}$.

This theorem may be extended to nonstationary systems.

The result concerned with the controllability of the ordinary dynamical systems may be expended to the systems with time--lags in some cases. Let the system be given

$$\frac{dx}{dt} = Ax + Bx(t-h) + Cu(t), \tag{3}$$

where h is a constant delay; an initial state is $x_0(\cdot)$:

$$x_0(\cdot) = \left\{ x(t) \equiv \varphi(t),\ t_0 - h \leq t < t_0,\ x(t_0) = x_0 \right\}, \tag{4}$$

here $\varphi(t)$ is a continuous function and x_0 is given.

For the system (3) and the initial state (4) can be used the definitions I,2 but "controlled" must be replaced by "relatively controlled". For exemple, the system (3) is called relatively controlled, if for each initial state (4) from some neighborhood of zero of continuous functions spase there exists such a piecewise continuous control that $x(T) = 0$, $T < +\infty$.

For the investigation of the relative controllability of the systems (3) we forme the determinant equation in the terms of the right hand part of the equation (3):

$$q_p(k) = Aq_p(k-1) + Bq_{p-1}(k).$$

The determinant equation we shall call nondegenerated if

$$\operatorname{rank}\left\{q_k(s),\ k = 1, \ldots;\ s = 1, \ldots\right\} = n.$$

Theorem 3. In order for the system (3) to be relatively controlled it is necessary and sufficient that the determinant equation be nondegenerated.

This result can be easily extended to nonstationary systems, tosystems with a variable delay, to nonlineary systems with time-lags.

We must note that the relative controllability of linear systems (3) is reduced to the algebraic problem. The same situation occurs in the theory of stability for linear systems.

Since a state of the system (3) for any t is defined by the trajectory range it is naturelly to give the following definition.

Definition 3. The system (3) is called completely controlled if for each initial state (4) there exist finite t_I and piece-wise continuous control $u(t)$, $t_0 \leq t \leq t_I$ such, that for the trajectory $x(t)$ we have $x(t) \equiv 0$, $t_I - h \leq t \leq t_I$.

It can be shown that in general the complete controllability does not follow from the relative controllability. There exist however several classes of systems for which the complete controllability follows from the relative controllability. For example

$$\frac{dx}{dt} = Bx(t - h) + Cu$$

or

$$\frac{dx}{dt} = Ax + Bx(t - h) + Cu,$$

C is nonsingular matrix.

2. The Existence of the Optimal Control

The dynamical systems controllability is closely related to the problem of admissible controls existence under the boundary conditions.

We have to consider next the problem of the optimal controls existence.

Given for example the performance index

$$J(u) = \varphi(x(t_1)) \tag{5}$$

along of the trajectories of the system (I) with $f = f(x,u,t)$ and given a class of mesurable functions $u(t)$, $u(t) \in U$.

Under what conditions there exists a control(from given class) which minimises (5)?

It is know, that even in simple cases the optimal control may be not exists.

Example I.

$$\begin{cases} \dot{x}_1 = u, & x_1(0) = 0, \quad U = \{u : u = \pm 1\}, \\ \dot{x}_2 = x_1^2, & x_2(0) = 0, \quad t \in [0,1], \end{cases}$$

$$J(u) = x_2(1) \to \min_u$$

The method of scalar functions increments allows to prove a number of new theorems of optimal controls existence without the constraints ofthe convex type. For example, the optimal controls exist for the problem

$$\frac{dx_i(t)}{dt} = a_{ij}(t)\, x_j(t) + a_{ij}^h(t)\, x_j(t-h) + b_i(u(t), t),$$

$$x_i(t) = \Phi_i(t), \quad t_0 - h \le t \le t_0, \quad u \in U,$$

$$J(u) = \varphi_1(x(t_1)) + \int_{t_0}^{t_1} \left[f_{n+1}^1(x,y) + f_{n+1}^2(u,t) \right] dt \to \min_u$$

if $\varphi_1(x)$, $f_{n+1}^1(x,y)$ are concave with respect to $\{x,y\}$, U is compact, $y(t) = x(t - h)$.

For the systems with delay

$$\frac{dx}{dt} = f\big(x(t),\, x(t - h(x,u,t)),\, u(t), t\big) \tag{6}$$

$x(t)=\Phi(t),\ t_0\in S_0,\ S_0$ is an initial set,

it is possible to extend Philippov's theorem.

3. Maximum Principle.

A new form of the maximum principle is proposed. This form is available for a large class of the control systems.

Let the control systems be described by any one of the following equations

a) $\frac{dx_i}{dt}=f_i(x,u,t),\quad x_i(t_0)=x_{i0};$

b) $\frac{dx_i}{dt}=f_i(x(t),x(t-h),u(t),t),\quad h=h(x,u,t),$

$x_i(t)=\Phi_i(t),\quad t\in S_0,\quad S_0$ is an initial set;

c) $\frac{dx_i}{dt}=\int_\alpha^\beta\int_\gamma^\delta f_i(x(t),x(t-\tau),u(t),u(t-\tau),t,\tau,\sigma)\,d\sigma\,d\tau,$

$x_i(t)=\Phi_i(t),\quad t_0-\beta\le t\le t_0,\quad u_\nu(t)=\Phi_\nu(t),\quad t_0-\delta\le t\le t_0;$

d) $\eta_{ij}^N(D,t)x_j(t)=f_i(x(t),\dot{x}(t),\ldots,x^{(k)}(t),u(t),t)$

$\eta_{ij}^N(D,t)\equiv a_{ij}^0(t)D^N+\ldots+a_{ij}^N(t),\quad D=\frac{d}{dt};$

e) $x_i(t)=\int_{t_0}^{t_1}f_i(x(\tau),u(\tau),\tau,t)\,d\tau.$

We shall write these equations in the form

$$X_i(x(\cdot),\dot{x}(\cdot),\ldots,x^{(l)}(\cdot),u(\cdot),t)=0,$$

where X_i denote functionals defined over functions

$$x(\cdot)=\{x_i(t),t\in T\},\ T=[t_0,t_1],\ u(\cdot)=\{u_\nu(t),t\in T\}$$

and their derivations $\dot{x}(\cdot),\ldots,x^{(j)}(\cdot)$.

Consider the problem of minimizing of the functional

$$J(u) = \int_{t_0}^{t_1} f_{n+1}(x(t), u(t), t)\,dt$$

over controls u(t) and trajectories x(t), $t \in T$. The controls are piecewise continuous functions, their values belong to a bounded set U:

$$u(t) \in U, \quad t \in T. \tag{7}$$

Let us take the functional

$$\Pi(x,\psi,u) = \int_{t_0}^{t_1} \psi_i(t) X_i(x(\cdot), \dot{x}(\cdot), \dots, x^{(\varepsilon)}(\cdot), u(\cdot), t)\,dt - \int_{t_0}^{t_1} f_{n+1}(x,u,t)\,dt,$$

where $\psi_i(t)$ are same additional functions.

Theorem 4. For the optimal control $u^0(t)$ and the solutions $x^0(t)$, $\psi^0(t)$ of the system

$$\frac{\delta \Pi(x^0, \psi^0, u^0)}{\delta \psi_i(t)} = 0, \quad \frac{\delta \Pi(x^0, \psi^0, u^0)}{\delta x_i(t)} = 0, \tag{8}$$

is carried out the maximum condition

$$\frac{\delta_{u*} \Pi(x^0, \psi^0, u^0)}{\delta u(t)} \leq 0, \ t \leq T, \ u^* \in U. \tag{9}$$

Here $\frac{\delta \pi}{\delta x_i(t)}$ is variational derivation of the function Π with respect to the functions $x_i(t)$, $t \in T$; $\frac{\delta_{u*}\Pi}{\delta u(t)}$ is variational derivation of the second type which is defined by the needle variations of the control u(t):

$$J(u + \Delta^* u) - J(u) = \varepsilon \frac{\delta_{u*} J(u)}{\delta u(\theta)} + o(\varepsilon),$$

$$\Delta^* u = \begin{cases} u^* - u(t), & t \in [\theta, \theta+\varepsilon), \\ 0, & t \notin [\theta, \theta+\varepsilon). \end{cases}$$

In general $x^0(t)$, $\psi^0(t)$ are defined by the equations (8) only for (t_0, t_1). A boundary conditions must be added, exept for the integral form e).

If there are linear differential operators (r) then it is more convenient to rewrite (8) in the form:

$$\tau_{ij}^{N}(\mathcal{D},t)\,x_j(t) = X_i'\left(x(\cdot),\dot{x}(\cdot),\dots,x^{(\ell)}(\cdot),u(\cdot),t\right),$$

Then the necessary optimality conditions take the form

$$\tau_{ij}^{N}(\mathcal{D},t)\,x_j^{\circ}(t) = \frac{\delta\pi'(x^{\circ},\psi^{\circ},u^{\circ})}{\delta\psi_i(t)},$$

$$\overset{*}{\tau}{}_{ij}^{N}(\mathcal{D},t)\,\psi_j^{\circ}(t) = \frac{\delta\pi'(x^{\circ},\psi^{\circ},u^{\circ})}{\delta x_i(t)},$$

$$\frac{\delta_{u^*}\pi'(x^{\circ},\psi^{\circ},u^{\circ})}{\delta u(t)} \le 0, \quad t\in T, \quad u^*\in U.$$

Here

$$\pi'(x,\psi,u) = \int_{t_0}^{t_1}\psi_i(t)X_i'\left(x(\cdot),\dot{x}(\cdot),\dots,x^{(\ell)}(\cdot),u(\cdot),t\right)dt - \int_{t_0}^{t_1} f_{n+1}\left(x(t),u(t),t\right)dt$$

and

$$\overset{*}{\tau}{}_{ij}^{N}(\mathcal{D},t)\,\psi_j(t) \equiv \sum_{m=0}^{N}(-1)^m\frac{d^m}{dt^m}\left[a_{ij}^{N-m}(t)\,\psi_j(t)\right]$$

are the adjoint differential operators.

The necessary optimality conditions in the form (8),(9) can be extended to optimisation problems for equations in partial derivations. For example consider the functional to be minimized

$$J(u) = \iint_R f_0\left(x(s,t),u(s,t),s,t\right)dt\,ds$$

for the equation

$$\frac{\partial^2 x_i(t,s)}{\partial t\,\partial s} = f_i\left(x(t,s),\frac{\partial x(t,s)}{\partial t},\frac{\partial x(t,s)}{\partial s},u(t,s),t,s\right)$$

Then t must be replaced by t,s in the results above.

4. Maximum Principle for Extremals

The problem of optimisation of

$$J(u) = c_i x_i(t_1), \quad x_i(t_0) = 0 \tag{10}$$

for the system (I),(7) by the maximum principle is reduced to the finding extremals that is to the finding functions u(t) under conditions

$$H(x(t),\psi(t),u(t),t) = \sup_{u\in U} H(x(t),\psi(t),u,t) \tag{II}$$

$$H(x,\psi,u,t) = \psi_i f_i(x,u,t),$$

$$\dot{x}_i = \frac{\partial H}{\partial \psi_i}, \quad x_i(t_0)=0; \quad \dot{\psi}_i = -\frac{\partial H}{\partial x_i}, \quad \psi_i(t_1) = -c_i \tag{I2}$$

It is very difficult to find the optimal control when the number of extremals is infinite. Such a situation may arise even for simple problems.

Example 2. Given a system

$$\dot{x}_1 = u, \quad x_1(0)=0 \qquad U = \{u : |u| \le 1\}; \quad T=[0,1];$$
$$\dot{x}_2 = -x_1^2, \quad x_2(0)=0;$$
$$J(u) = x_2(t_1) \to \min.$$

An extremal of the problem is

$$u^p(t) = \operatorname{sign} \cos \frac{(2p+1)\pi}{2} t, \quad 0 \le t \le 1, \quad p=0,1,\dots$$

Denote by $H^*(t)$ the function $H(x^*(t),\psi^*(t),u^*(t),t)$ for extremal control $u^*(t)$ and corresponding trajectories $x^*(t),\psi^*(t)$ due to the equation (I2). Now we can formulate the necessary conditions of optimality.

Theorem 5. The optimal control $u^0(t)$ for processes with a short duration $T=t_1-t_0$ is such that

$$H^0(t_1) = \max_{u^*\in U} H^*(t_1). \tag{I3}$$

The function $H^*(t)$ is constant along extremals for stationary systems therefore the condition (I3) can be verified at any t, $t \in T$ for such systems. It is easly to see that the control $u^p(t)$, $p \geq 1$, are not optimal (see ex. 2).

The conditions (II),(I3) are also sufficient for the control optimality if the system (I) and functional (IO) have the form

$$\dot{x}_1 = f_1(u),$$
$$\cdots\cdots$$
$$\dot{x}_{n-1} = f_{n-1}(u),$$
$$\dot{x}_n = f_n(x_1, \ldots, x_{n-1}, u), \quad x_i(0) = 0,$$

$$f_n(\lambda x_1, \ldots, \lambda x_{n-1}, u) = \lambda^m f(x_1, \ldots, x_{n-1}, u); \; J(u) = x_n(t_1), \; m > -1.$$

Consequently in the example 2 the control $u \equiv 1$ is optimal.

5. The Singular Controls

Up to now the necessary conditions of the singular controls optimality havebeen found either for some special cases or in the assumption that the set U is open. The increments method allows to find the necessary conditions of the singular controls optimality for general problem of the minimization of (5) for the system (I) with $f = f(x,u,t)$.

Definition 4. The control $u(t)$, $t \in T$ is called the singular control of the first order, if

$$H(x(t), \psi(t), u, t) - H(x(t), \psi(t), u(t), t) \equiv 0, \; t \in T, \; u \in U,$$

that is the function H independs on u_v, $u \in U$ along $u(t)$, $t \in T$ and functions $x(t)$, $\psi(t)$ due to (I) and

$$\dot{\psi}_i(t) = -\frac{\partial f_j(x(t), u(t), t)}{\partial x_i} \psi_j(t), \quad \psi_i(t_1) = -\frac{\partial \varphi(x(t_1))}{\partial x_i} \tag{I4}$$

Theorem 6. For the optimal singular control of the first order $u^0(t)$, $t \in T$ is satisfated the unequality

$$[\psi_{ij}^0(t) + \dot{\psi}_{ji}^0(t)] \Delta_{u^*} f_i(x^0(t), u^0(t), t) \Delta_{u^*} f_j(x^0(t), u^0(t), t) + \tag{I5}$$
$$+ \psi_{j_1}^0(t) \frac{\partial \Delta_{u^*} f_{j_1}(x^0(t), u^0(t), t)}{\partial x_j} \Delta_{u^*} f_j(x^0(t), u^0(t), t) \leq 0$$

for all $t \in T$, $u^* \in U$.

Here $x^{\circ}(t)$ is optimal trajectory, $\psi_i^{\circ}(t)$ are the solutions of the system (I4), $\psi_{ij}(t)$ are the solutions of the equations

$$\dot{\psi}_{ij}^{\circ} = -\frac{\partial f_{j_1}(x^{\circ}(t),u^{\circ}(t),t)}{\partial x_i}\psi_{j_1 j}^{\circ}(t) - \frac{\partial f_{i_1}(x^{\circ}(t),u^{\circ}(t),t)}{\partial x_j}\psi_{i j_1}^{\circ}(t)$$
$$-\frac{1}{2}\frac{\partial^2 f_{j_1}(x^{\circ}(t),u^{\circ}(t),t)}{\partial x_i \partial x_j}\psi_{j_1}^{\circ}(t); \quad \psi_{ij}^{\circ}(t_1) = -\frac{1}{2}\frac{\partial^2 \varphi_1(x^{\circ}(t_1))}{\partial x_i \partial x_j}$$

The same method may be applied for a study of the singular control of the second order for which left hand part of (I5) is vanish at any t, $t \in T$, $u^* \in U$.

6. The optimization problem with parameters

Given sets V and W in p-dimensinal and q-dimensional vector spaces respectively. It is required to find such parameters v, w that

$$\mathcal{I}(v^{\circ}, w^{\circ}) = \min_{v \in V,\, w \in W} \mathcal{I}(v, w), \tag{I6}$$

where $\mathcal{I}(v,w) = c_i x_i(t_1)$ is the function defined over the trajectories of the system

$$\frac{dx_i}{dt} = f_i(x_i, v),\ t \in T = [t_0, t_1],\ v \in V \tag{I7}$$

with initial conditions

$$x_i(t_0) = g_i(w),\quad w \in W. \tag{I8}$$

Denote by $\sigma(z, Z)$ the star neighborhood of the point z with respect to the set Z: $y \in \sigma(z, Z)$ if there exists a sequence of numbers ε_i, $i = 1,2,\dots$, $\varepsilon_1 = 1$, $\varepsilon_i \to 0$, $i \to \infty$ such that $(1-\varepsilon_i)z + \varepsilon_i y \in Z$ for all $i \geq I$. Introduce functions

$$H(x, \psi, v) = \psi_i f_i(x, v),\quad h(\psi, w) = \psi_i g_i(w).$$

Denote by $x^c(t)$ and $\psi^c(t)$, $t \in T$ the solutions of the system (I7) and the adjoint system

$$\frac{d\psi_i}{dt} = -\frac{\partial H(x(t),\psi(t),v)}{\partial x_i}, \quad \psi_i(t_1) = -C_i,$$

which correspond to optimal parameters v^c, w^c.

Denote by $f(x,V)$ the set

$$\{z : z = f(x,v),\ v \in V\},$$

Theorem 7. The following assertions hold for the problem (I6) - (I8):

I. a) $H(x^c(t),\psi^c(t),v^c) \geq \frac{1}{t_1-t_0}\int_{t_0}^{t_1} H(x^c(t),\psi^c(t),v)dt$

for all v such that $f(x^c(t),v) \in \sigma(f(x^c(t),v^c), f(x^c(t),V))$, $t \in T$;

b) $h(\psi^c(t_0),w^c) \geq h(\psi^c(t_0),w)$ for all w such that

$g(w) \in \sigma(g(w^c), g(W))$.

2. The maximum principle. If for each x sets $f(x,V)$, $g(W)$ are convex then

$$\text{a) } H(x^c(t),\psi^c(t),v^c) = \max_{v \in V} \frac{1}{t_1-t_0}\int_{t_0}^{t_1} H(x^c(t),\psi(t),v)dt \tag{I9}$$

$$\text{b) } h(\psi^c(t_0),w^c) = \max_{w \in W} h(\psi^c(t_0),w). \tag{20}$$

3. If functions $f_i(x,v)$, $g_i(w)$ are differentiable with respect to v, w then

a) $$\int_{t_0}^{t_1} \frac{\partial H(x^c(t),\psi^c(t),v^c)}{\partial v_\alpha} dt\, v_\alpha \geq \int_{t_0}^{t_1} \frac{\partial H(x^c(t),\psi^c(t),v^c)}{\partial v_\alpha} dt\, v_\alpha^c$$

for all $v \in \sigma(v^c; V)$.

b) $$\frac{\partial h(\psi^c(t_0),w^c)}{\partial w_\beta} w_\beta^c \geq \frac{\partial h(\psi^c(t_0),w^c)}{\partial w_\beta} w_\beta$$

for all $w \in \sigma(w^c; W)$

Here $\alpha = 1, \dots, p$; $\beta = 1, \dots, q$

4. If functions $f_i(x,v)$, $g_i(w)$ are differentiable with respect to v,w, and sets V,W are convex then

$$\text{a)} \quad \int_{t_0}^{t_1} \frac{\partial H(x^0(t), \psi^0(t), v^0)}{\partial v_\alpha} dt\, v_\alpha = \max_{v \in V} \int_{t_0}^{t_1} \frac{\partial H(x^0(t), \psi^0(t), t)}{\partial v_\alpha} dt\, v_\alpha \quad (21)$$

$$\text{b)} \quad \frac{\partial h(\psi^0(t_0), w^0)}{\partial w_\beta} w_\beta^0 = \max_{w \in W} \frac{\partial h(\psi^0(t_0), w^0)}{\partial w_\beta} w_\beta \quad (22)$$

5. If in 4. we add that the functions $H(x,\psi,v)$, $h(\psi,w)$ are concave with respect to v,w then for v^0, w^0 are carried out (I9), (20).

For example the conditions of the latter assertion hold for the system (I7) where functions $f_i(x,v)$ independ on x_n , functions $f_1(x,v), \dots, f_{n-1}(x,v), g_1(w), \dots, g_{n-1}(w)$ are linear with respect to v, w functions $f_n(x,v), g_n(w)$ are convex with respect to v, w and $C_1 = \dots = C_{n-1} = 0,\ C_n = 1$.

6. If functions $f_i(x,v)$, $g_i(w)$ are differentiable with respect to v, w and sets V,W are open then

$$\text{a)} \quad \int_{t_0}^{t_1} \frac{\partial H(x^0(t), \psi^0(t), v^0)}{\partial v_\alpha} dt = 0 \quad (23)$$

$$\text{b)} \quad \frac{\partial h(\psi^0(t_0), w^0)}{\partial w_\beta} = 0 \quad (24)$$

7. If functions $f_i(x,v)$, $g_i(w)$ are differentiable with respect to v, w then gradients of the function $J(v,w)$ at the point v^1, w^1 are equal

$$\text{a)} \quad \frac{\partial J(v^1, w^1)}{\partial v_\alpha} = -\int_{t_0}^{t_1} \frac{\partial H(x^1(t), \psi^1(t), v^1)}{\partial v_\alpha} dt$$

$$\text{b)} \quad \frac{\partial J(v^1, w^1)}{\partial w_\beta} = -\frac{\partial h(\psi^1(t_0), w^1)}{\partial w_\beta}$$

where $x^1(t), \psi^1(t)$ are the solutions of the system (I7) and the adjoint system for v^1, w^1.

8. If $f_i(x,v) = a_{ij}x_j + b_i(v)$ then conditions (I9),(20) are the necessery and sufficient for the optimality of v^0, w^0.

9. The ε-maximum principle;

a) $$H(x^0(t),\psi^0(t),v^0) \geq \frac{1}{t_1-t_0}\int_{t_0}^{t_1} H(x^0(t),\psi^0(t),v)dt - \varepsilon_1, \quad (25)$$

$\varepsilon_1 \geq 0$,

for all $v \in V$

b) $$h(\psi^0(t_0), w^0) \geq h(\psi^0(t_0), w) - \varepsilon_2, \ \varepsilon_2 \geq 0, \quad (26)$$

for all $w \in W$.

If sets V,W are bounded then for any $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ there exists a number τ snch that the conditions (25),(26) hold for (I6)-(I8).

Consider

Exemple 3.

$$\dot{x}_1 = v_1, \quad x_1(0) = w_1, \quad 0 \leq t \leq \tau, \quad C_1 = C_2 = 0, \ C_3 = 1$$
$$\dot{x}_2 = v_2, \quad x_2(0) = w_2$$
$$\dot{x}_3 = x_1^2 + x_2^2, \quad x_3(0) = 0$$

$$V = W = \{u_1, u_2 : (u_1-1)^2 + (u_2+1)^2 \geq 8, \ (u_1-2)^2 + (u_2+2)^2 \leq 18$$

First let be $w_1 = w_2 = 0$. We have $x_1(t) = v_1 t$, $x_2(t) = v_2 t$, $x_3(\tau) = (v_1^2+v_2^2)\frac{\tau^3}{2}$. Consequently, $v_1^0 = -1$, $v_2^0 = 1$. Then

$H(x,\psi,v) = \psi_1 v_1 + \psi_2 v_2 + \psi_3(x_1^2+x_2^2)$, $\dot{\psi}_1 = -2x_1\psi_3$, $\dot{\psi}_2 = -2x_2\psi_3$,

$\dot{\psi}_3 = 0$; $\psi_1(\tau) = \psi_2(\tau) = 0$, $\psi_3(\tau) = -1$, $x_1^0(t) = -t$, $x_2^0(t) = t$,

$\psi_3^0(t) = -1$, $\psi_1^0(t) = \tau^2 - t^2$, $\psi_2^0(t) = t^2 - \tau^2$, $\int_0^\tau H(x^0(t),\psi^0(t),v)dt = \frac{2}{3}\tau^3(v_1 - v_2) - \frac{2\tau^3}{3}$.

For the latter function the point $v_1 = -1, v_2 = 1$ is the point of the absolute minimum in V. Consequently, the condition (I9) is not valid for this example. Properties (2I),(22) are not valid

Now put $v_1 = v_2 = 0$. We have $x_1(t) = w_1$, $x_2(t) = w_2$, $x_3(\tau) = \tau(w_1^2 + w_2^2)$, and $w_1^0 = -1$, $w_2^0 = +1$.

The necessary conditions of optimality:

$$h(\psi, w) = \psi_1 v_1 + \psi_2 v_2,$$

$$\dot\psi_1 = -2x_1\psi_3, \ \dot\psi_2 = -2x_2\psi_3, \ \dot\psi_3 = 0, \ \psi_1(\tau) = \psi_2(\tau) = 0, \ \psi_3(\tau) = -1,$$

$$x_1^0(t) = -1, \ x_2^0(t) = 1, \ \psi_3^0(t) = -1,$$

$$\psi_1^0(t) = 2(\tau - t), \ \psi_2^0(t) = 2(t-\tau), \ h(\psi^0(0), w) = 2\tau(w_1 - w_2).$$

The latter function does not satisfy properties (20),(22),(24).

Since properties (19),(20) are not valid for any $\tau > 0$ we can conclude that the statment 9 in general can not be improved.

7. The optimization of the discrete systems

The general theory of this questinn is very complicated. The following example shows that some forms of the necessary conditions of the optimality are not correct.

Example 4. The equations of the plant; $x(t+1) = u(t)$, $y(t+1) = v(t)$, $z(t+1) = z(t) + x^2(t) + y^2(t)$, $0 \le t \le 2$, $x(0) = y(0) = z(0) = 0$;

control $\{u, v\} \in U$, where U is the set: $U = \{u, v: (u-2)^2 + (v+2)^2 \le 18, (u-1)^2 + (v+1)^2 \ge 8\}$.

It is required to minimize the functional $J(u,v) = z(2)$.

It can be seen that for the optimal control at t=0 the function $H(x, \psi, u, t)$ does not have the stationary value and the local maximum.

The usual forms of the necessary conditions of the optimality have a serious disadvantage. Namely, these forms do not transfer to the maximum principle. when the discrete systeme is going to its continuous analogy.

Forthe discrete systems is valid the following result without this drawback.

Theorm 8. If $u^0(t)$ is the optimal control for the system

$$x_i(t+h) = x_i(t) + h f_i(x,u,t),\ x_i(t_0) = x_{i0},\ t \in T_h = [t_0, t_{1h}]$$

and the functional

$$J_h(u) = \varphi(x(t_{1h})),\ u(t) \in U$$

where U is bounded set, then for each $\varepsilon > 0$ there exists $h = h(\varepsilon) > 0$ such that the condition of the ε-maximum holds:

$$H(x(t), \psi(t), u(t), t) \geq H(x(t), \psi(t), u, t) - \varepsilon$$

for all $u \in U$, $t \in T_h$. Here

$$H(x, \psi, u, t) = \psi_i x_i + \psi_i h f_i(x, u, t),$$

$$\psi_i(t-h) = h \frac{\partial f_j(x(t), u(t), t)}{\partial x_i} \psi_j(t) + \psi_i(t),\ \psi_i(t_{1h} - h) = -\frac{\partial \varphi(x(t_{1h}))}{\partial x_i}$$

In general this theorem can not be improved.

Example 5.

$$x_1(t+h) = x_1(t) + h u(t),$$

$$x_2(t+h) = x_2(t) + h[x_1^2(t) - u^2(t)],$$

$$x_1(0) = x_2(0) = 0,\ |u| \leq 1,\ \varphi(x_1, x_2) = x_2,\ t_0 = 0,\ t_{1h} = 2.$$

Here the optimal control does not satisfy to discrete analogy of the maximum principle for any $h > 0$.

8. The optimal processes with a number players.

Theorems of the optimal controls existence, the necessary and sufficient conditions of the optimality are derived using the methods presented above [5].

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7.5

ON THE MULTISTAGE GAMES

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U S S R

I. In the present paper there are given some results of the theory of multistage games (in which the behavior of players is described by difference equations).

The different statements of games and correlations between them are considered. The necessary optimality conditions for deter mining of the low and upper costs of the game quality are given.

The particular case of existing a saddle-point is considered. These results are also formulated for differential games (in which the behavior of players is described by differential equations).

2. Let the behavior of players is described by the following differerence equation

$$x_{k+1} = f(x_k, u_k, v_k) \qquad (k=0,1,\dots,N-1), \tag{2.I}$$

where vector $x_k = \{x_k^1, \dots, x_k^n\}$ defines the state of the game, and vectors $u_k = \{u_k^1, \dots, u_k^2\}$, $v_k = \{v_k^1, \dots, v_k^s\}$ define the control variables of players I and II resp. The player I can choose the values u_k from th fixed set U:

$$u_k \in U \qquad (k=0,1,\dots,N-1), \tag{2.2}$$

and the player II - v_k - from the set V:

$$v_k \in V \qquad (k=0,1,\dots,N-1). \tag{2.3}$$

The number of stages N is supposed to be fixed. The game quality (perfomance index) is following

$$J = \varphi(x_N) + \sum_{k=0}^{N-1} f_0(x_k, u_k, v_k), \tag{2.4}$$

and the player I tries to maximize the value of (2.4), but the player II - to minimize it.

Further we shall supposed that the sets U and V are closed and bounded, the functions $\varphi(x)$, $f_j(x,u,v)$ $(j=0,1,\dots,n)$ are assumed to be continuously differentiable on $E_n \times U \times V$.

We shall call sequances $u = \{u_0, u_1, \dots, u_{N-1}\}$, $v = \{v_0, v_1, \dots, v_{N-1}\}$, $x = \{x_0, x_1, \dots, x_N\}$ the control of player I,II and the game trajectory, respectively. (This type of control is often called "the open loop control"). Obviously, $J = J(x_0, u, v)$.

Besides, we shall introduce sequances of functions $u(k,x)=\{u_0(x_0),\dots, u_{N-1}(x_{N-1})\}$, $v(k,x)=\{v_0(x_0),\dots, v_{N-1}(x_{N-1})\}$, which we shall call the synthesis of the player I and II respectively (the closed loop control). These sequances also define the process: $J=J(x_0, u(k,x), v(k,x))$.
In connection with the information of each players about the course of the game we shall consider the following problems.

Problem I. Let's suppose that the player II may learn all the control u of the player I beforehand (as a function of time). In this situation the player I does his best if he chooses the control u to obtain

$$\max_u \min_v J(x_0, u, v) = \omega_1^-(x_0). \tag{2.5}$$

Choosing his control in such a way the player I guarantees himself the value of $J \geqslant \omega_1^-$,
whatever control v the player II would choose. $\omega_1^-(x_0)$ is the low cost of the game quality for player I in open loop control (i.e. when the information about the current state of the game is not available for the player I).
Similary we define the upper cost of the game quality:

$$\min_v \max_u J(x_0, u, v) = \omega_1^+(x_0).$$

Problem 2. Let's suppose that the player II do not know anything beforehand but he may learn the current values of state x_k and control variables u_k of his opponent. In this situation it may be proved by induction that the player I must choose his control to yield

$$\max_{u_0} \min_{v_0} \dots \max_{u_{N-1}} \min_{v_{N-1}} J(x_0, u, v) = \omega_2^-(x_0). \tag{2.6}$$

If on the contrary the player I may learn the current values v_k of the player II control variables than he chooses his control to obtain

$$\min_{v_0} \max_{u_0} \dots \min_{v_{N-1}} \max_{u_{N-1}} J(x_0, u, v) = \omega_2^+(x_0). \tag{2.7}$$

Problem 3. Now let's suppose that the player II may learn all the synthesis of the player I beforehand, that is he knows the rules by which the player I chooses his control variables u_k in dependance on the current game state x_k and probably on the number k of current stage.
In this case the player I must choose his synthesis to yield

$$\max_{u(k,x)} \min_{v(k,x)} J(x_0, u(k,x), v(k,x)) = \omega_3^-(x_0). \tag{2.8}$$

$\omega_3^-(x_0)$ is the low cost for the player I in closed loop control (i.e. when the current information about the state of the game is available for the player I).

3. Now we shall establish the correlations between the problems I-3. For this purpose we make use of following results in the theory of static games (see, for examle, [I]):

$$\max_{u(x)} \min_{v(u,x)} \varphi(x,u,v) \leq \min_{v(x)} \max_{u(v,x)} \varphi(x,u,v) \qquad (3.I)$$

$$\max_{u(x)} \min_{v(u,x)} \varphi(x,u,v) = \min_{v(u,x)} \max_{u(x)} \varphi(x,u,v) \qquad (3.2)$$

The inequality (3.I) means that the gain of the player I when his choice is known for the player II cannot be less his gain when vice versa the player I may learn the choice of player II. The equality (3.2) means that operators max and min are commutative when the information of each players about the game do not change. Using the connections (3.I),(3.2) to problems I-3 we may prove the following.

<u>Theorem 3.I</u>. For any initial state x_0 the following results are true

$$\omega_1^-(x_0) \leq \omega_3^-(x_0) \leq \omega_3^+(x_0) \leq \omega_1^+(x_0), \qquad (3.3)$$

$$\omega_3^-(x_0) = \omega_2^-(x_0), \quad \omega_3^+(x_0) = \omega_2^+(x_0). \qquad (3.4)$$

The equalities (3.4) show that the problems 2 and 3 are only the different statements of the same problem. Therefore there is need to distinquish two main games in the theory of dynamic games. In the first game which we shall denote by $\Gamma_1(x_0, u, v)$ the player I chroooses control beforehand and does not use in any way (or does not have) the current information about the game course; it is a static game in essence.

In the second game $\Gamma_2(x_0, u(k,x), v(k,x))$ the players already define their behavior in connection with the course of the game. The results (3.4) show that the player I for example is able to increase the quality of his control using the closed loop control .

From the theorem 3.I we may easy to obtain.

<u>Theorem 3.2.</u> If the game $\Gamma_1(x_0, u, v)$ has the saddle-point, i.e.

$$\max_u \min_v J = \min_v \max_u J,$$

then the game $\Gamma_2(x_0, u(k,x), v(k,x))$ has the saddle-point too,i.e.

$$\max_{u(k,x)} \min_{v(k,x)} J = \min_{v(k,x)} \max_{u(k,x)} J.$$

The inverse is not true in general.

This theorem shows that the open loop control is equivalent to the closed loop control only when the game Γ_1 has a saddle-point

This situation is probably not frequent in real dynamic games. The more likely when ω_3^- is considerably larger than ω_1^-; naturally in these games the closed loop control is compulsory. For more detailed analysis of these questions see paper[2].

4. Before formulating optimality conditions for problems I-3 we define some notions.

Consider the problem of finding

$$\max_u \min_v \varphi(u,v), \tag{4.I}$$

where $u \in \mathcal{U}$ and $v \in V$ are compacts, and $\varphi(u,v)$ is continuously differentiable function on $\mathcal{U} \times V$.

It was proved in [3-5] that the function $\Phi(u) = \min_{v \in V} \varphi(u,v)$ is only continuous but differentiable in any direction δu and

$$\delta\Phi(\bar u) = \min_{v \in V(\bar u)} \left[\frac{\partial \varphi(\bar u, v)}{\partial u}\right]^T \delta u, \tag{4.2}$$

where $V(\bar u)$ is the set of optimal responses v for $\bar u \in \mathcal{U}$:

$$V(\bar u) = \{v \,/\, \varphi(\bar u, v) = \min_{v \in V} \varphi(\bar u, v)\}.$$

Besides, we define an adjoint system

$$p_k = \frac{\partial f_0(x_k, u_k, v_k)}{\partial x_k} + \left[\frac{\partial f(x_k, u_k, v_k)}{\partial x_k}\right]^T p_{k+1} \quad (k = N-1, \ldots, 2, 1) \tag{4.3}$$

with boudary condition

$$p_N = \frac{\partial \Phi(x_N)}{\partial x_N}, \tag{4.4}$$

where vectors $p_k = \{p_k^1, \ldots, p_k^n\}$; $\frac{\partial f}{\partial x} = \left[\frac{\partial f^i}{\partial x^j}\right]_{ij}$; $\frac{\partial \Phi}{\partial x} = \left[\frac{\partial \Phi}{\partial x^j}\right]_j$; $\frac{\partial f_0}{\partial x} = \left[\frac{\partial f_0}{\partial x^j}\right]_j$; $(i,j = 1, \ldots, n)$.

Define also the Hamilton function

$$H(p_{k+1}, x_k, u_k, v_k) = f_0(x_k, u_k, v_k) + p_{k+1}^T f(x_k, u_k, v_k) \tag{4.5}$$

and denote by $\delta_u H$ and $\delta_v H$ the feasible differentials of this function

$$\delta_u H = \left[\frac{\partial H}{\partial u}\right]^T \delta u; \quad \delta_v H = \left[\frac{\partial H}{\partial v}\right]^T \delta v,$$

where $\delta u \in K(u)$, $\delta v \in K(v)$, and $K(u)$, $K(v)$ are the cones of feasible variations at the points $u \in \mathcal{U}$ and $v \in V$[6,7]

5. Now we can formulate the optimality conditions for the problemI. Using the notions of p.4 and the usual reasonings in the theory of optimal discrete control (see for example [7]) we can prove the following.

<u>Theorem 5.I.</u> Let $\Omega_v(u^*)$ is the set of optimal solutions v^* of the problem

$$\{\min_v J,\ x_{k+1} = f(x_k, u_k^*, v_k),\ v_k \in V \quad (k = 0, 1, \ldots, N-1)\}. \tag{5.I}$$

Then the following inequalities

$$\min_{v\in\Omega_v(u^*)} \delta_u H(p_{K+1}, x_K, u_K^*, v_K) \le 0, \qquad \delta u_K \in K(u_K^*) \tag{5.2}$$

are hold for the optimal control of the player I, where the optimal values $\{p_K, x_K\}$ are found from (2.I),(4.3) and (4.4) for $v \in \Omega_v(u^*)$.

We consider some particular cases now. Only for simplicity of designations we shall assume that $f_0(x,u,v) \equiv 0$, i.e. $J = \varphi(x_N)$.

Theorem 5.2. Let's the set $R_1^I(x,v) = \{z/z = f(x,u,v), u \in \mathcal{U}\} = f(x,\mathcal{U},v)$ is convex for all x and $v \in V$.

Then the equality

$$\max_{u_K\in\mathcal{U}} \min_{v\in\Omega_v(u)} [H(p_{K+1}, x_K, u_K, v_K) - H(p_{K+1}, x_K, u_K^*, v_K)] = 0 \tag{5.3}$$

is true for the optimal process.

The justice of this theorem is followed from the inequality (5.2) and the convexity of the set $R_1^I(x,v)$.

(Compare [4,7]).

Theorem 5.3. Let the solution of the problem (5.I) is unique. Then the following inequalities

$$\delta_u H(p_{K+1}^*, x_K^*, u_K^*, v_K^*) \le 0 \tag{5.4}$$

$$\delta_v H(p_{K+1}^*, x_K^*, u_K^*, v_K^*) \ge 0 \tag{5.5}$$

are hold for all $\delta v_K \in K(v_K^*)$ and $\delta u_K \in K(u_K^*)$.

The inequality (5.4) is directly followed from (5.2); the inequality (5.5) is the optimality condition of control v^*.

Theorem 5.4. Let: I) the sets $f(x,\mathcal{U},v)$ and $f(x,u,V)$ are convex for all x and $u \in \mathcal{U}$, $v \in V$;

2) the solution of the problem (5.I) is unique.

Then it is necessary that the Hamiltonian has a saddle-point:

$$\max_{u_K\in\mathcal{U}} \min_{v_K\in V} H(p_{K+1}^*, x_K^*, u_K, v_K) = \min_{v_K\in V} \max_{u_K\in\mathcal{U}} H(p_{K+1}^*, x_K^*, u_K, v_K) = H(p_{K+1}^*, x_K^*, u_K^*, v_K^*) \tag{5.6}$$

for the $\{u_K^*, v_K^*, x_K^*, p_K^*\}$ to be optimal.

The similar results are true for the general type of perfomance index (Comp. [7]).

Proof. From the inequality (5.4) and the convexity of the set $f(x,\mathcal{U},v)$ is followed[7]

$$\max_{u_K\in\mathcal{U}} H(p_{K+1}^*, x_K^*, u_K, v_K^*) = H(p_{K+1}^*, x_K^*, u_K^*, v_K^*). \tag{5.7}$$

From the inequality (5.5) and the convexity of the set $f(x,u,V)$ is followed

$$\min_{v_K\in V} H(p_{K+1}^*, x_K^*, u_K^*, v_K) = H(p_{K+1}^*, x_K^*, u_K^*, v_K^*). \tag{5.8}$$

The inequalities (5.7) and (5.8) are equivalent to $(5.6)^I$.

The similar results may be obtained for finding $\min_v \max_u J = \omega_1^+$.

The more detailed analysis of the above problems may be found in [8].

6. Let's come to optimality conditions for the problems 2,3. It may be proved that the original problem 2 (or 3) is equpivalent to the problems

$$\{\max_u J,\ x_{k+1} = f(x_k, u_k, v_k^*(x_k, u_k)),\ u_k \in \mathcal{U} \quad (k=0,1,\dots,N-1)\}, \tag{6.I}$$

$$\{\min_v J,\ x_{k+1} = f(x_k, u_k^*(x_k), v_k),\quad v_k \in V \quad (k=0,1,\dots,N-1)\}, \tag{6.2}$$

where $\{u_k^*(x_k)\}$ and $\{v_k^*(x_k, u_k)\}$ are optimal solutions of the problem 2 or 3.

Here we shall consider a singular case only. Namely we shall assume that the functions $u_k^*(x)$ and $v_k^*(x, u)$, which define a solution of the problem, are differentiable in any directions δx or δu. In this case it may be proved that the problems (6.I) and (6.2) have unique solutions.

Denote the optimal values of control variables by $u_k^* = u_k^*(x_k^*)$ and $v_k^* = v_k^*(x_k^*, u_k^*)$ for a given initial state x_0.

Theorem 6.I. Let: I)the functpions $u_k^*(x)$, $v_k^*(x, u)$ are differentiable in any directions δx and δu;
2) the sets $f(x_k^*, u_k^*(x_k^*), V)$ and $f(x_k^*, \mathcal{U}, v_k^*(x_k^*, \mathcal{U}))$ are convex for $k=0,1,\dots, N-1$.

Then

$$\max_{u_k \in \mathcal{U}} \min_{v_k \in V} H(p_{k+1}^*, x_k^*, u_k, v_k) = H(p_{k+1}^*, x_k^*, u_k^*, v_k^*) \tag{7.I}$$

is hold at the optimal process $\{u_k^*, v_k^*, x_k^*, p_k^*\}$, where the optimal values of $\{p_k^*\}$ are found from the adjoint system (4.3) with the boundary condition (4.4).

The proof of this theorem may be found in [9].

7. Now we consider the case when the game Γ_1 or Γ_2 has a saddle-point. In this case the game problem reduces to the pair of optimal control problems.

Namely, let for example there exist such controls u^* and v^* that the game $\Gamma_1(x_0, u, v)$ has a saddle-point, i.e.

$$\max_u \min_v J(x_0, u, v) = \min_v \max_u J(x_0, u, v) = J(x_0, u^*, v^*). \tag{7.I}$$

Then the game Γ_1 is equivalent to the following optimal control problems:

$$\{\max_u J,\quad x_{k+1} = f(x_k, u_k, v_k^*),\quad u_k \in \mathcal{U} \quad (k=0,1,\dots,N-1)\} \tag{7.2}$$

$$\{\min_v J,\quad x_{k+1} = f(x_k, u_k^*, v_k),\quad v_k \in V \quad (k=0,1,\dots,N-1)\} \tag{7.3}$$

Using to the (7.2),(7.3) the optimality conditions for discrete processes [7] we obtain directly.

Theorem 7.I. It is necessary that

$$\delta_u H(p_{k+1}^*, x_k^*, u_k^*, v_k^*) \le 0, \qquad \delta u_k \in K(u_k^*) \tag{7.4}$$

$$\delta_v H(p_{k+1}^*, x_k^*, u_k^*, v_k^*) \geqslant 0, \quad \delta v_k \in K(v_k^*) \tag{7.5}$$

for controls u^* and v^* to be saddle-point of the game.
Here the optimal values of $\{p_k^*\}$ are found from (4.3) and (4.4).
Corollary. Let the sets $f(x, \mathcal{U}, v)$ and $f(x, u, V)$ are convex for all $x \in E_n$ and $u \in \mathcal{U}, v \in V$. Then if the perfomance index $\mathcal{J}$ has the saddle-point, then the Hamilton function (4.5) has a saddle-point too.
This is directly followed from usual in the theory of discrete control reasonings [7].
Consider the game Γ_2 now.
Let the functions $\{u_k^*(x), v_k^*(x)\}$ exist, i.e.

$$\max_{u(k,x)} \min_{v(k,x)} \mathcal{J}(x_0, u(k,x), v(k,x)) = \min_{v(k,x)} \max_{u(k,x)} \mathcal{J}(x_0, u(k,x), v(k,x)) = \mathcal{J}(x_0, u^*(k,x), v^*(k,x)). \tag{7.6}$$

Then the game reduces to the problems

$$\{\min_{v} \mathcal{J},\ x_{k+1} = f(x_k, u_k^*(x_k), v_k),\ v_k \in V\ (k=0,1,\dots,N-1)\}, \tag{7.7}$$

$$\{\max_{u} \mathcal{J},\ x_{k+1} = f(x_k, u_k, v_k^*(x_k)),\ u_k \in \mathcal{U}\ (k=0,1,\dots,N-1)\}. \tag{7.8}$$

Theorem 7.2. Let: I) The functions $u_k^*(x)$ and $v_k^*(x)$ are differentiable in any direction δx (in this case the solutions of the problems (7.7),(7.8) are unique);
2) the sets $f(x, \mathcal{U}, v)$ and $f(x, u, V)$ are convex on $E_n \times \mathcal{U} \times V$.
Then if the perfomance index $\mathcal{J}$ has a saddle-point /(7.6) is hold /, the Hamiltonian (4.5) has a saddle-point too,i.e.

$$\max_{u_k \in \mathcal{U}} \min_{v_k \in V} H(p_{k+1}^*, x_k^*, u_k, v_k) = \min_{v_k \in V} \max_{u_k \in \mathcal{U}} H(p_{k+1}^*, x_k^*, u_k, v_k) = H(p_{k+1}^*, x_k^*, u_k^*, v_k^*),$$

where the optimal values $\{x_k^*, p_k^*\}$ are found from (2.I),(4.3),(4.4).
This theorem is directly followed from the theorem 6.I.
In conclusion we note that, the otpimal controls of players are not coincided for the games Γ_1 and Γ_2 in general case.
8. At last we shall formulate the similar results for differential games. In this case a game is discribed by differential equations of following type

$$\dot{x}(t) = f(x(t), u(t), v(t)), \quad 0 \leq t \leq T, \tag{8.I}$$

with initial state $x(0) = x_0$ and T to be fixed, where

$$u(t) \in \mathcal{U}, \qquad v(t) \in V, \tag{8.2}$$

and the perfomance index

$$\mathcal{J} = \varphi(x(T)) + \int_0^T f_0(x(t), u(t), v(t))\,dt \tag{8.3}$$

Further we shall assume that the functions $\varphi(x), f_j(x, u, v)\ (j=0,1,\dots,n)$ are continuously differentiable with respect to all their arguments, and the sets $\mathcal{U}$ and V are compacts.

As in the multistage games we introduce two games: $\Gamma_1(x_0, u, v)$, which is connected with the problems

$$\omega_1^-(x_0) = \max_u \min_v J(x_0, u, v), \tag{8.4}$$

$$\omega_1^+(x_0) = \min_v \max_u J(x_0, u, v), \tag{8.5}$$

and $\Gamma_2(x_0, u(t,x), v(t,x))$, which is defined from the problems

$$\omega_2^-(x_0) = \max_{u(t,x)} \min_{v(t,x)} J(x_0, u(t,x), v(t,x)), \tag{8.6}$$

$$\omega_2^+(x_0) = \min_{v(t,x)} \max_{u(t,x)} J(x_0, u(t,x), v(t,x)). \tag{8.7}$$

These problems are completely similar to problems I and 3 in multistage games. As for the prolem 3 we consider the following problem: find a control u^*, which yields the best value of functional (8.3), if the player II may learn the current values of $x(t)$ and $u(t)$ (it is supposed, of course, that the player II " must" do the worst for the player I).

This problem is apparently similar to problems 2 of p.2 and is called the game with discrimination [IO,II] or minórant (majorant) game [I2]. Note, that formally this problem may be written only as a limiting case of the problem 2. Also by limiting transition we can establish the correlations between the differential games Γ_1 and Γ_2, similar to the theorems 3.I and 3.2.

Define the adjoint system

$$\dot{p}(t) = \frac{\partial f_0(x(t), u(t), v(t))}{\partial x(t)} - \left[\frac{\partial f(x(t), u(t), v(t))}{\partial x(t)}\right]^T p(t), \tag{8.8}$$

with the boundary condition

$$p(T) = -\frac{\partial \varphi(x(T))}{\partial x(T)}, \tag{8.9}$$

and the Hamiltónian

$$H(p(t), x(t), u(t), v(t)) = -f_0(x(t), u(t), v(t)) + p^T(t) f(x(t), u(t), v(t)). \tag{8.IO}$$

Theorem 8.I. Let u^* is an optimal control of the plaeyer I in the problem (8.4) and $\Omega_v(u^*)$ is the set of optimal solutions of the problem (the set of optimal controls of player II):

$$\{\min_v J;\quad \dot{x}(t) = f(x(t), u^*(t), v(t)),\ v(t) \in V,\ t \in [0,T]\}. \tag{8.II}$$

Then the equality

$$\min_{u(t) \in \mathcal{U}} \max_{v \in \Omega_v(u^*)} [H(p(t), x(t), u(t), v(t)) - H(p(t), x(t), u^*(t), v(t))] = 0 \tag{8.I2}$$

is hold for optimal process, where the optimal values $x(t), p(t)$ are found from (8.I), (8.8), (8.9).

This theorem evidently is similar to the theorem 5.2.

Theorem 8.3. Let $u^*(t) = u^*(t, x^*(t))$ and $v^*(t) = v^*(t, x^*(t), u^*(t))$ are optimal controls for problem (8.6) for a given initial state x_0 and let the functiuons $u(t,x), v(t,x,u)$ are differentiable in any directions δx or δu (in this case the solutions of the problems:

$$\{\max_u J,\ \dot{x} = f(x(t), u(t), v^*(t, x(t), u(t))),\ u(t) \in \mathcal{U},\ t \in [0,T]\},$$

$$\{\min_v J,\ \dot{x} = f(x(t), u^*(t, x(t)), v(t)),\ v(t) \in V,\ t \in [0,T]\}$$

are unique).

Then

$$\min_{u \in \mathcal{U}} \max_{v \in V} H(p^*(t), x^*(t), u, v) = H(p^*(t), x^*(t), u^*(t), v^*(t))$$

is true for the optimal process $\{u^*(t), v^*(t), x^*(t), p^*(t)\}$ where the optimal values of $\{x^*(t), p^*(t)\}$ are found from (8.I),(8.8),(8.9). The proof of this theorem see in [I3].

Theorem 8.4. Let the pair $u^*(t,x)$, $v^*(t,x)$ is a saddle-point for the game $\Gamma_2(x_0, u(t,x), v(t,x))$.

Then, if the functions $u^*(t,x)$ and $v^*(t,x)$ are differential in any direction δx, $t \in [0,T]$, the Hamilton function has a saddle in the same point, i.e.

$$\max_{v \in V} \min_{u \in \mathcal{U}} H(p^*(t), x^*(t), u, v) = \min_{u \in \mathcal{U}} \max_{v \in V} H(p^*(t), x^*(t), u, v) = H(p^*(t), x^*(t), u^*(t), v^*(t))$$

This theorem is directly followed from the theorem 8.3. Note, that the similar result was obtained inpaper [I4] for the case of "regular" synthesis and in the paper [I5] for the case of "smooth" synthesis.

9. In conclusion, we note, that the results obtained in the present paper permit to develope efficient numerical methods in the theory of dynamic games.

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8.1

MAGNETIC ADAPTIVE COMPONENTS FOR AUTOMATIC CONTROL SYSTEMS

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Development of adaptive automatic control systems often calls for introduction of components with a variable transfer coefficient performing the following function

$$z = x\,F(y,t) \tag{1}$$

where x - the input variable, and y - the control (adapting) signal causing the change of the component transfer coefficient for the variable x in accordance with the adaptation function $F(y,t)$.

Selection of the adaptation function depends on the principle of building of the specific control system. Some typical expressions of this function are as follows:

$$F(y,t)_{t>t_0} = F_1[y(t_0)] \tag{2.I}$$

$$F(y,t) = F_2\left[\int_0^t y\,dt\right] \tag{2.2}$$

$$F(y,t)_{t>t_n} = F_3\left[\sum_{j=1}^{n} y(t_j)\right] \tag{2.3}$$

$$F(y,t)_{t>t_n} = F_4\left[\sum_{j=1}^{n} \operatorname{sign} y_1(t_j)\cdot y_2(t_j)\right]. \tag{2.4}$$

In all four cases the adaptation function is some monotonous but not necessarily single-valued function of the control signal y. The latter not seldom constitutes the difference between reference and actual value of some controlled parameter of the control system.

In some instances it is necessary that

$$F_1[y(t_0)] = k_1\,y(t_0) \tag{3.I}$$

or

$$F_2\left[\int_0^t y\,dt\right] = k_2\int_0^t y\,dt. \tag{3.2}$$

In the first case the adaptation function (2.I) reduces to the memorization of the adaptation signal value at some moment t_0 (the adaptation moment). Each new act of adaptation "erases" the preceeding condition and sets up a new transfer coefficient value independent of the preceeding values. The remaining three adaptation functions determine the transfer coefficient value for the variable x not only according to the adapting signal in the given mement but also with regard to the preceeding values of the signal or transfer coefficient. The adaptation functions (2.2) - (2.4) are commonly used in systems adapted by the method of consecutive search or learning. It should be noted that the control signal y in the expression (2.2) can be both a continuous value and pulses of random form.

The adaptation function (2.4) contains two independant control signals y_I and y_2. The change of this function only occurs at simultaneous action of both signals. Such function is used for matrix access of the address of the component chosen for adaptation in learning systems containing a large number of adaptive components [I].

A common feature for four above mentioned kinds of adaptation functions is that each of them posseses the ability of memory. Realization of these functions calls for analog (or multi-level) storing components.

In recent years there were many suggestions concerning the development of adaptive components, that is components with a variable transfer coefficient and with memory [I]. But only two groups of components, electrochemical and magnetic, found practical application. The advantage of electrochemical components is that they represent controlled active resistance [I, 2] and therefore have a large bandwidth for the adapted signal. They can be used both for continuous and binary variables x. The disadvantages are time instability of their characteristics, long adaptation time (setting up a new transfer coefficient value), galvanic connection between control and input (output) circuits, and a relatively low resistance that limits their sphere of possible applications.

Magnetic adaptive components on cores with rectangular hysteresis loop are greatly favoured because of their small dimentions, low cost, high time stability, high speed of operations, and other advantages. Any of the above mentioned adaptation functions (2.I) - (2.4), (3.I) or (3.2) can be achieved with magnetic components.

The disadvantage of the most known magnetic adaptive components is that they preserve the above mentioned advantages only for binary and ternary variables X. This derives from the fact that the action of the input variable reduces practically to switching on (by X = I) and switching off (by X = 0) or to the change of 180^0 of the read current phase of the analog memory component [I]. At the same time it is desirable to have adaptive components capable of changing the transfer coefficient of continuous signals. This paper deals with the problem of building magnetic adaptive components for continuous variables.

Methods of achieving the desirable adaptation function, e.g. (2.I) - (2.4), do not depend on the transferred variable being binary or continuous. The main difference between magnetic adaptive components for the two types of the variable X consists of the method of carrying out multiplication of the variable X by adaptation function F(Y,t) in accordance with the expression (I). Possible methods of performing this operation for continuous variables depend greatly on the used method of reading the data stored in accordance with the adaptation function in the NDRO magnetic analog memory component.

Before getting down to the analyses of the different methods of reading and their characteristics it should be noted that if there is an adaptive component for binary signals then in accordance with the expression (I) it would be possible to develop an adaptive component for continuous signals with addition of a multiplication device in accordance with the Fig.I. One of the factors on the input of the multiplication device represents the adapted continuous signal X, the second factor being the signal V = F(Y,t) from the output of the adaptive component for binary signal.

Such a method of developing the adaptive components for continuous variables is provided in the system of magnetic computing components developed in the Institute of Automatics and Telemechanics (engineering cybernetics) [4]. This system includes magnetic analog memory realizing the function (3.I) with the error not exceeding 0,5% and a magnetic integrator with memory realizing the function (3.2) with the error not exceeding I%. With a diode multiplier these components realize the equation (I) for continuous signals with a summary error not more than I-2 %.

Adaptive components which comprise the adaptation and multiplication functions in one device are of great interest. In this case it is possible to reduce dimentions and cost of the component and improve its characterists. There are the following basic principles of NDRO the information from magnetic memory components [I.3] :

a) Generation of even harmonics of magnetic induction depending on the residual flux state in the magnetic core;

b) using the dependance between the reversible permeability and the level of the residual magnetization;

c) using a magnetic probe indicating the level of residual magnetization in the core of the memory component.

The essence of the first method is that if ferromagnetic core is exited with two currents of different frequences:

$$i_1 = I_1 \sin \omega_1 t , \quad i_2 = I_2 \sin \omega_2 t$$

there develop combination induction harmonics of the type

$$B_{m,n} \sin[(m\omega_1 \pm n\omega_2)t + \varphi_{m,n}].$$

At the absence of residual magnetism of the core the $m + n = 2k + I$ is an odd number [3]. The even value of the $m + n$ can be only at the presence of dc component of the induction B_0 in the core. Then alongside with other components there is also an induction component:

$$B_{1,1} \sin[(\omega_1 + \omega_2)t + \varphi_{1,1}].$$

Combination frequences appears only at the current amplitude i_I or i_2 being sufficient for developing non-linear distortion. On the other hand, these amplitudes have an upper limit as the currents i_I and i_2 should not influence the residual flux. Taking I_2 = const and $I_1 \ll I_2$ we get $B_{1,1} \equiv I_1$.

Besides $B_{1,1}$ is the monotonous function B_0. Therefore if the amplitude of the current i_1 corresponds to the variable x then separating by means of suitable filter the component of the output voltage with the frequency $\omega_1 + \omega_2$ we get an adaptive component for a continuous signal $x = CI_1$ (Fig. 2,a):

$$E_{1,1} = W(\omega_1 + \omega_2) S B_{1,1} = C_1 x \varphi(B_0) \quad (4)$$

where $E_{1,1}$ is the e.m.f. amplitude with the frequency $\omega_1 + \omega_2$ induced in the output winding W, S - cross-section of the core, $\varphi(B_0)$ - monotonous function of the residual induction B_0 of the core set up by the adaptation signal iy.

The shown principle was used in Japan for combining in one component function of an analog memory and a multiplier [6]. To prevent a direct voltage transformation of the main frequences ω_1, ω_2 at the output of the circuit there are four instead of one core with the windings interconnected in accordance with Fig.2,b. Points indicate the beginning of each winding.

If the core in Fig.2 is excited with a.c. current of only one frequency ($I_2 = 0$) then even induction harmonics $2k\omega_1$ occur as before only at assymetric core magnetization due to the residual induction B_0. At small amplitudes of the current i_I nonlinear distortion and even harmonics do not occur whereas at high amplitudes of i_I the stored information is erased. Therefore separation of the even harmonics (or the 2nd harmonic $2\omega_1$) of the voltage induced in the output winding is used only for building adaptive components for binary signals [I,3]. In this case as before $x = cI_1$. Basic circuit of such a component is shown at Fig.2,c. To prevent the appearance of odd harmonics in the output of the circuit two cores are used here which often eliminates the need for the filter shown in Fig.2,a.

Use of the second harmonic memory element (Fig.2,c) to build an adaptive element for continuous signals is possible with the help of a multiplier of Fig.I. In this case I_I=const and the output voltage of the double frequency ($E_{2\omega}$) is preliminary rectified.

There are two ways to build adaptive elements for continuous signals on the basis of the circuit of Fig.2,c and on the basis of the similar circuits in which it is impossible to change the readout current amplitude (I_1) within wide range and to get a proportional change of the output voltage amplitude.

The first way is to change the readout current frequency ω in proportion to the variable x (Fig.3,a). Here we use the well-known dependence of the output voltage amplitude on the readout current frequency when I_1 = const. and B_0 = = const. (suppose that within the given changes of frequency we may ignore the influence of losses in the core). If an autonomous source of readout current is required, we get $\omega \sim \sim x$ while using a magnetic-transistor d.c. to a.c. converter.

The second way is to use pulse-width modulation of the readout current with the frequency $\omega_M \ll \omega$ (Fig.3,b). If T is an interval during which in each period $\left(T_m = 2\frac{\pi}{\omega_M}\right)$ the readout current i_1 is applied to a memory element the mean value of the output voltage is $E = C\omega T/T_m \varphi(B_0)$.

If the pulse-width modulator gives $T/T_m = X$, we get the requaired characteristic (1). Note that a magnetic amplifier with self-saturation [3] can simultaneously perform both the function of the signal amplifier X and a modulator of such a type. The switch K in the circuit (Fig.3,b) may be either a diode or a transistor.

The first shortcoming of the circuit introduced in Fig.2,c with double frequency is that it is rather difficult to avoid transformation of the readout current onto the circuit output.

The second way of the readout is to use the well-known experimentally established fact that the reversible magnetic permeability μ_r of the core with a rectangular hysteresis loop is a function of the core residual magnetization [7].

This dependence is not simple. For example, if the core is preliminary negatively saturized, and there is a monotonous increasing of the level of the residual induction B_0 than we shall get the dependence $\mu_r(B_0)$ shown in Fig.4,a (curve I). If we preliminary demagnetize the core, we shall get curve 2, while monotonously changing B_0. If we introduce a d.c. bias which after the core saturation in the negative direction sets the induction B_b corresponding to the maximum of μ_r on the curve I we may get the monotonous dependence $\mu_r(B_0)$ for the positive values of the control (adaptive) signal. If changing of B_0 is obtained with short current pulses, in this case we can get the dependence μ_r on the number of pulses n given in Fig.4,b.

There is a number of ways to build adaptive elements using characteristics shown in Fig.4. The simpliest of them is given on Fig.5,a. The mutual inductance between the windings W_1 and W_2 is

$$M = \frac{\mu_0 \mu_r W_1 W_2 S}{l}, \tag{5}$$

where l is a mean core length and μ_0 is the magnetic constant.

If the magnetic field strength from the current i_1 does not exceed some threshold value then μ_r does not depend on the value i_1 and for EMF in the output winding W_2 (if we ignore the influence of the currents in other windings) we have

$$e = M \frac{di_1}{dt} \tag{6}$$

If $i_1 = I_1 \sin \omega t$ then $e = \omega M I_1 \cos \omega t$. (7)

Thus if I_1 or ω are proportional to the variable x, we get an adaptive element for continuous signals.

The shortcoming of the circuit given in Fig.5,a is that μ_r and consequently the circuit transfer coefficient change in rather small range which usually not exceeds four. This shortcoming may be eliminated by using a compensation core according to the circuit given in Fig.5,b. Here B is a compensation core and the output EMF is

$$e = \frac{\omega \mu_0 W_1 W_2 \cos \omega t}{l} (\mu_{rB} - \mu_{rA}) I_1 \tag{8}$$

where μ_{rA} and μ_{rB} are the values of the reversible permeability of the corresponding cores. $\mu_{rA} = \mu_{rB}$ and

$e \equiv 0$ in the presetted state of the cores before giving an adaptive signal. If the adaptive signal (i_y) is applied to the core B in such a way that creates a magnetizing field of the opposite direction relatively to the bias field the circuit becomes "reversible", i.e. we get a possibility to change the transfer coefficient sign.

Instead of the controlled mutual inductance we may use the controlled inductance L, for example, in the bridge circuit (Fig.5,c). The circuit becomes balanced for some value of L by selecting values of L_o, r_o, R_1 and R_2. The control signal i_y changes the circuit transfer coefficient U_o/U which in general has a complex character. If however in all the branches of the bridge we use inductances, the transfer coefficient will not practically depend on the voltage frequency ω.

The circuit given in Fig.5,d has the same property. The constant inductance L_o which value is much greater than that of the inductance of the winding W_1 is put in series with the winding W_1. Therefore for the current i_1 we have

$$U = L_o \frac{di_1}{dt}.$$

Taking into account (6) we find for the output voltage

$$e = \frac{M}{L_o} U,$$

i.e. the output voltage e differs from the input voltage U corresponding to the variable x by the constant coefficient which does not depend on the frequency or the wave form of the voltage U. Thus to increase the range of transfer coefficient changes we may use a simple voltage divider (Fig.5,e) to get the voltage U_k which compensate circuit output voltage for some value of the coefficient of the mutual inductance M [8].

Frequency changes ω of the input signal U in the circuits introduced in Fig.5,d and e are limited by the fact that the amplitude of the current i_1 which can change with changing of the frequency must not exceed the permissible threshold value otherwise μ_r will be changed in the function of i_1.

Readout in all memory elements considered above is achieved by way of a reversible change of the core magnetization state near some value corresponding to the set value of adaptation function.

Physical properties of one and the same ferromagnetic are used both for storing information and for nondestructive readout of this information. In contrast to this while reading with the help of the sonde method functions of storing and reading of the information are divided between different components. The core with the rectangular hysteresis loop fulfils the first function while a special sonde-component fulfils the function of nondestructive readout indicating the level of the core residual magnetization. Such a function division often gives essential advantages and permits in particular to increase the maximum power taken off from the analog memory element.

Different types of galvanomagnetic elements and magnetic modulators including magnetic amplifiers may be used as sondes.

The principal circuit of the adaptive element with Hall element mounted in the split of a toroidal core used as a memory element is given in Fig.6,a. EMF appearing on the Hall element output is directly proportional to the product of the mean value of the core residual inductance B_0 and the current i in the element taken as an independent variable $(i \sim X)$ [9]: $e = c B_0 i$, i.e. Hall element combines the function of a readout sonde and that of a multiplier.

Existance of the split with the width of δ in the core for placing the Hall element into it creates a demagnetizing field which value under uniform magnetization of the core is [3].

$$H_D = B_0 \frac{\delta}{\mu_0 l}.$$

This field must not cause self-erasing of the data written in the core in the most unfavourable case when $B_0 = B_r$, i.e.

$$H_{D.max} = B_r \frac{\delta}{\mu_0 l} < H_T \leq H_c \qquad (10)$$

where B_r is residual inductance on the maximum ferromagnetic hysteresis loop and H_T is a threshold field. When $H > H_T$ the inductance of a core whose initial value is $-B_r$ will also increase. Fulfilment of this condition requires using film Hall elements to get small values of δ and using magnetic materials with a large relation of H_c / B_r which in its turn permits to increase the maximum permissible value of δ.

Hall element in the circuit shown in Fig.6,a can be replaced by a magnetoresistor, i.e. a semiconducter element which active resistance changes in the magnetic field [9-II]. There are elements which under the influence of a field in the range of 1 T increase the resistance in IO-20 times. Usually $R_M = R_o (I + c B_o^2)$ in wide range of induction variations.

Use of such devices requires keeping the condition (IO).

Changes of adaptive element characteristics and getting a zero transfer coefficient in particular can be achieved by putting a magnetoresistor R_M into one of the branches of the bridge circuit (Fig.6,b). Selecting temperature coefficients of other bridge resistors we may partially compensate the influence of temperature on the magnetoresistor and on adaptive elements characteristics. Note that magnetoresistor characteristics may be widely varied by using of the resistor R shown in Fig.6,b with a dotted line between points from which Hall EMF is usually taken off [IO].

Several typical circuits of magnetic analog memory devices in which readout takes place by using a sonde which is a magnetic modulator (MM) or a magnetic amplifier (MA) are given in Fig.7. In such devices the information store can be made of either another magnetic material than the sonde core (Fig.7,a,b,c) or of the same material and represent the uniform constructive unit with the sonde (Fig.7,d and Fig.8)[3]. In the first case data store core is usually made of a more highcoercive material than the sonde core. This decreases an opportunity of occasional eraseing accumulated information and increases a range of changing of the element transfer coefficient. However this is usually achieved by increasing required writing currents and the element cost.

It should be noted that magnetic modulators or amplifiers with ferromagnetic cores used as sondes allow to remove completely the demagnetizing field which appears when galvanometric devices are used (Fig.6).

A toroidal core (Fig.7,a) made of a soft magnetic material is in principle a very simple magnetic transducer (MT) controlled by a "permanent" magnet which at the same time functions as a data storage device (DSD). At the supply voltage $u = U_m \sin \omega t$ where ω = const. and $U_m \sim x$ the amplitude value I_m of the load current i rises with the increase of the residual flux value Φ_o of the storage core. However the amplitude, r.m.s. or mean value of the current i changes in proportion to U_m only when the a.c. induction component B_m in toroidal core is relatively small as compared with the saturation induction. When the values of U_m are considerably high, the dependence $I(U_m)$ becomes nonlinear though it retains monotone character. The readout in the circuit (Fig.7,a) may also be performed by means of selecting either the combination frequency $(\omega_1 + \omega_2)$ (as it is done in the circuit of Fig. 2,a) or the double frequency (Fig.2,b) (see ref.[3], page 486). In this case the sonde is in principle a magnetic modulator.

It is also possible to use the core made of a soft magnetic material (Fig.7,a) for building a controlled coil of mutual inductance or a transformer (Fig.5). The circuit of such an element is shown in Fig.7,b where another design approach of the storage device is represented. In comparison with the circuit shown in Fig.7,a the circuit shown in Fig.7,b provides for a wider range of changing the mutual inductance coefficient of the windings W_1 and W_2. Besides the bias signal is not required in this circuit and it also allows for a wider range of changing the amplitude of the current i_1. It is also possible to apply the circuitry (Fig.5,b,g) to the element shown in Fig.7,b.

The sonde in the circuit (Fig.7,c) functions as a magnetic amplifier with self-saturation controlled by the residual flux of the storage core. This circuit is used as a rule for attaining high output power of the memory element. Even when both cores (the storage core and the sonde one) are combined in construction and are made of the same material (Fig.7,d) it is practically impossible to erase the data

(Fig.7,c) written in the part of the core functioning as a memory device [3]. If the combined core with rectangular hysteresis loop is used as it is shown in Fig.7,d, the load current I_L is not a linear function of the voltage amplitude U. Thus for building an adaptive element for analog signals it is necessary to use one of the circuits shown in Fig.I or Fig.3.

If the readout from the core (Fig.7,d) is carried out according to the circuit shown in Fig.7,b and the amplitude of the readout current i_1 is permanent, there is an incident part in the dependence of the output voltage mean value E from Φ_o (see curve I, Fig.8,c). The application of three-hole cores (Fig.8) helps to remove this incident part (Fig. 8,c, curve II)[3]. The D.C. bias current I_B in the winding W_B (Fig.8) prevents the magnetic flux in this arm from variating.

Now let us consider some possibilities of designing adaptive elements for realizing the adaptation function according to the circuit shown in Fig.I. The components shown in Fig.7,c,d and Fig.8,a,b are used for this purpose. In the circuit (Fig.9,a) the analog memory element made of one core (see the circuit in Fig.7,c) functions at the same time as a pulse-width modulator. For this reason the voltage readout is taken from the rectangular waveform voltage source with the permanent amplitude. The readout current in the resistor R has a waveform shown in Fig.9,b. While the voltage U_R impressed to the winding W_1 calls for the change of the magnetic flux of the arm, the current i_r in this winding is small. As soon as the arm is saturated the current i_r is increased jumpwise. The value $\omega t = \alpha$ at which the arm is saturated depends on the residual flux Φ_c and can be changed in the range from 0 to π. The current i_r is partially used for controlling the transistor switch. If the values of i_r are low, the transistors are switched off by the bias current. They are switched on only in the intervals $k\pi + \alpha \leq \omega t \leq (k+1)\pi$ where $k = 0, I, 2...$ The mean value of the load voltage is

$$U_L = \frac{\pi - \alpha}{\pi} U = F(\Phi_o)\, U,$$

where $u \sim x$ is an analog bipolar signal. The adaptive element (Fig.9,c) is based on the same principle [I2]. The application of the three-hole core and special compensating windings W_c (which make the erasing of the residual flux impossible) provides for a more stable behaviour of the circuitry. The output voltage of the memory element applied to the emitter-base of the transistors is shown in Fig.9,d. Thus for the circuit in consideration.

$$u_L = \frac{\alpha}{\pi} u = F(\Phi_o) u$$

In the multiplier according to Fig.I good linearity is required only for the channel X. This property permits to simplify the multiplier considerably and in particular to use different types of controlled resistors for performing the multiplying function.

The thermistor T in Fig.IO,a controlled by a memory element is used as a controlled resistor. Here r_h stands for the resistance of the heater and R is controlled resistance of the thermistor. The basic advantages of this circuit are its simplicity and considerable bandwidth of the controlled channel X. The shortcomings are: great inertia of the controlling channel Y and significant dependence of the resistance R on the environment temperature. It is evident that the current in resistor R must not change the resistors temperature very much. A partial temperature compensation may be attained by putting R into the bridge circuit (Fig.6,b) and by choosing the correspondent temperature coefficients of the other bridge resistances. Another design approach of temperature compensation is the series connection of two thermistors as it is shown in Fig.IO,b.

It is possible to apply other types of controlled resistors [I3]. Field-effect transistors (FET) are of great interest in this respect [I4].

From the above mentioned principles of design and the circuitry of adaptive elements for continuous signals it follows that the properties of toroidal and multi-aperture magnetic cores with rectangular hysteresis loop are such that it is possible to build adaptive elements with different characteristics. The simpliest adaptive elements are attained in the cases when the continuous transferred signal is either the a.c. current (or voltage) of fixed frequency and of changing amplitude or the a.c. current of fixed amplitude and of changing frequency or at last periodic width-modulated pulses. In such cases the functions of the analog memory and of the multiplier necessary for building adaptive elements are performed on the same cores.

More universal adaptive elements suitable for analog signals of any type may be built by using the output signal of the magnetic analog memory element for changing the active resistance of the controlled resistor.

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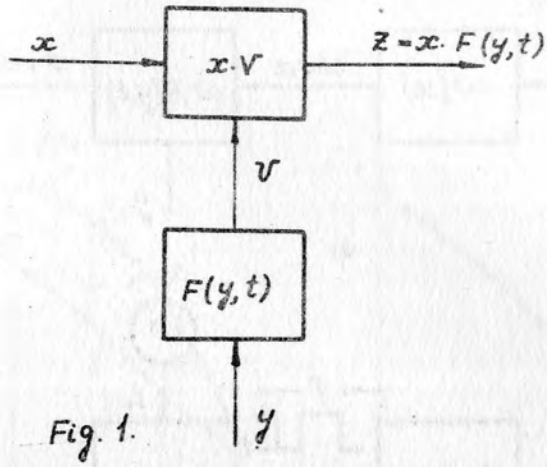
x
x·V
z = x·F(y,t)
V
F(y,t)
y

Fig. 1.

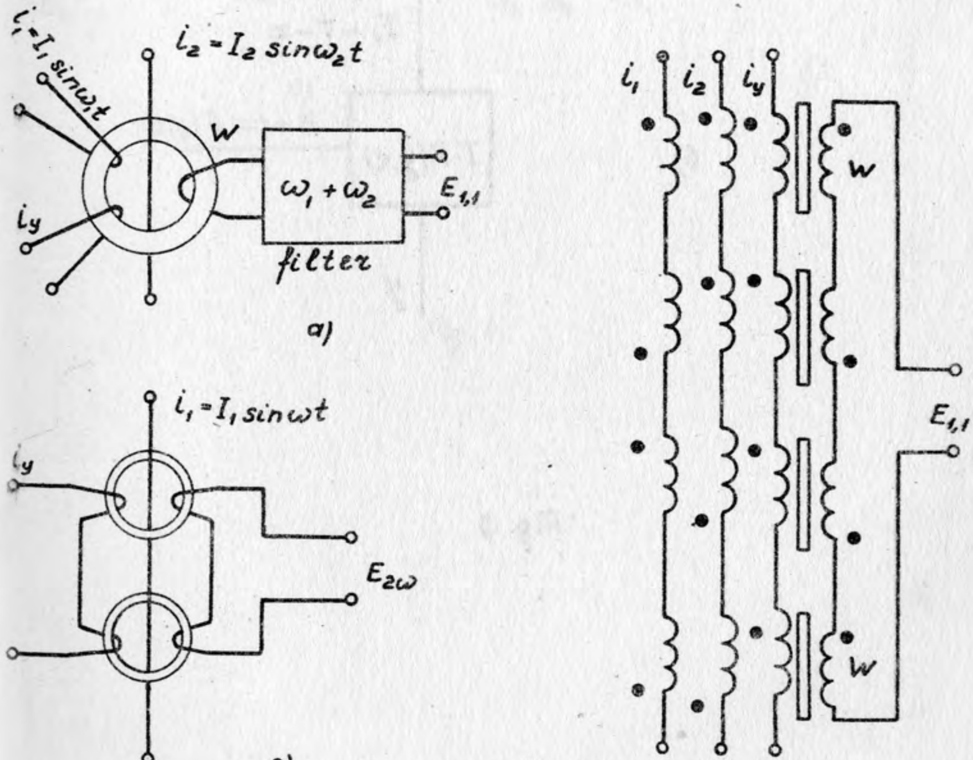
i₁ = I₁ sinω₁t
i₂ = I₂ sinω₂t
W
iy
ω₁ + ω₂
E₁,₁
filter
a)
i₁ = I₁ sinωt
iy
E₂ω
c)
i₁
i₂
iy
W
E₁,₁
W
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Fig. 2

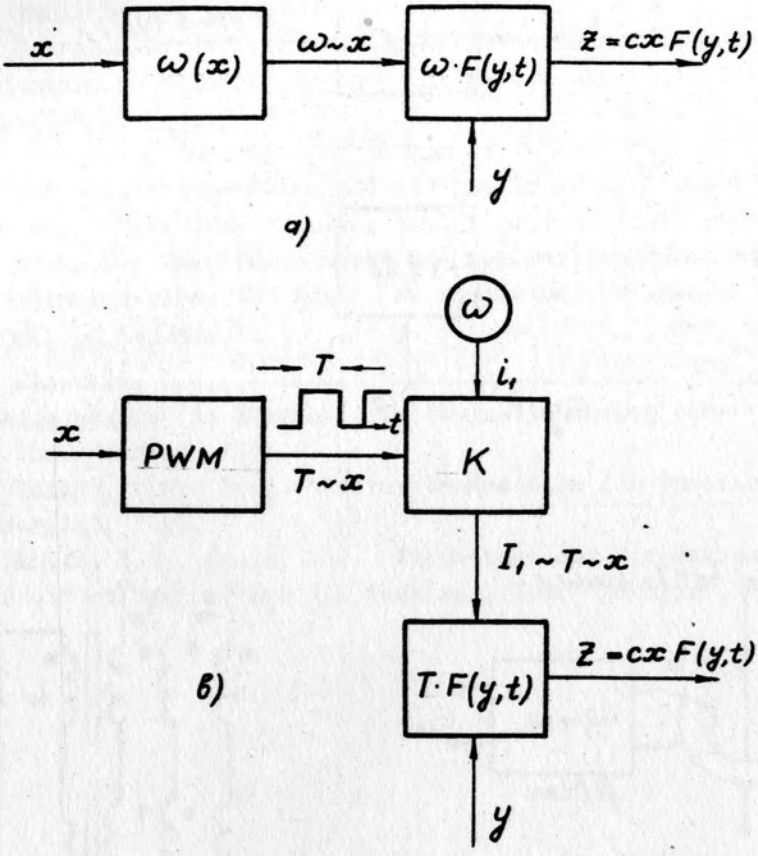

Fig. 3

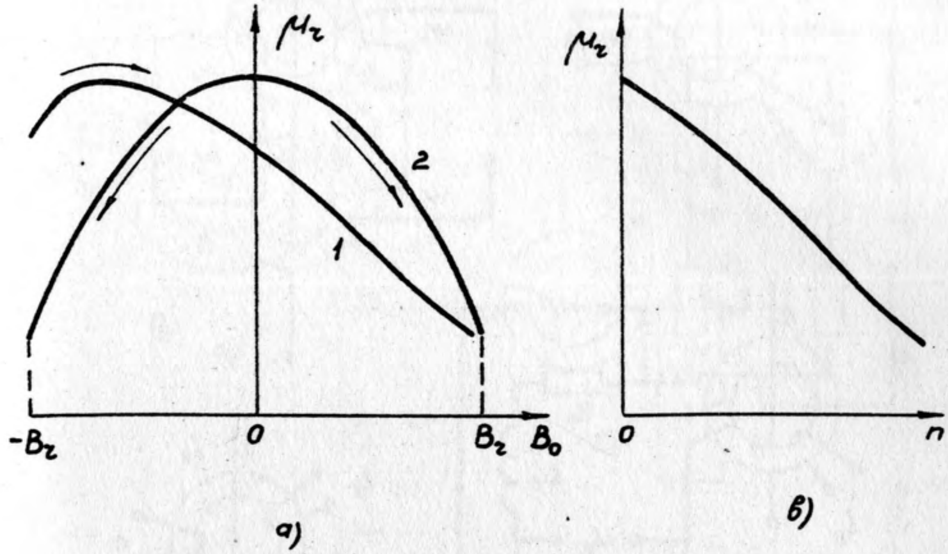

Fig. 4

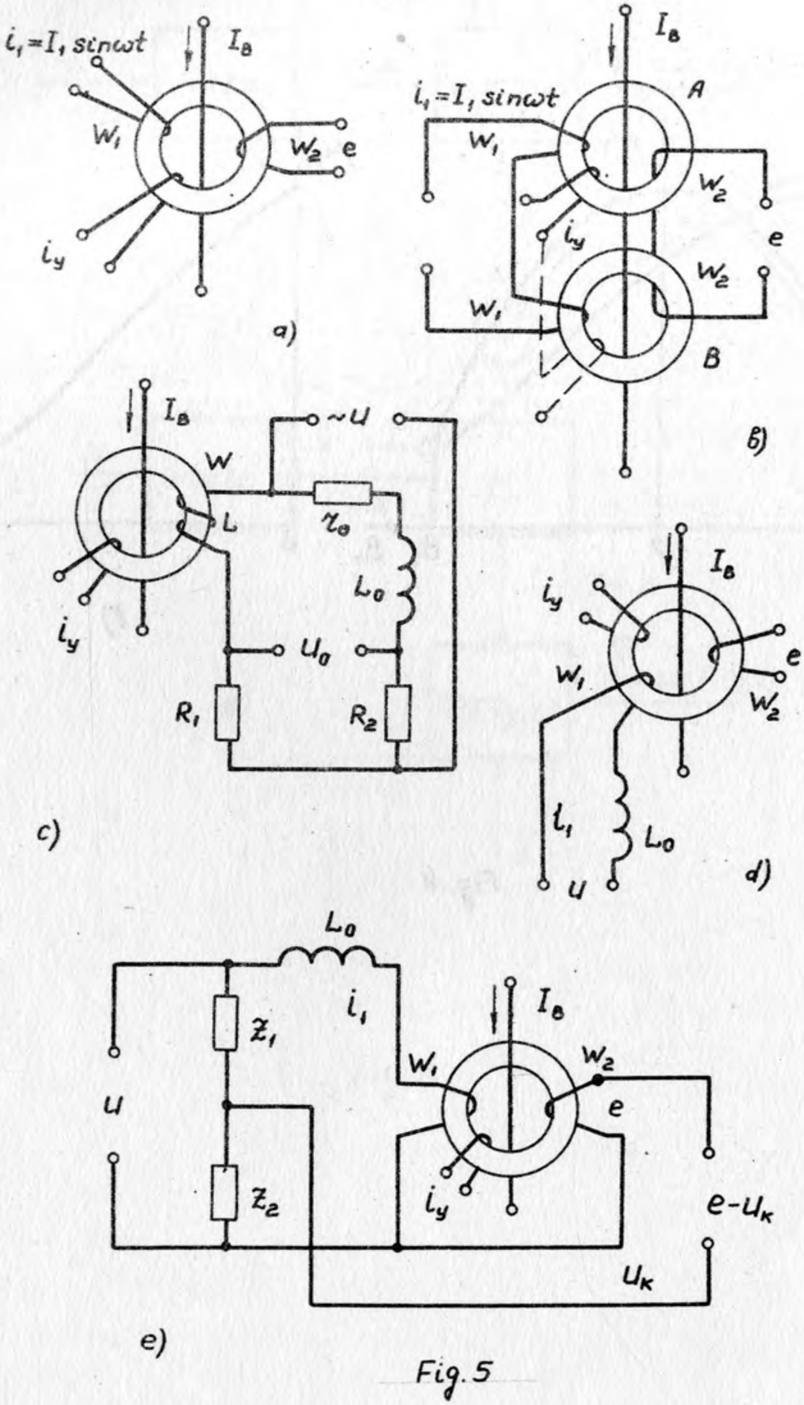
i₁=I₁ sinωt
I_B
W₁
W₂
e
i_y
a)
A
B
б)
W
L
~U
r₀
L₀
U₀
R₁
R₂
c)
i₁
U
d)
Z₁
Z₂
e-U_K
U_K
e)

Fig. 5

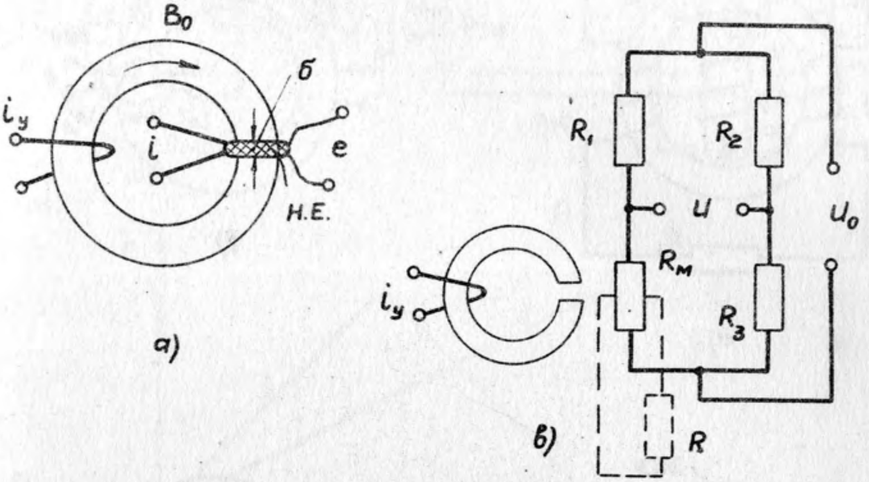

Fig. 6

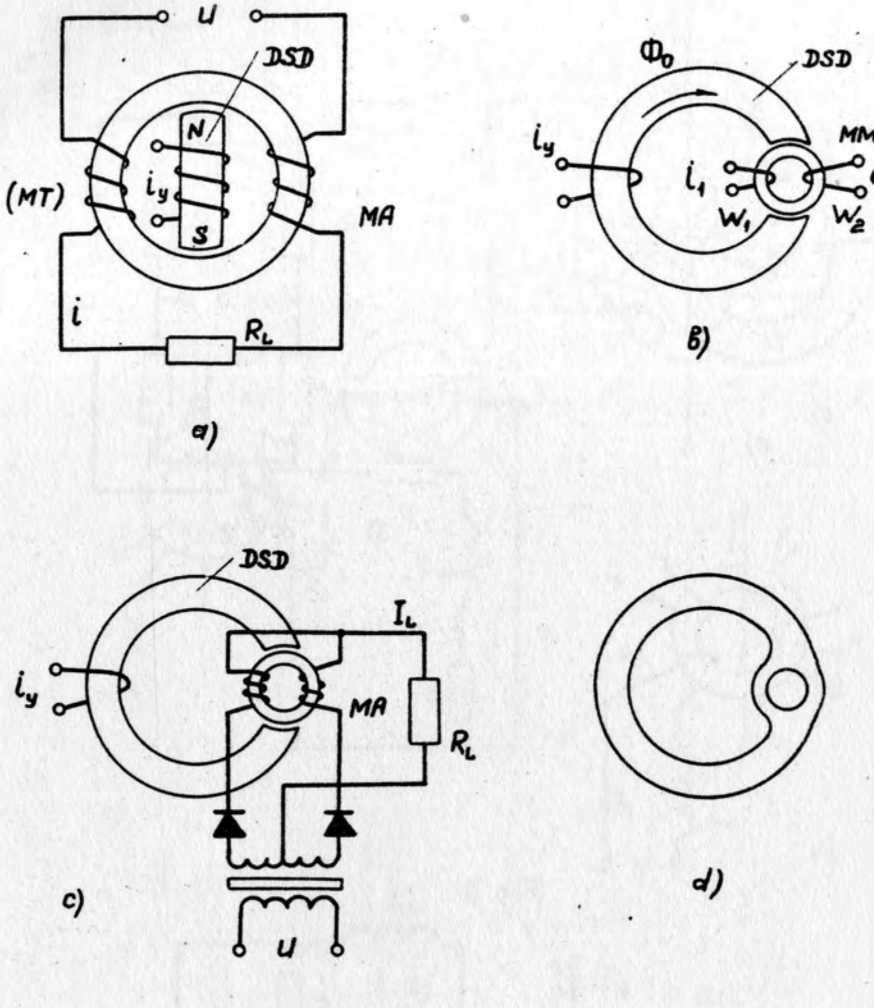

Fig. 7

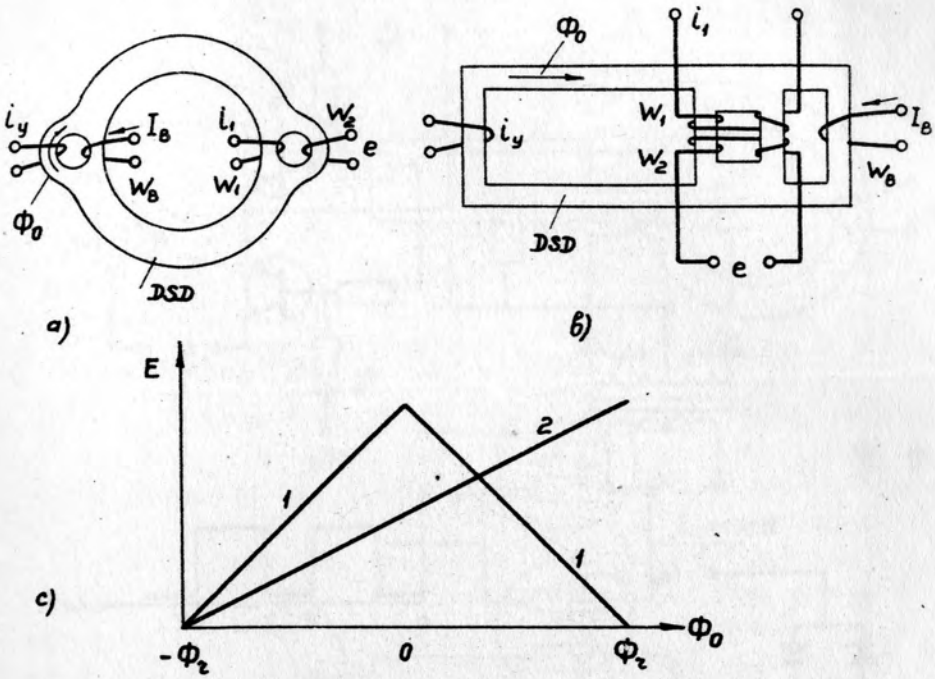
i_y
I_B
i_1
W_2
e
W_B
W_1
Φ_0
DSD
a)
Φ_0
i_1
W_1
i_y
W_2
I_B
W_B
DSD
e
b)
E
2
1
1
c)
$-\Phi_r$
0
Φ_r
Φ_0

Fig. 8

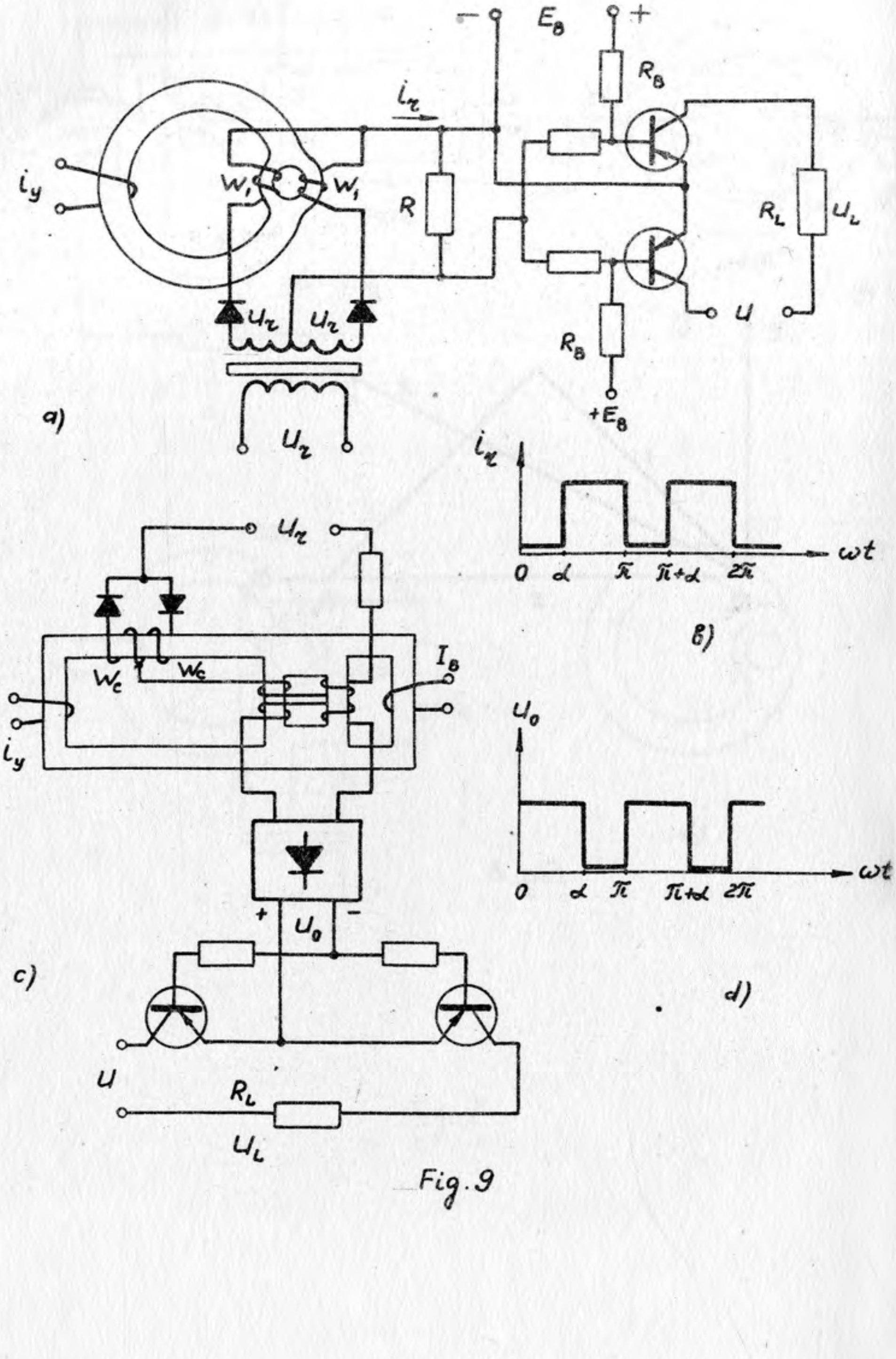
a)
E_B
R_B
i_r
i_y
W_1
W_1
R
R_L
U_L
U_r
U
$+E_B$
i_r
ωt
0
α
π
$\pi+\alpha$
2π
b)
U_r
W_c
W_c
I_B
i_y
U_0
c)
R_L
U_L
d)

Fig. 9

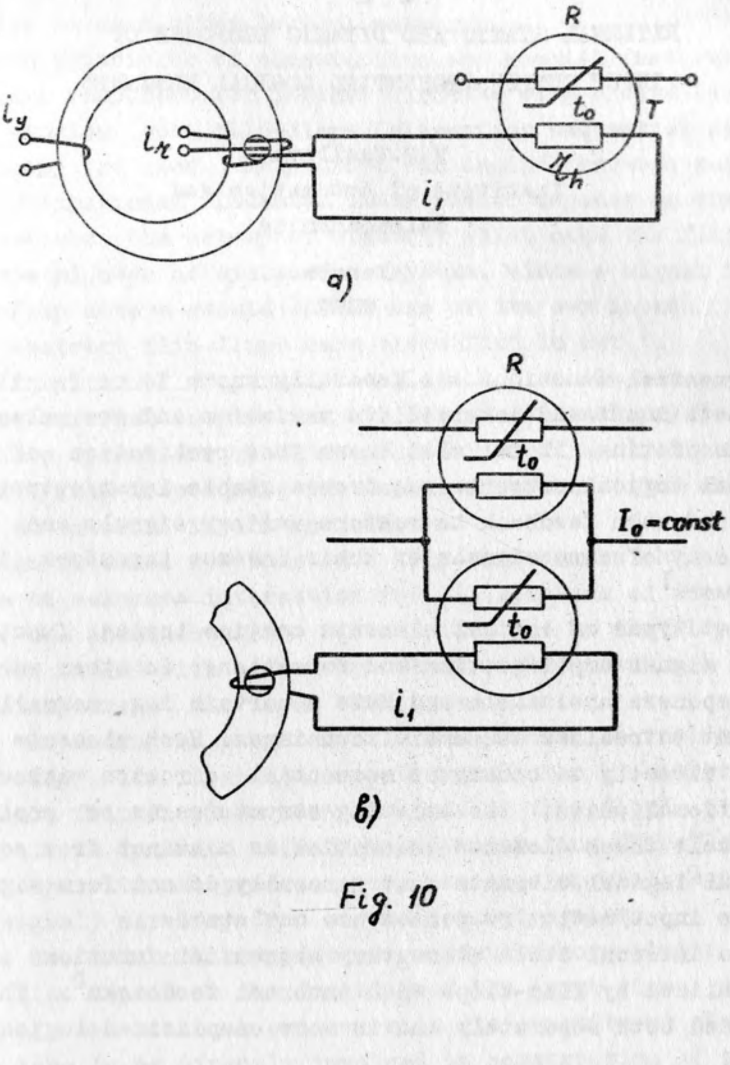

Fig. 10

8.2

EXTERNAL STATIC AND DYNAMIC RESPONSE OF INPUT OUTPUT SEQUENTIAL LOGICAL ELEMENTS

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Sequential functions are generally known to be functions of both input and intermediate variables and are also functions of time. It is also known that realization of sequential logical networks requires a stable input/output response of each feedback to restore unitary signals and ensure decay of zero signals or their inverse transform in that network[1].

Most types of logical elements comgine logical functions with signal amplification and formation; in other words their responses are stable and have a certain lag, normally sufficient to realize sequential functions. Such elements can be used directly to construct sequential circuits without any additional units; the existing structures can be employed. We will call these elements sequential as distinct from combinational logical elements that generally do not form signals, or whose input/output responses are not stable.

Two internal state elementary sequential functions are often realized by flip-flops with internal feedbacks[2]. These can be used both separately and in more complicated logical circuits.

Because a flip-flop can be included into any other sequential circuit; the overall stability of this can only be found if the input/output response is known which will be termed external in contrast to internal input/output response of its elements in an open loop. External responses are important to determine stable operation not only of circuits with flip-flop but of other sequential circuits with closed loops incorporated between other logical networks.

The principles of construction and special features of external responses for logical circuits with closed loops are best studied when flip-flops (elementary sequential feedback circuits) are used. Flip-flops can include several sequential and combinational elements. Their number depends on the function realized, the set-up of elements which make the flip-flop and the number of cycles in a system, since a signal from a flip-flop output should act at one of its own inputs.

Abstract flip-flops were classified in Ref [3].

An external response of a flip-flop is the relation of one of its external outputs and one of its external inputs in closed-loop operation. The number of external input/output responses of a flip-flop equals the number of combinations made by signals at external inputs and outputs.

An external response of a flip-flop can be obtained by analysis of response interaction for all elements of the constituents. One should bear in   that one of the flop-flop inputs is loaded with a its own output signal (feedback signal) which appears at the input with a certain lag.

Depending on a system of elements used to realiz the flip-flop , its input and output signals can be either pulsed (dynamic flip-flop) or constant (static flip-flop).

In an dynamic, asynchronously operating flip-flop the time of transitional process (response to step-wise change of input signal) may vary from one to several cycles.

The external response of an asynchronous static flip-flop will also depend on the duration of the input signal and the form of transitional processes in its constituent elements that have to be directly involved in construction of the external characteristic.

In sunchronous flip-flops the maximal response time equals the number of cycles in the system while the transition process in flip-flop switching, when the signals are synchronous, is pure lag. By synchronous signals we understand synchronous pulses not only of a certain length but of a certain magnitude required for complete synchronous switching of a flip-flop .

Analysis of a sunchronous flip-flop operation and representation of its external responses require the knowledge of its internal characteristics both dynamic and static.

Let us illustrate representation of a flip-flop external responses by known input/output responses of its constituent elements.

Let a flip-flop which realizes a memory function $x_{\tau}=(a+x)b$ consist of two sequential two-cycles elements OR and AND (Fig. 1 a). whose responses are given as graphs of Figs 1a and b. They are identical with respect to each of two inputs a and x, b and y :

$$y=\varphi_1(a)=\varphi_1(x)=\varphi_1(a+x)$$
$$x=\varphi_2(b)=\varphi_2(y)=\varphi_2(b+y)$$

Then their overall internal response in terms of the relation between input of OR and output of AND will be

$$x=\varphi_2[\varphi_1(a)]=\varphi_2[\varphi_1(x)] \qquad \text{(Fig.1d)}$$

Let us consider two directly opposite cases of signal inter - action in an OR element which consists of a combinational part which realizes logical summation and the amplifying part which has an iterative (I) response . In the first case the signals are summed on diodes and in the second case they are summed on resistors of windings of cores. In both cases the function AND is realized in diodes with further level amplification as is done in OR elements by amplifiers of any type with an I-res - ponse.

The two cases differ in the nature of summation. The diode summation is essentially closser to logical summation since the output signal of a diode circuit equals the greatest (by modulo) input signal. In summation on resistors or in summation of m.m.f. on cores normal arithmetic summation takes place which then, owing to the amplifier non-linear response (L-response) turns into logical summation. In thus case, however, the response have different slope at different numbers of signals summed.

Difference between in these two ways of summation is of no substantial importance in combinational circuits though it does affect the response curve slope. We will show below that for sequential circuits the two ways of summation result in two different types of external responses.

1. Flip-flop with diode summation of signals.

In representation of the external response $x_\tau = F(a)$ the response $x = \varphi_2[\varphi_1(a)]$ (Fig. 1d) serves as internal response.

At $b = 0$ $x_\tau = F(a) = x_0 = const$ the flip-flop is switched of irrespective of a and $x_{t=0}$.

At $b=1; x_0=0; a \le a_{\kappa p}; x_\tau = \varphi_2[\varphi_1(a)]$ since while $a < a_{\kappa p}$, $x < a$. Consequently does not affect the output potential of the diode circuit OR. At $b = 1$ and $a > a_{\kappa p}$ feedback starts to act and the output signal x_τ . exceedes the signal a . When the signal reaches the input afain it is amplified until the highest point of the response stable equilibrium is reached (see Fig. 1e). The transitional process of switching in a flip-flop may take several cycles. This duration depends on the magnitude of the signal a and can be found by the initial response of the flip-flop. The transitional process will complete in one cycle only at signals very close to a, which represent the highest point of stable equilibrium in the response of an open loop. The static part of the flip-flop response in the point $a_{\kappa p}$ has a discontinuity shown in Fig.1e as thick dash line.

Thus in this case the external dynamic response of a flip-flop depends on the dynamic response of its open loop or; in other words, its internal dynamic response. The presence of a feedback affects only the slope of the external dynamic response curve and does not affect that of the external dynamic response which is equal to the slope of the internal response.

Let us consider other external response of a flip-flop: $x_\tau = \varphi(b)$, in this case in terms of the signal b, $\tau = t_T$. Let us recall that the function AND which is first realized in diodes and then amplified; therefore it is the smaller signal of the two input signals that is amplified.

At $a = 1$ and $y = 1$, $x_\tau = \varphi(b) = \varphi_2(b)$ irrespective of the value $x = 0 \div 1$, i.e. the output characteristic in this case coincides with the internal characteristic of the element AND (Fig. 1 without horizontal branch Fig. 1f).

At $a = 0$, $x_{t=0} = 1$; $x_\tau = \varphi(b) = \varphi_2(b)$ while the signal $y = \varphi_1[\varphi_2(b)]$ exceeds b , because the diode circuit AND transmitt the least of its inputs potentials y and b . When

$y=\varphi_1[\varphi_2(b)]$ becomesless than b, i.e. at $b<b_{\kappa p}$ of the upper curve in Fig. 1g, x_τ becomes a function of y; $x_\tau=\varphi_2(y)$ and at constant b $y_\tau=\varphi_1[\varphi_2(y)]$ begins to diminish at each cycle since at $b<b_{\kappa p}$ a feedback acts and the flip-flop switches of in several cycles. Its dynamic characteristic for signals $b<b_{\kappa p}$ depends on the internal response of the pair AND-OR. The static characteristic has a discontinuity in the point $x_\tau=\varphi_2(b)$ where $b=b_{\kappa p}$ is determined by the response. b_κ $y_\tau=\varphi_1[\varphi_2(b)]$ and generally does not coincide with the point of unstable equilibrium of the response $\varphi_2(b)$. Thus at signals $b>b_{\kappa p}$ the output response of a flip-flop for the output x is found by the internal response of the element AND $x_\tau=\varphi_2(b)$ and for the output y by the internal response of the pair AND-OR $y_\tau=\varphi_1[\varphi_2(b)]$. At signals $b<b_{\kappa p}$ where $b_{\kappa p}$ is the point of unstable equilibrium of the response of that pair; the flip-flop static response has a discontinuity due to the action of feedback. The slope of the flip-flop dynamic response does not increase. However, for the transitional process not to exeeed one cycle, the signal for disconnection of b should fall to a value close to b_0.

Comparison of all external response of a flip-flop obtained, Fig. 1 d,f,g shows that the smallest slope and stability margin are the properties of the response which is formed by $\varphi_2(b)$ yust one element AND when the disconnectiпg signal b is controlled. It is this characteristic which determines the stability margin at the input b. If the output signal y of a flip-flop is also used separately, the worst response could be $y_\tau=\varphi_1(a)$ (Fig. 1b).

2. A flip-flop with arithmetic summation of signals (Fig. 1a).

The external response of that flip-flop, $x_\tau=F(a)$, is also reconstructed by the internal response of the pair $x=\varphi_2[\varphi_1(a)]$; however, the signal x should also be taken into consider ation which is summed with a by positive feedback law:

$$x_\tau=\varphi(a+x)$$

Fig. 2 shows the known graphical method for obtaining the characteristic $x_\tau=F(a)$ if the response $x=\varphi(a+x)$ is known which is in this case the internal response of the pair OR-AND:

$$x = \varphi_2\,[\varphi_1(a+x)]$$

The slope of this characteristic increases rapidly in comparison with the initial one and is even negative. At signal $x = a$ such that $\frac{dy}{dx} = 1$ the static response (dash line in Fig.2) has a discontinuity. The magnitude a_1 is always smaller than the magnitude of $a_{кр}$ therefore the external response is shifted much to the left of the initial one. The margin of zero stability of such a flip-flop is much less than that of its constituent elements . It may even be negative.

The effect of the response of the flip-flop discussed relative inputs b does not differ at all from the external characteristics of a flip-flop with diode summation of signals because at $a = 1$ (Fig. 1f). the signal $b<y$ because $y = \varphi_1(a+x) > 1$ Thus, the magnitude of the output signal of the diode part of AND is determined by the magnitude of b . At $a = 0$, Fig. 1g, the way in which signals are summed in element OR does not affect the form of characteristics at all. Comparison of a flip-flop external responses with arithmetic signal summation of Fig.2 and Fig. 1f and g leads to the conclusion that the smallest stability margin can be either in a response with positive feedback; Fig. 2, or that characteristic at $a = 1$, i.e. responses of the element AND. A flip-flop should evidently be calculated and monitored by critical points of these two responses.

An ther example is representation of external responses of a flip-flop which is memory on two inverters $x_\tau = \overline{(\overline{a+x})+b}$ The schematics is shown in Fig. 3a. The case of diode summation is prezented in Fig. 3b,c,d; of arithmetic summation in Figs 4 a,b.

3. External characteristics of a synchronous flip-flop.

Let us discuss representation of external responses of a synchronous flip-flop ;more specifically, a counting flip-flop type $x_\tau = ax + \bar{a}\bar{x}$. This, as other flip)flops, can be made of most various elements to obtain different responses. We will reconstruct the characterustics of a flip-flop made of most various elements to obtain different responses. We will reconstruct only the characteristics of a flip-flop made of magnetic-diode equivalence elements, because their representation and form are typical cases o diode logic with pulse amplifiers of different forms. Counting flip-flops are intended for counting each pulse which appears at its counting input (or passing pulses at each control cycle). Therefore a dynamic counting flip-flop should pperate only in synchronous condition and static response is meaningless for it. Dynamic characteristics are the relations between the magnitude of an output pulse and the magnitude of input signals in one cycle. But a feedback signal appears at the input only in a cycle which follows the one where the initial input signal was active. Therfore external dynamic responses of flip-flops are despite the presence of a feddback circuit responses of open loops whose one input signal depends on the flip-flop state in the previous cycle.

The schematics of the flip-flop discussed is shown in Fig; 5 a. The upper group of hatched diodes perform the function of logical multiplication; the potential at its output is always equal to the smallest (by modulo) potential of the input a and x.

By the expression $x_{\tau} = ax + \bar{a}\bar{x}$ if the signals a and x are equal the potentials of the upper and the lower groups of diodes are equal, the cores are not remagnetized and the output equals 1. If one of the signals a or x equals zero the potential of the upper group of diodes also equals zero, the potential of the lower group is maximal, the core remagnetizes and the output is zero.

The value of the input signal x can be considered as a parameter for each characteristic which will reoresent the relation between the input signal and the output signal at constant value of the feedback signal.

We can also treat the response as the relation b/etween the value of the output signal and the feedback signal x at constant value of the signal a; in other words a can be treated as a parameter because the signals a and x are completely equal and symmetrical.

Thus $x_\tau = f(a, x)$

Let x be equal to zero then at $a = 0$, x_τ is equal to 1 1. When the signal a increases (each time we take a new pulse with an increased space while x is each time assumed zero) the demagnetizing potential will increase and x_τ will decrease.

The shape of the input/output response depends on the response of the chain: inverse response of the core and the iterative response of the element OR included in the flip-flop, Fig. 5 b.

At $x = 0.5$ the output reaches the value 1 when a=0;5; at the values of signals $a = 0$ and $a = 1$ equal demagnetizing potentials will appear that are half the maximum. Therefore x_τ is also 0.5 (the central response curve of Fig. 5b).

At $x = 1$ and $a = 0$, x_τ is zero; when the signal a increases; x_τ also increases while at $a = 1$ x_τ is one; this means that we obtain the iterative response.

Thus depending on the magnitude of the feedback signal the flip-flop input/output response from inverse turns int/o iterative, passing all intermediate stages as shown in Fig.5b.

By the family of dynamic responses obtained we can analyses all possible cases of the flip-flop operation and malfunctions. E.g. let at $x = 1$ the flip-flop receive the signal $a = 0.5$, then, by Fig. 5b in the following half-period $x \approx 0.5$. at the output. Suppose the signal $a = 0.5$ is again received; but now we have $x = 0.5$, therefore $x_\tau = 1$, etc. Consequently; if a series of signals of the magnitude L.5 are fed to the flip-flop we will have a series of cycles 0.5 ; 1; 0.5; 1, etc. at the output.

Treating the input parameter a as a parameter we can obtain a family of characteristics $x_\tau = f(x)$ at various values of a (Fig. 5c). This family of characteristics is convenient in that a minimal signal a can be found at which the flip-flop operated without malfunctions, the evidence of which is its stable characteristic. In Fig. 5c the response where a =0.85 is also such a response.

It is evident that the relation between the flip-flop output signal and the state of its inputs can most illustratively be represented as a surface where the height of each point depends on the values of a and x , Fig. 5d. This surface is bounded in space by a cube with sides equal to +1. The family of response discussed above was obtained when that surface intersected with planes parallel to $x_\tau x$ (Fig.5 c) and parallel to $x_\tau a$ (Fig.5 b).

For operation without malfuctions at zero and one as signals all sections of that surface by faces of a cube should have the form of stable response curve, while for operation without malfunctions at input signals greater than zero and less than one they should have a stable form of section by planes that go along the appropriate axes at appropriate di distances. These conditions can be extended to dynamic flip-flops with the number of vatiables above 2 whose input/output responses will be hypersurfaces in an n-dimensional space where (n-1) is the number of the flip-flop inputs. The section of the hyperplanes bu faces of n-dimensional cubes with coordinates of the vertices 0,1 should be stable input/output responses (meet the conditions of stability), while the sectionsof these hypersurfaces by planes parallel to planes of n-dimensional cubes and going through these cubes enable to determine the permissible errors between the signals and zeroes and ones when the operation of flip-flops is still stable.

The methods of representing the external responses in flip-flop can evidently be extended to other types of sequential circuits which are closed loops of elements connected in series.

This is also true for the properties introduced by the way in which signals are summed in feedback loops. These properties can perhaps be assumed independent of the number of states in elements connected in series and will affect equally the external responses istics of both flip-flops and other types of sequential circuits with feedback loops whose signals are summed with external signals.

C o n c l u s i o n s

The analysis which involved several flip-flop typical of sequential circuits most wifely used in industry seems to show that:

1. To determine the stability of sequential circuits it is generally essential to know, apart from the responses of open loops (internal responses) of elements, the characteristic of clossed loops of elements relative the external input and output signals, i.e; the external responses.

2. The form of dynamic and static responses of flip-flops and other sequential circuits is substantially dependet on the way in which summation is performed in feedback loops. In logical elements there are two kinds of summation which make two kinds of feedback.

a) Diode summation (akin to logical summation) at which the output signal of the element summing part is equeal to the magnitude of the greatest input signal and does not depend on the magnitude of other input signals.

b) Arithmetic summation.

3. In diode summation of input signals in flip-flops and other sequential circuits feedback signals do not increase the gain of external dynamic responses in comparison with the gain in internal of these circuits.

In external static responses of flip-flops and other sequential circuits a relay discontinuity can appear where This discontinuity appears in the internal dynamic characteristic unstable equilibrium point in a chain of elements that make the circuit which receives the control signal.

The stability margin of sequential sustems is determined by the worst of the internal responses, or responses of separate sequential elements and decade-type chains which make the circuit.

4. In arithmetic summation of input signals inflip-flops and other sequential circuits the feedback signals increase drast drastic ally the gain of their external dynamic responses so that an unstable region may appear in the responses as well as negative slope and a relay discontinuity. This decreases the stability margin of the circuit in zero and one.

Static responses of such flip-flops have a relay discontinuit at signals which are far less than the abscissa of flip-flops initial internal responses instable equilibrium point. Therefore the worst responses of flip-flops with arithmetic summation are their external responses whose critical points must be used in flip-flops monitoring.

5. In practical operation of flip-flops with diode summation, the summation of signals can be closer to arithmetic summation than to the ideal diode summation. This happens due to high resistance in signals sources and non-ideal properties of diodes. Therefore in such flip-flops the external responses are to be monitored equally with internal responses.

6. Static and dynamic responses of synchronous flip-flops should not differ more than the feasible values of zero and one signals. Therefore not only pulses of a certain duration whose rhytm coincides with that of the flip-flop operation can be treated as synchronous signals, but also pulses of a certain magnitude sufficient for complete switching of flip-flops in one cycle.

7. External dynamic responses of dynamic flip-flops used only in synchronous operation, in particular of counting flip-flops coincide with internal dynamic responses of these flip-flops which are functions of all flip-flop inputs including the feedback input.

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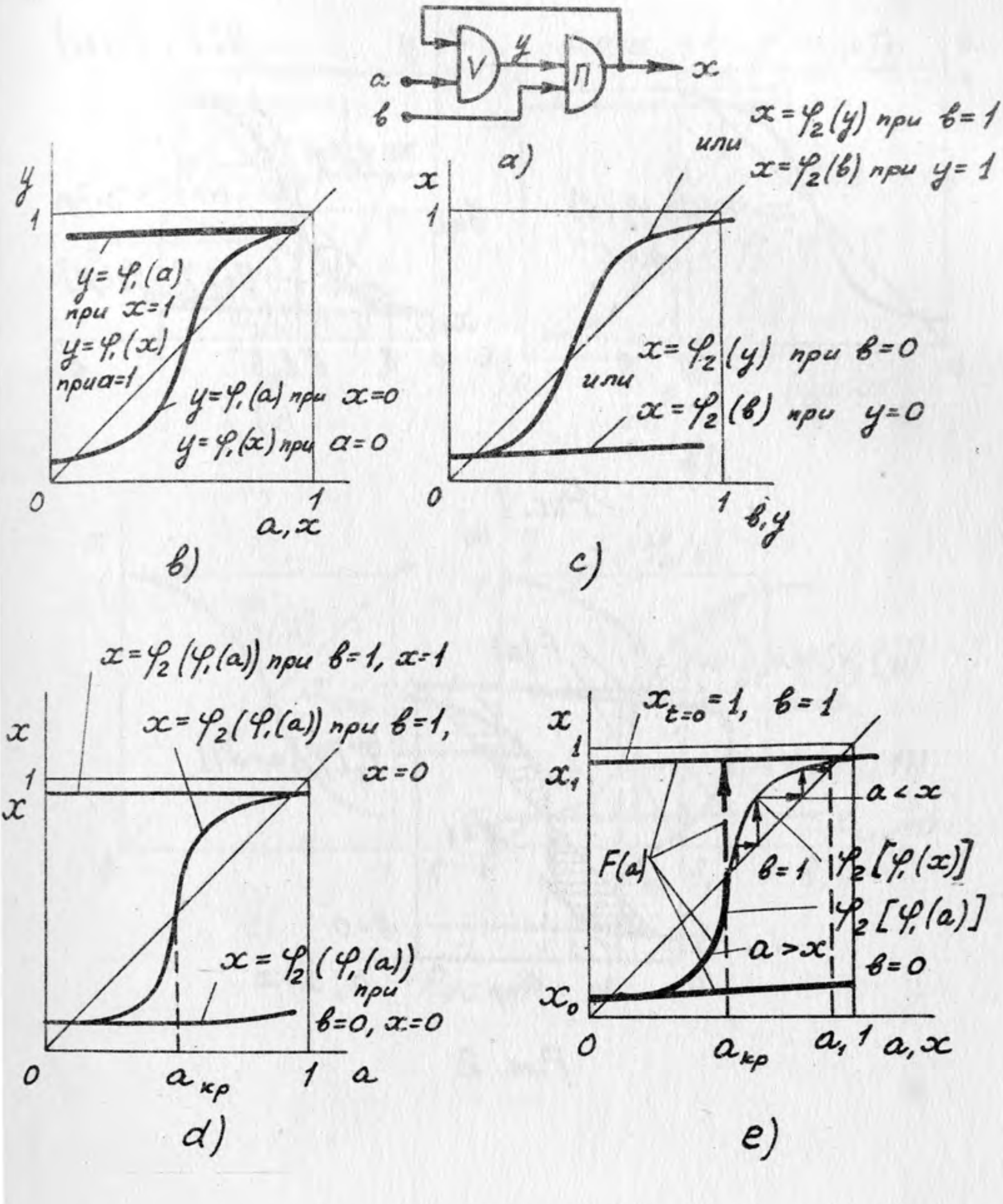

y
a
в
V
П
x
a)
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или
$x=\varphi_2(в)$ при $y=1$
1
$y=\varphi_1(a)$
при $x=1$
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при $a=1$
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в)
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или
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в, y
c)
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$x=\varphi_2(\varphi_1(a))$
при
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$a_{кр}$
a
d)
$x_{t=0}=1$, $в=1$
x_1
$a<x$
F(a)
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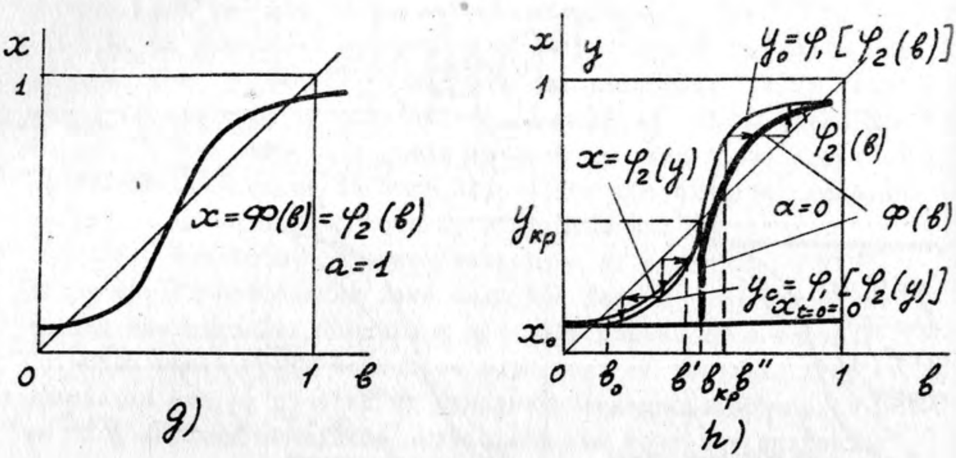
x_0
a_1
e)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в
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x=φ2(y)
α=0
Φ(в)
yкр
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в'
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в''
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в
h)

Рис. 1

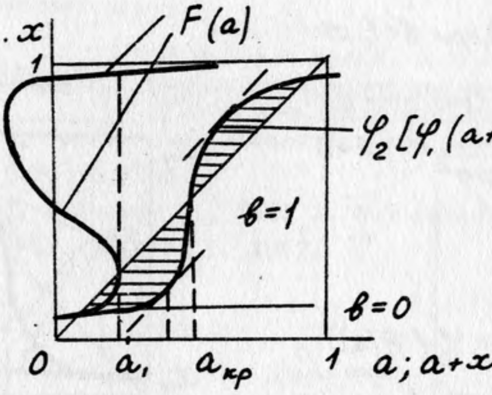
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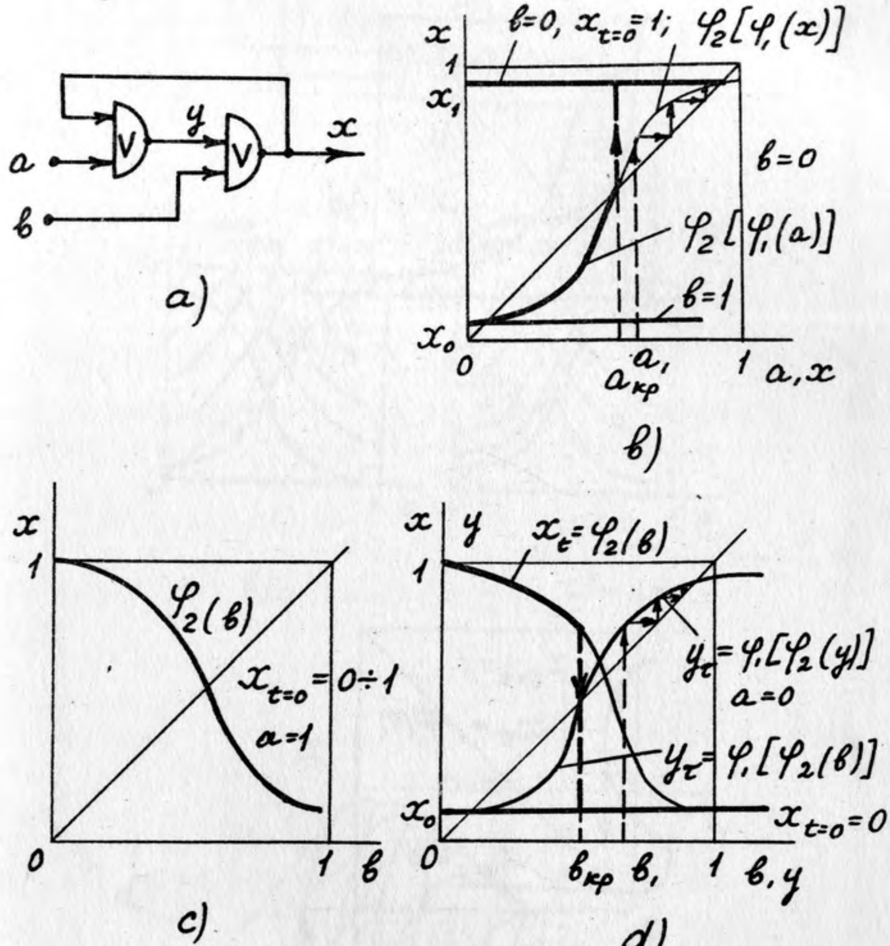
Рис. 2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b
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a)
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b=1
x₀
0
a₁
a_кр
1
a, x
в)
x
1
φ₂(b)
x_{t=0}=0÷1
a=1
0
1
b
c)
x
y
x_t=φ₂(b)
1
y_t=φ₁[φ₂(y)]
a=0
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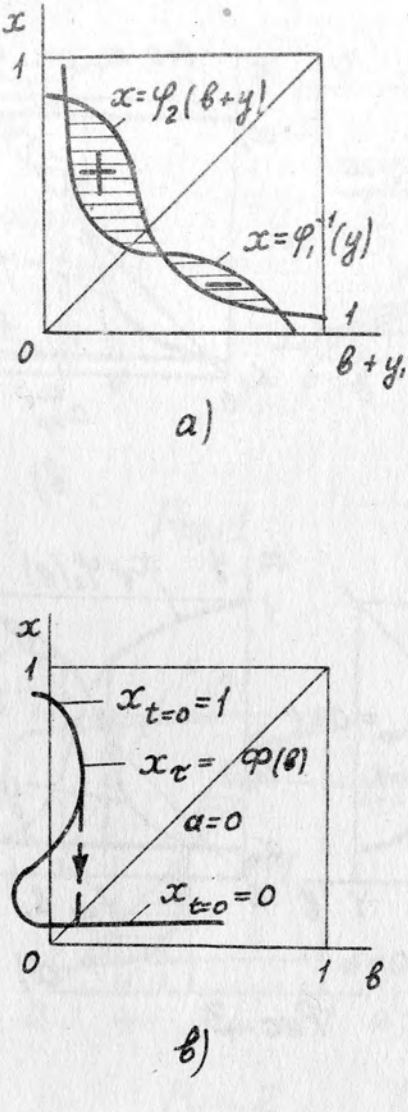
b₁
1
b, y
d)

Рис. 3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−
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a)
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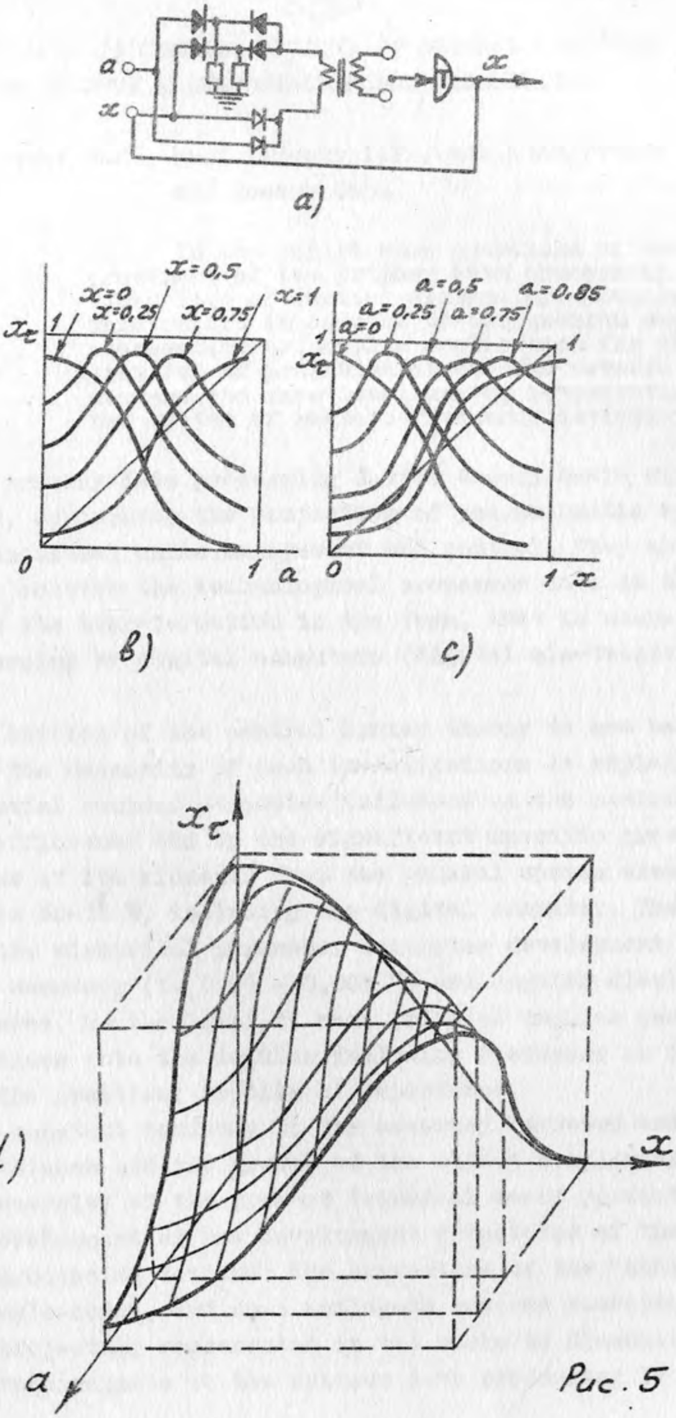
0
1
b
b)

Рис. 4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x
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a)
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x_τ
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0
1
a
в)
a=0,5
a=0,85
a=0
a=0,25
a=0,75
x_τ
1
1
0
1
x
c)
x_τ
x
d)
a

Рис. 5

8.3

PRIMARY DATA PROCESSING DEVICES OF CONTROL COMPUTERS ON QUANTUM MAGNETOMEASUREMENT PRINCIPLES

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In the report some questions of the improvement of the primary data processing devices loop of control systems are considered. This report is devoted to the quantum magnetomeasurement principles application for the creation of precision direct and reverse converters and functional units, representing a new system of magneto-frequency devices.

The primary data processing device theory deals with the questions, concerning the projecting of the automatic control system functional units management and control. They are designed to receive the technological processes data in the objects and its transformation in the form, that is convenient for processing by digital computers (digital electronic computer).

The section of the control system theory is now being developed. The necessity of such investigations is explained by the essential contour parameter influence on the controlled devices efficiency and by the significant specific gravity of the number of its elements from the general system element number, up to 60-70 %, including the digital computer. The necessity of the electrical parameter converter development with the high accuracy (to 0,01 - 0,005 %) and angular displacements, for instance, on the level of some units of angular seconds or its fractions into the impulse following frequency or into the code is the practical problem of importance.

The constant tendency to the accuracy increase and control trustworthiness and the growth of the object complexity leads to the necessity of the present technical means perfection and to the development of new development principles of the primary data processing devices: the converters of the "analog-code" type, "angle-code", and some analogous reverse converters.

In projecting represented in the works by Glushkov V.M., general requirements to the contour data processing is defined by the requirements to the systems of higher order. In accordance with it some controlled parameters sets of the object, the general view of the control system work algorithm and its structure scheme are considered to be known (see the first steps of the synthesis algorithm in fig[1,2,3].

According to it some definite possibilities are contained in the synthesis approach of data processing devices on the basis data converter complex projecting, communication lines, comparefors and commutators. However the most radical decisions are the basis of the main functional block construction principles. In improving these construction principles the main attention was paid to their accordance with the system control frequency requirements, which have some essential advantages. For the analysis and comparison the generalized block-diagramm of the automatic control system with a digital computer and unified parameter in the form of the impulse following frequency was taken. The use of precision frequency converters in such control systems permits to decide the following problems:

1. To eliminate too complex comparing devices on the direct current, which reduce the accuracy because of the ageing of the component parameter and standard.

2. To increase the control system reliability by the application of noncontact commutators; they do not reduce the control accuracy in using impulse signals in communication lines.

3. To increase considerably the control system potential accuracy, because the standard frequency stability is higher some orders the standards stability of emf.

4. To simplify the object coupling with the controlled machine, because the frequency data is efficiently converted and coded for the input to the digital computer.

This implies of the considerable influence of the construction frequency method of data-processing devices on all synthesis steps of primary data processing devices (fig.1). The optimal synthesis sequency is the special research object.

The development of the nuclear spectroscopy principles is the natural direction of the construction principle improvement of frequency converters of the "analog-code" and "angle-code" type, which are the basis of the primary data processing devices loop. The initial devices of the quantum type have been originated from the nuclear spectroscopy (lasers and masers with the radiotechnical range).

The nuclear spectroscopy deals with atomic constants, for instance, in the form of the gyromagnetic relationship of a nuclear or g- atom factor.[3,9] The use for the convertion process of such constan is the logical development of the frequency convertion technology. It has been earlier reflected in the works on the precision electromechanic converters development with camerton, string and quartz generators[3,4,7].

The analysis of the substance spin particle interaction of different types, which is in various aggregate states with the magnetic and electric field has shown that it is expedient to use some types of the nuclear magnetic resonance nonstationary methods for the precision transformer construction. They include a double optical resonance (at normal temperatures) and free precession (in a cryogenic version), as far as they give the precision measurement possibility of weak and supperweak magnetic fields. The latter promotes frequency quantum converter creation with the rather high input resistance, small line width and the absolute output performance linearity:[3,9]

$$f = K \cdot H_{\Sigma} , \qquad (1)$$

where H_{Σ} - the strength modulus of the magnetic field made by controlled signals in the spin system zone;

f - the frequency on the converter output.

The signal frequencies received at the spin system output are in the range of some kHz to 1-1.5 mHz[2,10] . This range is convenient for the quick-acting automatic control system designing.

The figure 2 shows the block-diagramm of the quantum converter, which is steady to the outer disturbances. The converter is constructed according to the differential scheme, based on two frequency sensors (1 and 7).

The double optical resonance atom phenomenon of one of alkali metals (rubidium, cesium, potassium) is used in quantum frequency sensors. These metals vapours are in the closed thermostatic absorption chamber 4 (Fig. 3)[9,11]. The absorption chamber is radiated by the radiation source I on the resonance wavelength with the polarized light through the polarizator 2. The light flux intensity fixed by the photodiode 6 at the absorption chamber exit is changed, in case the strength with the frequency (I) is given on the auxiliary solenoid 5.

The converter differential diagram allows the Earth magnetic field influence to be excluded and the requirements to the system screening factor should be considerably reduced. In some cases the screen may be excluded completely[11].

Each frequency sensor may be covered by the positive feedback network with amplifiers 13 and 15, and as a result of it the harmonic stresses of quite definite, equal frequencies are generated in the outputs with the work steady-state and in the absence of the input signal.

The rest blocks are not used in the regime of electric signal conversion.

According to the indicated scheme the frequency measuring instrument of phase shifts can also be designed, it each sensor is exited by the signals with compared phases, from the block 19 (Fig. 2).

In feeding the magnetic loop in the shape of the Gelmgoltz rings, 4 or 3 of one of the sensors to the controlled current source the frequency of the sensor is changed. The constant of the exitation magnetic loop in using the Gelmgoltz rings on quartz frames can have the stability of 2.10^{-5} order and even better. As a result of it the sensor frequency deviation and the difference frequency in the differential scheme output 17 is defined, in general, by the controlled signals value[6,11,12].

In controlling stresses the main error is introduced by the additional resistance instability, successively included with the Gelmgoltz rings. The use of precision manganin resistances of 0.005 class made in "Vibrator" limits this error by the value $6 \div 7 \cdot 10^{-3}\%$ during some months of the continuous work.

When both frequency sensors are exited by the Gelmgoltz rings 4 and 9 by applying some current, then the turn of one of it relative to its initial state gives the difference frequency signal again. This results from the fact that the auxiliary magnetic fields H_0 and $H_{0\gamma}$ together with the Earth T field form the vector sum $\bar{B}_1$, $\bar{B}_2$ ($\bar{B}_\gamma$) (Fig.4); the vector modulus (such as $\bar{B}_1$ and $\bar{B}_2$) at their mutual displacements appear to be different in each frequency sensor zone.

Since the attained quantum magnetometer sensitivity is not less than 0.01γ($1\gamma = 10^{-5}$ э.), the angle sensitivity at the field of 50000 γ order appears to be equal to 2.10^{-7}, that is the hundreds' of the angle second[5,12].

Thus, the block-diagram of differential type in Fig. 2, in case of another construction, transfers into the relative angle displacement precision measurer of two neighbouring sensors placed side by side.

According to the vector diagram in Fig. 4 the vector modulus change $\bar{B}$ at the turn of one sensor is:

$$\Delta B^{\alpha}=\Delta(B_1-B_2)=\sqrt{\left[H_r^2+H_0^2\left(1-\frac{H_0\Delta\alpha}{H_r+H_0\Delta\alpha}\right)^2\right]\frac{(H_r+H_0\Delta\alpha)^2}{H_r^2}+H_B^2}-B_2, \quad (2)$$

where $\Delta\alpha$ - vector turn angle increment H_0 at the horizont plane.

$Hr = T \cos I$, $H_B = T \sin I$, I - angle inclination of the vector T to the plane xoy.

As the assumption to the drawing of the formulae (2) (and below for 3 and 4) it is accepted, that we may have the equality of vectors $\bar{B}_1$ and $\bar{B}_2$ by alignment the system in its initial position (The field tuning is made by the current source 14).

The additional angular displacements of shafts I and 2 on the angles $\Delta\beta$ and $\Delta\gamma$ also give the modulus value change, which is found similarly:

$$\Delta B^{\beta}=\sqrt{\left[H_B^2+H_0^2\left(1-\frac{H_0\Delta\beta}{H_B+H_0\Delta\beta}\right)^2\right]\frac{(H_B+H_0\Delta\beta)^2}{H_B^2}+H_r^2}-B_2 \quad (3)$$

$$\Delta B^{\gamma}=\sqrt{\left[H_B^2+H_{0\gamma}^2\left(1-\frac{H_{0\gamma}\Delta\gamma}{H_B+H_{0\gamma}\Delta\gamma}\right)^2\right]\frac{(H_B+H_{0\gamma}\Delta\gamma)^2}{H_B^2}+H_r^2}-B_{2\gamma} \quad (4)$$

where $H_{0\gamma}$ - field strength made by loops 3 and 8 (the loops 3 and 8 are used only at the γ - angle transformation).

The Fig. 5 shows that in real precision devices when the low angular displacements is transformed, disadvantages of shaft bearings and the presence of insignificant angles $\Delta\beta$ and $\Delta\gamma$ largely influence the angular transformation accuracy $\Delta\alpha$ into the code. Particularly, at $T = 5.10^4$ gamma, $H_0 = 10^4$ gamma, $I = 72^\circ$, the molulus difference values of vectors on the formulae 2 and 3 (the curves α and β) and the modulus difference value and the displacement as over the angle α , as over the angle β (the curves $\alpha + \beta$) at $\alpha = \beta$ and $\alpha = 0.5\beta$.

The extent of the value change ΔB^{α} in terms of β , and ΔB^{β} in terms of γ at the precision measurements is rather high; therefore for the signal separation for angles α and β (or β and γ) it is necessary to apply a special technique.

Using the relationships 2 and 3 we may find, that under the angular control α and β , defined values H_r and H_B

$$\Delta B_1^{\alpha} = \left(\frac{\partial B}{\partial \alpha}\right)_1 \Delta\alpha + \left(\frac{\partial B}{\partial \beta}\right)_1 \Delta\beta \tag{5}$$

and at the changed meaning of one of the support field component H_B (by the current supply to the magnetic system 5).

$$\Delta B_2^{\alpha} = \left(\frac{\partial B}{\partial \alpha}\right)_2 \Delta\alpha + \left(\frac{\partial B}{\partial \beta}\right)_2 \Delta\beta . \tag{6}$$

The meanings of angular increments α and β are taken unchanged.

Correspondingly at the angle control γ :

$$\Delta B_1^{\gamma} = \left(\frac{\partial B}{\partial \alpha}\right)_3 \Delta\alpha + \left(\frac{\partial B}{\partial \gamma}\right)_1 \Delta\gamma \tag{7}$$

$$\Delta B_2^{\gamma} = \left(\frac{\partial B}{\partial \alpha}\right)_4 \Delta\alpha + \left(\frac{\partial B}{\partial \gamma}\right)_2 \Delta\gamma \tag{8}$$

where $\frac{\partial B}{\partial \alpha_i}$, $\frac{\partial B}{\partial \beta_j}$, $\frac{\partial B}{\partial \gamma_K}$ - partial derivatives of the expressions 2,3,4 on the coordinates α , β , γ at the initial and changed value of H_B component.

Differentiating the function (2) over α , and the functions 3 and 4 according over β and γ ., we obtain, that at the partial field compensation the required angle α is defined in general by frequencies Δf_1 and Δf_2 $(\Delta f_i = K\Delta B_i^{\gamma})$ in the following way:

$$\alpha = \frac{1}{K} \cdot \frac{\Delta f_1 \left(\frac{\partial B}{\partial \beta}\right)_2 / \left(\frac{\partial B}{\partial \beta}\right)_1 - \Delta f_2}{\left(\frac{\partial B}{\partial \alpha}\right)_1 \left(\frac{\partial B}{\partial \beta}\right)_2 / \left(\frac{\partial B}{\partial \beta}\right)_1 - \left(\frac{\partial B}{\partial \alpha}\right)_2} \quad (9)$$

The 50 % compensation of the vertical component, if to substitute the derivative meanings from the formula 2 and 3 into 9, gives:

$$\alpha = (3{,}7\Delta f_1 - 2{,}64\Delta f_2) \cdot 10^{-3} \quad (10)$$

If the construction is made thoroughly enough and the shaft beat approaches 0 ($\Delta\beta = \Delta\gamma = 0$), the conversion process of the mutual angular position into the code is substantially simplified. In this case it consists of the measurement of the vector modulus increment ΔB_1 . This measurement is made by the single frequency difference definition of two quantum sensors when the compensative magnetic systems are switched off.

The investigation of the sensor effectiveness increased possibility of the proton-precision type (PPS) by deep cooling of the working substance and receiving elements shows that in such constructions we may have significant advantages over the common sensors of proton-precision magnetometers.

The increase of "signal-to-noise", the quick-action rise, and the sharp power decrease used by the polarization is attained. This is connected with the thermal noise decrease and the increase of the working substance magnetic receptivity in the form of liquid gases[5,13].

The Fig. 6 shows three types of cryogenic measures, the work conditions of which agree with that of creotrons: duar, cooled reception winding, polarizing liquid gas. The mentioned measurers while freezing the field provide the control of instantaneous electrical values[14], the stable oscillation generation[13], and the measurement of the magnetic field components[14].

For calculating of the precision magnetic systems of the above mentioned devices it was developed the common computation algorithm at the arbitrary form of the exited contour envelope $y=f(x)$, based on the common expression for the circular contour strength through Lagandr' polynoms:

$$H_x=\frac{I\sin^2\varphi}{\rho}\sum_{n=1}^{\infty}\left(\frac{r}{\rho_0}\right)^{n-1}P_n'(\cos\psi)\left[\cos\theta P_n(\cos\theta)+\frac{\sin^2\theta}{n}P_n'(\cos\theta)\right] \quad (11)$$

$$H_y=f\left[y,\rho,P_n(\cos\psi),P_n(\cos\theta)\right].$$

The winding discrete state - its step and the unequal distribution over the cross-section - are specified in the algorithm. The design examples show, that by using of the given algorithm we may specify the meaning of the strength and gradient, because the field design accuracy must agree with the high gyromagnetic relation accuracy.

Using the known equations, for defining "signal-to-noise" ratio of spin systems[10], the concrete values for some quantum sensors constructions were defined.

The solenoids, the Gelmgoltz rings and some kinds of optical systems in various combinations, characterised by a different construction complexity are used in mentioned constructions.

The converter construction on the tiny double Gelmgoltz rings has the minimum size and best qualities.

For this construction the frequency shift indicatrix was defined because of disturbances $\Delta B_n^{\gamma}=f\left(|\bar{B}+\bar{\Pi}|\right)$, where B_1 - the field strength vector modulus in the sensor area, Π - the disturbance signal, which acts under α_0 - angle to this field.

It was proposed a converter design algorthm, according to the technical requirements, based on the received data (Fig. 7).

The table I shows various magnetic-frequency devices, analysed for some realization types.

CONCLUSIONS

I. The basis of the development of the DC primary data processing loop converters were received on the quantum-magnetometry principles. They allow to increase the electrical parameters conversion accuracy and angular displacements into the code. New types of precision converting devices, using nonstationary methods of the nuclear magnetic resonance and radioscopy constants, known with $2.10^{-3}\%$ accuracy are developed.

II. The possibility of the increased convertion efficiency in using new physical methods is given theoretically and experimentally. These methods have not been applied to the control systems to data. As the key of the main converters block, the magnetic masers with a higher sensitivity to the smallest magnetic fields are used. The deep cooling effect was used to increase the convertion efficiency, and also new methods of the nuclear signal reception in creagenic devices.

III. The analytical design technique of the electrical parameters convertion channel into the code according to the given requirements to the convertion accuracy, to the continuous stability and quick-action is developed.

IV. The magnetometric principles construction possibilities of the primary data processing loop devices are illustrated by the set of new functional block devices widely used (of the phase converter type, amplifier, reverse frequency converter, standard generator of the alternating frequency and etc.) and which characterise the new system of magnetic-frequency devices.

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The list of figures to the report "Primary data processing devices of control computers on quantum magnetomeasurement principles" by Bach. of Science, assistant-professor KUDRJAVTSEV V.B...

Fig. 1. The synthesis stages of the primary data processing loop devices.

Fig. 2. The block-diagram of a quantum converter:
1 and 7 - quantum sensors, 3,4,6, and 8,9, - orthogonal Gelmgoltz rings.
5 and 10 - feed sourse of the radiator
12 - low frequency filter.
13 and 15 - amplifiers in the reverse sensor network.
14 - phasometer of the synchronisation system.
14^1 - the block for the modulus tuning B_1 and B_2; the former and the definition block of two sensors frequency difference.
17 - corrector
18 - low frequency filter.
19 - low frequency generator.
20 - program block.
21 - recording device.
22 - the calculator.

Fig. 3. The converter devices elements.

Fig. 4. The vector diagram to the measurer action principle of angular displacements.

Fig. 5. The dependence of the magnetic field vector modulus increment on the angle turning.

Fig. 6. The cryogenic strength measurer.
(1 - the thermal isolation,
2 - cooled working winding.
3 - signal precession preamplifier
4 - amplifier
5 - frequency meter.
6 - liquid nitrogen.
7 - liquid hydrogen.
8 - internal duar.)

Fig. 6 a. The instantaneous meanings meter of electrical signals
(1 - duar,
2 - superconducting cylinder.
3 - the solenoid, flowed by the controlled current.
5 - commutator.
6 - amplifier.
7 - frequency meter.
8 - controlled signal source.
9 - cylinder driving.
10 - driving control scheme.)

Fig. 6 b. The frequency sensor of "angle-frequency" type.
1 - code disk driver with the minimized discharge
2 - reader.
3 - code disk.
4 - controlled current source.
5 - shaft.
6 - the sensor framework.
7 - solenoid.
8,9 - internal duars.
16 - movable magnetic loop.
15 - the scheme of comparison for the 1-driving switch.
(in case the accurate frequency meaning is not correspondent to the given code in 4 - block input.)

Fig. 7. The converter design algorithm according to the technical requirements.

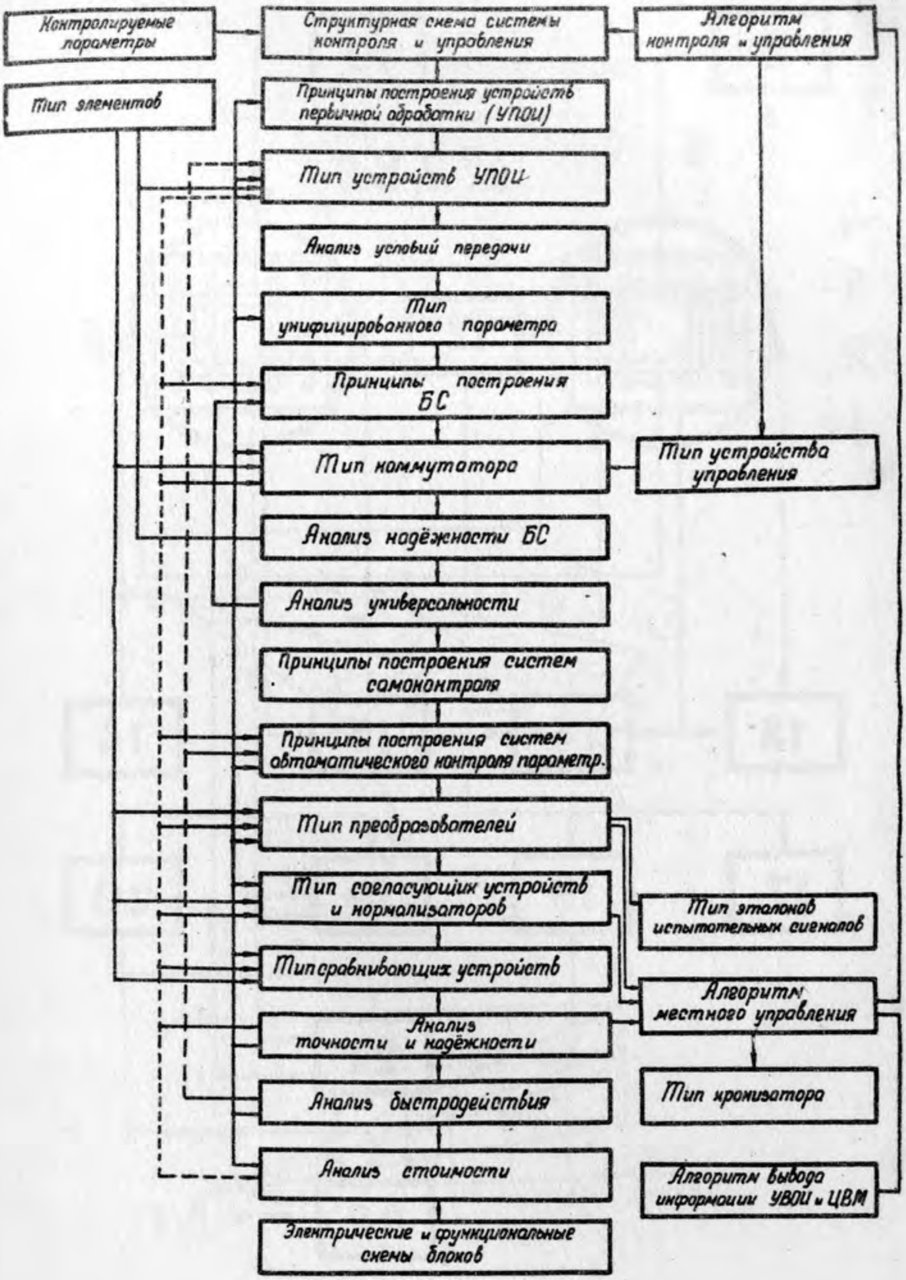

Контролируемые параметры
Структурная схема системы контроля и управления
Алгоритм контроля и управления
Тип элементов
Принципы построения устройств первичной обработки (УПОИ)
Тип устройств УПОИ
Анализ условий передачи
Тип унифицированного параметра
Принципы построения БС
Тип коммутатора
Тип устройства управления
Анализ надёжности БС
Анализ универсальности
Принципы построения систем самоконтроля
Принципы построения систем автоматического контроля параметр.
Тип преобразователей
Тип согласующих устройств и нормализаторов
Тип эталонов испытательных сигналов
Тип сравнивающих устройств
Алгоритм местного управления
Анализ точности и надёжности
Тип хронизатора
Анализ быстродействия
Анализ стоимости
Алгоритм вывода информации УВОИ и ЦВМ
Электрические и функциональные схемы блоков

Fig. 1

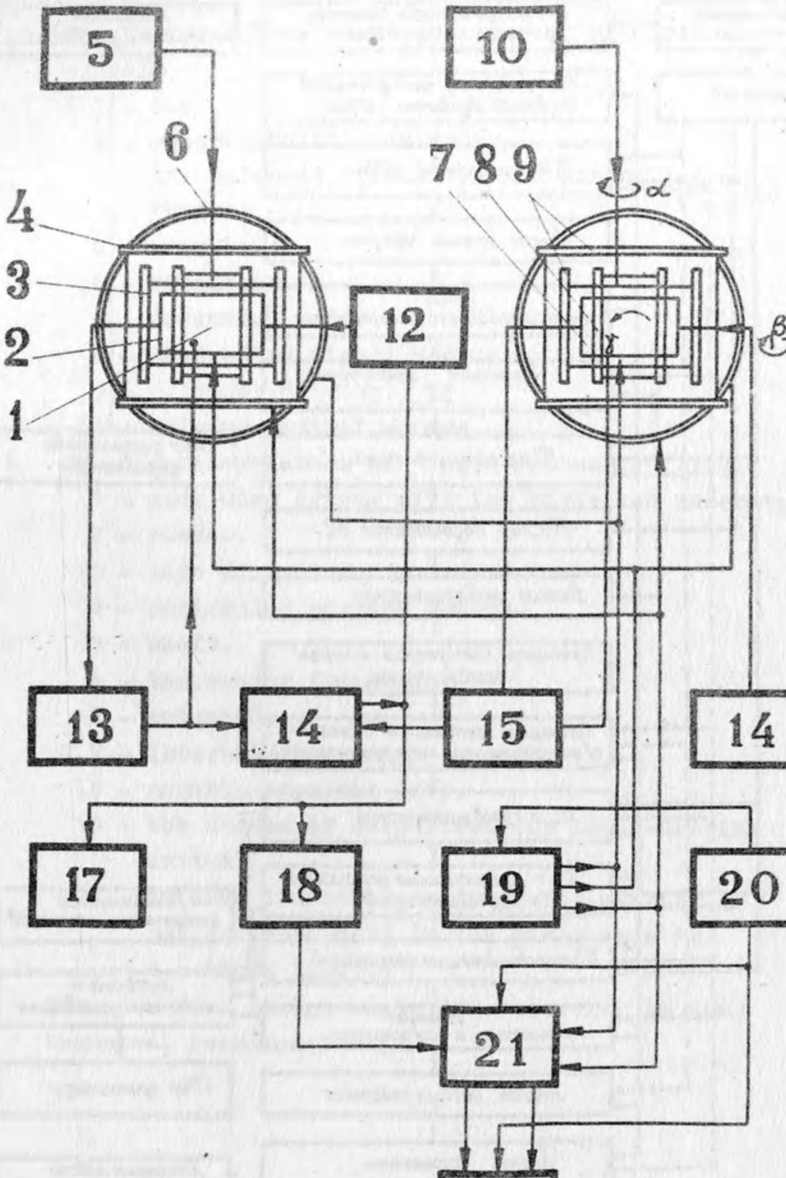


Fig. 2

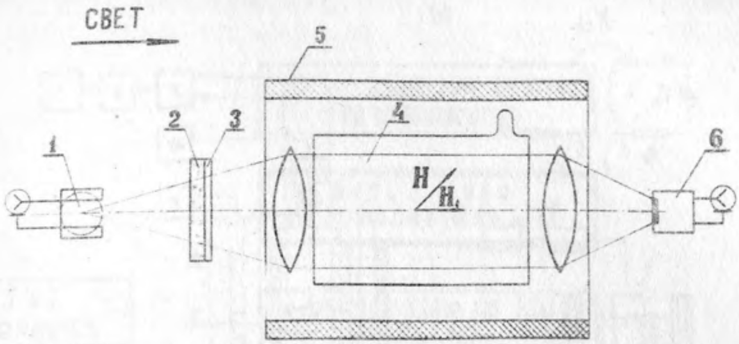


Fig. 3

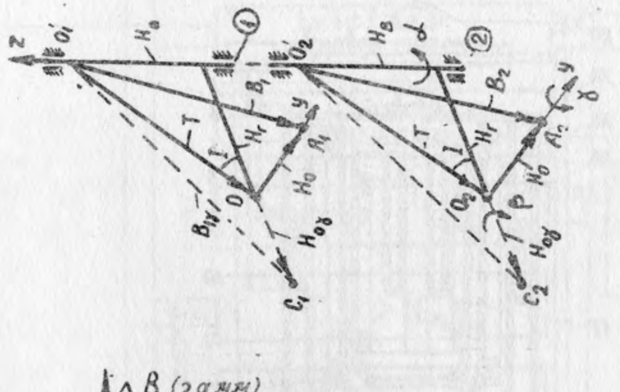


Fig. 4

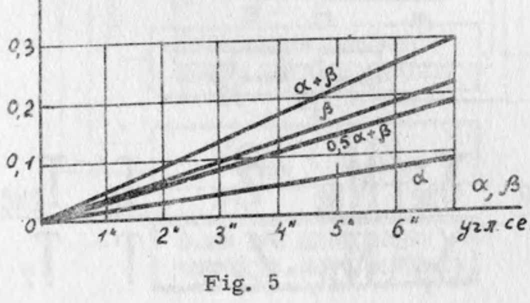
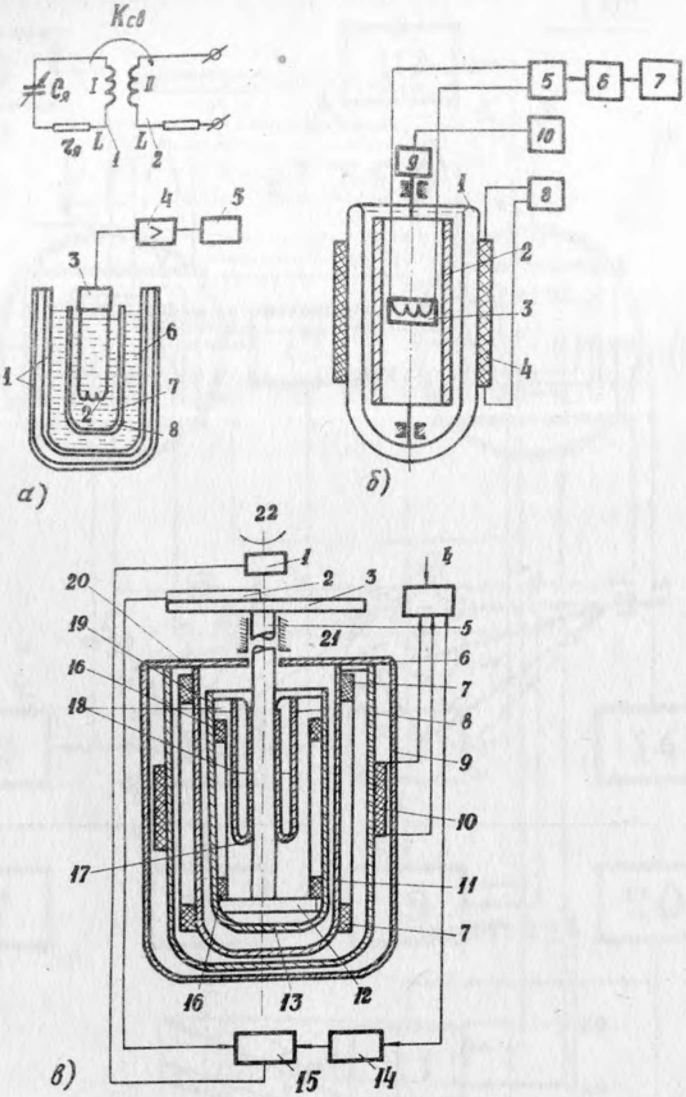


Fig. 5



$$\frac{\beta_{He}}{\beta} = \frac{\gamma_{He}}{\gamma_{p}} \left(\frac{N_{He}}{N_{p}} \cdot \frac{\tau_{к}}{\tau_{к\,He}} \cdot \frac{T_{0}}{T} \cdot \frac{T_{2He}}{T_{2}} \right)^{\frac{1}{2}}$$

Fig. 6

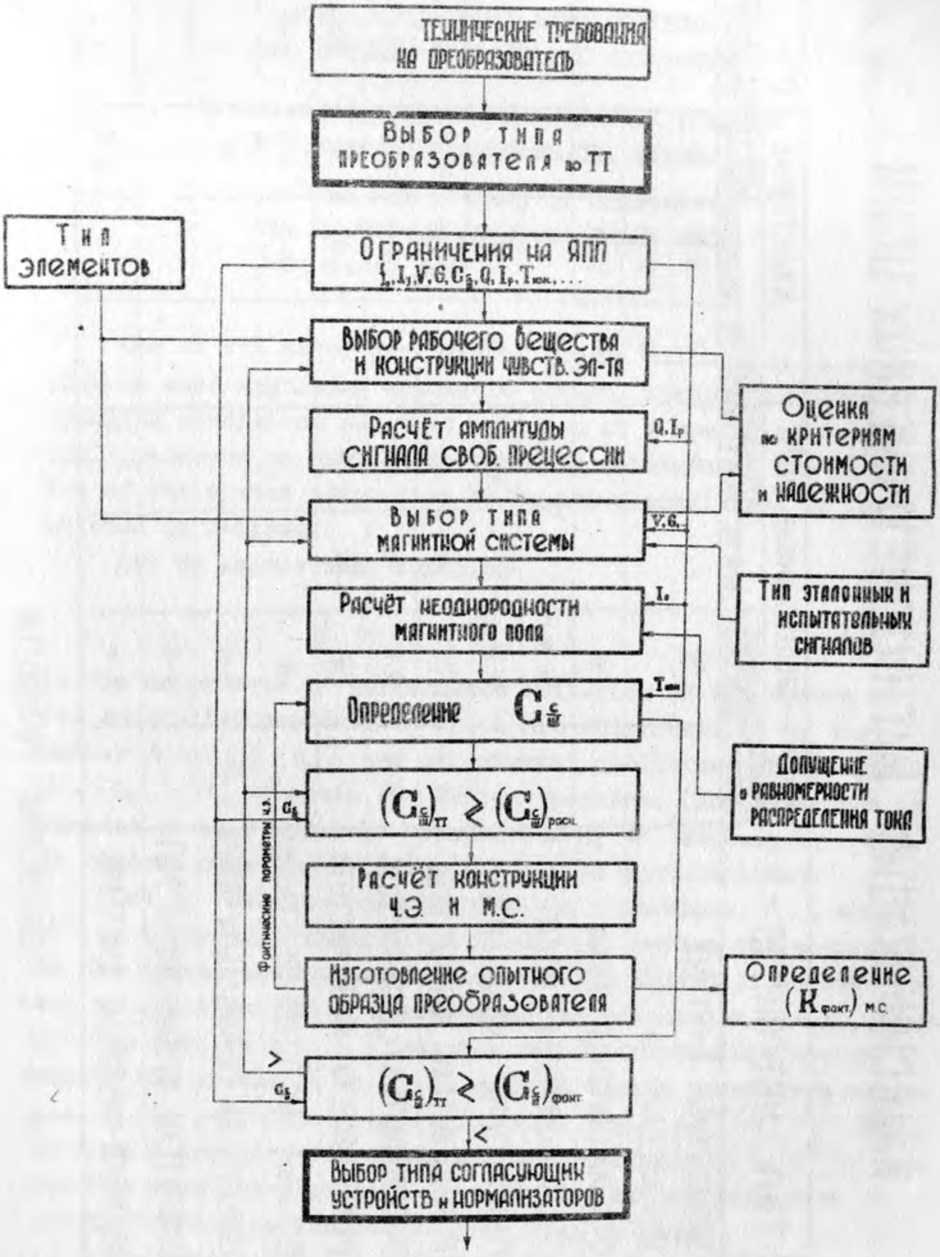

ТЕХНИЧЕСКИЕ ТРЕБОВАНИЯ НА ПРЕОБРАЗОВАТЕЛЬ
ВЫБОР ТИПА ПРЕОБРАЗОВАТЕЛЯ по ТТ
Типа элементов
Ограничения на ЯПП
Выбор рабочего вещества и конструкции чувств. эл-та
Расчёт амплитуды сигнала своб. прецессии
Оценка по критериям стоимости и надежности
Выбор типа магнитной системы
Расчёт неоднородности магнитного поля
Тип эталонных и испытательных сигналов
Определение
Допущение о равномерности распределения тока
Расчёт конструкции Ч.Э. и М.С.
Изготовление опытного образца преобразователя
Определение
Фактические параметры Ч.Э.
Выбор типа согласующих устройств и нормализаторов

Fig. 7

КЛАССИФИКАЦИЯ МАГНИТОМЕТРИЧЕСКИХ УПОИ

ТИП УСТРОЙСТВ	ЯДЕРНО-ПРЕЦЕССИОННЫЕ ($C^{Я}$)				КВАНТОВЫЕ ($C^{А}$)			
	ДИФФЕРЕНЦИАЛЬНЫЕ		ЭКРАНИРОВАННЫЕ				ДИФФЕРЕНЦИАЛЬНЫЕ	
	НОРМ °C	КРИОГЕННЫЕ	НОРМ °C	КРИОГЕННЫЕ	НОРМ °C	КРИОГЕННЫЕ	НОРМ °C	КРИОГЕННЫЕ
УСИЛИТЕЛИ = ТОКА		*		*	*	**	*	**
УСИЛИТЕЛИ ≈ ТОКА					*	**	*	**
СТАБИЛИЗАТОРЫ ТОКА		*			*	**	*	
СТАБИЛИЗАТОРЫ ПОЛЯ		**		*			*	
ГЕНЕРАТОРЫ КОЛЕБАНИЙ		**		**	*	**	*	
ПРЕОБРАЗОВАТЕЛИ „ТОК-ЧАСТОТА"	**	*	*	*	*	**	**	*
ПРЕОБРАЗОВАТЕЛИ „УГОЛ-ЧАСТОТА"	**	**	*	*	*	**	**	*
ПРЕОБРАЗОВАТЕЛИ „ФАЗА-ЧАСТОТА"		*	*	*	*	*	*	*
ОБРАТНЫЕ ЧАСТОТНЫЕ ПРЕОБРАЗОВ.		*			*	**	**	**
ИЗМЕРИТЕЛИ СВЕРХСЛАБЫХ ПОЛЕЙ	*	**					*	
ИЗМЕРИТЕЛИ СОСТАВЛЯЮЩИХ МП		**						
СИНХРОННО-СЛЕДЯЩИЕ СИСТЕМЫ		**					**	

** – С ПОВЫШЕННОЙ ТОЧНОСТЬЮ

12.1

MULTIDIMENSIONAL EXTRAPOLATION FOR OPTIMAL CONTROL AND DESIGNING

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The Institute of Electronics and
Computers, USSR

One of the major problems of modern automation is creation of self-adjusting automatic control systems adapting to changing conditions so, that extremum of a specified functional (performance criterion) could be maintained. The problem of the system adaptation to changing conditions is formulated as follows.

Let us assume that function

$$I = I\,(X, A) \tag{I}$$

is the dependence of performance criterion of the system on its controlled parameters, which are characterized by the vector $X = (x_1, \dots, x_n)$, and on external conditions or situation $A = (a_1, \dots, a_m)$ wherein the system operates. (The situation is assumed to be completely represented by m numbers $a_1, \dots, a_m$). In general case the relation (I) may be nonformalized.

Let S be a set of all possible situations A, while U be a set of permissible controls X. Then the problem of the system optimal adaptation will be solved if we have found an algorithm or an operator of the conversion $S \to U$, that give to each vector A from the set S of possible system states the vector $X^* \in U$ of optimal system parameters extremizing its performance criterion (I), e.g. $I[X^*(A), A] = min$. In such a way, determination of the dependence $X^* = X^*(A)$ solves the adaptation problem (the problem of the situation A identification is assumed to have been solved).

The most versatile method of the system adaptation is a search [I,2], which allows to extremize a specified criterion (I) by collecting the information about the object behaviour for the purpose of finding its optimal response X^* for new conditions.

However, if the dependence (I) is not formalized, the search should be performed directly on the object in real time scale what considerably reduced the efficiency and operativeness of adaptation. It very often happens that search on the object is prohibited or considerably limited because of technological considerations. In this cases it is advantageous to solve the problem of the system adaptation applying searchless methods. As a such method of the system adaptation to a new situation one may use learning from the results of previous experience.

Let us assume that for a number of different situations A_i $(i=1,\dots,k)$ we have obtained the corresponding optimal parameters by some definite way:

$$\left.\begin{array}{l} A_1 \to X_1^* \\ \dots\dots\dots \\ A_k \to X_k^* \end{array}\right\} \qquad (2)$$

Let us name this set of correspondencies by instructive sequence. It is evident that (2) can be considered as being the results of observation of an unknown functional dependence

$$X^* = F(A) \qquad (3)$$

Then the problem of determination of optimal system parameters in a particular new situation A_{k+1} reduces primarily to restoration of said dependence (3) according to given k observations, and then to determination of a value of X^*_{k+1} in a point A_{k+1}: $X^*_{k+1} = F(A_{k+1})$. In such formulation the problem can be solved, for example, with the help of potential functions method [3], stochastic approximation methods [4], or by means of iterative procedures [5] similar to the abovementioned methods.

But these methods provide satisfactory restoration of an unknown dependence (3) only with a sufficient number of observations. Besides, the efficiency here greatly depends on the choice of function system, linear combination of which approximates an unknown functional dependence. This means that to ensure functioning of the mentioned procedures one should hav a considerable a priori information about the function's behaviour (3). But in practical situations and especially in problems of optimal designing the information about the behaviour of the mentioned function is very poor and is not able to provide the basis for the choice of function system by which the function (3) is expanded. Moreover, a number of observations is practically always very small what likewise hinders the choice specified above.

Below is suggested and analyzed a method conventionally called a method of multidimensional linear extrapolation with the help of which it is possible to find sufficiently valid evaluations for optimal system parameters in a new situation on the basis of rather limited previous experience. Specifically, a number of observations k can be smaller than the dimension of a space of situations:

$$k \leq m \qquad (4)$$

<u>Multidimensional linear extrapolation method</u>. According to the suggested method, values for optimal system parameters X^* in a new situation are found by the following way [6]. Across the vectors, being a part of instructive sequence, in a space of situations S and in a space of optimal solutions U correspondingly, hyperplanes S' and U' are drawn. Evidently, any element $A' \in S'$ in an assumption about the space of situations linearity may be represented in the form of linear combination:

$$A' = A_1 + \sum_{i=1}^{k-1} \lambda_i (A_{i+1} - A_1) \qquad (5)$$

wherein - coordinates in a base constructed from the elements of instructive sequence. If , then in this situation it is suggested to use vector specified by expression:

$$X = X_1^* + \sum_{i=1}^{k-1} \lambda_i (X_{i+1}^* - X_1^*) \qquad (6)$$

as an estimate for the vector of optimal system parameters. That is, a linear relation between hyperplanes S' and U' is formed. In such a way, the required functional dependence is linearized on subspaces S' and U' what allows to find the value of a vector of optimal system parameters by linear extrapolation for any situation $A' \in S'$.

If the new situation A does not belong to the hyperplane S' it is easily identified with the nearest, in a certain sense, situation lying in S'.

To do this, a metric is introduced in the space of situations, which allows to bring in correspondence to each pair of situations a number ρ, specifying the measure of their closeness, e.g. the distance between them:

$$\rho(A, A') = |A - A'|^2 \qquad (7)$$

In this case the situation $A \in S$ will be identified with its orthogonal projection A' on a hyperplane S'.

Identification of A and A' essentially means that properties of the subspace S' are spread over or extrapolated on a surrounding and close to it region of a space S.

Extrapolation algorithm can be represented in the form of

$$A \equiv A' \rightarrow X \equiv X^* \qquad (8)$$

Directly from this algorithm follow conditions of its application. Thus, given method is reasonable to carry out only in case when the following conditions are fulfilled:

I. An area S of the situation parameters change is sufficiently small, so that to maintain quality of the system within the desired range it is enough to correct optimal system parameters (without changing its structure).

2. There is some experience of optimal adaptation of the system to some situations within the area S, the instructive sequence being based on it.

3. An unknown functional dependence (3) may be linearized within the area S with reasonable accuracy.

The latter condition with $k \leqslant m$ is not valid at all, as with absence of a priori information about the function (3) a number of observations k is not sufficient for unique determination of linear transformation $S \rightarrow U$. If the length of instructive sequence is more than the dimension of the situa -

tion space the latter condition for the method application is weakened: it should be assumed that an unknown functional dependence (3) can be linearized with sufficient accuracy within the environment of $q < k$ points ($q \leq m$) from an area S. In this case, a value for X^* is determined by the nearest q (in a sense of introduced metric) to the situation under investigation without using all available observations.

The choice of representation for the proximity function and optimal meaning of q can be made by the following way. Let us remove the i-th element of instructive sequence and extrapolate solution according to A_i. The vector obtained, generally speaking, does not coincide with existing X_i^*. Discrepancy

$$\Delta = \sum_{i=1}^{k} / X_i^* - X_{ie}^* / \qquad (9)$$

determines efficiency of the choice of proximity function and q optimum. That is why optimal choice of proximity function must minimize this discrepancy.

Theoretical investigation of the effect of the relation between the length k of instructive sequence and the dimension m of the situation space on the value of average integral loss δ_F of the system performance within the range of its possible states was carried out for the cases of linear and non-linear dependence (3). It turned out, that in both cases the loss steadily decrease as k approaches m. With $k > m$ the linear function is restored correctly and the loss equals to zero. In case of non-linear function local-linear approximation is accomplished.

For comparison, average integral loss δ_F^* of the system performance on restoring the same functions by "association" method was determined. According to this method a new situation is identified with the nearest one forming a part of instructive sequence. As shown in Fig.I, extrapolation method of linear dependence (3) restoring leads to substantially lesser integral loss of the system performance. The methods are equivalent only in case when information about the function to be restored is negligibly small ($k \ll m$, i.e. $N = \frac{m}{k} \to \infty$). Analogous picture is observed in case of non-linear dependence (3). However here the relative advantage of the specified extrapolation method is reduced so far as linearization results in additional loss of the system performance.

Estimates for optimal system parameters found by extrapolation are random by virtue of accidental nature of previous experience. Investigations of statistical properties of these estimates by plurality of instructive sequencies showed that estimates are unbiassed and their dispersion decreases with increase of length of instructive sequence[7].

Some applications of the method. Some practical problems were considered in respect to application of the suggested multidimensional linear extrapolation method.

I. The problem of optimal system designing is known[8,9] to be connected with laborious procedure of finding optimal parameters with which the system meets most efficiently economic, technical and time requirements. Inevitable variations of individual initial conditions, characteristics, limitations and the like, arising in the process of system designing, lead to necessity of multiple repeating of given laborious calculations to correct optimal system parameters. Difficulty of these calculations and, therefore, time of the system designing can be considerably reduced using the suggested extrapolation method.

To do this, a definite initial number of system parameters corrections is performed applying known direct methods of multiparameter optimization. These calculations form the instructive sequence. Subsequent corrections are performed by extrapolation method based on this experience. It should be mentioned that instructive sequence can be formed only from the results of optimization of the stable structure system(see first condition of the method application). Vector of situation should completely determine all the conditions for which optimal solution was obtained.

Checking of the latter condition for the extrapolation method application is easily performed by the value of minimum discrepancy (9), which represents the method efficiency for given problem.

2. The problem of the function behaviour prediction (the function being preset in m points of observation range)

amounts to the choice of an analytical expression which in a certain sense most efficiently approximates the function observed. Let us assume that from the previous experience it is known, that functions similar to those observed may be in the best way approximated by polynomials of degree n where

$$n \geq m \tag{IO}$$

It is known, that a problem of plotting a polinomial of degree n across $m \leq n$ points admits a great number of solutions. From all possible solutions one should choose the only least contradicting to results of the previous experience. To do this, the following instructive sequence is formed according to [8]. Components of vecter of situation A_j are the values of previously observed j-th function in marked points of observation range; components of the optimal parameters vector X_j^* are values of coefficients of polynomial approximating the j-th function in the best way. Now, let us apply the multidimensional linear extrapolation method for finding estimates for coefficients of the polynomial sought and thus for solving the problem of prediction behaviour of the function being observed.

The following condition should be observed in this case: functions constituing a part of instructive sequence and observed anew should be close in the sense that they describe analogous processes occuring in analogous conditions.

3. <u>The problem of a dynamic object identification</u> consists in construction of mathematical model simply isomorphic to an object by behaviour. Structure and parameters of such a model are selected under condition, that with like input signals of the object and the model, unbalance of their output signals should be the least in a certain sense. If structure of the model is determined, the problem of optimal adjustment is reduced to finding the vector of optimal parameters extremizing the index of output signals unbalance.

However, in real conditions properties of the object do not remain constant with time. Then necessity of continuous or periodic adjustment of optimal parameters of the model arises. It had been shown [II] that in case of quasi-stationary object the problem of model adaptation is conveniently solved by using multidimensional linear extrapolation method. A priori information about the object at a stage of instruction of a predictor can be obtained with the help of search methods of the model self-adjustment [IO,II].

As applied to the object identification problem, instructive sequence should set up a correspondence between a characteristic defining its current state and parameters of a mathematical model of an object in this state. In the process of normal use either directly recordings of input and output signals, or their statistical properties can serve as a characteristic of the object state. It is known [IO] , that cross-correlative function of output and input signals of the object represents the most informative characteristic. In specific cases, it is allowed to observe the change of the object by the change of auto-correlative function of its output signal.

Employment of any of the mentioned functions as a situation requires their previous parametrization. Considering exceptional difficulty of conventional method of parametrization of correlation functions [IO], the work [II] suggested simple method of representation of the function in the form of a vector: direct values of the function are its components, e.g. corresponding to certain m moments of the variable τ_i $(i=1, ..., m)$ values of $K_{yx}(\tau_i)$.

Experimental investigation of a process of adjustment of optimal parameters of the model by extrapolation method was carried out on a digital computer for the class of objects having monotonous transfer characteristic. The results obtained with statistical evaluations of cross-correlative functions confirmed possibility of suggested approach to a solution of the operational model adaptation problem. The least error is achieved when observing the cross-correlative function of an object.

Conclusion :

I. A method of searchless adaptation of a complex system to multidimensional situation changes has been suggested and investigated.

2. The method is based on extrapolation of a previous be-

haviour accumulated in a process of optimal operation of the object under similar conditions. The distinctive feature of the method is that it allows to make well-founded decisions with rather limited a priori information.

3. Experimental investigation of the method in problems of optimal designing, predicting, identification and control showed efficiency of developed approach to the solution of adaptation problem.

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13.5

ON THE THEORY OF OPTIMAL CONTROL WITH BOUNDED STATE SPACE VARIABLES

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U S S R

I. Introduction. The report is an extension of the theory presented in [1,2] to linear control problems with bounded state space variables as well as to a number of related infinite-dimensional control problems. Problems of such kind had been considered in general setting and investigated from various points of view, (e.g. see [3-9]). For the linear problem necessary conditions of optimality had been obtained on base of general functional methods in paper [10]. It should be noted, however, that the properties of the Lagrange multipliers (which may be considered here as elements of corresponding functional spaces), and the detailed structure of the solutions had not been discussed.

A notable feature of the problem considered here is such that even having found the Lagrange multipliers, we cannot be sure that it is always possible to determine the optimal control directly from the maximum principle. On certain time-intervals (those, in general, when the trajectory runs along the constraint) the computation of the optimal control requires some additional suggestions to be applied. This is due to singularities in the maximum principle which occurr in the intervals mentioned above.

In this report we make use of the approach [1,2] in order to study the properties of the Lagrange multipliers as well as to indicate certain means of investigating the problems concerned by approximating or even reducing them to corresponding finite-dimensional problems and to discuss the possibility of computing the controls throughout the whole time interval.

II. The optimal control problem with bounded phase coordinates.

1. Statement of the problem.

Given is a linear control system described by the differential equation

$$dx/dt = A(t)x + B(t)u + \omega(t) \qquad (2.I)$$

Here x is an n-dimensional vector of the phase coordinates, u is the r-dimensional control, $A(t)$, $B(t)$, are continuous matrices of corresponding dimensions, $\omega(t)$ is a given n-dimensional vector function which is the disturbance at the input of the system. Suppose that also given are: the initial vector $x_\alpha = x(t_\alpha)$, a convex terminal lattice $\mathcal{M}$ and constraints

$$u \in \mathcal{U}, \quad x(t) \in \mathcal{X} \qquad (2.2)$$

on the instantaneous values of $u(t)$ and $x(t)$. Sets $\mathcal{U}$ and $\mathcal{X}$ are convex and are supposed to contain the origin as an interior point. The set $\mathcal{X}(t)$ varies semicontinuously in time t.

Problem 2.I

Among the controls $u(t)$ which transfer system (2.I) from state x_α to the lattice $\mathcal{M}$ ($x_\beta \in \mathcal{M}$) one is to specify the optimal control which ensures that $t_\beta^\circ - t_\alpha = \min$.

2. Conditions of solvability. The maximum principle.

Suppose $\{t_i\}$ is the set of points $t_\alpha + p(N)(t_\beta - t_\alpha)/2^N$, where $p(N) \le 2^N$, $(N = 1, \dots, m, \dots)$. Set $\{t_i\}$ is dense in $[t_\alpha, t_\beta]$. The problem 2.I may hence be reduced to a countable moment problem

$$\int_{t_\alpha}^{t_\beta} h_k(t_\beta, \tau)\, u(\tau)\, d\tau = c_{k\beta} + x(t_\beta)$$

$$\int_{t_\alpha}^{t_\beta} h_k(t_i, \tau)\, u(\tau)\, d\tau = c_{ki} + z_{ki} \qquad (2.3)$$

Here

$$x(t_\beta) \in \mathcal{M};\ z^{[i]} = \{z_{1i}, \dots, z_{ni}\} \in \mathcal{X}(t_i);\ u(t) \in \mathcal{U},\ t_\alpha \le t \le t_\beta \quad (2.4)$$

functions $h_k(t,\tau)$ are the k-th components of the n-vector

$$h(t,\tau) = B'(\tau) S(\tau,t),\ (dS(\tau,t)/d\tau = -S(\tau,t)A(\tau),\ S(t,t) = E) \quad (2.5)$$

and $h_k(t,\tau) \equiv 0$ if $\tau \ge t$. Numbers c may be found as the components of the vector

$$c(t_i)=\int_{t_\alpha}^{t_i} S(\tau,t_i)\omega(\tau)d\tau + S(t_\alpha,t_i)x_\alpha\ ,\ c_\beta = c(t_\beta) \qquad (2.6)$$

Denote

$$\rho[h] = max_u \int_{t_\alpha}^{t_\beta} h'(t)u(t)dt\ ,\ u\in\mathcal{U};\ \gamma_1 = max_u l'u\ ,\ u\in\mathcal{U};$$
$$\gamma_2[l] = max_x l'x\ ,\ x\in\mathcal{M};\ \gamma_t[l] = max_x l'x\ ,\ x\in\mathcal{X}(t) \qquad (2.7)$$

Suppose t_β is a fixed number.The necessary and sufficient conditions for the solvability of problem (2.3)-(2.5) are given in functional analysis [12] .It should be noted,however, (and this is most important in the discussion) that a specific property of problem (2.3)-(2.5) - the continuity of functions $h(t,\tau)$ -makes it possible to present the conditions of solvability in the form of an inequality

$$\rho[\lambda' h(t_\beta,t) + \int_t^{t_\beta} l'(\tau)h(\tau,t)d\tau] - \lambda' c_\beta - \int_{t_\alpha}^{t_\beta} l'(t)\omega(t)dt +$$
$$+ \int_{t_\alpha}^{t_\beta}\gamma_t[l(t)]dt + \gamma_2[\lambda] \geq 0 \qquad (2.8)$$

which must hold for an arbitrary n-vector $\lambda=\{\lambda_1,...,\lambda_n\}$ and an arbitrary square-summable n-vector function $l(t)=\{l_1(t),...,l_n(t)\}$. (Sign / denotes the transpose).

Conditions (2.8) may be written in another form which follows from (2.5)-(2.8) and is equivalent to (2.8).With t_β being fixed,the problem (2.3)-(2.5) is solvable if and only if for an arbitrary n-vector λ and an arbitrary n-vector function such that

$$\lambda'\lambda + \int_{t_\alpha}^{t_\beta} l'(t)l(t)\,dt \leq \tau\ ,\ \tau = const > 0$$

we have (2.9)

$$inf_{\lambda,l}\{ max_{u\in\mathcal{U}} \int_{t_\alpha}^{t_\beta} u'(t)B'(t)s(t)dt + s'(t_\alpha)x_\alpha +$$
$$+ \int_{t_\alpha}^{t_\beta}\omega(t)s(t)dt + \int_{t_\alpha}^{t_\beta}\gamma_t[l(t)]dt + \gamma_2[s(t_\beta)] = 0$$

Here $s(t)$ is a solution of the adjoint equation

$$ds(t)/dt = -A'(t)s(t) + l(t)\ ,\ s(t_\beta) = s_\beta \qquad (2.10)$$

The condition (2.9) is similar to the problem of finding the saddle -point for the function of Lagrange in the theory of concave programming [13].

The optimal time is the least of numbers $t_\beta - t_\alpha$ for which (2.9) holds.We assume here that the inf in (2.9) is attained at a nonzero element $\eta = (\lambda^\circ, \ell^\circ(t))$ of the Hilbert space $\mathcal{H} = \{\lambda, \ell\}$, $\lambda \in E_n$, $\ell^\circ(t) \in \mathcal{L}_2$. It is to this regular case that the further treatment is confined. Note however, that in general the minimizing function $\ell^\circ(t)$ should be considered as a distribution of the first order, as it may be formed of δ-functions as well as of square-integrable components. Clear that in this case the function $s^\circ(t)$ is discontinuous.

It follows immediately from (2.9) that the optimal control satisfies a necessary condition of optimality-the maximum principle

$$\max_{u \in \mathcal{U}} \int_{t_\alpha}^{t_\beta} u'(t)B'(t)s^\circ(t)dt = \int_{t_\alpha}^{t_\beta} u^{\circ\prime}(t)B(t)s^\circ(t)dt \tag{2.11}$$

or in other words

$$\max_{u \in \mathcal{U}} u'(t)B'(t)s^\circ(t) = u^{\circ\prime}(t)B'(t)s^\circ(t) \tag{2.12}$$

where (in terminology of[2]) $s^\circ(t)$ is the minimal motion of system (2.10), that is the one which delivers a minimum to the functional

$$\begin{aligned}\Psi[s_\beta, \ell] = \int_{t_\alpha}^{t_\beta} \gamma_1[B'(t)s(t)]dt + s'(t_\alpha)x_\alpha + \int_{t_\alpha}^{t_\beta} s'(t)\omega(t)dt + \\ + \int_{t_\alpha}^{t_\beta} \gamma_t[\ell(t)]dt + \gamma_2[s(t_\beta)] \geq 0\end{aligned} \tag{2.13}$$

with

$$\|s_\beta\|^2 + \int_{t_\alpha}^{t_\beta} \|\ell(t)\|^2 dt \leq 1 \tag{2.14}$$

Symbol $\|x\|$ denotes the euclidian norm of vector x . The functional $\Psi(s_\beta, \ell)$ is nonnegative and convex.

Note. Let $\mathcal{M}$ be a hyperplane $x_k(t_\beta) = x_k$, $k = 1, \dots, i$. Then the set G of all points $x = \{x_1, \dots, x_i\}$ which satisfy the inequality (2.13) with any $s_\beta, \ell(t)$, satisfying (2.14), is (with respect to the first i coordinates) the set of points attainable at moment t_β from state $x(t_\alpha)$ under restrictions (2.2).

III. Properties of the solutions.

1. The minimal function. Suppose the solution $h^\circ = B's^\circ$ of problem (2.13), (2.14) is already determined. Let $x^\circ(t)$ denote the optimal trajectory of the system (2.10). It follows from the solution of problem (2.3)-(2.5) that $\ell^\circ(t) \equiv 0$ if

the trajectory $x^\circ(t)$ is within the interior of $\mathcal{X}(t)$. If the components h_i° of function h° are non-zero almost everywhere, the control $u^\circ(t)$ is determined according to (2.I2). However it is another kind of situation that is of importance here. Dropping in our discussion the most irregular cases of the problem, we shall assume that each of $h_i^\circ(t)$ may be an identical zero only on a finite number of time intervals $[\tau_{k_1}, \tau_{k_2}]$, $k=1,\dots,m$, whose total measure is less than $t_\beta^\circ - t_\alpha$. Assume also that system (2.I) is completely controllable [14] in the strong sense [2] and that its trajectories constructed under control $u: \gamma[u]=\mu$ cannot intersect with the boundary of $\mathcal{X}(t)$ except at a set of points $\{t\}$ of measure zero. Then function $h^\circ(t)$ will vanish if and only if $x^\circ(t)$ runs along the boundary of $\mathcal{X}(t)$. The moments of time when $x(t)$ runs on or off the boundary of $\mathcal{X}(t)$ thus coincide with points $\{\tau_{k_i}\}$. Note that the points τ_{k_i} are determined directly by minimizing the functional (2.I3), i.e. without applying in explicit form the Weierstrass-Erdman conditions.

2. Computation of the minimal function.

It follows from above that in order to determine the multipliers s_β° , $l^\circ(t)$ one ought to solve the variational problem (2.I3), (2.I4) of minimizing functional Ψ under a convex limitation. The solution of the specific problem (2.I3), (2.I4) may, however, be simplified. Discussing briefly the essence of the problem concerned, we presuppose that (2.I) is a stationary, completely controlled system with scalar control u and with restriction on the first coordinate only : $|x_1(t)| \le f(t)$. The path $x^\circ(t)$ is supposed to run along the constraint within a sole interval of time: $\tau_1 \le t \le \tau_2$. We have

$$h^\circ(t) = b' s^\circ(t) = s_\beta^{\circ\prime} h(t_\beta, t) + \int_t^{t_\beta} l_1^\circ(\tau) h_1(\tau, t)\, d\tau \equiv 0 \quad (3.I)$$

Making use of a still stronger property of the functions $h(\tau,t)$ -their differentiability- we arrive at a differential equation for the function $l_1^\circ(t)$. Indeed, by a number of differentiations and a transformation we obtain from

(3.I) the differential equation of the $(n-1)$-st order

$$b'\{A'^{n-1}\ell(\tau)+\sum_{j=1}^{n-1}\Big((-1)^j A'^{n-j-1}\frac{d^j\ell(\tau)}{d\tau^j}+d_j\sum_{i=0}^{j-1}(-1)^{i+1}A'^{j-i-1}\frac{d^i\ell(\tau)}{d\tau^i}\Big)\}=0 \quad (3.2)$$

where $\ell(\tau)=\{\ell_1^\circ(\tau),0,\dots,0\}$ is an n-vector function, d_j $(j=1,\dots,n-1)$ are the coefficients in the expansion of vector $b'A'^n$ with respect to the basis $\{b'A^k;\ k=0,\dots,n-1\}$: $b'A'^n=\sum_{k=0}^{n-1}d_k b'A'^k$. The conditions of continuity for function $h^\circ(t)$ and its derivatives at point τ_1 give us (n-1) boundary conditions which form a system of algebraic equations between $\tau_1,\tau_2,s_\beta^\circ;\ d^k\ell_1/d\tau^k|_{\tau=\tau_1}$. Solving (3.2) with the aid of the boundary conditions, we obtain $\ell_1^\circ(t)=$ $=f_1(t,\tau_1,\tau_2)s_\beta^\circ$ where $f(t)$ is a nonlinear n-vector function of τ_1,τ_2. Substituting $\ell_1^\circ(t)$ into (2.I3), we obtain further on that with τ_1,τ_2 being fixed numbers, $\Psi[s_\beta,\ell^\circ]=$ $=\Psi(s_\beta,\tau_1,\tau_2)$ is a convex function of s_β. The limitation (2.I4) is now transformed as follows: $v(s_\beta,\tau_1,\tau_2)\le\tau$ where $v(s_\beta,\tau_1,\tau_2)=\tau$ is the equation of an ellipsoid if τ_1,τ_2 are fixed. The computational procedure thus consists in minimizing $\Psi(s_\beta,\tau_1,\tau_2)$ with respect to s_β $(v(s_\beta,\tau_1,\tau_2)\le\tau)$ for any fixed pair $\{\tau_1,\tau_2\}$, and further on in finding the pairs $\{\tau_1^\circ,\tau_2^\circ\}$ which deliver a minimum to the nonlinear function $\Psi(s_\beta^\circ(\tau_1,\tau_2),\tau_1,\tau_2)$. If $|x_1(t)|=f(t)$ within m intervals of time $[\tau_{k1},\tau_{k2}]$, $k=1,\dots,m$, then $\ell_1^\circ(t)$ is a solution of (3.2) within each interval. Function $\Psi(s_\beta^\circ,\tau_{11},\tau_{12},\dots,\tau_{m1},\tau_{m2})$ is then convex in s_β and nonlinear in τ_{k_i}. Suppose there is an estimate for m. The computation of $\ell_1^\circ(t)$ may now be fulfilled, assuming first then $m=2$, etc., till we obtain $\Psi(s_\beta^\circ,\tau_{k_i}^\circ)=0$.

An equation similar to (3.2) (but, of course, of a more complicated form) may be derived for the problem 2.I in general. It must be supposed, however, that matrices $A(t)$, $B(t)$ are differentiable.

3. The approximation of the solutions.

The following is the statement of an important property for the functional $\Psi[s_\beta,\ell]$ and the minimal function $h^\circ(t)$[15]. Suppose the restriction (2.2) on the coordinates holds only in discrete moments of time $t_{i_\varkappa}$ $(i_\varkappa=1,\dots,2^\varkappa)$. A consequence of this assumption is a finite-dimensional moment problem which is solvable if and only if the inequality

$$\min_{s_{\varkappa}(t)} \Psi(s_{\beta}, l_{\varkappa}(t)) = 0 \qquad (3.3)$$

holds for any solution $s_{\varkappa}(t)$ of the adjoint system

$$ds_{\varkappa}(t)/dt = -A'(t)s_{\varkappa}(t) + l_{\varkappa}(t),$$

$$s_{\beta\varkappa}(t_{\beta}) = s_{\beta\varkappa}, \; l_{\varkappa}(t) = \sum_{i_{\varkappa}=1}^{2^{\varkappa}} l(t_{i_{\varkappa}})\delta(t_i - t_{i_{\varkappa}})$$

with the restriction $s'_{\beta\varkappa}s_{\beta\varkappa} + 2^{-\varkappa}\sum_{i_{\varkappa}=1}^{2^{\varkappa}} l'(t_{i_{\varkappa}})l(t_{i_{\varkappa}}) = \tau$. Functions $s_{\varkappa}(t)$ are continuous on the right.

The optimal control $u^{\circ}_{\varkappa}$ in the discrete problem satisfies the maximum principle

$$\max_{u \in \mathcal{U}} \int_{t_{\alpha}}^{t_{\beta}} u'_{\varkappa}(t)B'(t)s^{\circ}_{\varkappa}(t)dt = \int_{t_{\alpha}}^{t_{\beta}} u^{\circ\prime}_{\varkappa}(t)B'(t)s^{\circ}_{\varkappa}(t)dt$$

for the minimal function $h^{\circ}_{\varkappa} = B'(t)s^{\circ}_{\varkappa}(t)$ of condition (3.3). Assuming problem 2.I to be regular, we may chose the sequence $s^{\circ}_{\varkappa}(t)$ so as to satisfy the following relations

$$t^{\circ}_{\beta\varkappa} \to t^{\circ}_{\beta}, \; \Psi[s^{\circ}_{\beta\varkappa}, l^{\circ}_{\varkappa}(t)] \to \Psi[s^{\circ}_{\beta}, l^{\circ}(t)]$$
$$s^{\circ}_{\varkappa}(t) \to s^{\circ}(t), \; x^{\circ}_{\varkappa}(t) \to x^{\circ}(t) \qquad (3.4)$$

uniformly in $t \in [t_{\alpha}, t^{\circ}_{\beta} - \varepsilon]$ for an arbitrary $\varepsilon > 0$.

$$u^{\circ}_{\varkappa}(t) \dashrightarrow u^{\circ}(t) \text{ (in the weak convergence)} \qquad (3.5)$$

IV. Computation of the optimal control.

If $h^{\circ}_{i}(t) \equiv 0$ in the intervals $[\tau_{k_1}, \tau_{k_2}]$, then the optimal control $u^{\circ}(t)$ cannot be determined directly from the maximum principle. Its computation requires an additional discussion. With $\tau_{k_1}, \tau_{k_2}, s^{\circ}, l^{\circ}(t)$ being computed, problem 2.I may sometimes be reduced to an optimal problem with a special restriction $x(t) \in \bar{\mathcal{X}}(t)$, $\tau_{k_1} \le t \le \tau_{k_2}$, where $\bar{\mathcal{X}}$ is the boundary of $\mathcal{X}$. In case $\mathcal{X}(t)$ is given by inequalities $|x_k| \le \nu_k$, such a reduction may simplify the computation in whole.

In general the situation may be rather complicated. In order to compute $u^{\circ}(t)$ when $\tau_{i_1} \le t \le \tau_{i_2}$, we may use the relation (3.5), as functions $h^{\circ}_{i\varkappa}(t)$ are nonzero almost everywhere. It should be noted that (3.5) gives a sufficient condition for the optimality of $u^{\circ}(t)$. In general the controls are discontinuous inthe discrete version of the problem. There is an increasing number of discontinuities in $u_N^\circ(t)$, as $N \to \infty$. In view of this, there is only a weak convergence of the controls $u_N^\circ(t)$ towards $u^\circ(t)$. The computation of $u^\circ(t)$ by (3.5) is rather difficult. The procedure may be modified by averaging the functions

$$u_\delta^\circ(t) = \frac{1}{\delta} \lim_{N\to\infty} \int_0^\delta u_N^\circ(t+\vartheta)d\vartheta \qquad (4.1)$$

Functions $u_\delta^\circ(t)$ are continuous. If $\mathcal{X}(t)$ is continuous in the intervals $[\tau_{k1}, \tau_{k2}]$, the functions $u_\delta^\circ(t)$ converge uniformly ($\delta \to 0$) to the optimal control $u^\circ(t)$.

The procedure (4.1) is based on a direct utilization of the limit transition from III.3. It is desirable, however, to compute $u^\circ(t)$ avoiding the limit transition. Suppose s_β°, $\ell^\circ(t)$ are already computed and $\ell^\circ(t) \equiv 0$ on the set $\Omega = \bigcup_k \{t: [\tau_{k1} \le t \le \tau_{k2}]\}$. Divide Ω into a finite number of nonintersecting intervals $[\tau, \tau+\varepsilon], \tau\in\Omega$ of length ε. Now consider problem 2.1 in modified form with constraints (2.2) being applied to $x(t)$ everywhere except the interval $[\tau', \tau'+\varepsilon]$. Denote the solution for the new problem as $[u^\circ(t)]_\varepsilon$, $h_\varepsilon^\circ(t)$. Clearly $h_\varepsilon^\circ(t) \not\equiv 0$, $\tau' \le t \le \tau'+\varepsilon$
Denote also

$$\frac{1}{\varepsilon}\int_{\tau'}^{\tau'+\varepsilon} [u^\circ(t)]_\varepsilon dt = u_\varepsilon^\circ(t), \quad \tau' \le t \le \tau'+\varepsilon$$

Suppose $u^\varepsilon(t)$ is the control determined for the interval $\tau' \le t \le \tau'+\varepsilon$, according to the maximum principle

$$u^{\varepsilon\prime}(t) h^\varepsilon(t) = \max_u u'(t) h^\varepsilon(t), \quad u \in \mathcal{U}$$

where $h^\varepsilon(t) = B'(t) s_\varepsilon^\circ(t)$ and $s_\varepsilon^\circ(t)$ is determined by minimizing the functional Ψ in λ_1, λ_2 under the following limitations

$$ds_\varepsilon/dt = -A'(t)s_\varepsilon + \ell_\varepsilon^*(t) + \lambda_1\delta(t-\tau') + \lambda_2\delta(t-\tau'-\varepsilon)$$

$$\ell_\varepsilon^*(t) = \begin{cases} \ell^\circ(t), & t \bar\in [\tau', \tau'+\varepsilon] \\ 0, & t \in [\tau', \tau'+\varepsilon] \end{cases}; \quad s_{\varepsilon\beta} = s_\beta^\circ \qquad (4.2)$$

$$\lambda_1^2 + \lambda_2^2 = \int_{\tau'}^{\tau'+\varepsilon} \ell^{\circ\prime}(t)\ell^\circ(t)dt$$

Denote $u^{\varepsilon 0}(t) = \frac{1}{\varepsilon}\int_0^\varepsilon u^\varepsilon(\tau'+\vartheta)d\vartheta$. Then
$\|u^{\varepsilon 0}(\tau') - u_\varepsilon^\circ(\tau')\| \to 0$, $\|u_\varepsilon^\circ(\tau') - u^\circ(\tau')\| \to 0 \quad (\varepsilon \to 0)$
Hence the function $u^{\varepsilon 0}$ may be considered as an approxima-

tion of the optimal control u° . With $\lambda^\circ_\beta , \ell^\circ(t)$ being given, $u^{\varepsilon\circ}$ is computed by minimizing a function of two variables on each step of ε -length.

V. Examples.

1. Consider a model system, which, however, is a good illustration for the main procedures of the method and which may be solved of course by simple mechanical suggestions. System (2.I) is the following:

$$\dot{x}_1 = x_2 \ , \quad \dot{x}_2 = -x_1 + u$$

We are to specify the time-optimal control $u^\circ(t)$ which transfers the path $x^\circ(t)$ from $x(0) = \{0,0\}$ to $x(t_\beta) = \{2,0\}$ in minimal time under the restrictions $|u(t)| \le 2$, $|x_2(t)| \le 1$ of type (2.2). Problem (2.I3)-(2.I4) has the form

$$\min_{\lambda,\ell} \{2\int_0^{t_\beta} |\lambda_1 \sin(t_\beta - \tau) + \lambda_2 \cos(t_\beta - \tau) +$$

$$+ \int_t^{t_\beta} \ell(\tau)\cos(\tau - t)d\tau|dt - 2\lambda_1 + \int_0^{t_\beta} |\ell(t)|dt = 0 \qquad (5.I)$$

$$\lambda_1^2 + \lambda_2^2 + \int_0^{t_\beta} \ell^2(t)dt \le \tau$$

The solution of (5.I) is given by

$$\lambda_1^\circ = 1/2 \ ; \ \lambda_2^\circ = -tg(t^\circ_\beta - \tau_2)/2 \ ; \ \ell^\circ = 0 \ , \ 0 \le t < \tau_1 \ ;$$

$$\ell^\circ = -\cos(t^\circ_\beta - \tau_2)/2 \ , \ \tau_1 \le t < \tau_2 \ , \ \ell^\circ = 0 \ , \ \tau_2 \le t \le t^\circ_\beta \ ;$$

$$\tau_1 = \frac{\pi}{6} \ ; \ \tau_2 = \frac{\pi}{6} + \sqrt{15} + \sqrt{3} - 4 \ ; \ t^\circ_\beta = \tau_2 + \arcsin\frac{1}{4} \ ; \ h^\circ \equiv 0 \ , \ \tau_1 \le t \le \tau_2$$

(here the equation (3.2) is simply $d\ell/dt = 0$ and the boundary condition is a linear equation). Making use of the procedure given in IV, we obtain

$$u^\circ(t) = 2 \ , \ 0 \le t < \tau_1 \ ; \ u^\circ(t) = 2 - \sqrt{3} - \frac{\pi}{6} - t \ , \ \tau_1 \le t < \tau_2 \ ;$$

$$u^\circ(t) = -2 \ , \ \tau_2 \le t \le t^\circ_\beta .$$

2. The second example deals with the irregular case in problem 2.I. Consider the system

$$\dot{x}_1 = x_2 \ , \quad \dot{x}_2 = u$$

with $x_\alpha = \{0,0\}$, $x_\beta = \{5,0\}$; $|u| \le 1$, $|x_2| \le at + b$,
$a = 0$, $b = 1$ if $0 \le t \le 2$; $a = 2$, $b = -3$ if $t > 2$.
Computing (I.9)-(I.10) with $\tau = 8$,we obtain

$$\ell_1^\circ = 1 , \ell_2^\circ = -2 ; \; \mathcal{P}^\circ(t) = \ell^\circ(t) + \alpha\delta(t-\tau_2) , \; t_\beta^\circ = 5$$

so that

$$h^\circ(t) = \ell_1^\circ \cdot (5-t) + \ell_2^\circ + \int_\tau^5 \ell^\circ(t)dt$$

Here $\ell^\circ(t) = -1$ if $\tau_1 \le t \le \tau_2$ and $\ell^\circ(t) = 0$ if $0 \le t \le \tau_1$, $\tau_2 < t \le 5$; $\alpha = -1, t_1 = 1, t_2 = 2$ .Computing $u^\circ(t)$ as in IV,we obtain

$$u^\circ(t) = 1 , \; 0 \le t < 1 ; \; u^\circ(t) = 0 , \; 1 \le t < 2 ;$$

$$u^\circ(t) = 1 , \; 2 \le t < 3 ; \; u^\circ(t) = -1 , \; 3 \le t \le 5$$

Note.With the assumptions of II the main procedures of II-IV may also be applied with slight modifications to the irregular case.Function $\ell^\circ(t)$ is presented in general as the sum

$$\ell^\circ(t) = \ell(t) + \sum_k \alpha_k \delta(t - \tau_k)$$

The dimension of problem (I.9)-(I.10) now increases due to additional variables α_k in which (I.9) is to be minimized.

VI. Optimal controls in systems with time delay.

Suppose that the control plant operates in the presence of an after-effect [16] and is described by the equation

$$dx(t)/dt = Ax(t) + Gx(t-\tau) + Bu \; ; \; \tau \ge 0 \qquad (6.I)$$

whose coefficients are assumed to be constant.

Problem 6.I

Given are: a time interval $t_\alpha \le t \le t_\beta$,the initial state $x_{t_\alpha}(\vartheta)$, $(-\tau \le \vartheta \le 0)$ and the constraints (2.2) on the control u and the coordinates $x_i(t)$ .One is to determine the control function u° so as to minimize

$$\varepsilon^\circ = \rho \, [x_{t_\beta}(\vartheta)] + \| x_\beta \| = \min_u$$

Symbols $\rho[x(\vartheta)]$, $\|x\|$ denote certain norms in the space of n-vector functions and inthe finite-dimensional space respectively.

The approach tothe solution of problem 2.I given in II-IV may be applied im modified version to problem 6.I as well. The result is that we obtain a necessary condition of optimality for the control u° -the maximum principle

$$\int_{t_\alpha}^{t_\beta} h^\circ(t)u^\circ(t)dt = \max_{u\in U} \int_{t_\alpha}^{t_\beta} h^\circ(t)u(t)dt \tag{6.2}$$

where $h^\circ(t)=B'(t)s^\circ(t)$ and $s^\circ(t)$ is a nontrivial solution of the adjoint equation

$$\begin{aligned} ds/dt &= -A's - Gs(t+\tau) + \ell(t)\ , \quad t_\alpha \le t \le t_\beta - \tau \\ ds/dt &= -A's + p(t)\ , \quad t_\beta - \tau \le t \le t_\beta \\ s(t_\beta) &= s_\beta\ ;\ s(t_\alpha) = s_\alpha \end{aligned} \tag{6.3}$$

which delivers a minimum

$$\Psi(s_\beta^\circ, p^\circ, \ell^\circ) = \min_{s_\beta, p, \ell} \Psi(s_\beta, p, \ell) = 0 \tag{6.4}$$

to the nonnegative convex functional

$$\begin{aligned} \Psi(s_\beta, p, \ell) = & \int_{t_\alpha}^{t_\beta} \gamma_1[B'(t)s(t)]dt + \int_{t_\alpha}^{t_\beta-\tau} \gamma_t[\ell(t)]dt + \\ & + k_1\rho^*[p(t)] + k_2\|s_\beta^*\| + s_\alpha' x(t_\alpha) + \\ & + \int_{t_\alpha-\tau}^{t_\alpha} x'(t)G's(t+\tau)dt\ ;\ k_1 \ge 0,\ k_2 \ge 0 \end{aligned} \tag{6.5}$$

under the constraint

$$s_\beta' s_\beta + \int_{t_\alpha}^{t_\beta} \ell'(t)\ell(t)dt \le 1$$

Here $\|s\|^*$ and $\rho^*[p(t)]$ are conjugate norms with respect to $\|x\|$ and $\rho^*[x(\vartheta)]$. The value ε° is determined as the sum of numbers k_1°, k_2° which are a solution of (6.4).

The further process of solution is similar to the procedure for problem 2.I. It should be noted that a convenient approximation for problem 6.I may be obtained by replacing (6.I) with the finite-dimensional system

$$dy^\circ/dt = Ay^{(0)} + Gy^{(m)} + Bu\ ;\ dy^{(i)}/dt = \frac{m}{\tau}(y^{(i-1)} - y^{(i)}) \quad (i=1,\dots,m) \tag{6.6}$$

according to [17] . This, in particular, is due to the fact that with $m \to \infty$ and $t \geq 2\tau$, the solution of th homogeneous equation (6.I) may be approximated by functions $y^{(0)}(t), \ldots, y^{(m)}(t)$ of the homogeneous equation (6.6) uniformly in $t \in [t-\tau, t]$.

VII. Minimization of the maximal deviation of the system. The approach described above may be applied to the problem of minimizing the maximal deviation of the system [19] . Here is an example of the problem.

Problem 7.I

Boundary values $x_\alpha = x(t_\alpha)$, $x_\beta = x(t_\beta)$ being given, one is to determine the control $|u_k(t)| \leq C$ under the restriction u° , so that

$$\max_t |x_1(t)| = \min_u \tag{7.I}$$

The solution of problem 7.I is the control $u^\circ(t)$, the multipliers l°, $l^\circ(t)$ and the constant $k_1^\circ = \max_t |x_1^\circ(t)|$ which deliver a saddle-point to the functional $\mathcal{L}(u, l, l_1(t))$:

$$\inf_{l, l(t)} \max_{u \in \mathcal{U}} \Big\{ \int_{t_\alpha}^{t_\beta} B'(t) s(t) u(t) dt + s'_\alpha x(t_\alpha) + $$
$$+ k_1^\circ \int_{t_\alpha}^{t_\beta} |l(t)| dt \Big\} = 0 \quad ; \; l(t) = \{l_1(t), 0, \ldots, 0\}$$

where $s^\circ(t)$ is a solution of the adjoint system with the restriction (2.I4). The singularities are to be treated here as in problem 2.I.

A similar problem arises in the computation of a control programm when system 2.I is to track a given motion $\xi(t)$, so as to minimize the deviation $\| x(t) - \xi(t) \|$. The solution is an analogue of the procedure in II-IV. Here the end $x_\beta = x(t_\beta)$ of the trajectory $x^\circ(t)$ is free and it satisfies the restriction $\| x_\beta \| \leq k_2^\circ$, where k_2° is the optimal value for $\| x^\circ(t) - \xi(t) \|$.

A related problem is when the criterion of optimality in problem 7.I has the form

$$\min_t \| x(t) - \xi(t) \| = \min_u = k_3^\circ$$

and the end condition for the trajectory is $\| x_\beta - \xi(t_\beta) \| \geq k_3^\circ$. This problem also lies within the framework of the given method.

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13.6

Finding of Initial Values of the Auxiliary Variables in Optimal Control of a Class of Linear Systems

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1. Introduction

In cases when the principle of maximum is employed to determine the minimum time optimal control of a System whose motion is described by the following vector-form equation

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} \qquad /1.1/$$

the adjoint variables $\psi_1(t), \psi_2(t), \ldots, \psi_n(t)$ are introduced with the help of the vector equation

$$\dot{\vec{\psi}} = -A'\vec{\psi} \qquad /1.2/$$

and the condition of the extremal control can be written as

$$\vec{\psi}(t) B \vec{u}(t) = \max_{\vec{u} \in U} \vec{\psi}(t) B \vec{u}(t) \qquad /1.3/$$

In /1.1/ $\vec{x}(t)$ is a vector in the phase space X_n, $\vec{u}(t)$ – vector belonging to the set $U = \{ \vec{u} : -1 \leq u_i(t) \leq 1 , i = 1,2,\ldots r \}$ of the space of permissible control, $A = \{a_{ij}\}$ – matrix of order $n \times n$ with real non-positive ei x/, $B = \{b_{ij}\}$ matrix of order $n \times r$, A' in /1.2/ nsposed A matrix of

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x/ This condition is not essential

The vector $\vec{\Psi}(t)$ is defined by means of /1.2/ with the accuracy up to the constants $\Psi_1^0, \Psi_2^0, \ldots, \Psi_n^0$ /these are the coordinates of initial value of $\vec{\Psi}(0)$ /. It hence results that the Eqs. /1.3/ represent an infinite set of extremal control , $U_{ext} \subset U$.

Let an additional restriction $\vec{u}(t) \in U_{ext}$ be imposed on /1.1/. This means that an infinite set of $\phi_{ext}[\vec{x}^0]$ of trajectories uniquely defined by members U_{ext} is ascribed with the aid of /1.1/ to a fixed initial state $\vec{x}^0 = (x_1^0, \ldots, x_n^0)$ from the control region Ω . According to the sufficient condition of the minimum time optimalization /1.3/ [1] , motion along these trajectories is executed with the minimum time lags and, therefore, it is reasonable to call the members $\phi_{ext}[\vec{x}^0]$ the extremal trajectory

It follows from the theory of existence and uniqueness of optimal control [1, 2] that there exists a unique extremal trajectory which passes through an origin of coordinates. Unfortunately, these theorems fail to answer the question how to construct the vector $\vec{\Psi}(0)$ which determines $\vec{u}_{opt}(t)$ Therefore, it is necessary to seek a solution to the following problem:

With arbitrary $\vec{x}^0 \in \Omega$, such vectors $\vec{\Psi}(0)$ should be found that, if they are the initial values of /1.2/, Eq. /1.3/ defines , $U_{ext}(t)$ for which the corresponding extremal trajectory passes through the origin of coordinates, that is, $\vec{u}_{ext}(t) = \vec{u}_{opt}(t)$.

This is one of basic but not solved yet problems of optimal control. Many approximate methods for numerical solution to this problem are known in literature but they do not, however, provide an exact relation between $\vec{x}^0$ and $\vec{\psi}^0$ [3, 4, 8 thru 13].

In the present work, we have obtained a complete solution to the problem considered for a narrow but practically important class of linear systems

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \dots & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & & -\frac{a_{n-1}}{a_n} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \frac{b}{a_n} \end{bmatrix}$$

with one-dimensional domain $\mathcal{U} = \{ |u(t)| \leq 1 \}$

In the matter of fact this is a class of control systems whose motion is described by the following linear differential equation with constant coefficients

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \dots + a_1 \frac{dx}{dt} + a_0 x = b\, u(t) \qquad /1.4/$$

2. The Decomposition of n-dimensional Space ψ^0 into Sub-domains Containing those Initial $\vec{\psi}(0)$ Corresponding to Extremal Control Have the same Number of Breaking Points /Switchings/

The systems investigated have one control: $-1 \leq u(t) \leq 1$ and the extremal Eqs. /1.3/ can be written as

$$u_{ext}(t) = \operatorname{sign} \psi_n(t) \qquad /2.1/$$

Solutions will contain the known distinction depending on whether or not the matrices A have a zero eigen-value.

However, this distinction contributes nothing new to investigation and, therefore, we shall consider only a case when:

The eigen-values of the matrix A are real numbers $\lambda_1 < \lambda_2 < < \lambda_3 < .. \lambda_m < 0$ and their multiplicities $k_1, k_2, \dots, k_m$ $(k_1 + k_2 + \dots + k_m = n)$ [xx1]

$$u_{ext}(t) = \operatorname{sign}\left\{ -\sum_{i=1}^{n} \sum_{q=1}^{k_i} C_q^i \left[\sum_{r=1}^{q-1} \frac{t^r}{r!} \right] e^{-\lambda_i t} \right\} \qquad /2.2/$$

The constants $C_q^i \begin{pmatrix} i = 1,2,\dots,m \\ q = 1,2,\dots,k_i \end{pmatrix}$ are determined uniquely [xx2] from relations

$$\left.\begin{aligned} \psi_k^0 &= \sum_{i=1}^{m} \sum_{q=1}^{k_i} C_q^i \left\{ \sum_{j=1}^{k-1} \frac{a_j}{a_n} \left[\sum_{l=0}^{q-1} \binom{k+l-j-1}{k-j-1} \frac{1}{\lambda_i^l} \right] \frac{1}{\lambda_i^{k-j}} \right\} \\ k &= 1, 2, \dots, n-1 \\ \psi_n^0 &= -\sum_{i=1}^{m} \sum_{q=1}^{k_i} C_q^i \end{aligned}\right\} \qquad /2.3/$$

If Δ_1 denotes a determinant of the coefficients of variables $C_q^i \begin{pmatrix} i = 1,2,\dots,m \\ q = 1,2,\dots,k_i \end{pmatrix}$ in /2.3/, d_{qs}^{1i} is a minor corresponding to an element in Δ_1 located in th row and $k_1 + k_2 + .. + k_{i-1} + q + s$ column, then

$$C_q^i = \frac{1}{\Delta_1} \sum_{s=1}^{n} (-1)^{k_1 + k_2 + .. + k_{i-1} + q + s} d_{qs}^{1i} \psi_s^0 , \quad \begin{matrix} i = 1, 2, \dots m, \\ q = 1, 2, \dots, k_i \end{matrix}$$

The function: $\operatorname{sign} \psi_n(t)$ changes its sign only for those values of t which represent the roots of non-even multiplicity in equation

$$\psi_n(t) = 0 \qquad /2.5/$$

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xx1 If λ is an eigen-value of the matrix A, $-\lambda$ is an eigen-value of matrix A'

xx2 Determinant of the coefficients of variable C_q^i is not equal to zero [6]

For an arbitrary initial vector $\vec{\Psi}(0)$, Eq /2.5/ determines uniquely a countable set $T[\vec{\Psi}(0)]$ of those values of t which are its roots of odd multiplicity. Since /2.5/ is related with a real process we assume $t \geq 0$.

It results from the Lemma [xx3] that the set $T[\vec{\Psi}(0)]$ is finite and a number of its members $N_T[\vec{\Psi}(0)]$ does not exceed $n-1$ [xx4/]. This important conclusion enables to decompose the n-dimensional vector space of initial vectors Ψ^0 into n mutually not-intersecting sub-domains $\Psi_l^0\,(l=0,1,2,\dots n-1)$ related to all vectors $\vec{\Psi}(0)$ which are such that $T[\vec{\Psi}(0)]$ contains $N_T[\vec{\Psi}(0)]=l$ members.

Theorem 1. The sets Ψ_l^0 are l-parametric systems of sets Ψ^0 each one of which is a union of a finite number of linear sub-spaces. These sub-spaces does not include their intersections with l hyper-planes. If $M[n;l]$ denotes a number of all set-ups l of odd non-negative numbers whose sum does not exceed $n-1$, the union corresponding to an arbitrary set-up l of parameters is uniquely defined and contains $M[n;l]$ sub-sets.

Proof.

The aggregate Ψ_l^0 consists of all vectors $\vec{\Psi}(0)$ for which the equation

$$\sum_{i=1}^{m}\sum_{q=1}^{k_i}\Big[\sum_{s=1}^{n}(-1)^{k_1+k_2+\dots+k_{i-1}+s+q}\, d_{qs}^{1i}\,\psi_s^0\Big]\Big[\sum_{r=0}^{q-1}\frac{t^r}{r!}\Big]e^{-\lambda_i t}=0 \qquad /2.7/$$

---

xx3/ Lemma: Let $\lambda_1,\dots,\lambda_m$ be real numbers different in purs $f_1(t),\dots,f_m(t)$ be polynomials with real coefficients of powers $k_1,\dots,k_m$ correspondingly. Then the function

$$f_1(t)e^{\lambda_1 t}+\dots+f_m(t)e^{\lambda_m t} \qquad /2.6/$$

has not more than $k_1+k_2+\dots+k_m+m-1$ real roots.

xx4/ The function $\Psi_n(t)$ in/2.5/ can rewritten into the form/2.6/.

has l different roots with odd multiplicity.

Let $t_1, t_2, \ldots, t_l$ be arbitrary non-negative values of t satisfying the inequalities $0 < t_1 < t_2 < \ldots < t_l$ and simultaneously they are roots of Eq. /2.7/ with odd multiplicities $\tau_1, \tau_2, \ldots, \tau_l$, correspondingly. In other words after denoting $f_{qi}(t) = \left[\sum_{\tau=0}^{q-1} \frac{t^\tau}{\tau!}\right] e^{-\lambda_i t}$ and ν -th derivative by $f_{qi}^{(\nu)}(t)$, the numbers t_j should fulfil the conditions

$$\left.\begin{aligned}
&\sum_{i=1}^{m}\sum_{q=1}^{k_i}\left[\sum_{s=1}^{n}(-1)^{k_1+k_2+\ldots+k_{i-1}+s+q}\, d_{qs}^{1i}\,\psi_s^0\right] f_{qi}(t_j)=0\\
&\sum_{i=1}^{m}\sum_{q=1}^{k_i}\left[\sum_{s=1}^{n}(-1)^{k_1+k_2+\ldots+k_{i-1}+s+q}\, d_{qs}^{1i}\,\psi_s^0\right] f_{qi}^{(1)}(t_j)=0\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\sum_{i=1}^{m}\sum_{q=1}^{k_i}\left[\sum_{s=1}^{n}(-1)^{k_1+k_2+\ldots+k_{i-1}+s+q}\, d_{qs}^{1i}\,\psi_s^0\right] f_{qi}^{(k_j-1)}(t_j)=0\\
&\sum_{i=1}^{m}\sum_{q=1}^{k_i}\left[\sum_{s=1}^{n}(-1)^{k_1+k_2+\ldots+k_{i-1}+s+q}\, d_{qs}^{1i}\,\psi_s^0\right] f_{qi}^{(\tau_j)}(t_j)=0
\end{aligned}\right\}$$

Thus to a defined set-up of odd numbers satisfying the condition $\tau_1 + \tau_2 + \ldots + \tau_l \leq n-1$ and to a fixed set-up of values of parameters $t_1, t_2, \ldots, t_l$, there is ascribed a linear system with conditions

$$\left.\begin{aligned}
&\sum_{s=1}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\ldots+k_{i-1}+q}\, d_{qs}^{1i}\, f_{qi}(t_1)\right](-1)^s\psi_s^0=0\\
&\sum_{s=1}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\ldots+k_{i-1}+q}\, d_{qs}^{1i}\, f_{qi}^{(-1)}(t_1)\right](-1)^s\psi_s^0=0\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\sum_{s=1}^{m}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_1}(-1)^{k_1+k_2+\ldots+k_{i-1}+q}\, d_{qs}^{1i}\, f_{qi}^{(\tau_1-1)}(t_1)\right](-1)^s\psi_s^0=0\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\sum_{s=1}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\ldots k_{i-1}+q}\, d_{qs}^{1i}\, f_{qi}(t_l)\right](-1)^s\psi_s^0=0
\end{aligned}\right\} \quad /2.8/$$

$$\sum_{s=1}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_1}(-1)^{k_1+k_2+\ldots+k_{i-1}+q}\,dq_s^{1i}\,f_{q_i}^{(\tau_1-1)}(t_1)\right](-1)^s\psi_s^o=0$$

$$\sum_{s=0}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\ldots+k_{i-1}+q}\,dq_s^{1i}\,f_{q_i}^{(\tau_1)}(t_1)\right](-1)^s\psi_s^o=0$$

$$\sum_{s=0}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\ldots+k_{i-1}+q}\,dq_s^{1i}\,f_{q_i}^{(\tau_2)}(t_2)\right](-1)^s\psi_s^o=0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\sum_{s=0}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\ldots+k_{i-1}+q}\,dq_s^{1i}\,f_{q_i}^{(\tau_l)}(t_l)\right](-1)^s\psi_s^o=0$$

According to the Lemma cited, the number of Eqs /2.8/ does not exceed $n-1$, thus, indeed, all vector solutions $\vec{\psi}(0)$ of the first $\tau_1+\tau_2+\ldots+\tau_l$ equations represent a linear sub-space $\mathcal{T}$ from which there are excluded all vector solutions of the following linear system:

$$\sum_{s=1}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\ldots+k_{i-1}+q}\,dq^{1i}\,f_{q_i}^{(\tau_1)}(t_1)\right](-1)^s\psi_s^o=0$$

$$\sum_{s=1}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\ldots+k_{i-1}+q}\,dq^{1i}\,f_{q_i}^{(\tau_2)}(t_2)\right](-1)^s\psi_s^o=0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\sum_{s=1}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\ldots+k_{i-1}+q}\,dq^{1i}\,f_{q_i}^{(\tau_l)}(t_l)\right](-1)^s\psi_s^o=0$$

The number of possible sets-up of odd numbers $\tau_1, \tau_2, \ldots \tau_l$ is equal to $M[n:l]$.

Therefore, if an aggregate of all vectors $\vec{\psi}(0)$ satisfying /2.8/ is denoted by $\mathcal{T}_{ep}^o[t_1,t_2,\ldots,t_l,\tau_1,\tau_2.\tau_l]$ it results from the given reasoning that

$$\mathcal{T}_l^o=\bigcup_{\{0<t_1<t_2\ldots<t_l\}}\left\{\bigcup_{p=1}^{M[n:l]}\mathcal{T}_{ep}^o[t_1,t_2,\ldots,t_l;\tau_1,\tau_2,\ldots\tau_l]\right\}$$

The set $\mathcal{T}^o$ is also uniquely defined since

$$\mathcal{T}_0^o=\mathcal{T}^o\setminus\bigcup_{l=1}^{n-1}\mathcal{T}_l^o$$

3. Parametric Representation of Permissible Extremal Trajectory

An aggregate of permissible extremal trajectories $\Phi_{ext}[x^{s}]$ is uniquely defined by the vector differential equation

$$\dot{\vec{x}} = A\vec{x} + Bu_{ext}(t) \qquad /3.1/$$

$u_{ext}(t)$ involved in /3.1/ are piece-wisely constant functions /see Section 2/ because $u_{ext}(t)$ are completely given by $\sigma = u_{ext}(0)$ and discontinuity time moments $0 < t_1 < \ldots < t_l \quad (l \leq n-1)$. For values $t \neq t_1, t_2, \ldots t_l$ there is

$$u_{ext}(t) = \begin{cases} \sigma & \text{if } \; 0 \leq t < t_1 \\ (-1)^s \sigma & \text{if } \; t_s < t < t_{s+1} \\ & \quad s = 1, 2, \ldots, l-1 \\ (-1)^l \sigma & \text{if } \; t_l < t \end{cases} \qquad /3.2/$$

Eqs. /3.2/ show that for $t \neq t_1, t_2, \ldots, t_l$ the differential equation /3.1/ is a non-homogeneous linear differential equation.

Solution to /3.1/ for $t \neq t_1, t_2, \ldots, t_l$ can be written in the following vector form

$$\vec{x}(t) = \begin{cases} \sum\limits_{i=1}^{m} \sum\limits_{q=1}^{k_i} D_q^{1i}(0) \vec{x}_q^{1i}(t) + \dfrac{b}{a_0 a_n} \sigma \vec{l}_1 & \text{for } \quad 0 \leq t < t_1 \\ \sum\limits_{i=1}^{m} \sum\limits_{q=1}^{k_i} D_q^{1i}(s) \vec{x}_q^{1i}(t) + (-1)^s \dfrac{b}{a_0 a_n} \sigma \vec{l}_1 & \text{for } \quad t_s < t < t_{s+1} \\ & \qquad s = 1, 2, \ldots l-1 \\ \sum\limits_{i=1}^{m} \sum\limits_{q=1}^{k_i} D_q^{1i}(s) \vec{x}_q^{1i}(t) + (-1)^l \dfrac{b}{a_0 a_n} \sigma \vec{l}_1 & \text{for } \quad t_l < t \end{cases}$$

where $\mathcal{D}_q^{1i}(s)$ $s=0,1,2,\dots l$ are arbitrary constants corresponding to s-th interval

$$\vec{x}_q^{1i}(t) = \left[\sum_{\tau=0}^{q-1} \frac{t^\tau}{\tau!} \vec{h}_{q-\tau}^{(i)} \right] \quad \text{where, if} \quad \vec{h}_{q-\tau}^{(i)} = \left(h_{q-\tau,1}^{(i)}, h_{q-\tau,2}^{(i)}, \dots h_{q-\tau,n}^{(i)} \right)$$

then $h_{q-\tau,1}^{(i)} = 1$, $h_{q-\tau,k}^{(i)} = \lambda_i^{k-1} \left[\sum_{l=0}^{m-1} \binom{k-1}{l} \frac{1}{\lambda_i^l} \right]$ $k=2,3,\dots n$

and $\vec{l}_1 = (1, 0, 0 \dots 0)$ is n-dimensional vector [6].

Lemma 1. Extremal trajectories $\phi_{ext}[x^\circ]$ are also continuous at switching points corresponding to $u_{ext}(t)$

Proof.

Lemma 1 follows directly from representation of trajectory in the following vector form

$$\vec{x}(t) = \sum_{\gamma=1}^{n} \varphi_\gamma(t) \left[x_\gamma^\circ + \int_0^t \psi_\gamma^n(x) u_{ext}(x) dx \right]$$

where ψ_γ^n denotes the γ-th component of $\vec{\psi}^n$ [1,7]

The continuity of extremal trajectory also at the switching moments $t_1, t_2, \dots, t_l$ corresponding to $u_{ext}(t)$ fully determines the constants $\mathcal{D}_q^{1i}(s), s=0,1,2,\dots l$ $i=1,2..m$, $q=1,\dots k_i$ by means of $x_1^\circ, x_2^\circ, \dots x_n^\circ, \sigma$ and $t_1, t_2 \dots t_s$

Appendix 1 explains how the following formulae

$$\mathcal{D}_q^{1i}(s) = \frac{1}{\prod\limits_{\substack{i=1 \\ m \geq j > l}} (\lambda_j - \lambda_1)^{k_j k_l}} \left[W_q^{1i}(x_1^\circ, \dots, x_n^\circ, \sigma) + (-1)^{\sum_{j=1}^{i-1} k_j} (-1)^{1+q} 2\sigma \frac{b}{a_0 a_4} \sum_{u=1}^{3} \Delta_{1q}^{1i}(t_u) e^{-\lambda t_u} \right] \tag{3.3}$$

are derived for $\mathcal{D}_q^{1i}(s)$

The symbols $W_q^{1i}(x_1^\circ, x_2^\circ, \dots, x_n^\circ, \sigma)$ and Δ_{1q}^{1i} are determinants given in Appendix 1.

4. Determination of Initial Values of Adjoint, Unknowns for which Extremal Control is Optimal one.

The problem stated in Section 1 is completely solved by proving Lemma 3 and Theorem 2.

Lemma 3. If eigen-values of the matrix A are real numbers $\lambda_1, \lambda_2, \ldots, \lambda_m$ with their corresponding multiplicities $k_1, k_2, \ldots k_m$, $(k_1+k_2+\ldots+k_m = n)$ and they satisfy the condition $\lambda_1 < \lambda_2 < \ldots < \lambda_m < 0$, in this case, out of 2n transcedental systems xxxx1/

$$(-1)^{\sum_{j=1}^{i-1} k_j}(-1)^{1+q}\frac{a_0 a_n}{b} W_q^{1i}(x_1^0, \ldots x_n^0, \sigma) + (-1)^i \Delta_{1q}^{1i}(t^*)\, e^{-\lambda_i t^*} = 0 \qquad /4.1/$$

$$i = 1, 2, \ldots, m \qquad q = 1, 2, \ldots k_i$$

$$(-1)^{\sum_{j=1}^{i-1} k_j}(-1)^{1+q}\frac{a_0 a_n}{b} W_q^{1i}(x_1^0, x_2^0, \ldots x_n^0, \sigma) + 2\sum_{u=1}^{s}(-1)^{u-1}\Delta_{1q}^{1i}(t_u)\, e^{-\lambda_i t_u} +$$

$$+ (-1)^s \Delta_{1q}^{1i}(t^*)\, e^{-\lambda_i t^*} = 0$$

$$i = 1, 2, \ldots m \qquad q = 1, 2, \ldots k_i$$

only one has the solution $t_1, t_2, \ldots t_s, t^*$ which satisfies the condition $0 < t_1 < t_2 < \ldots < t_s < t^*$ and this solution is unique.

Proof.

The Lemma cited in the foot-note No 3 enables to decompose the aggregate of extremal trajectories $\Phi_{ext}[\vec{x}^0]$ into not-intersecting sub-aggregates $\Phi_{ext}^p[\vec{x}^0]$ $(p=0, 1, \ldots n-1)$ in dependence on a number of breaking points of $u_{ext}(t)$ determining them. According to the theorem of existence, there exists such an optimal trajectory that passes through

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xxxx1/ The last system of /4.1/ has been obtained for the particular case $m=n$, $k_1=k_2=\ldots k_n=1$, by Yu.S. Antomonov in [5].

the origin of coordinates and belongs to one of sets $\Phi^{\rho}_{ext}[\vec{x}^{\circ}]$

If this extremal process belongs to $\Phi^{\rho}_{ext}[\vec{x}^{\circ}]$ this will be, according to the parametric equation /3.3/ a trajectory for which there exists such $t^{*}>0$ that

$$\left.\begin{aligned} &\sum_{i=1}^{m}\sum_{q=1}^{k_i} D_q^{1i}(p)\left[\sum_{\tau=0}^{q-1}\frac{(t^*)^{\tau}}{\tau!}\right]e^{\lambda_i t^*} + (-1)^{\rho}\frac{b}{a_0 a_n}\sigma = 0 \\ &\sum_{i=1}^{m}\sum_{q=1}^{k_i} D_q^{1i}(p)\left[\sum_{\tau=0}^{q-1}\frac{(t^*)^{\tau}}{\tau!}\left(\lambda_i^{k-1}\sum_{j=1}^{q-\tau-1}\binom{k-1}{j}\frac{1}{\lambda_i^{j}}\right)\right]e^{\lambda_i t^*} = 0 \\ &k = 2,3,\ldots n-1, n \end{aligned}\right\} \qquad /4.2/$$

The systems /4.2/ are linear with respect to $D_q^{1i}(p)$ and they are of the same type as the systems from which $D_q^{1i}(p)$ are determined in Appendix 1. They differ only by free terms.

This fact enables also in this case to use the determinant denotations $\Delta_q^{1i}(p)$ introduced in Appendix 1, that is

$$D_q^{1i}(p) = \frac{(-1)^{\sum_{j=1}^{i-1} k_j}}{\prod_{i=1}^{m}(\lambda_j-\lambda_i)^{k_j k_i}}(-1)^{1+q}\Delta_{1q}^{1i}(t^*)e^{\lambda_i t^*}(-1)^{\rho-1}\frac{b}{a_0 a_n}\sigma \qquad /4.2'/$$

The left-hand side of /4.2'/ involves the $D_q^{1i}(p)$ determined already by the formula /3.3/. The symbols σ and $t_1, t_2, \ldots t_p, t^*$ used in $D_q^{1i}(p)$ denote breaking points of $u_{opt}(t)$. It is obvious that $0<t_1<t_2<\ldots<t_p<t^*$

The relations /4.2'/ obtained after $D_q^{1i}(p)$ are inserted represent, in the matter of fact, the n equations of the ρ-th transcedental system of /4.1/.

Taking into account that it is not known to what $\Phi^{\rho}_{ext}[\vec{x}^{\circ}]$ the optimal trajectory is related and, moreover, the σ is also unknown, we, at the beginning, set out 2n systems of /4.1/ for all $p = 0, 1, 2, \dots n-1,$ and $\sigma = \pm 1$ and determine those for which one solution $t_1, t_2, \dots t_p, t^*$ satisfies the condition $0 < t_1 < \dots < t_p < t^*$ The uniqueness of this system and its solutions is conditioned by the theorem of uniqueness of $u_{opt}(t)$.

Let eigen-values $\lambda_1, \lambda_2, \dots \lambda_m$ of the matrix A in /1.1/ have the corresponding multiplicity $k_1, k_2, \dots k_m$ and $\lambda_1 < \lambda_2 \dots < \lambda_m < 0$ and $(k_1 + k_2 + \dots k_m = n)$

Let the system

$$(-1)^{\sum_{j=1}^{i-1} k_j} (-1)^{1+q} \frac{a_0 a_n}{B} \sigma W_q^{1i}(x_1^0, x_2^0, \dots, x_n^0, \sigma) + 2\sum_{l=1}^{p} (-1)^{e-1} \Delta_{1q}(t_l) e^{\lambda_i t_e} + (-1)^{p} \Delta_{1q}^{1i}(t^*) e^{-\lambda_i t^*} = 0 \qquad /4.3/$$

$$i = 1, 2, \dots, m \,;\; q = 1, 2, \dots, k_i$$

be, for arbitrary chosen and fixed state $\vec{x}^0 \in X_n$, one of three transcedental systems /4.1/ between the solutions of which there exists such solution $t_1, t_2, \dots, t_p, t^*$ that $0 < t_1 < t_2 < \dots < t_p < t^*$

Let $\tau_1, \tau_2, \dots, \tau_p$ be an arbitrary combinations of odd and non-negative integers fulfilling the inequality

$$\tau_1 + \tau_2 + \dots + \tau_p \leq n - 1 \qquad /4.4/$$

For fixed $\tau_1, \tau_2, \dots, \tau_p$ the solutions to /4.3/ uniquely determine the set $T^0_{onT}[\vec{x}^0; \tau_1, \tau_2, \dots, \tau_p] \subset \Psi_p^0$ composed of all initial vectors $\vec{\Psi}(0)$. This set is a solution to the system

$$\left.\begin{aligned}
&\operatorname{sign} \psi_n^0 - \sigma = 0 \\
&\sum_{s=1}^{n} \Big[\sum_{i=1}^{m} \sum_{q=1}^{k_i} (-1)^{k_1 + k_2 + \dots + k_{i-1} + q} d_q^{1i} f_{qi}(t_1) \Big] (-1)^s \psi_s^0 = 0 \\
&\sum_{s=1}^{n} \Big[\sum_{i=1}^{m} \sum_{q=1}^{k_i} (-1)^{k_1 + k_2 + \dots + k_{i-1} + q} d_q^{1i} f_{qi}^{(-1)}(t_1) \Big] (-1)^s \psi_s^0 = 0 \\
&\sum_{s=1}^{n} \Big[\sum_{i=1}^{m} \sum_{q=1}^{k_i} (-1)^{k_1 + k_2 + \dots + k_{i-1} + q} d_q^{1i} f_{qi}^{(\tau_1 - 1)}(t_1) \Big] (-1)^s \psi_s^0 = 0
\end{aligned}\right\} \qquad /4.5/$$

$$\sum_{s=1}^{n}\left[\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\dots+k_{i-1}+q}\, d_q^{i}\, f_{qi}(t_p)\right](-1)^s\varphi_s^0=0$$

$$\sum_{s=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\dots+k_{i-1}+q}\, d_q^{i}\, f_{qi}^{(\tau_p-1)}(t_p)\Big](-1)^s\varphi_s^0=0$$

$$\sum_{s=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{k_i}(-1)^{k_1+k_2+\dots+k_{i-1}+q}\, d_q^{i}\, f_{qi}^{(\tau_1)}(t_1)\Big](-1)\varphi_s^0\neq 0$$

$$\sum_{s=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{k}(-1)^{k_1+k_2+\dots+k_{i-1}+q}\, d_q^{i}\, f_{qi}^{(\tau_p)}(t_p)\Big](-1)^s\varphi_s^0\neq 0$$

Where $f_{qi}(t)$ and $f_{qi}^{\nu}(t)$ denote functions $\left[\sum_{\tau=0}^{q-1}\frac{t^{\tau}}{\tau!}\right]e^{-\lambda_i t}$ and their γ -th derivatives correspondingly. Possible combinations of p non-negative odd numbers $\tau_1, \tau_2, \dots \tau_p$ satisfying Eq. /4.4/ are a finite number $M[n:p]$

Theorem 2.

The set $\bigcup_{\{\tau_1\tau_2\dots\tau_p\}} \Upsilon^0_{opt}\left[\vec{x}^0 : \tau_1, \tau_2, \dots \tau_p\right]$ contains all initial vectors that define $u_{opt}(t)$ by means of Eq. /2.1/.

The proof of this theorem immediately follows from Theorem 1, Lemma 2 and sufficiency of the condition /2.1/.

Example.

We shall solve the problem stated in section 1 for a control system whose motion is described by the differential equation

$$\frac{d^2x}{dt^2} + 3\,\frac{dx}{dt} + 2x = u(t)$$

where $\lambda_1=-2 \quad \lambda_2=-1 \quad a_0=2 \quad a_1=3 \quad a_2=1 \quad b=1$

Let the point $(\frac{1}{2}, 0)$ be chosen as an initial state $\vec{x}^0$. The transcedental systems /4.1/ for the chosen $\vec{x}^0$ have the form

$$\sigma = 1 \qquad\qquad\qquad\qquad \sigma = -1$$

$$s = 0 \begin{cases} (-1)^0 e^{t^*} = 0 \\ \frac{1}{2}(-1)^1 e^{2t^*} = 0 \end{cases} \qquad\qquad s = 0 \begin{cases} 2 - (-1)^0 e^{t^*} = 0 \\ -1 - \frac{1}{2}(-1)^1 e^{2t^*} = 0 \end{cases}$$

$$s = 1 \begin{cases} 2(-1)^0 e^{t_1} + (-1)^1 e^{t^*} = 0 \\ -(-1)^0 e^{2t_1} + \frac{1}{2}(-1)^2 e^{2t^*} = 0 \end{cases} \qquad s = 1 \begin{cases} 2 - 2(-1)^0 e^{t_1} - (-1)^1 e^{t^*} = 0 \\ -1 + (-1)^0 e^{2t_1} - \frac{1}{2}(-1)^2 e^{2t^*} = 0 \end{cases}$$

It can be found that for $\sigma = -1$ and $s = 1$ the corresponding system has the solution $e^{t_1} = 3$ and $e^{t^*} = 4$ and $m.e.\ 0 < \ln 3 = t_1 < \ln 4 = t^*$

For the chosen $\lambda_1 = -2$ and $\lambda_2 = -1$, the system /4.4/ is written in the form of one equality and two inequalities

$$\left.\begin{array}{r} \psi_2^0 < 0 \\ \alpha :\ -44\psi_1^0 + 5\psi_2^0 = 0 \\ -20\psi_1^0 + 11\psi_2^0 \neq 0 \end{array}\right\}$$

the solutions to which lie on the thicker segment of the straight line α /see Fig.1/.

5. Appendix 1

The continuity of extremal trajectories also at the breaking points $t_1, t_2, \ldots, t_l \ (l \leq m-1)$ $u_{ext}(t)$ requires the agreement of constants $D_q^{1i}(s)$, $s = 0, 1, 2, \ldots, l$ in order to satisfy Eqs /3.3/ for values $t_1, t_2, \ldots, t_l$. The continuity of trajectories at $t_1, t_2, \ldots, t_l$ enables in /3.3/ to write, for any t_s the corresponding equality in the vector form

$$\sum_{i=1}^{m} \sum_{q=1}^{k_i} \left[D_q^{1i}(s) - D_q^{1i}(s-1) \right] \vec{x}_q^{1i}(t_s) = 2(-1)^{s-1} \sigma \frac{B}{a_0 a_n} \vec{l_1} \qquad /5.1/$$

The determinant at the front of unknown

$$D_q^{1i}(s)-D_q^{1i}(s-1), \qquad \begin{matrix} i=1,2,\dots,m \\ q=1,2,\dots,k_i \end{matrix} \qquad (k_1+k_2+\dots+k_m=n)$$

is a Wronskian $W(t_s)$ of the basic system of the homogeneous differential equation

$$\dot{\vec{x}}=A\vec{x}, \qquad \text{for } t=t_s$$

$$W(t_s)=W(0)e^{-\int_0^{t_s} S_A dx}$$

and thus the S_A is a trace of matrix A; and then

$$S_A=-(k_1\lambda_1+k_2\lambda_2+\dots+k_m\lambda_m)$$

It is easy to prove that

$$W(0)=\prod_{\substack{i=1\\ m\geq j>i}}^{m}(\lambda_j-\lambda_i)^{k_j k_i}$$

This result enables to write the formula

$$W(t_s)=\prod_{\substack{i=1\\ m\geq j>i}}^{m}(\lambda_j-\lambda_i)^{k_j k_i} e^{-\sum_{\nu=1}^{m} k_\nu\lambda_\nu t_s}$$

Since $W(t_s)\neq 0$ for arbitry t_s, the conclusion is that $D_q^{1i}(s)-D_q^{1i}(s-1)$ is uniquely determined from /5.1/ with the help of formulae

$$D_q^{1i}(s)-D_q^{1i}(s-1)=\frac{(-1)^{\sum_{j=1}^{i-1} k_j}}{\prod_{\substack{i=1\\ m\geq j>i}}^{m}(\lambda_j-\lambda_i)^{k_j k_i}}(-1)^{1+q}\, 2\sigma\frac{b}{a_0 a_n}\Delta_q^{1i}(t_s)e^{-\lambda_i t_s} \qquad /5.2/$$

where $\Delta_{1q}^{1i}(t_s)$ are determinants obtainable from a supplementary minor of the elements $\omega_{1r}^{i}(t_s)=\left[\sum_{r=0}^{q-1}\frac{t_s^r}{r!}\right]e^{-\lambda_i t_s}$

in the first row of $W(t_s)$ after the exponents $e^{\lambda_i t_s}$ are put from columns to outside the parentheses.

If t_s in /5.2/ is replaced by $t_{s-1}, t_{s-2}, \dots, t_1$ and the equalities thus obtained are summed up, we obtain the following formulae for $D_q^{1i}(s)$

$$D_q^{1i}(s) = D_q^{1i}(0) + \frac{(-1)^{\sum_{j=1}^{i-1} k_j}}{\prod\limits_{\substack{i=1 \\ m \ge j > i}}^{m} (\lambda_j - \lambda_i)^{k_j k_i}} (-1)^{1+q}\, 2\sigma \frac{B}{a_0 a_n} \sum_{v=1}^{s} \Delta_{1q}^{1i}(t_v)\, e^{-\lambda_i t_v} \qquad /5.3/$$

It is found that $D_q^{1i}(0)$ are linear functions of $x_1^0, x_2^0, \dots$ $\dots x_n^0, \sigma$. Their exact form is determined by the system

$$\left.\begin{aligned} &\sum_{i=1}^{m} \sum_{q=1}^{k_i} D_q^{1i}(0) + \sigma \frac{B}{a_0 a_n} = x_1^0 \\ &\sum_{i=1}^{m} \sum_{q=1}^{k_i} \left[\lambda_i^{k-1} \sum_{j=0}^{q-1} \binom{k-1}{j} \frac{1}{\lambda_i^{j}}\right] D_q^{1i}(0) = x_k^0 \end{aligned}\right\}$$

by means of the formulae

$$D_q^{1i}(0) = \frac{W_q^{1i}(x_1^0, x_2^0, \dots, x_n^0, \sigma)}{\prod\limits_{\substack{i=1 \\ m \ge j > i}}^{m} (\lambda_j - \lambda_i)^{k_j k_i}}$$

The term $W_q^{1i}(x_1^0, x_2^0, \dots, x_n^0, \sigma)$ is obtained by substituting the numbers $x_2^0, x_3^0, \dots x_n^0, x_1^0 - \sigma \frac{B}{a_0 a_n}$ into the i-th group of $W(0)$.

15.1

INVESTIGATION OF DYNAMIC BEHAVIOURS OF CONTROLLED THYRISTOR ELECTRIC DRIVES

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(U.S.S.R)

The advancement of semiconductor technique has openedup new prospects for the development and creation of controlled A.C. and D.C. thyristor electric drives.

Though a number of papers concerning the investigation of the "Static frequency converter - induction motor" system [1-7] has been published in recent years, the phenomena taking place in these systems are still not clearly understood.

New difficulties due to the presence of valves cousing in particular the occurence of circuits with periodic commutation are added to the common difficulties encountered while studying the transient processes in an induction motor.

One of the important features of static frequency converters is that the output voltage of these converters is represented by staircase waveform. This being so, the motor current is usually evaluated either by sequential consideration of the processes at different inverter states [3,6] or using the method of harmonic analysis [7] .

For quasi-stationary and transient electromagnetic processes one can obtain a general analytical expression for the currents and the torque of an induction motor with the use of the sampled-data systems theory and the discrete Laplace transform.

Let us consider one of the efficient inverter circuits, i.e. a three-phase bridge circuit with commutating capacitances ,with intermediate D.C. supply [1]. .

It is assumed ideal commutation,the capacity of the smoothing -out condenser is so high that the D.C. supply voltage remains constant regardless of the load. Under such idealization the inverter voltage is of a staircase function depending on the nature of the load and the way of connection.

Analytically, the voltage may be described in terms of staircase functions taking the following form :

$$U_k = A_k \cos\frac{\pi}{3}n + B_k \sin\frac{\pi}{3}n . \quad (k=1,2,3) \tag{1}$$

The sampling period of these functions depends on the frequency, i.e.

$$T = \frac{1}{6f} = \frac{2\pi}{6\omega} \tag{2}$$

Table I gives voltage expressions for the case when the load is star and delta connected.

Table I

VOLTAGE	LOAD CONNECTION CIRCUIT	
	Delta	Star
$U_1[n]$ Phase I $U_2[n]$ Phase II $U_3[n]$ Phase III	$U_1[n]$ $U_1[n-2]$ $U_1[n-4]$	$U_1[n]$ $U_2[n-2]$ $U_1[n-4]$
At active load	$U_1[n] = U_d \cos\frac{\pi}{3}n$	$U_1[n] = \frac{\sqrt{3}}{3} U_d \sin\frac{\pi}{3}(n+1)$
	$0 \leq \omega t_g \leq \frac{\pi}{3} \quad 0 \leq \varepsilon \leq \gamma$	
$U_1[n]$	$U_1[n] = U_d[\cos\frac{\pi}{3}n - \frac{\sqrt{3}}{3}\sin\frac{\pi}{3}n]$	$U_1[n] = \frac{2}{3} U_d \cos\frac{\pi}{3}n$
At active-inductive load	$0 \leq \omega t_g \leq \frac{\pi}{3} \quad \gamma \leq \varepsilon \leq 1$	
	$U_1[n] = U_d \cos\frac{\pi}{3}n$	$U_1[n] = \frac{\sqrt{3}}{3} U_d \sin\frac{\pi}{3}(n+1) \cdot 1[n-\gamma]$
	$\frac{\pi}{3} \leq \omega t_g \leq \frac{\pi}{2}$	
	$U_1[n] = \frac{2\sqrt{3}}{3} U_d \sin\frac{\pi}{3}(n+2)$	$U_1[n] = \frac{2}{3} U_d \cos\frac{\pi}{3}n$

The "Static frequency converter - induction motor" system being so presented may be considered as a sampled-data system consisting of a zero-order hold and linear plant corresponding to the equivalent circuit of the induction motor.

For the quasi-stationary process the transfer function of the linear plant appears as

$$K(p) = \frac{k}{1 + T_1 p} \tag{3}$$

where $T_1 = \frac{L}{r}$; here r and L are the equivalent resistance and inductance of the motor.

Introducing a new transform parameter, $q = pT$ we determine the transfer function $k^*(q,\varepsilon)$ 8.

For the current function

$$i^*(q,\varepsilon) = k^*(q,\varepsilon) \cdot u^*(q,0) \tag{4}$$

where $u^*(q,0)$ is determined from the respective values of the voltage originals given in Table I .

From the inverse discrete Laplace transform the current function originals are determined.

Below is a detailed representation of current function expressions for the case when the motor is placed in the delta $(0 \leq \omega t_q \leq \frac{\pi}{3})$

for $0 \leq \varepsilon \leq \gamma$

$$i_I^*(q,\varepsilon) = \frac{U_d}{r}\left[A^*(q,\varepsilon) - B_I^*(q,\varepsilon)\right] \tag{5}$$

for $\gamma \leq \varepsilon \leq 1$

$$i_{II}^*(q,\varepsilon) = \frac{U_d}{r}\left[A^*(q,\varepsilon) - B_{II}^*(q,\varepsilon)\right] \tag{6}$$

where

$$A^*(q,\varepsilon) = \left(1 - \frac{e^q - 1}{e^q - e^{-\beta}} e^{-\beta\varepsilon}\right) \frac{e^q(e^q - 0.5)}{e^{2q} - e^q + 1} ;$$

$$B_I^*(q,\varepsilon) = \frac{1}{2}\left(1 - \frac{e^q - e^{-\beta(1-\gamma)}}{e^q - e^{-\beta}} e^{-\beta\varepsilon}\right) \frac{e^q}{e^{2q} - e^q + 1} ;$$

$$B_{II}^*(q,\varepsilon) = \frac{1}{2} \cdot \frac{(1 - e^{-\beta\gamma})}{e^q - e^{-\beta}} \frac{e^q}{e^{2q} - e^q + 1} e^{-\beta(\varepsilon - \gamma)} \tag{7}$$

The values of the originals are determined from the inverse discrete Laplace transform.

When studying the electromagnetic transient process in a system of axis rotating at a synchronous speed, the expression for Laplace transform of the stator current takes the following form:

$$i_{ss}(p) = \frac{e_{ss}(p)}{x_s'} K_T(p) \tag{8}$$

where

$$K_T(p) = \frac{p + \alpha_r + js}{(p+j)(p+\alpha_r'+js) + \alpha_s'(p+\alpha_r+js)} \tag{9}$$

By determining the transfer function $K^*(q,\varepsilon)$, from equation (9), and the value of $e_{ss}^*(q)$ from the phase voltages, we obtain the expression for the current function $i_{ss}^*(q,\varepsilon)$.

If the active resistance of the stator is neglected one can also obtain the analytical expression for the flux-linkage. Detailed expressions for both the current and the flux-linkage are given in paper[4].

When studying the electromechanical transient processes it is reasonable to use either an analog or a digital computer to perform the necessary calculations.

To carry out calculations with the use of a digital computer the equations for the motor were recorded in the system of variable-speed axes α and β [9].

In the case of the start the equivalent impedance is of an inductive nature. In this case the expressions for the voltages U_α and U_β as related to the rotating axes take the form:

$$U = \sqrt{\frac{2}{3}}\, U_d \cos\left(\frac{\pi}{3}n - \omega_0 \vartheta t\right);$$

$$U_\beta = \sqrt{\frac{2}{3}}\, U_d \sin\left(\frac{\pi}{3}n - \omega_0 \vartheta t\right). \tag{10}$$

The initial phase angle has been taken to be zero.

For the case of frequency control at a constant load torque (constant air-gap flux) we have the following system of differential equations:

$$\sqrt{\tfrac{2}{3}}\,\omega_o \nu U_d \cos\left(\tfrac{\pi}{3}n-\omega_o \nu t\right)=x_1\frac{di_{1\alpha}}{dt}+x_m\frac{di_{2\alpha}}{dt}+r_1\omega_o i_{1\alpha}-x_1\omega_o \nu i_{1\beta}-x_m\omega_o \nu i_{2\beta};$$

$$\sqrt{\tfrac{2}{3}}\,\omega_o \nu U_d \sin\left(\tfrac{\pi}{3}n-\omega_o \nu t\right)=x_1\frac{di_{1\beta}}{dt}+x_m\frac{di_{2\beta}}{dt}+r_1\omega_o i_{1\beta}+x_1\omega_o \nu i_{1\alpha}+x_m\omega_o \nu i_{2\alpha};$$

$$0=x_2\frac{di_{2\alpha}}{dt}+x_m\frac{di_{1\alpha}}{dt}+r_2\omega_o i_{2\alpha}-x_2\omega_o \nu s i_{2\beta}-x_m\omega_o \nu s i_{1\beta}; \qquad (11)$$

$$0=x_2\frac{di_{2\beta}}{dt}+x_m\frac{di_{1\beta}}{dt}+r_2\omega_o i_{2\beta}+x_2\omega_o \nu s i_{2\alpha}+x_m\omega_o \nu s i_{1\alpha};$$

$$-\frac{ds}{dt}=\frac{1}{2\nu H}\left(M_e-M_s\right);$$

$$M_e=x_m\left(i_{2\alpha}i_{1\beta}-i_{1\alpha}i_{2\beta}\right);$$

where $\nu=\frac{f_1}{f_{nom}}$ is the relative frequency of the stator current,

$\nu_2=\frac{f_2}{f_{nom}}$ is the absolute slip, $s=\frac{\nu_2}{\nu}$, $\varphi_1=\omega_o \nu t$,

ω_o = 157 rad/s , and H - is the inertia constant, s

Based on the system of equations (11), calculations were made concerning the electromechanical transient process of the induction motor having an output power of 2,8 kw, U = 380 v, and n_o = 1500 rpm.

To compare the trends of the transient processes for the cases of staircase and sine voltage calculations were made with respect to these two versions; in both cases the Runge-Kutta method was used.

In the case of sine voltage a standard program was used, step length is automatically adjusted. In the case of staircase waveform of voltage a standard subprogram to solve the set of differential equations was used in the general program for the solution of the system (11). The block diagram of the solution program is shown in Fig. 1. The step length h was taken to be $h=\frac{1}{4T}$. While programming the following designations were introduced: $i_{\alpha 1}=c$, $i_{1\beta}=b$, $i_{2\alpha}=y$, $i_{2\beta}=g$, and $S=\mathcal{R}$.

Several motor starting versions were considered both in the no-load state and at a constant load torque for different frequency values in the range from $\nu = 1$ to $\nu = 0,4$.

Figures 2 and 3 show graphs of $M_e=f(\tau)$ at a rated torque ($M_s = 1$) at $\nu = 1$ and $\nu = 0,4$ for both the sine and the staircase waveform of voltage. Fig. 4 presents curves of $M_e=\varphi(s)$ for staircase waveform of voltage with $M_s = 1$ at $\nu = 1$ and $\nu = 0.4$.

An analysis of the curves obtained has shown that pulsations appear in the torque curves when staircase waveform of voltage is supplied to the motor. The motor starting time increases as compared to that in the case of sine voltage power supply. When starting the motor under load at $\nu = 1$ the length of the starting period increased 2.85 times, while at $\nu = 0.4$ it doubled. When starting the motor under idling conditions the starting time increased 1.25 and 1.57 times, respectively.

A closed-loop system of automatic speed control for the induction motor at constant power (P = const) has been developed and studied. The block diagram of this system is presented in Fig. 5. The motor is operated from a static frequency converter. The load is provided by a D.C. generator. Owing to a specially developed circuit [10, 11] the generator output is automatically maintained at a constant level.

There are two control loops (one of them depending on frequency and the other one on voltage) to put into effect the frequency control law. The frequency control is based on the current of the induction motor. To control the voltage according to the required relationship $u=\sqrt{\nu}$, a functional transducer has been included in the circuit.

In conducting analytical studies on this system, a linearization of the equations was made and consideration was given only to small deviations.

In evaluating the transfer function of the motor the higher harmonics in the voltage curves were neglected [6].

Linearization of both the dynamic balance and torque equations defines the following relationships:

$$\Delta M_m - \Delta M_s = \theta p \Delta\omega_r ; \tag{12}$$

$$\Delta M_m = W_{1m}(p)\Delta U_i + W_{2m}(p)\Delta\varphi + W_{3m}(p)\Delta\varphi_r ; \tag{13}$$

where

$$W_{1m}(p) = \frac{3}{2} P U_{1m}^2 \frac{b_3 p^3 + b_2 p^2 + b_1 p + b_0}{a_4 p^4 + a_3 p^3 + a_2 p^2 + a_1 p + a_0} ;$$

$$W_{2m}(p) = \frac{3}{2} P U_{1m}^2 \frac{c_3 p^3 + c_2 p^2 + c_1 p + c_0}{a_4 p^4 + a_3 p^3 + a_2 p^2 + a_1 p + a_0} ;$$

$$W_{3m}(p) = \frac{3}{2} P U_{1m}^2 \frac{d_4 p^4 + d_3 p^3 + d_2 p^2 + d_1 p + d_0}{a_4 p^4 + a_3 p^3 + a_2 p^2 + a_1 p + a_0} .$$

Here, P is the number of pole pairs, U_{1m} - the amplitude value of voltage, ΔU_i - the relative increment of the voltage amplitude, $\Delta\varphi$ - the increment of the angle of rotation of the stator field, and $\Delta\varphi_r = \frac{\Delta\omega_r}{P}$ is the increment of the rotor angle of rotation.

The coefficients in the transfer functions are determined from the parameters of the motor.

The equations for the control loops based on frequency and voltage may be presented as:

$$\Delta f_i = - B(p)\Delta I_m ; \qquad \Delta U_i = - A(p)\Delta I_m . \tag{14}$$

The relationship between the increments of current ΔI_m and velocity $\Delta\omega_r$ is obtained on the basis of the frequency control law at P_s = const :

$$\Delta\omega_r = -K\Delta I_m \tag{15}$$

where $K = \dfrac{\omega_{nom}}{I_{nom}}$.

Simultaneous solution of equations (12-15) rezulted in a characteristic equation of the system

$$K\theta p^2 - W_{1m}(p)A(p)p - 2\pi W_{2m}(p)B(p) - KW_{3m}(p) = 0 \tag{16}$$

Stability studies were conducted at different frequencies from 50 to 20 cycles. The experimental studies have shown that the system makes it possible to maintain a constant output at a stable speed control of the induction motor with the frequency varying from 60 to 20 cycles.

Fig. 6 demonstrates oscillograms characterizing the change from one speed to another with varying frequency.

A new 9 kw D.C. thyristor electric drive system with the half-controlled three phase rectifier has been devised for wrapping machines used in tyre manufacturing. The speed control range of the system is 1 to 60 .

The motor speed control is accomplished as a fuction of the rate of the cord tyre and the drum diameter. A non-contact selsyn is the control element of the system.

The block diagram of this system is shown in Fig. 7. The system has been studied in detail both theoretically and experimentally.

The transient process calculations and the stability analysis were performed with the use of an electronic digital computer.

The analysis of the electromagnetic processes in the "Three phase half-controlled rectifier - motor" system was performed by the method of difference equations[12]. Solutions were obtained for two cases, i.e. with regard and with disregard to the commutation angle.

If no account is taken of the commutation angle the repetition period involves two subintervals.

In this case the expression for the current at the moments of commutation takes the following from:

$$J[n]=R_1\frac{1-e^{-n\frac{2\pi}{3}ctg\varphi}}{1-e^{\frac{2\pi}{3}ctg\varphi}}+J_o e^{-n\frac{2\pi}{3}ctg\varphi} \tag{17}$$

where $R_1=f(\alpha,\varphi)$, α - is the angle specified by the control system, and φ is the load parameter.

With the commutation angle being taken into account, dependences of commutation angles on the values of current at discrete moments of time were obtained which allowed to eliminate the values of commutation angles from the difference equations. In this case the expressions for current take the following form:

$$J[n]=\frac{R_2}{2x+3}\left\{\left[1-\left(\frac{x+1}{x+2}\right)^{2n}\right]+J_o\left(\frac{x+1}{x+2}\right)^{2n}\right\} \tag{18}$$

where $R_2=\varphi(\alpha,x)$, and x - is the load inductance.

Using equations (17-18) one can calculate the transient processes for digferent versions of changes in such parameters as the angle of control in the range of $0-\pi$, the e.m.f. at the input and output of the converter, etc.

An electronic digital computer was used to perform computations concerning the electromechanical transient processes in the "Rectifier - D.C. motor" system of a wrapping machine at fixed values of the angle α and at a variable moment of inertia. The system under investigation is given by the equations:

$$L\frac{di}{dt}+Ri+Cn=\frac{U_{do}}{2}(1+\cos\alpha) \tag{19}$$

$$M_m-M_s=M_d$$

where

$$M_m=C_m i\,;\quad M_s=\frac{30v(F_c+F_g)}{\eta\pi}\cdot\frac{1}{n}\,;$$

$$M_d=\frac{2\pi}{60}\left[J_m+\eta^2 J_d+\eta^2\frac{\pi b}{2g}(\gamma_c+0.7\gamma_g)\left(\frac{81\cdot10^4}{\eta^4\pi^4}\cdot\frac{v^4}{n^4}-R_o^4\right)\right]\frac{dn}{dt}$$

J_m and J_d are moments of inertia of the motor and drum, respectively; R_o is the initial radius; $\eta = \frac{n_d}{n_m}$; γ_c and γ_g are specific gravities of the cord and gasket, respectively; b is the cord width; and F_c and F_g are tensile forces for the cord and gasket, respectively.

Based on the machine computation results, a law has been derived concerning the variation of the control angle α under starting conditions; this law provides a limited value of starting current:

$$\alpha = Arccos\left[\frac{\sqrt{2}\pi}{3}\left(1-e^{-\frac{t}{T}}\right)\right] \tag{20}$$

In Fig. 8 are shown design curves obtained for the variation of current and speed in the course of starting, with the angle α changing according to equation (20).

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LIST OF FIGURES

for the paper "INVESTIGATION OF DYNAMIC BEHAVIOURS OF CONTROLLED THYRISTOR ELECTRIC DRIVES"

by

A. A. EFENDIZADE, B. A. LISTENGARTEN, S. M. BAGIROV, T. A. ZAIROVA, and Y. M. KURDUKOV

Figure 1. Block diagram of program.

Figure 2. Graph $M_e = f(\tau)$ at $M_s = 1$ and $\nu = 1$.

Figure 3. Graph $M_e = f(\tau)$ at $M_s = 1$ and $\nu = 0.4$.

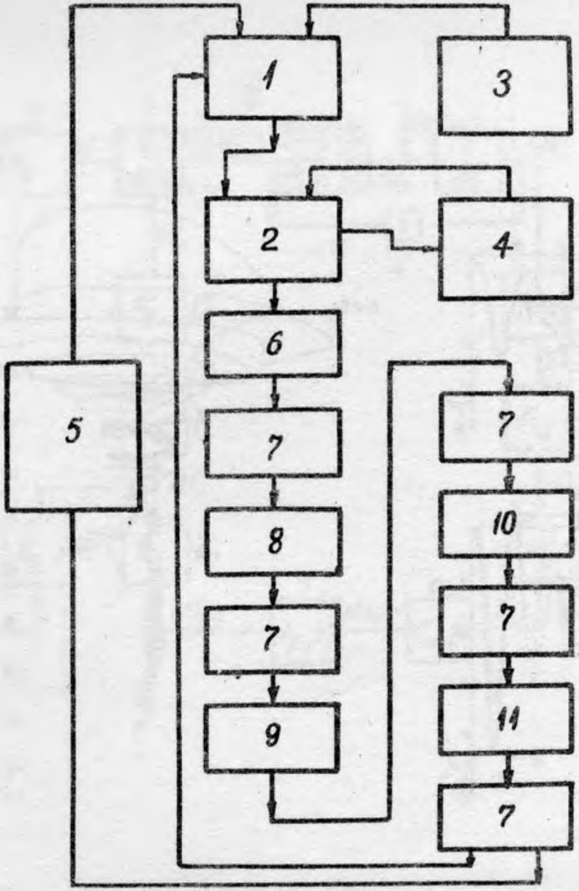
Figure 4. Torque-slip curves $M_e = \varphi(s)$ for $M_s = 1$, $\nu = 1$ and $\nu = 0.4$.

Figure 5. Block diagram of closed-loop system of automatic speed control for induction motor at constant shaft power.

Figure 6. Oscillograms of frequency control of speed of induction motor at $P_s =$ const.

Figure 7. Block diagram of D.C. thyristor electric drive system for rimming machines.

Figure 8. Graph $i = f(t)$ and $n = \varphi(t)$.



1. Cycles computer for calcul. of torque ($N_{max} = 4$).
2. Solution for organisation variation of „h" cycles computer ($N_{max} = 16$)
3. Calculation of torque, print results, drop to zero in computer.
4. Programme of change „n". Print to zero in computer.
5. Solution printing in point „t_i", sending values of c, b, y, g, π from (81+N) cells to (33+N) cells.
6. Calculation of right part of eq. 1.
7. Standard subprogramme.
8. Calculation of right part of eq. 2
9. Calculation of right part of eq. 3
10. Calculation of right part of eq. 4
11. Calculation of right part of eq. 5

Fig. 1

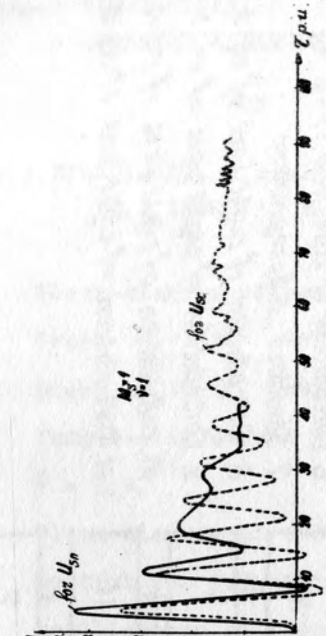


Fig. 2

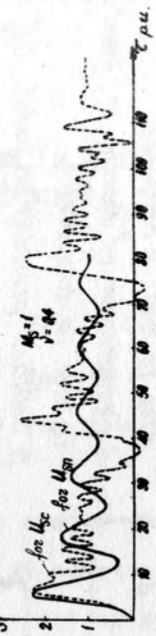


Fig. 3

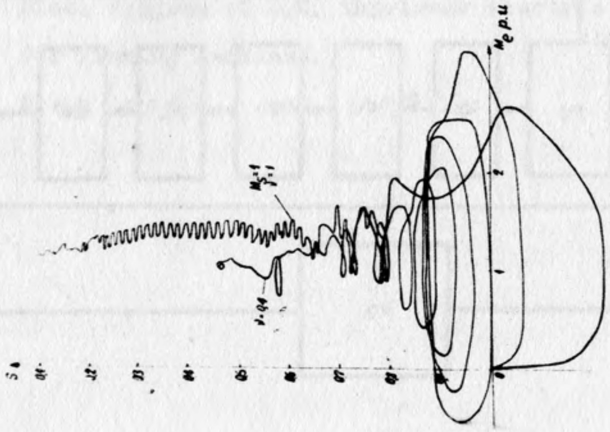
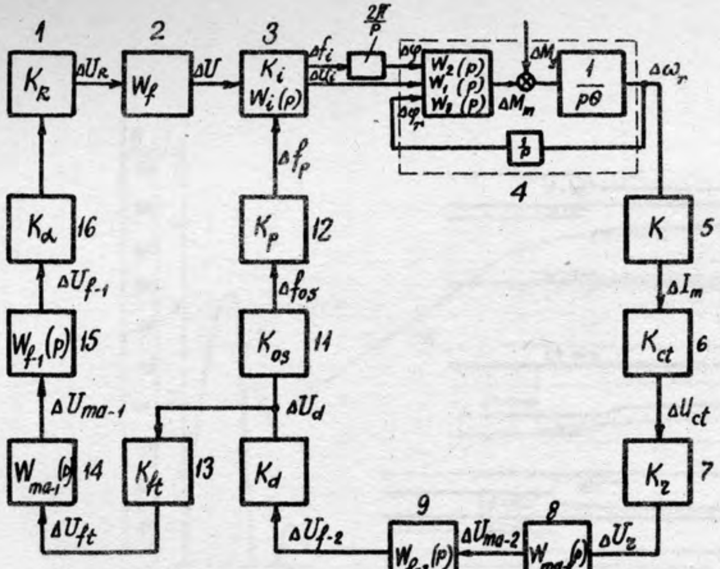


Fig. 4



1. Bridge controlled rectifier.
2. Filter L-C.
3. 3-phase bridge inverter
4. Induction motor.
6. Current transformer
7. Rectifier.
8. Magnetic ampflifier.
9. Hum filter.
10. Voltage divider
11. Oscillator
12. Recalculating circuit of system controlling inverter.
13. Functional transducer
14. Magnetic ampflifier
15. Hum filter
16. System for controlling of rectifier.

Fig. 5

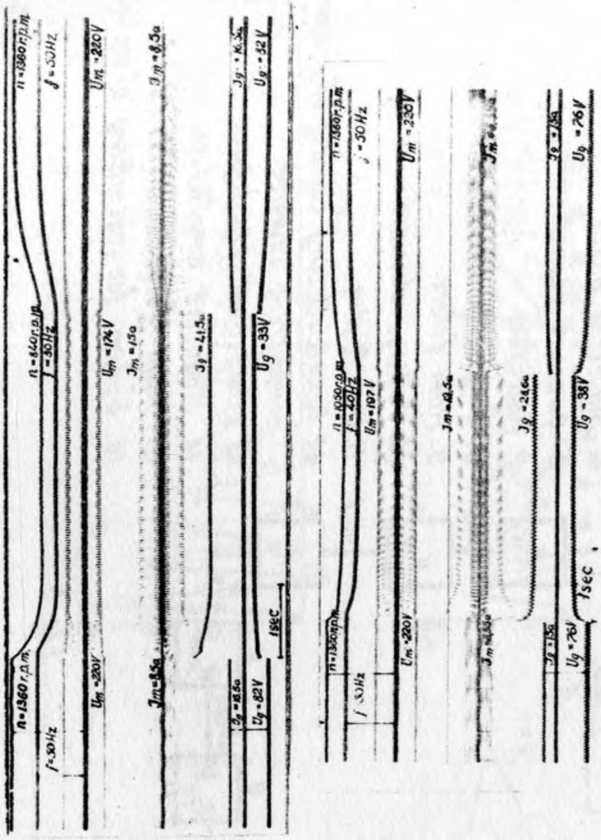


Fig. 6

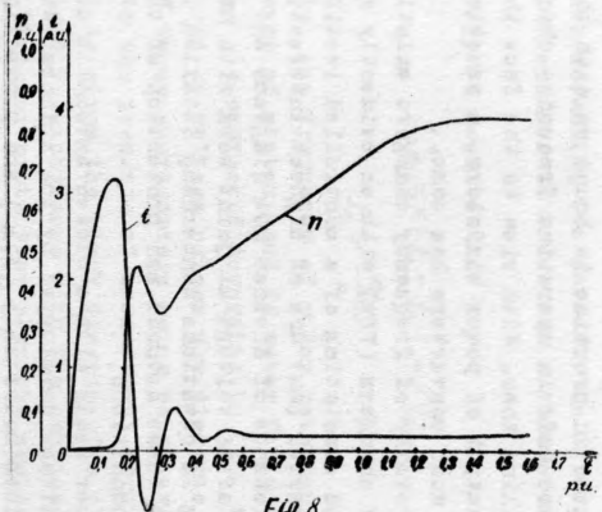
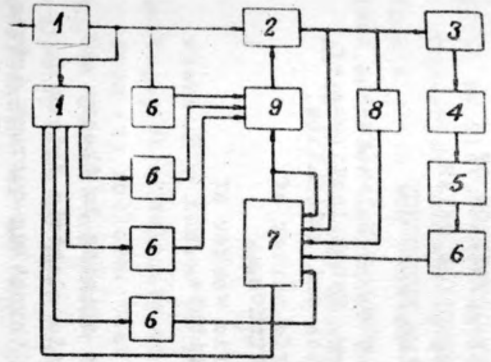


Fig. 8



1. Transformers
2. Controlled rectifier
3. D-c. motor
4. Drum
5. Non-contact selsyn
6. Rectifiers
7. Magnetic amplifier
8. Stabilized transformer
9. System of firing angle control

Fig. 7

15.2

SYSTEM OF AUTOMATIC GOVERNING THE CONDITION OF ABSOLUTE SLIP CONSTANCY OF AN INDUCTION DRIVE WITH TIRISTOR CONVERTER CONTROLLED FREQUENCY

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One of the most significant problems arising in theory and practice of constructing modern automatic elektric drives is the creation of frequency controlled alternating-current drives, where short-circuited induction motors (АД) are used. The realization of such drives in practice is bound up with the elaboration of economic and safe in operation frequency changers and a system for controlling them. With view to the fact that industry has mastered the output of power tiristors, a practical possibility of producing such converters has come.

Out of a multiformity of frequency changers existing today, tiristor frequency changers (ТПЧ) with an evidently marked direct current circuit and consisting of a controlled rectifier (В) with self-contained inverter (АИ) are of highest interest.

Despite the existence of diverse ТПЧ diagrams and methods for automatic control of АД voltage at joint work with ТПЧ, there is a considerable lag in questions concerning detailed investigation and realization of these methods and protection of the system against abnormal conditions.

The present paper deals with a closed automatic control system (CAP) of АД voltage developped by the authors, which secures constancy of absolute motor slip ($S_{абс}$) and protection of ТПЧ tiristors against excess currents.

Automatic control through condition $S_{абс} = const$ in the ТПЧ-АД system can be realized by means of three closed circuits: frequency, voltage and capacity of commutating АИ condensers.

At sufficient frequency stability of the master generator it is possible to control the АИ output frequency over an open-loop circuit. The capacity (C) control circuit as well can be realized over an open-loop circuit in case that variation of C is considered as disturbance action upon the voltage control circuit.

Converter voltage is regulated according to deviation of $S_{абс}$ from a given and constant value in dependence upon frequency, load and value of commutating capacity.

As is known, the absolute АД slip is expressed

$$S_{абс} = \frac{60}{p} f_1 - n = K f_1 - n \ , \qquad (1)$$

where P - number of pole pairs; f_1 - frequency of supply voltage; n - velocity of АД rotor rotation.

From (1) follows, that for realization the condition $S_{абс} = const$ the difference of stator rotating field velocity and АД rotor velocity must be kept constant. But in such case for practical realization of CAP analogous measuring devices should be required. However, measurement of rotor velocity by tachometric methods causes errors amounting two percent, being commensurable with value $S_{абс}$. Therefore we have used a discrete velocity measuring method.

Formula (1) we rewrite that way

$$S_{абс} = K(f_1 - f_2) = K f_3 \ , \qquad (2)$$

where
$$f_2 = \frac{n}{K} \qquad (3)$$

- physically, the frequency of rotor rotation velocity will be the difference of stator- and АД rotor field frequencies.

When $S_{абс}$ is expressed according to (2), the values f_1 and f_2 to be measured as voltage pulses are discrete ones and they are deprived of those shortcomings being peculiar to uninterrupted values as to the presence of measurement errors.

In fig.1 is given the block diagram of a closed CAP of АД voltage, realized in accordance with the mentioned principle (2), as well as of tiristor protection elements against excess currents. Here it means: ДС - smoothing choke; РМТ - overcurrent relay; ФД - photo-electric pulse transmitter; СУВ , СУИ - В and АИ control systems, respectively; ЗГ - master generator; ВЧ - deductor of f_1 and f_2 frequencies; ПЧН - converter of frequency f_3 into voltage U_{oc} ; ФС - phase shifter; УН , УМ - voltage- and power amplifiers, respectively; СЭ - comparing element; $U_з$ - master voltage; $U_э$ - standard voltage; БФА$_1$, БФА$_2$ - pulse generator blocks $Д_1$ and $Д_2$, respectively; ЛЭ - logical element; БВ - contactless switch.

To obtain an АД rotor rotation velocity signal, it has been worked out an ФД, in the design of which the АИ diagram and the number of АД pole pairs have been taken into account.

The measurement f_1 can be accomplished by two ways - directly at the АД terminals or with the СУИ. From the standpoint of convenience, precision of measurements and diminuation the number of device elements, f_1 rather should be taken with the help of СУИ. At this the frequency increases m times, where m may be equal to 6 or 3, what depends on the АИ diagram. In the case under consideration can be two variants of obtaining f_1, what depends on the ЗГ phase number.

If ЗГ is a single-phase one with following frequency division by means of a conversion ring, f_1 is taken off the ЗГ output. In case that ЗГ is a three-phase one, f_1 by means of a diode assembly must be taken off the output of the impulse generator.

We have worked out a ВЧ and ПЧН electronic diagram shown in fig.2. After amplification in valve $Л_6$ (6Н1П) frequency f_2 is given to $Л_7$ (6Н20П) - ВЧ; to that value as well is given frequency f_1 from ЗГ. For normal work of ВЧ it is necessary, that duration of pulse f_1 exceeds that of f_2. After amplification in valve $Л_8$ (6Н1П) the ВЧ output value is converted into an uninterrupted value $U_{ос}$ by means of waiting multivibrator $Л_9$ (6Н1П) and chain R_1C_1. Voltage $U_{ос}$ is then compared with voltage $U_з$ and $U_з$. The differential voltage $U_у$ is amplificated through voltage amplifier $Л_{10}$ (6Н2П) and power amplifier $Л_{11}$ (6П7С). The anode load of the latter is choke ДМ control winding ОУ of phase shifting bridge СУВ.

B and АИ control system. The bridge circuit of the converter is one of widely-disseminated ones for rectifying and inverting. But a tiristor control system of indicated diagrams is rather complicated, because it will entail the necessity of generating double pulses and strict match of pulse sequence at assembly and adjustment.

As has shown the analysis of B and АИ bridge circuit tests, the system for controlling them can be simplified to a high extent by reducing the number of doubling control pulses.

In fig. 3 and 4 are shown СУВ and СУИ basic diagrams. СУВ includes phase shifting elements, converting single-phase voltage into three-phase one, and pulse generating devices. For obtaining from the three-phase input six symmetrical bridge circuit control pulses, differential transformers (*ТД*) are used.

The diagram *СУИ* (fig.4) includes *ЗГ* , inversion ring and control pulse generator.

A distinctive feature of both control circuits is that they generate one doubling pulse. In circuit *СУВ* the doubling pulse is generated by one diode Д_1 , being switched between the pulse generator channels. Analogously, in circuit *СУИ* the doubling pulse is generated by diode Д_2 .

Such construction of B and *АИ* control circuits combined with preswitched resistance $R_{\text{п}}$(fig.3) results in reduction of control elements, simplifying of assembly and adjustment, decrease of control system power requirements and discharge of electrode - cathode transition of B and *АИ* tiristors.

The main object of applying $R_{\text{п}}$ consists in the following: One of the basic conditions of switching *ТПЧ* to the mains is the cophasal supply of control pulses to the tiristors of B and *АИ* . If this condition is not followed, "inertia" or a "dead zone" will show up, what depends upon the frequency ratio of B and *АИ* tiristor control. If time of "inertia" at switching is overcome automatically by the system itself, for overcoming the "dead zone" it is necessary to change *АИ* control frequency or to shift the phase of B control pulses.

For eliminating the indicated effects it is suggested to connect parallel between B and *АИ* an active resistance $R_{\text{п}}$ before *ДС* so, that independent of the moment of entering *АИ* control pulses $R_{\text{п}}$ will ensure a current which is equal to the tiristor retaining current.

Protection of the system against excess currents. The main form of normal work disturbance of *АИ* is its reversal with short circuit condition as a result, thus creating unfavourable conditions for action of B and *АИ* tiristors.

Existing protection methods (fusible.anode cutouts, short contactors a.o.) have substantial shortcomings[1]. For this reason we have developped a contactless protection of tiristors against excess currents, the basic diagram of which is shown in fig.5.

The working principle of this protection is based on the comparision of pulse A_1 with pulse A_2 (fig.1). At reversal of AU pulse A_1 is not generated and logical element $ЛЭ$ delivers a signal A_3, which acts on СУВ in such manner, that the rectified voltage could not decrease to zero, and that when sd it remains to a certain degree a fixed angle α_φ, which through tiristors B and AU proviedes a flow of current with lower value than their rated current.

For creation of α_φ the direct current in circuit $CУВ$ to be overlapped consists of fixed U_φ and adjustable U_3 voltage. These voltages are matched so, that they will provide the required α_φ if U_3 is switched off.

For obtaining pulse A_1 a pulse transformer (TU) is used (fig.4). All TU pulses are collected by a diode assembly ($CД$) and amplified in valve $Л_1$ (6Н1П). The amplified pulse is delivered to the waiting multivibrator $Л_2$ (6Н1П), where a pulse of required bandwith is taken from and given to $ЛЭ$ -second grid of $Л_5$ (6Ж2ОП). Pulse A_2 from $CУU$ input transformer TP_1 and through waiting multivibrator $Л_3$ (6Н1П) and amplifier $Л_4$ (6Н1П) is given to $Л_5$ - first grid. At lack of A_1 pulse A_2 opens valve $Л_5$ and on the output of transformer TP_2 appears a signal A_3. This signal switches the tiristor T, by the current of which U_3 is switched off. The contactless switch is a triode type П202 with a current flowing through it, being conditioned by U_3. Current for opening the triode is taken from the same source U_3 through a resistance R_1. Furthermore, into circuit U_3 is switched a normally closed contactor $K1$, who switches off from PMT (fig.1); the latter protects B and AU tiristors against overload currents. Contacts $K1$ in circuit T and anode voltage circuit of valves $Л_1-Л_5$ serve for return of the circuit into its initial condition.

Here are the main equations of its elements in linear approximation, required for stability analysis of the considered system. For this tolerances practically assumed for such systems have been adopted[2].

<u>Equation of node B -AC</u> with a short-circuit winding against sustained vibrations in AC (fig.1) according to [2] is written

$$(\beta T_2 p + 1)\Delta U_0 + (T_2 p + 1) R_э \Delta i_0 = (\beta_1 T_2 p + \rho) R_э \Delta\alpha \;, \tag{4}$$

where $\beta = \frac{T_{кз}}{T_2}$, $T_{кз} = \frac{L_{кз}}{R_{кз}}$, $T_2 = T_э + T_{кз}$,

$T_э = \frac{L_э}{R_э}$, $\beta_1 = \beta\rho$, $\rho = \frac{K_B}{R_э}$,

$R_э$, $L_э$, i_0 - active resistance, self-induction factor and current of $ДС$ operating circuit; $R_{кз}$, $L_{кз}$ - active resistance and self-induction factor of $ДС$ short-circuit winding; U_0 - $АU$ input voltage; K_B, α - transmission factor and control angle of B; P - sign of differentiation.

Equation of voltage regulator. The main elements of the voltage regulator are ФД , ВЧ , ПЧН , УН , УМ and СУВ (fig.2).

The equation of ВЧ and ФД is defined from (2) and (3). The dynamic condition of ПЧН in the main is defined by parameters of chain R_1C_1. Its derivated equation as well will be

$$(T_c p + 1)\Delta U_{oc} = K_c \Delta U_1 \;, \tag{5}$$

where $K_c = \frac{R_н}{R_1 + R_н}$ and $T_c = K_c R_1 C_1$

- transmission factor and time constant of chain R_1C_1, respectively; $R_н$ - ПЧН load resistance; U_1 - output voltage of ПЧН waiting multivibrator.

In comparing element СЭ voltage U_{oc} from the ПЧН output is compared with master voltage $U_з$ and permanent standard voltage $U_э$. СЭ output voltage will be equal to

$$\Delta U_y = \Delta U_з - \Delta U_{oc} \;. \tag{6}$$

The equation of УН we write

$$\Delta U_a = K_н \Delta U_y \;, \tag{7}$$

where $K_н$ - voltage amplification factor of amplifier $Л_{10}$ (6Н2П).

Voltage U_a is amplificated by power amplifier $Л_{11}$ (6П7С), the anode load of which is the control winding of the phase shifting bridge (fig.3). Then, symbolling the control winding current of this choke by i_n, we obtain

$$(T_n p + 1)\Delta i_n = K_м \Delta U_a \;, \tag{8}$$

where $T_n = \frac{L_n}{R_a}$ - anode circuit time constant; R_a - its active resistance; L_n - inductance of control winding ДМ; K_M - power amplification factor.

The parameters of elements included in the composition of СУВ - converter of single-phase voltage into three-phase one, pulse generator a.o. - practically have no influence on output co-ordinates of the phase shifting bridge (voltage phase). Therefore equations of the grid arrangement B in the whole are defined by the following equation of phase shifting bridge

$$\Delta\alpha = K_\alpha \Delta i_n \ , \tag{9}$$

where α - phase shift between input- and output voltages of phase shifting bridge - regulation angle B.

Equation of node АИ-АД we obtain proceed from the equation of drive moment

$$\mathcal{M} = \mathcal{M}_c + \frac{GD^2}{375} K \frac{df_1}{dt} - \frac{GD^2}{375} \frac{dS_{абс}}{dt} , \tag{10}$$

where M - АД torque; $\mathcal{M}_c$ - moment of resistance of operating gear, given to the АД shaft.

Moment M is defined by the ratio

$$\mathcal{M} = \frac{U^2 Б}{6{,}54 \pi f_1} = \varphi(U, f_1, C, S) , \tag{11}$$

where U - АД supply voltage, which in the considered system is defined by the expression[3]

$$U = \frac{\pi U_0 z_3 z(S)}{3\sqrt{6} \, K_T \tau_э} = \varphi(U_0, f_1, C, S, i_0) , \tag{12}$$

where $Б$, $z_э$, $z(S)$, $\tau_э$ - equivalent circuit parameters of node АИ-АД[3]; K_T - transformer factor of inverter transformer.

Linearizing equations (10) - (12) at C = const and taking into account, that $S = \frac{S_{абс}}{K f_1}$, we obtain the transfer function of node АИ-АД

$$(K T_3 p - K_4)\Delta f_1 - K_5 \Delta U_0 - K_6 \Delta i_0 + K_7 \Delta M_c = (T_3 p + 1) S_{абс} , \tag{13}$$

where $T_3 = \frac{GD^2}{375 K_3}$ - electromechanical time constant of the drive; $K_3 - K_7$ - linearisation factor[4].

The total transfer function of closed САР is defined in conference by solution of equations (2) - (9) and (13).

An experimental check on workability of the developped system has been carried out on a type УПВК-50 tiristorized converting assembly. As adjustable motor an АД 2.8 kw has been used. Results, obtained in time of theoretic investigation have shown good coincidence with the experiment.

As illustration we bring some of the oscillograms obtained. In fig. 6a are shown working dynamics of the protection circuit. From there it can be seen, how ancde current i_1 or i_2 who enters into action of the tiristor, ascends in the moment of Au reversal. The time for ~~b~~being the separate tiristor under short-circuit current at the level of working current makes 0.01 sec. Fig. 6b shows one of rated load charge and removing oscillograms, where control frequency is equal to 60 c.p.s. From that it can be seen that the quality of the system's transient response is satisfactory.

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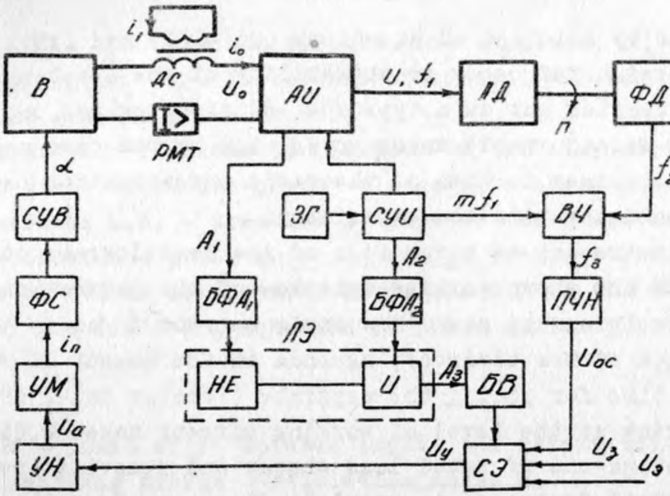

Fig. 1. Block diagram CAP of АД voltage and elements of ТПЧ tiristor protection.

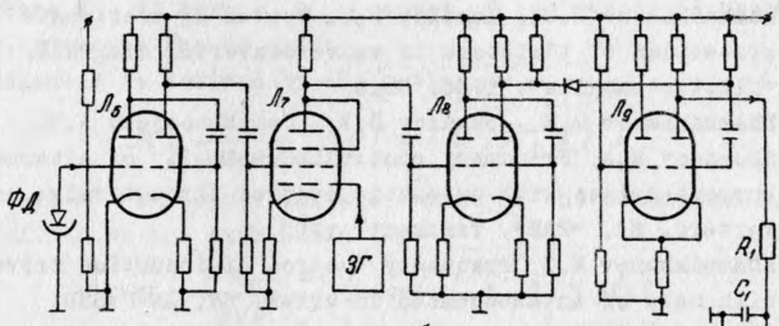

Fig. 2. Basic diagram of feedback-circuit of the system.

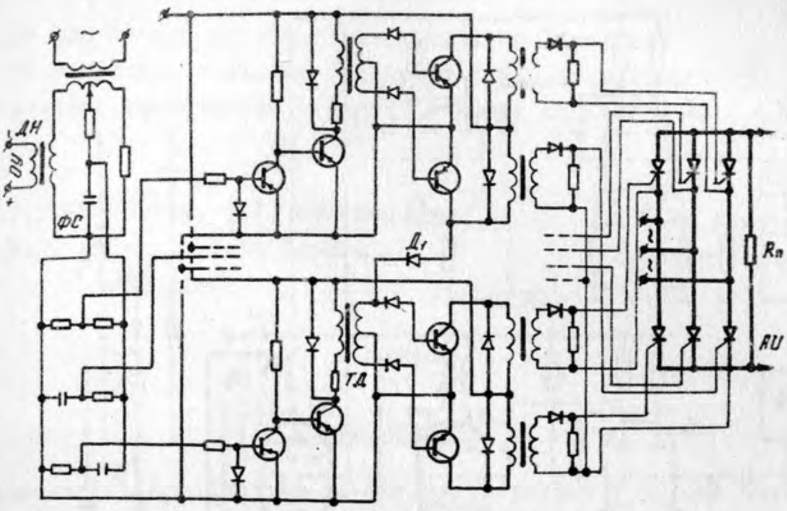

Fig. 3. Basic diagram of rectifier control system.

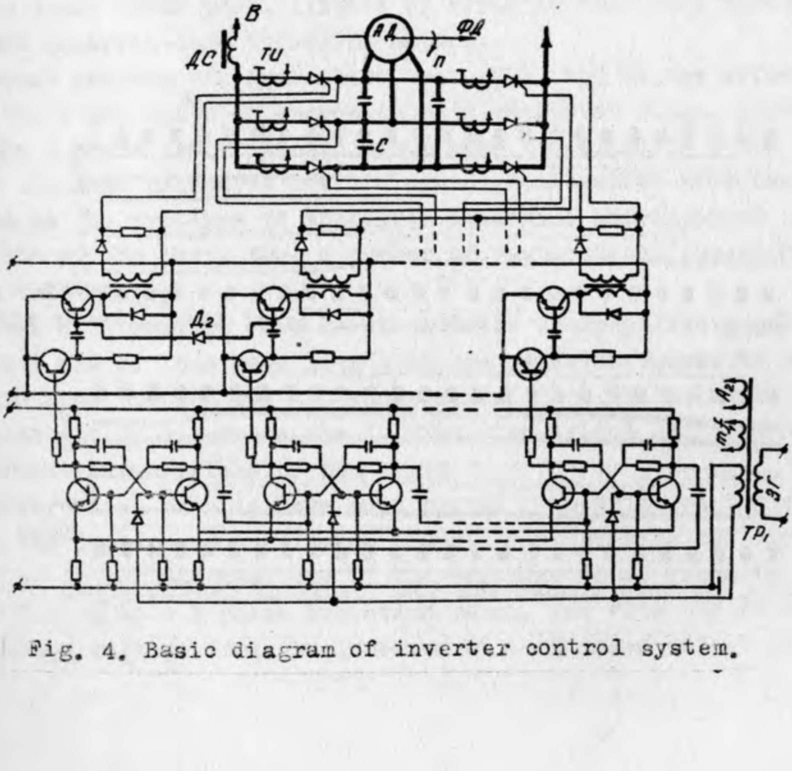

Fig. 4. Basic diagram of inverter control system.

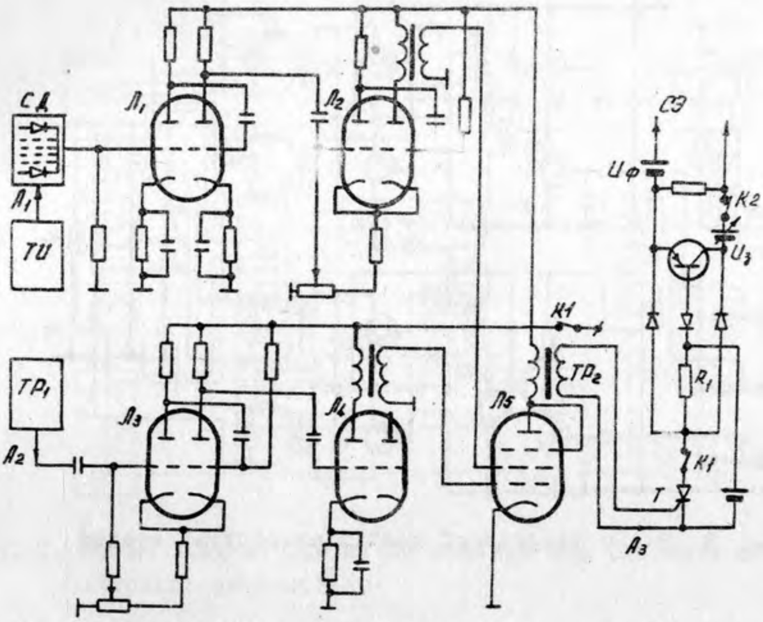

Fig. 5. Basic diagram of ТПЦ tiristor protection

a

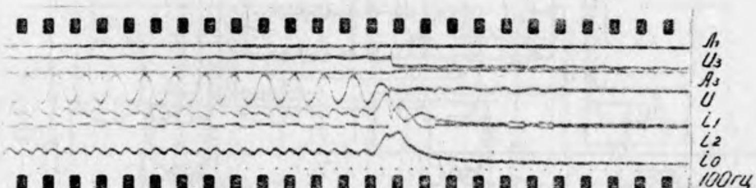

Fig. 6a. Oscillogram of working dynamics of the tiristor protection circuit.

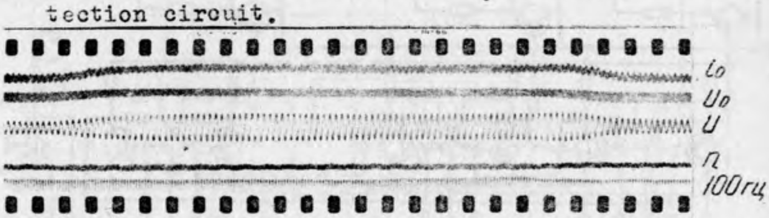

Fig. 6b. Load charge and load removing oscillogram.

15.3

On the Theory of concerning Problems Invariant Servosystems with the Thyristor Variable - Frequency Control by Squirrel - Cage 3 - Phase Induction Motors

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I. Introduction

Industrial application of the servosystems with the thyristor variable-frequency by induction motors proves to be of great economic importance. These servosystems will have higher efficiency and reliability comparing with the existing systems with more than 3 rotating motors. When using thyristors there is no doubt about possibilities of variable-frequency control by the squirrel-cage induction motors.

Great numbers of papers have been published to the effect. Now there are units in operation with thyristor r.p.m. control of the 3-phase squirrel-cage motor with the capacity up to some thousand kilowats. However not much attention have been given to the problems of such high precision servosystems despite of the fact, that a number of institutions investigation this theory.

Some difficulties have been met while studying the problem and not all of them have been overcome yet. This paper is considers some theoretical and experimental date, allowing to realize and to eliminate the difficulties arised and to design invariant servosystems of the type.

Theoretical data is have been proved by testing on the stand with the 3-phase squarrel-cage induction motor with the capacity of IO kw. Block-scheme of the servosystem is shown in fig. I. *Д* is a 3-phase induction motor, fed with the variable frequency voltage from the inverter *И* . *Д* makes object *О* rotate with the help of reductor P . A.c. provides the controled rectifier $УВ$. Further the rectified voltage is supplied to the filter $Ф_2$ and through the current transmitter $ТОД$ to the invertor $И$. Output voltage $УВ$ is controlled by the block $БУВ$, producing the opening angles Ψ of thyristor rectifier. The block $БУВ$ is fed by the amplifier $У$. By the other channel amplifier $У$ supplies the friquency oscillator $ЗГЧ$ through the filter $Ф_1$. Output pulses $ЗГЧ$ are supplied to the reversive closed loop $РК$, controlling order of phase turning. The servo blocks $БВВ$, $БВГ$, $БВК$ give the pulses, the necessary shope and deliver them to the invertor $И$. Thus, variable voltage with variable frequency is supplied to the motor $Д$, which causes smooth control of r.p.m. and produces the reverse effect.

Voltage $U_{вх}$ from sum block $СУ$ and the signal from the current limitation block $БОТ$ are supplied the amplifier $У$. The voltage proportioned to the angle of mismatch $\theta = \alpha_д - \alpha_о$ between the transmitting selsyn $СД(\alpha_д)$ and the receiving selsyn $СО\ (\alpha_о)$ provers to be the main control signal.

Voltage, generated by the tachometer $ТО$ and the differentiator $Д\varphi о$ (feed backs) provide the high stability. Transmitters of invariant signals tachometer $ТД$ rotated by the controlling installation $ЗД$ and differentiator $Д\varphi д$ compensate the error stipulated by the controlling signal $\alpha_д$. The current transmitter $ТОД$ and differentiator $Д\varphi м$ compensate the error stipulated by the perturbation torque $M_в$. The above system has same nonlinear and pulse blocks. Such servosystem is a complicated pulse nonlinear system, that is not practically calculated. Therefore while deriving the equation, the above mentioned "unit nonlinear function" is used. Unit nonlinear function is determined by the ratio of meanings nonlinear magnitude to the corresponding linearized values of it.

These values are selected either graphically or analytically and they make it possible to derive the equation true for both linearized and nonlinearized variants. Initially the linearized variant was calculated provided the high stability (including real negative roots of characteristic equation) and high precision of following. Afterwards the nonlinear variant with the known constants values of the selected installations is being checked.

With the lack of recuperation of energy the constant linear equation coefficients of the servosystem prove to be different for both the working méchanical characteristics of the induction motor and for the brake characteristics of it. The simpler case is being discussed there, when the recuperation of energy takes place that is when rectifier $УВ$ is bosed on symmetrical thyristors.

2. Initial equations

Initial equation of fig. I is

$$U_{Bx} = K_1 i_n \Theta - (\tau_6'' P^6 + \ldots + \tau_1'' P)\alpha_o + (\tau_7' P^7 + \ldots + \tau_1 P)\alpha_\partial + (\tau_{6M} P^5 + \ldots + \tau_{1M} P + P_{oM})[M_B + i_o^2 J_c (Z_j - 1) P^2 \alpha_o], \quad (1)$$

where K_1 - is the slope of the characteristic of CO transmitter output [v/rad],

i_n - transfer value of the fine transmitting selsyn TO up to the axis of the object

$$\tau_i'' = \frac{U_i''}{P^i \alpha_o} \frac{v \cdot sec^i}{rad}; \quad \tau_i' = \frac{U_i'}{P^i \alpha_o} \frac{v \cdot sec^i}{rad}; \quad \tau_{iM} = \frac{U_{im}}{P^i M_B} \frac{v \cdot sec^i}{K_g m}$$

- the slope of the characteristics of the diffirentiator output of order i, which belongs correspondingly to the rotation angle of the object α_o to the rotation angle α_o of the transmitting selsyn and to the perturbation torque M_B.

U_i'', U_i', U_{im} - voltage, generated by these differentiators [v]

Voltage $i^2 J_c (Z_j - 1) P^2 \alpha_o$ serves for the compensating influence on the dynamics of servosystem of variable consistent of the torque of inertia of the object. The expression in square brackets is the value of the transmitter discussed in [2].

Z_j - unit nonlinear function, involving the changes of the torque of inertia of the object.

Sum signal U_{bx} feeds the nonlinertia amplifier $У$. Further on this signal splits into two channals.

Output amplifier voltage $K_{y2} U_{bx}$ operates the work of the rectifier through the block $БУВ$ by means of changing the opening angles of thyristors Ψ.

Fig.2 (chart 1) shows the relationship between this angle ψ and the input voltage $K_{y2} U_{\text{вх}}$.

This relationship is nonlinear and the form of the nonlinearity is influenced by the block-scheme features. .

It could be expressed in the following way

$$K_{y2} U_{\text{вх}} = K_\delta Z_\delta (1 + T_\delta P)(\pi - \psi), \qquad (2)$$

where $K_\delta = \frac{K_{y2} U_{\text{вхн}}}{\pi}$ - a constant,

$Z_\delta = \frac{K_{y2} U_{\text{вх}}}{K_\delta (\pi - \psi)}$ - a unit nonlinear function

$\psi_H = \pi$

Z_B - is shown by the curve 3 (fig.2).

T_δ - is this block time constant.

Voltage pulses, corresponding to the angle ψ are supplied to the driven thyristor rectifier $УВ$.

Relationship of the output rectifier U_B operational voltage from the angle ψ can be seen in fig.3 (graph 1).

The linearized curve is reprisented by the straight line 2. If

$$K_B = \frac{\pi - \psi_0}{U_{BH}}; \quad Z_B = \frac{1}{U_B}\left(U_{BH} - \frac{\psi - \psi_0}{K_B}\right),$$

then $\psi = \psi_0 + K_B (U_{BH} - Z_B U_B)$, $\qquad (3)$

where U_B and U_{BH} - current and nominal values of the operational voltage at the rectifier output.

Z_B - a unit nonlinear function (chart 3 in fig.3) with the maximum increment equal to Z_{BH}

Assuming that $\pi - \psi_0 = K_B U_{BH}$ from the expressions (2) and (3), we have

$$K_{y2} U_{\text{вх}} = K_\delta Z_\delta K_B Z_B (1 + T_\delta P) U_B \qquad (4)$$

The voltage U_B is supplied on the T-section filter φ_2.

If we neglect the influence of the electromotive force on the invertor the relationship between the input U_B and output U_φ voltages could be written in a such form:

$$U_B = K_{\varphi_2}\left(P^2 + A_{12}P + A_{02}\right)U_\varphi = K_{\varphi_2}(P-\lambda_2)(P-\lambda_3)U_{\varphi_1} \quad (5)$$

where λ_2 and λ_3 are the roots of the characteristic equation, that corresponds the expression (5).

The factors A_{12}, A_{02} could be determined by the characteristic equation roots λ_2 and λ_3 this characteristic equation corresponds to the expression (5) in accordance with the Vieta relationships.

From the expressions (4) and (5) we have

$$K_{y2}U_{\text{вx}} = K_{\varphi_2}K_\delta Z_\delta K_B Z_B(1+T_\delta P)(P^2+A_{12}P+A_{02})U_\varphi \quad (6)$$

The voltage U_φ is supplied the invertor $И$.

Across another channel where the variable frequency are created the amplifier voltages $K_{y1}U_{\text{вx}}$ are supplied on the φ_1 -section filter. Its function is to sinchronize in the dynamics the frequency f_1 and the voltage U_φ that are supplied on the invertor. This is why this filter constants are chosen (if possible) to be equal to the filter φ_2 constants.

The filter φ_1 differential equation would be of the following form

$$K_{y1}U_{\text{вx}} = K_{\varphi_1}(P^2+A_{11}P+A_{01})U_{\varphi_1} = K_{\varphi_1}(1+T_{\varphi_1}P)(1+T_{\varphi_2}P)U_{\varphi_1} \quad (7)$$

where U_{φ_1} - this filter output voltage and $K_{\varphi_1} = K_{\varphi_2}$. Such kind of equation corresponds to Γ -section filter, sssembled using the inductance L_φ and the capacitance C_φ where $T_{\varphi_1} = T_{\varphi_2}$ are this filter time constants.

The voltage U_{φ_1} is supplied on the master frequency oscillator. The relationship of the frequency f_1 this voltage could be expressed by the following formula:

$$U_{\varphi_1} = K_F Z_F (1+T_F P) f_1 \quad (8)$$

where $K_F = \dfrac{U_{\varphi 1H}}{f_{1H}}$ - a constant,

$Z_F = \dfrac{U_{\varphi_1}}{K_F f_1}$ - a unit nonlinear function.

We assume that the blocks PK, БВГ, БВВ , БВК and ГИ do not bring the dynamic errors into the relationship (8) , but only distribute the signals of the required frequency among the phases ; and give the pulses that open the invertor thyristors the necessary shapes, that make the thyristors to be easily opened. As a result the a.c. voltage of the controlled frequency would be supplied on the induction motor winding, but the sinusoidal shapes of the voltage curves still would have the wellknown form of the rectangular approximation.

For the three-phase squirell-cage induction motor from the statics point of view there is a special relationship between the number of the revolutions per minute n and the motor shaft moment namely:

$$n = n_c - \frac{K_M \, Z_M \, M_\partial}{Z_\delta^2 \, Z_\gamma^2 \, Z_B^2}, \tag{9}$$

where n_c - sincronous number of revolutions per minute ,

$K_M = \dfrac{n_c - n_H}{M_{\partial H}}$ - a constant, defining the relationship between the moment and the number of revolutions per minute.

$Z_M = \dfrac{n_c - n}{K_M M_\partial}$ - a unit nonlinear function,

$Z_\gamma = \dfrac{U_\partial}{U_{\partial_1}}$ - a unit nonlinear function defining the decline on the motor of the factual voltage from the linearized one.

U_∂ and U_{∂_1} - actual and linearized voltages.

During the variable-frequency control of this motor speed it is necessary with the frequency decreasing to minimise the voltage supplied on its stator winding. The academician M.P. Kostenko was the first who proposed the voltage-frequency relationship. But this dependance does not provide the constant slope of the motor mechanical feature and freedom of the maximum moment from the frequency. The required conditions would be best satisfied by the relationship between these values that are defined by the equations (2-68)[1]. This relationship is applied for the family of the stable mechanical characteristics of the operational motor; and fig.4 shows these values.

If we limit the relationship (2-68)[1] with the permissible maximum number of errors in absolute slipping, i.e. $\beta = 0$ and $\beta = \alpha$, where α is the voltage relative frequency supplied on the motor, then the range of the relative voltage γ changes would be limited by the curves, shown in fig.5. Let us linearize it by the straightline 1. Then the value of relative linearised voltage on the motor would satisfy the conditions of the following equation

$$\gamma = \gamma_0 + K_\gamma Z_\gamma \alpha \qquad (10)$$

and a unit nonlinear function defining the relationship between the nonlinear voltage and the corresponding linearized values, is equal to

$$Z_\gamma = \frac{\gamma}{\gamma_1} \qquad (11)$$

where γ - accoding to the equation (2-68)[1] should be equal to the following expression:

$$\gamma = \sqrt{\frac{2\tau_1 \frac{\tau_2'}{\beta}\alpha + (b^2 + c^2\alpha^2) + (d^2 + e^2\alpha^2)\frac{\tau_2'^2}{\beta^2}}{2\tau_1 \frac{\tau_{12}'}{\beta} + (b^2 + c^2) + (\alpha^2 + e^2)\frac{\tau_2'^2}{\beta^2}}} \qquad (12)$$

The definition of all the units that are in the equation (12) is given in the paper[1].

The equation (9) is valid for the statical conditions of the motor operation. In the dynamics this relationship would be changed by the influence of electromagnetic inside stator and rotor processes. Approximately the influence could be expressed by two motor time constants T_{∂_1} and T_{∂_2}. Then the equation (9) would be of the following form :

$$n = K_i(P^2 + A_{1\partial} P + A_{0\partial})\left(\frac{f_1}{K_f} - \frac{K_M}{Z_\delta^2 Z_B^2 Z_\gamma^2} M_\partial\right) \qquad (13)$$

where $K_f = \frac{f_{1H}}{n_{CH}}$ - a constant.

f_{1H} and n_{CH} - a nominal frequency and the revolution number per minute.

$$A_{1\partial}=\frac{1}{T_{1\partial}}+\frac{1}{T_{2\partial}}; \quad A_{0\partial}=\frac{1}{T_{1\partial}T_{2\partial}}, \quad K_i=\frac{1}{A_{0\partial}}.$$

The moment, developed by the object, and applied to the motor shaft if we neglect the influence of the drive effeciency would be equal to

$$M\partial=\frac{M_B}{i_o}+i_o\, J\, P^2 \alpha_o \tag{14}$$

Here $M_B=\pm M_{CT\partial} i_o + M_B'$ kgm is the resulted perturbation torque, applied to the object.

$M_{CT\partial}=M_{xx}+\frac{M_{CT}}{i_o}$ - is a torque of the statical re istors, including the idle run motor moment. M_{xx}, calculated for the motor shaft.

M_B' - is an inherent perturbation torque including the instable torque [kgm].

i_o - transfer value from the motor to the object.

$J=J_o+\frac{J_o}{i_o^2}$ - the inertia torque of the drive rotating parts; calculated for the motor shaft [kgm.sec^2].

If the inertia torque J_o is variable, then we can put it in the following form:

$$J=J_\partial+\frac{J_o}{i_o^2}=Z_j\, J_c \tag{15}$$

where $Z_j=J/J_c$ - a unit nonliner function, defining of the decline of from the constant value

3. Nonlinear differential equation

By the simultaneous solving of the equations (1)-(8),(10), (13)-(15), we have the following expression.

$$(a_7 Z_0 p^7 + A_6 p^6 + \dots + A_1 p + A_0)\,\theta(t) =$$
$$= (B_7 p^7 + \dots + B_1 p)\,\alpha_\partial(t) + (G_5 p^5 + \dots + G_1 p + G_0) M_B + B_0 ,$$

where
$$A_i = a_i + \sigma \tau''_{ni} , \quad i = 1, 2, \dots, 6 , \tag{16}$$
$$B_i = A_i - \sigma \tau'_{ni} , \quad i = 1, 2, \dots, 7 ,$$
$$G_i = g_i - \sigma \tau_{niM} , \quad i = 0, 1, \dots, 5 ,$$
$$B_0 = \frac{\pm M_{cт.\partial}}{i_0 J_c T_F T_{\varphi 1} T_{\varphi 2} T_{\partial 1} T_{\partial 2}} ,$$

$$A_0 = \frac{K_{y1} i_n K_1}{I_0} \frac{1}{sec^7} , \quad \sigma = \frac{K_{y1}}{I_0} = \frac{A_0}{K_1 i_n} ,$$

$$I_0 = K_f K_F K_M i_0 J_c T_{\partial 1} T_{\partial 2} T_F T_{\varphi 1} T_{\varphi 2} \frac{z_{\varphi 1} + z_{\varphi 2}}{z_H} \; v \cdot sec^7 ,$$

$$\tau''_{ni} = \tau''_i + \tau''_{i-1}(T_{\partial 1} + T_{\partial 2}) + \tau''_{i-2} T_{\partial 1} T_{\partial 2} \; \frac{v \cdot sec^i}{rad} ,$$

$$\tau'_{ni} = \tau'_i + \tau'_{i-1}(T_{\partial 1} + T_{\partial 2}) + \tau'_{i-2} T_{\partial 1} T_{\partial 2} \; \frac{v \cdot sec^i}{rad} ,$$

$$a_7 = \frac{1}{I_0}\left[\tau''_6 (T_{\partial 1} + T_{\partial 2}) + \tau''_5 T_{\partial 1} T_{\partial 2}\right] \frac{v \cdot sec^7}{rad} ,$$

$$a_6 = Z_0 \left(\frac{1}{T_F} + \frac{1}{T_{\varphi 1}} + \frac{1}{T_{\varphi 2}} + \frac{1}{T_{\partial 1}} + \frac{1}{T_{\partial 2}} \right) \frac{1}{sec} ,$$

$$a_5 = Z_0 \left[\frac{1}{T_{\varphi 1} T_{\varphi 2}} + \left(\frac{1}{T_{\varphi 1}} + \frac{1}{T_{\varphi 2}} \right)\left(\frac{1}{T_F} + \frac{1}{T_{\partial 1}} + \frac{1}{T_{\partial 2}} \right) + \right.$$
$$\left. + \frac{1}{T_{\partial 1} T_{\partial 2}} + \frac{1}{T_F} \left(\frac{1}{T_{\partial 1}} + \frac{1}{T_{\partial 2}} \right) \right] \frac{1}{sec^2} ,$$

$$a_4 = Z_0\left[\frac{1}{T_F}\left(\frac{1}{T_{\varphi 1}T_{\varphi 2}} + \frac{1}{T_{\partial 1}T_{\partial 2}}\right) + \left(\frac{1}{T_{\varphi 1}} + \frac{1}{T_{\varphi 2}}\right) \times\right.$$

$$\left.\times\left(\frac{1}{T_F T_{\partial 1}} + \frac{1}{T_F T_{\partial 2}} + \frac{1}{T_{\partial 1}T_{\partial 2}}\right) + \frac{1}{T_{\varphi 1}T_{\varphi 2}}\left(\frac{1}{T_{\partial 1}} + \frac{1}{T_{\partial 2}}\right)\right] + \frac{Z_F}{T_{\partial 1}T_{\partial 2}T_M}\ \frac{1}{sec^3},$$

$$a_3 = Z_0\left[\frac{1}{T_F T_{\varphi 1}T_{\varphi 2}}\left(\frac{1}{T_{\partial 1}} + \frac{1}{T_{\partial 2}}\right) + \frac{1}{T_F T_{\partial 1}T_{\partial 2}}\left(\frac{1}{T_{\varphi 1}} + \frac{1}{T_{\varphi 2}}\right) +\right.$$

$$\left. + \frac{1}{T_{\varphi 1}T_{\varphi 2}T_{\partial 1}T_{\partial 2}}\right] + \frac{Z_F}{T_{\partial 1}T_{\partial 2}T_M}\left(\frac{1}{T_F} + \frac{1}{T_{\varphi 1}} + \frac{1}{T_{\varphi 2}}\right)\frac{1}{sec^4},$$

$$a_2 = \frac{Z_0}{T_F T_{\varphi 1}T_{\varphi 2}T_{\partial 1}T_{\partial 2}} + \frac{Z_F}{T_M T_{\partial 1}T_{\partial 2}}\left[\frac{1}{T_{\varphi 1}T_{\varphi 2}} + \frac{1}{T_F}\left(\frac{1}{T_{\varphi 1}} + \frac{1}{T_{\varphi 2}}\right)\right]\frac{1}{sec^5},$$

$$a_1 = \frac{Z_F}{T_F T_{\varphi 1}T_{\varphi 2}T_{\partial 1}T_{\partial 2}T_M}\ \frac{1}{sec^6},$$

$$g_5 = \frac{Z_0}{i_0^2\ J_c}\ \frac{1}{kgm\ sec^2},\quad g_4 = \frac{g_5\ a_6}{Z_0}\ \frac{1}{kgm\ sec^3},$$

$$g_3 = \frac{g_5\ a_5}{Z_0}\ \frac{1}{kgm\,sec^4},\quad g_2 = \frac{g_5}{Z_0}\left(a_4 - \frac{Z_F}{T_{\partial 1}T_{\partial 2}T_M}\right)\frac{1}{kgm\ sec^5},$$

$$g_1 = \frac{g_5}{Z_0}\left[a_3 - \frac{Z_F}{T_{\partial 1}T_{\partial 2}T_M}\left(\frac{1}{T_F} + \frac{1}{T_{\varphi 1}} + \frac{1}{T_{\varphi 2}}\right)\right]\frac{1}{kgm\ sec^6},$$

$$g_0 = \frac{g_5}{T_{\partial 1}T_{\partial 2}T_{\varphi 1}T_{\varphi 2}T_F}\ \frac{1}{kgm\ sec^7},$$

$$Z_0 = \frac{Z_M Z_F}{Z_\delta^2 Z_\gamma^2 Z_B^2},\qquad T_M = \frac{\pi K_M J_c}{30}\ sec.$$

4. The examples of the calculation

This calculation was made for the operational model of the servosystems with the induction motor with the capacity of 10 kw (1350 r.p.m.). The object speed could be equal to $\pm$ 60°/sec. and the accelleration – $\pm$ 40°/sec^2 ; the object inertia torque equals to J_o=2740 $\pm$ 1300 kgm.sec^2 ; the statical servosystems errors being less than 1 x 10^{-3}, and dynamical ones – less than 3 . 10^{-3} rad.

The maximum nonlinearity values are as follows:

$$Z_m = \begin{cases} 1 \\ 0{,}85 \end{cases}, \quad Z_o = \begin{cases} 6{,}76 \\ 0{,}293 \end{cases} \quad Z_F = 1$$

The so-called method of the type equations has been used high values of the stability [3] .

There has been selected the linearised equation with the real negative roots of the characteristic equation. The amplifier increasing factor has become equal to K_{y_1} =462. In the linearied variant the overcorrection is equal to zero and the duration of the transition process – 0,14 sec.

The differential equation of the nonlinear variant is of the following form :

$$\begin{aligned}&[a_7 Z_o P^5 + 474{,}6 Z_o P^4 + (5{,}537\cdot 10^4 Z_o + 1{,}087\cdot 10^4) P^3 + \\ &+ (1{,}65\cdot 10^5 Z_o + 6{,}555\cdot 10^6) P^2 + 2{,}51\cdot 10^8 P + 3{,}46\cdot 10^9]\,\Theta(t) = \\ &= [Z_o P^5 + 474{,}6 Z_o P^4 + (5{,}537\cdot 10^4 Z_o + 1{,}087\cdot 10^4) P^3]\,\alpha_a(t) + \\ &+ Z_o (3{,}28\cdot 10^{-4} P^3 + 0{,}156 P^2 + 18{,}2 P + 51{,}6) M_B \pm 1{,}16\cdot 10^4 \qquad (17)\end{aligned}$$

When Zo=1 we get the linearized variant of this equation. The checking up by the computer of the transition process of non-linear variant has shown that the curve decline from the corresponding linearized variant is too small; and the servosystem errors are not beyond the given ranges.

5. Conclusion

1. The servosystems (described in the examples above) with the variable-frequency control of the squirrel-cage 3-phase induction motor is of great importance for the industrial applications and it is recomended to be used in industry as soon as possible.

2. The said servosystem is pulsed nonlinear system, and the proposed method of deriving the equation with the so-called unit nonlinear function gives the opportunity write the differential equation of the nonlinear variant, that could be easily transformed into the linearized one.

3. Using the voltage current pickup proportional to the perturbation torque at one and the same time we compensate the influence of the variable of inertiar torque object component on the dynamics of the servosystems.

4. The said nonlinearities influence only upon the natural components a_i and g_i of the factors A_i and G_i, and they have no influence upon the differential components of these factors. If in nonlinearized variant we would choose the high stability and the high amplification factor, than as a rule these systems in the nonlinear variant would be invariably stable.

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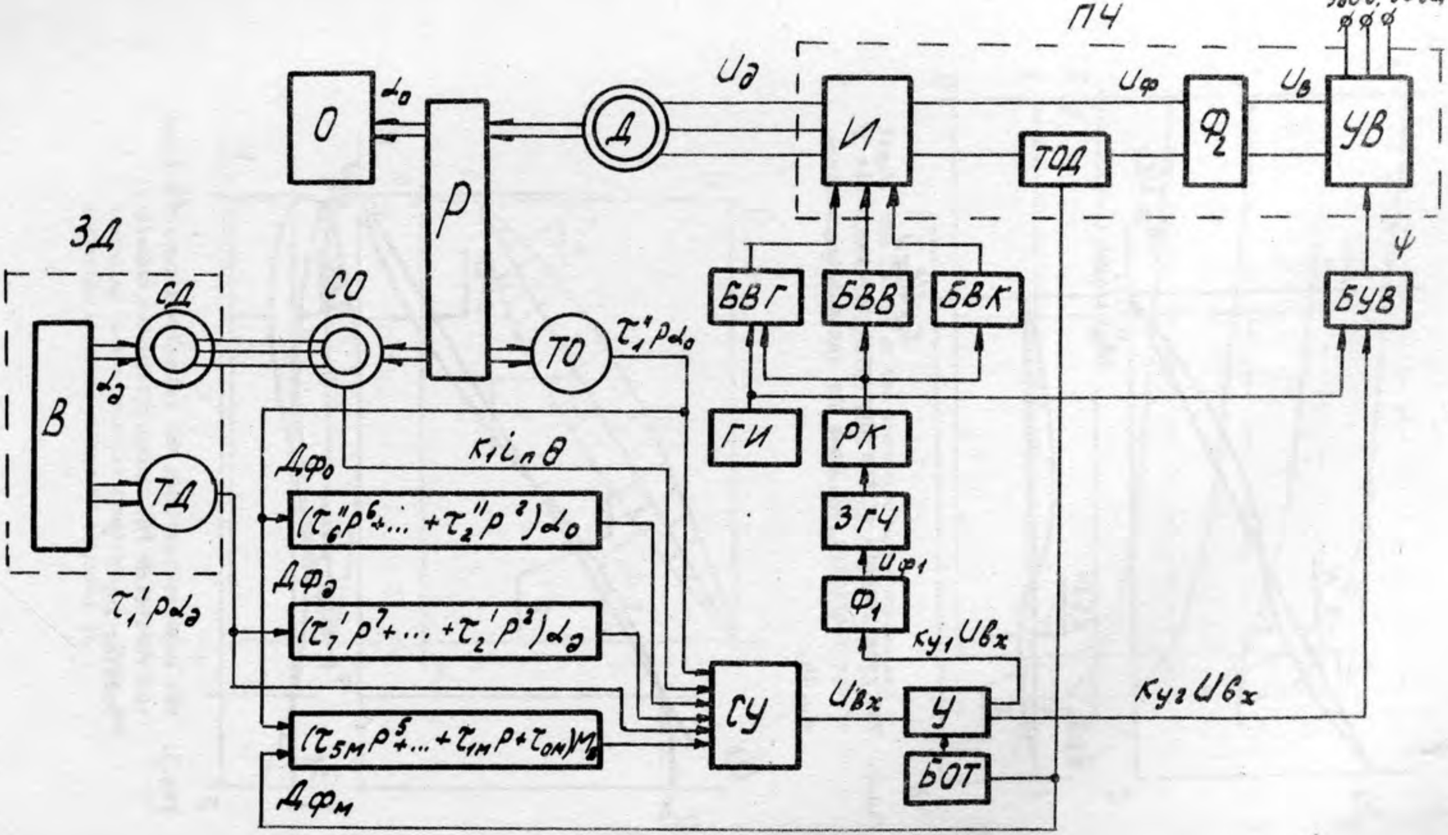

Fig.1. The block-scheme of the servosystem

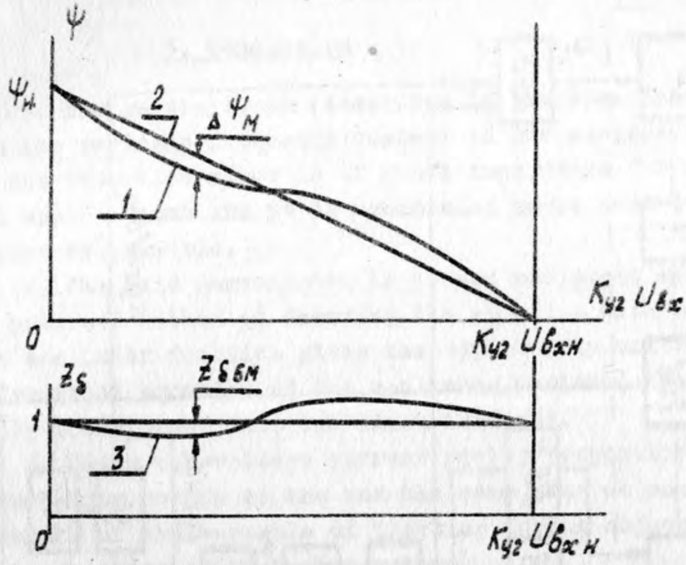

Fig.2. The relationship of the thyristor opening angle ψ of the rectifier УВ from the controlling voltage $K_{у2} U_{вх}$

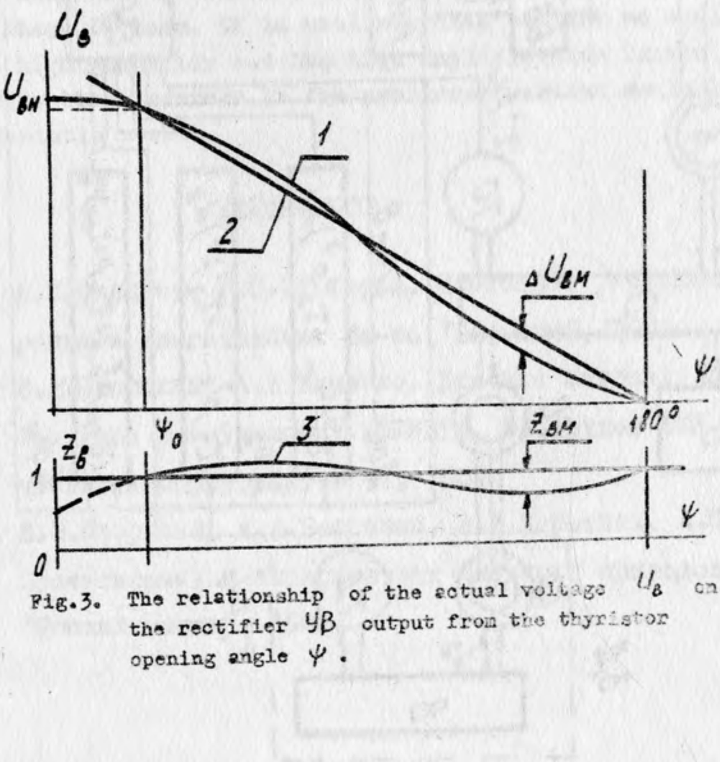

Fig.3. The relationship of the actual voltage $U_в$ on the rectifier УВ output from the thyristor opening angle ψ.

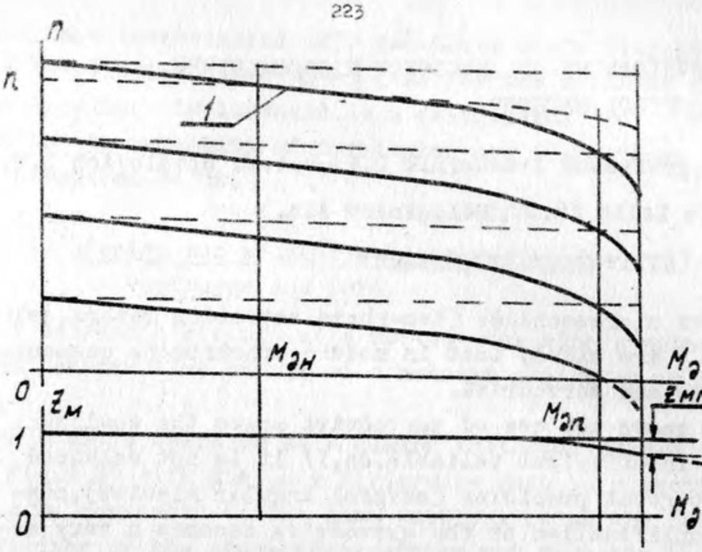

Fig.4. The family of the induction motor mechanical characteristics, that are dependent upon the frequency f_1 and the loading torque M_∂.

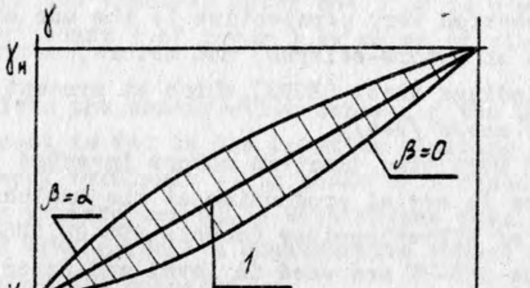

Fig.5. The family range of the required changes of the relative voltage γ , supplied on the motor ; depending on the relative frequency α under the different absolute values of slipping β.

15.6 DEVELOPMENT OF INDUCTION MICROMACHINES CONTROL METHODS.

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Induction micromachines (two-phase actuating motors,selsyns and s.o.) are widely used in modern synchronons communication systems and servodrive.

In some cases the use of servodrive makes the equipment to bulky and insufficient reliable.So,if it is not required to have a too great precision (several angular minutes),considerable simplification of the servodrive becomes a very actual problem.

The trend of reducing weight,size and cost of servodrive has led to the development of electric micromachines involving several functions [1].

In this connection very perspective is the use of motosyns (combination of motors-selsyns) and motors,controlled by space shift of windings axes. (DUPS),which at present are used in some practical cases [2+4].

Motosyns and DUPS type electric motors invented by D.V. Svecharnic [1] are in serial production at the Smolensky experimental plant of NIITeplopribor (models SDS-2M,SDS-3M).

Motosyns type SDS-2M are used in level and water head indicators type A5020,A5034 + A5038 installed on the lockes of the Kiev,Saratov and Kanev hydro-electric power stations. Hundreds of type SDS-3M motosyns are used in electromechanical manipulators.

Tables 1 and 2 give the main technical characteristics of motosyns and motors,controlled by space shift of windings axes,worked out by NIITeplopribor.

The new induction micromachines constructed on the basis of combination of a selsyn and an electric motor with a hollow rotor,cannot be investigated using the theoretical lay out made for selsyns and for machines with a hollow rotor because the later are investigated only for fixed angle displacement between the windings axes $\theta=\frac{\pi}{2}$; as for the motosyns and DUPS the angular displacement is a determining factor. This has led to the necessity of developing new theoretical principles discribed below.

Design and principle of operation of motosyns and DUPS.

The schematic circuit of a servosystem with a motosyn and the constructive scheme of a motosyn are given on fig. 1,a,b.

The motosyn operates as a common indicator selsyn circuit (fig.1,a) with the only difference that it's exciting winding in connected to the same supply line as the exciting winding of the synchrotransmitter, not directly but through a phase shift condenser.

In the clearance between the motosyn stator and rotor (fig.1,b) a current conducting cylinder 2 is placed which is the rotor of the electric motor and wich is connected with the output shaft 9 of rotor 3 by means of step-down gear 8.

Fig.1,b gives the constructive scheme of the motosyn in which the current is fed to the rotating exciting winding of rotor 3 (input terminals 4) by means of a flace or radial type ring transformer (5,6) with output terminals 7 from the fixed winding. Such a constructive scheme have type SDS-IMB and SDS-B-400 motosyns.

Currents flowing through the synchronisation winding of stator 1, create a pulsing field the axis of wich is defined by the position of the synchrotransmitter rotor T and in general case does not coincide with the synchrorepeates rotor winding axes (motosyn). The magnitisation forces of the synchronizing and exciting windings are shifted in space and time (by means of condenser C) this couses a rotating field in the motosyn R. The space angle θ of the windings shift varies according to the unbalance between the transmitter and the receiver rotor (when balanced $\theta=0$).

The actuating motor control is carried out by means of changing of the rotating field ellipticity degree,due to the change of not angly the value and the phase of the phase of the voltage supplied to the windings,but also to the space shift of the windings.The resulting rotating field interacts with the eddy currents,induced in the hollow rotor,and creates a torque. by rotation of the hollow rotor the controlled axis 9 is turned through the step-down gear and the unbalance decreases.

The torque on the output axis of the motosyn is:

$$M=(M_S+i.\eta.M_{S\partial})\cdot\sin\theta \; ; \qquad (1)$$

where:

$M_S, M_{S\partial}$ -selsyn and actuating motor torques;

i,η -gear ratio and gear efficiency;

θ -space angle of the shift betwwen the windings axes of the stator and the inner stator.

Depending on the gear ratio the motosyn increases the torque in hundreds and more times.

The motosyn replaces the actuating motor and the selsyn without any intermediat gain in the control circuit.

The motosyn enables the remote control of relatively powerful regulating units without use of additional electronic,magnetic an other amplifiers.

On the base of motosyn it is lasy build up an angle time integrator.

For selected signal coefficient and voltage phase a linear relation between the rotation speed of the hollow rotor and the space angle in the range up to 40° can be received with an error of ±0,5%.Connecting the hollow rotor with a revolution counter,we shall obtain on it the time integration of a quantity,depending on the shift angle θ ,predetermined directly or remotely.

Some motosyns are made by combining an actuating motor with a differential selsyn (for exemple motosyns SDS-2D and SDS-2M).

The possibilities of the motosyn in respect to control methods are very wide.

By realising a three-phase winding on the stator and the inner rotor different modes of control can be received by means of change of either of the parameters:

I)voltage value (signal coefficient);

2)voltages meantime phase,supplied to the windings;

3)position of the three-phase windings resulting axes;

4)space shift angles of the rotor and the stator.

As a result control modes[3] can be obtained insted of the three usually used for the actuating motors:amplitude, phase and amplitude-phase control method.

So,the possibilities of such a machine are made much wider.

In the case of absence of kinematic connection between the hollow rotor and the control axis and wohen the stator and rotor of the motosyn have a one-phase winding,the motosyn becomes an actuating motor,controlled besides the widely known methods(amplitude,phase and amplitude-phase methods) by the space shift of the control and exciting,windings axes.The virtue of such actuating motors consists in that the controlling value is inserted directly asa mechanical displacement.

It is intesesting to underline that the first induction micromachines in wich the method of space shift of the windings magnetizing forces axes was used,were the contact and the contactless selsyns [3].

Theoretical principles of motosyns and motors(DUPS) controlled by the windings axes space shift.

In the bibliography publiched works dedicated to theoretical and experimental investigations of motosyns and DUPS are listed.

Below,the principles of theory and design methods of the mentioned micromachines worked out by the authors are considered.

The starting torque of a motosyn interconnected in a servosystem.

As an example the most common case is analised: the differential type motosyn interconnection.

Fig.2 gives the schematic circuit of a differential

selsyn-motosyn remote transmission where:
1,2 and 3 are the windings phases of the synchrotransmitters "a" and "k" and of the receiver R; 4-is the hollow rotor of the motosyn; 5-the step-down gear; 6-the output shaft of the motosyn.

The synchronization winding phases of transmitter a are interconnected with the rotor winding phases, and the phases of transmitter "k" synchronization winding are interconnected with the motosyn R stator winding phases.

The exciting windings of the transmitters are connected to a common supply line. The exciting winding of transmitter "k" is connected to this supply line through a phase-shift condenser.

To simplify investigation the follow assumptions are made:

a) a sinusoidal change of flut linkages Ψ.
of the exciting flux Ψ_B with phases 1,2 and 3 of the transmitters synchronisation winding in changing the shift angles α and β of the transmitters rotors;

b) the full resistance in the circuit of each pair of seriesly connected transmitter and receiver phases and independent from the rotors position and flowing currents:

$$Z_{KR_1} = Z_{KR_2} = Z_{KR_3}\,; \quad Z_{aR_1} = Z_{aR_2} = Z_{aR_3}\,,$$

whereby

$$Z_{KRi} = (r_{Ki} + r_{li} + r_{Ri}) + j\,(X_{Ki} + X_{li} + X_{Ri}) = R_{KRi} + j\,X_{KRi} = |Z_{KRi}| \cdot e^{j\varphi_{KR}}\,; \qquad (2)$$

$$Z_{aRi} = (r_{ai} + r_{li} + r_{Ri}) + j\,(X_{ai} + X_{li} + X_{Ri}) = |Z_{aRi}| \cdot e^{j\varphi_{aR}}\,, \qquad (3)$$

where $Z_{KRi} = |Z_{KRi}| \cdot e^{j\varphi_{KR}}$ is the impedance in a complex form in the circuit of a pair of series connected phases of the transmitter "k" and the receiver "R";

$Z_{aRi} = |Z_{aRi}| \cdot e^{j\varphi_{aR}}$ -the same for the transmitter a and the receiver "R";

$\varphi_{KR}, \varphi_{aR}$ -arguments of the impedance in a complex form in the circuit of a pair of series connected phases of the transmitter "k" and the receiver "R" respectively;

$r_{Ki}, r_{ai}, r_{Ri}, r_{li}$ -the pure resistance of the synchronization winding phase of the transmitters "k","a",receiver "R" and communication line respectively;

$x_{ki}; x_{ai}; x_{Ri}; x_{\ell i}$ -the same for the inductanses;

c)by qualitative investigation of physical processes in a system the losses in exciting windings and in of the magnetic lead are not taken into account;quantitatively the losses are taken into account by becouse of using in the calculation the exciting voltage $\dot{E}_{B}$ main reactive component instead of supply voltage U_{B} in expressions for electromotive force of windings and currents in system circuits.

The starting torque of a asynchronous two-phase motor controlled by the chenge of the shift angle between the exciting and control windings magnetic axes as shown in /2/,is: $M = \frac{1}{\omega} \cdot K \cdot R_c^{\prime} \cdot |\dot{I}_{2}| \cdot |\dot{I}_{s}| \cdot \sin(\varphi_2 - \varphi_s) \cdot \sin\theta$, (4)

where M -the starting torque:,

ω -supplied voltage frequency;

K -transformation coefficient between the rotor and stator windings;

$R_r^{\prime}$ -additional resistance,briging in the stator circuit by the hollow rotor in the clearince;

I_2, I_s -the current in the rotor and stator winding respectively;

φ_2, φ_s -the rotor and stator current phase respectively;

θ -the space shift angle of the rotor and stator windings axes:

The investigation method is based on respecting the torque on the motosyn hollow rotor as a sum of torques generated by the interaction of currents in all motosyn rotor and stator winding pairs.

In a differential motosyn there are six windings, the interaction of the magnetomotive forces of which leads to the creation of a torque on the rotor of the machine: three stator and three rotor phases.

At first let us determine the torque on the hollow rotor hinder unbalance condition created by the interaction of the magnetomotive forces of the motosyn stator winding first phases with its rotor winding. For that, we have to find at first the currents in the circuits a,r and k,r, in a complex form.

The instanteneous current value in the transmitter "a" first phase synchronization winding circuit:

$$i_{1a} = \frac{e_{1a}}{Z_{an}} = \frac{E_{bam}}{Z_{aR}} \cdot \frac{W_{a_1} \cdot K_{Wa_1}}{W_{ba}} \cdot \sin\left(\omega t - \varphi_{ba} - \varphi_{aR} - \frac{\pi}{2}\right) \cdot \cos\alpha, \quad (5)$$

where:

e_{1a} -the electromotive force induced by the exciting flux in the first phase of the transmitter "a" synchronization winding;

W_{a_1}, K_{Wa_1} -the number of turns and the first phase winding coefficient of the transmitter "a" synchronization winding respectively;

W_{ba} -number of turns of the transmitter "a" exciting winding;

φ_{ba} -the transmitter "a" exciting winding impedance argument in a complex form;

The current in a complex form in the motosyn rotor first phase winding is:

$$\dot{I}_{1a} = |\dot{I}_{1a}| \cdot e^{-j(\varphi_{ba} + \varphi_{aR} + \frac{\pi}{2})} \cdot \cos\alpha =$$

$$= \cos\alpha \cdot \frac{W_{a_1} \cdot K_{Wa_1}}{W_{ba}} \cdot \left|\frac{\dot{E}_{bam}}{Z_{aR_1}}\right| \cdot e^{-j(\varphi_{ba} + \varphi_{aR} + \frac{\pi}{2})} \quad (6)$$

by analogy, the expression for the current in the first phase of the motosyn stator winding is:

$$\dot{I}_{1K} = \cos\beta \cdot \frac{W_{K_1} \cdot K_{WK_1}}{W_{bK}} \cdot \left|\frac{E_{bKm}}{Z_{KR_1}}\right| \cdot e^{-j(\varphi_{bK} + \varphi_{KR} + \frac{\pi}{2})}, \quad (7)$$

where W_K, W_{bK} -are numbers of turns in the first phase of the transmitter "k" synchronization winding and its exciting winding respectively.

After substitution in the expression (4) of the receiver current values I_{1a} and I_{1K} the value of the torque on the motosyn hollow rotor is obtained, which is created by the interaction of the stator and rotor windings first phases magnetomotive forces:

$$M_{R_1} = \frac{1}{\omega} \cdot K \cdot R_c' \cdot \frac{W_{a_1} \cdot K_{Wa_1}}{W_{ba}} \cdot \frac{W_{K_1} \cdot K_{WK_1}}{W_{bK}} \cdot \left|\frac{\dot{E}_{bam}}{Z_{aR_1}}\right| \cdot \left|\frac{\dot{E}_{bKm}}{Z_{KR_1}}\right| \times$$

$$\times \sin\left(\varphi_{ba} + \varphi_{aR} + \frac{\pi}{2} - \varphi_{bK} - \varphi_{KR} - \frac{\pi}{2}\right) \cdot \cos\alpha \cdot \cos\beta \cdot \sin\theta, \quad (8)$$

where:

α, β -are angles between the first phase axis of the "a" and "k" transmitters synchronization winding respectively and the transmitter exciting flux axis;

w_{ba}, w_{bk} -numbers of turns in the "a" and "k" transmitter synchronization windings respectively;

w_a, w_k -numbers of turns in the "a" and "k" transmitter exciting windings respectively.

The torque on the motosyn hollow rotor is the sum of torques wich are created by the unbalance due to the interaction of the windings pairs magnetomotive forces.

Taking this account and after doing some simple transformations we shall have

$$M_R = \frac{3}{\omega} \cdot R_c' \cdot \frac{W_{np} \cdot K_w}{W_{Rc}} \cdot \frac{W_a \cdot K_w}{W_{ba}} \cdot \frac{W_k \cdot K_w}{W_{bk}} \cdot \left| \frac{\dot{E}_{ba}}{Z_{aR}} \right| \cdot \left| \frac{\dot{E}_{bk}}{Z_{kR}} \right| \times \qquad (9)$$
$$\times \sin(\varphi_{ba} + \varphi_{aR} - \varphi_{bk} - \varphi_{kR}) \cdot \sin[\Theta - (\beta - \alpha)],$$

where M_R -the starting torque on the motosyn R hollow rotor;

W_{RR}, W_{RS} -numbers of turns in the motosyn "R" stator and rotor phases respectively;

K_w -the winding coefficient.

If the synchrotransmitters are identical and the motosyn stator windings,are identical too and in the transmitter "a" exciting circuit a phase shift condenser is interconnected,the capacitance of which is chosen to be

$$\varphi_{ba} - \varphi_{bk} = -\frac{\pi}{2},$$

$$\sin(\varphi_{ba} + \varphi_{aR} - \varphi_{bk} - \varphi_{kR}) = \sin\left(-\frac{\pi}{2}\right) = -1,$$

The torque of the motosyn R hollow rotor will be maximum; by connected the same capacitance in the transmitter "k" exciting circuit,torque M_R will also have its maximum value,but it will have an inversed opposit sign.The abovementioned assumptions are made for the type A5020 device in which type SDS-2D motosyn haring identical windings is connected to two type BD-50IA contactless synchrotransmitters.

The calculation of additional resistance R_c' created in the stator circuit by the rotor in the clearance,is carried out in accordance with the expressions given in /3/:

$$R_c' = \omega^2 \cdot \sigma \cdot \Delta \cdot Q \cdot (F_c)^2,$$

where: ω -the angular frequency of the supply voltage;

σ -the material conductivity of the hollow rotor;

Δ -thickness of the hollow rotor;

$$Q = 2\pi \cdot R^4 \cdot \left(\frac{\ell}{2R} - \frac{ch \frac{L-\ell}{3R} \cdot sh \frac{\ell}{2R}}{ch \frac{L}{2R}} \right);$$

$$F_\ell = K_1 \frac{\mu_0}{k_\delta \cdot \delta} \cdot \frac{K_{w\ell} \cdot W_\ell \cdot m}{\pi \cdot p},$$

where: R -the hollow rotor radins;

ℓ -the working length of the motosyn stator pack;

L -length of the hollow rotor;

$K_1 = \frac{F_\delta}{\Sigma F}$ -ratio of the magnetization force;

μ_0 -permeability in the clearance;

K_δ -air gap coefficient;

δ -rate of working clearens;

W_ℓ -number of spires of the stator winding;

$K_{w\ell}$ -the number of stator winding phases;

p -number of the motosyn pole pairs.

The calculation of R_c' can also be made in accordance with the expressions given in monographs [8 + 10].

By analogy with the above mentioned it is easy get an expression for the starting torque on the hollow rotor of a motosyn coupled with one synchrotransmitter.

This expression is:

$$M_{mR} = -\frac{3}{2} R_c' \cdot \frac{1{,}02 \cdot 10^4}{\omega} \cdot \frac{E_{\ell g} \cdot E_{\ell R}}{2Z} \cdot \frac{W_g \cdot K_w}{W_{\ell g}} \cdot \frac{W_n \cdot K_w}{W_{\ell R}} \times$$
$$\times \cos(\varphi_{\ell g} \cdot \varphi_{\ell R} - \frac{\pi}{2} - \varphi) \cdot \sin(\theta_R - \theta_g) \, [gcm], \quad (10)$$

where $E_{\ell g}, E_{\ell R}$ -effective value of the electromotive force of the transmitter and receiver exciting windings;

Z -impedance of seriesly connected of the transmitter and receiver synchronization windings;

$W_{\ell g}, W_{\ell R}$ -numbers of turns of the transmitter and receiver exciting winding respectively;

θ_g, θ_R -angles between the phase I axes of the transmitter and receiver synchronization winding and the respective exciting winding.

In this case the torque on the synchrotransmitter shaft will have the following expression:

$$M_g = -\frac{3}{2} \cdot \frac{1{,}02 \cdot 10^4}{\omega} \cdot \frac{E_{eg} \cdot E_{eR}}{2Z} \cdot \frac{W_g \cdot K_w}{W_{eg}} \cdot \frac{W_R \cdot K_w}{W_{eR}} \times$$
$$\times \sin(\Theta_g - \Theta_R) \cdot \cos(\varphi_{eg} - \varphi_{eR} - \tfrac{\pi}{2} - \varphi) \qquad (11)$$

The above received expressions (I0) and (II) are right for small unbalance angles (upto I5=20°),when the reaction of the synchronization winding on the exciting winding can practicaly be neglected.

In fig.3 the diagramm of dependence of the starting torque on the hollow rotor of a type SDS-2D differential motosyn when actuated by contactless type BD-50IA synchrotransmitters from the unbalance angle $/\theta - (\beta - \alpha)/$ is shown.

The calculation (dotted line) and the experimental (continuor line) coincide rather accurately.

The difference in specific synchronization torque does not exceed I0%.

The hollow rotor,windings axes space shift controlled motor investigation (DUPS).

Type DUPS motor is a particular case of motosyn.Fig.4 gives DUPS electrical diagram.Windings A and B are fed by phase shifted voltages.In the stator-inner rotor gap the hollow rotor (HR) is situated.The inner rotor (B) can turn about stator (A) on arbitrary angle θ .

In coincidence of windings axes A and B rotating magnetic field is absent in the gap and the hollow rotor is at stand-still.When there is an angle between the stator and inner rotor windings axes,elliptical rotating magnetic field in the gap is on and the hollow rotor begins to rotat,having speed maximum in angle θ range of 60 + I20 electrical degrees (depending on supplied voltages phase shift value).

The direct electromagnetic connection between windings A and B in motors type DUPS is chenged variation angle between the windings axes.

The mutual inductance between A and B windings,when the rotor is at stand-still changes in absolute value from its maximum, when the axes are parallel, to zero, when the axes are perpendicular.

According to fig.5, DUPS type motor has an electromagnetic system of three inductive linked windings: winding A, winding B and hollow rotor HR.

By assumptions generally accepted for inductive micromachines namely: the magnetic system is nonsat rated; steel losses are absent, induction distribution in the gap is sinusoidal; Kirhgof's equations in complex form for DUPS type motors will be:

$$\begin{aligned}
\dot{U}_A &= [R_A + j\omega(L_A - M) + j\omega M(1-\cos\Theta) + j\omega M\cdot\cos\Theta]\dot{I}_A + \\
&\quad + j\omega M\cdot\cos\Theta\cdot\dot{I}_B + j\omega M(1-\cos\Theta)\cdot\dot{I}_P + j\omega M\cdot\cos\Theta\cdot\dot{I}_P; \\
\dot{U}_B &= [R_B + j\omega(L_B - M) + j\omega M(1-\cos\Theta) + j\omega M\cdot\cos\Theta]\cdot\dot{I}_B + \\
&\quad + j\omega M\cdot\cos\Theta\cdot\dot{I}_A + j\omega M(1-\cos\Theta)\dot{I}_P + j\omega M\cdot\cos\Theta\cdot\dot{I}_P; \\
0 &= [R_P(1-\cos\Theta) + R_P\cdot\cos\Theta + j\omega(L_P - M)(1-\cos\Theta) + \\
&\quad + j\omega(L_P - M)\cdot\cos\Theta]\cdot\dot{I}_P + j\omega M(1-\cos\Theta)\cdot\dot{I}_A + \\
&\quad + j\omega M\cdot\cos\Theta\cdot\dot{I}_A + j\omega M(1-\cos\Theta)\dot{I}_B + j\omega M - \cos\Theta\cdot\dot{I}_B
\end{aligned} \qquad (12)$$

The hollow rotor dissipation inductance is much more less then its pure resistance, and it can be neglected, hence

$$L_P - M = 0$$

Equations (I2) are in accordance with the equivalent circuit diagram represented in fig.6.

The equivalent circuit diagram is designed for a common case, when windings A and B have a different turns number. All parameters of the equivalent circuit diagram are in accordance with winding A turns number.

Following designations are used for the equivalent circuit diagram:

R_A, R_B -windings A and B pure resistances;

X_A, X_B -windings A and A dissipation inductance;

X_{mA} -inductance corresponding to the flux flowing through gap, stator and inner rotor; in accordance with winding A turns number;

X_{mB} -the same, but in accordance with winding B turns number;

R_{pA} -the hollow rotor pure resistance in accordance with winding A turns number;

R_{pB} -the same,but in accordance with winding B turns number;

$K=W_A/W_B$ -effective windings A and B turns number ratio;

$\dot{U}_{A_1}, \dot{U}_{B_1}$ -voltages which form a circular rotating magnetic field of direct sequence,for a given Θ;

$\dot{U}_{A_2}, \dot{U}_{B_2}$ -voltages,which form a circular rotating magnetic field of reverse sequence for a given Θ.

Analitical parameters of the equivalent circuit diagram can be found with help of the known formulas from /8÷10/.

The direct sequence currents can are determinated by the follawing equations:

$$\dot{I}_{A_1} = \frac{\dot{U}_{A_1}(K^2 R_B + jK^2X_B + R' + jX') - K\dot{U}_{B_1}(R'+jX')\cos\Theta}{(R_A + jX_A + R' + jX')(K^2 R_B + jK^2X_B + R' + jX') - (R'+jX')^2\cos\Theta};$$

$$\dot{I}_{B_1} = \frac{K^2\dot{U}_{B_1}(R_A + jX_A + R' + jX') - K\dot{U}_{A_1}(R'+jX')\cdot\cos\Theta}{(R_A + jX_A + R' + jX')(K^2 R_B + jK^2X_B + R' + jX') - (R'+jX')^2\cdot\cos\Theta} \qquad (13)$$

To find reverse sequence currents $\dot{I}_{A_2}$ and $\dot{I}_{B_2}$ parameters $\dot{U}_{A_2}, \dot{U}_{B_2}, R'', X''$ have to besubstituted for $\dot{U}_{A_1}, \dot{U}_{B_1}$, R', X' .Electromagneticpower educed in the hollow rotor for direct sequence is:

$$P_{\partial_1} = 2\cdot I_{A_1}^2\cdot R'\cdot\sin^2\Theta \qquad (14)$$

for reverse sequence:

$$P_{\partial_2} = 2\cdot I_{A_2}^2\cdot R''\cdot\sin^2\Theta \qquad (15)$$

Resulting torque on the hollow rotor can be found as torque difference of direct and reverse sequences:

$$M = \frac{97400}{n_c}\left(2I_{A_1}^2 R' - 2I_{A_2}^2 R''\right)\sin^2\Theta\text{, гcm} \qquad (16)$$

here n_s -synchronous speed.

In fig.7 starting currents stator (A) and rotor (B) windings dependence from angle Θ for motors type DUPS based motosyn type SDS-3 are described.

Continuons lines represent the experimental characteristics dotted lines represent characteristics calculated from the equivalent circuit diagram.The difference between the calculated and the experimantal characteristics satisfy the regnirements of engineering practice.

At fig.8 mechanical characteristics of motor type DUPS-63 are given:

a)for space sift control;

b)for amplitude control.

Application of motors type DUPS and motosyns in automatic scheems.

The last gears motosyns and motors type DUPS find a rather wide application in automatic scheems het us mark the most important cases of there applications:

I.Synchro repeater with strengthen torque for synchro systems in level and water head indicators type A5020,A5034 + A5038(SDS-2D,SDS-2M).

2.Electromechanical manipulator drive (DUPS type SDS-3M and DUPS-63M).

3.Electrical executive mechanisms drive (DUPS-IO,DUPS-25,DUPS-63,DUPS-I60,DUPS-400).

Perspective workings out with that kind of micromotors are:

distance transmission angle device with synchro reapiter unloading [11];synchro-motors positioners [I2];synchro-motors type regulating devices [3];differentiating,integrating,multiplying-intergrating devices.

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Summary

The trend of reducing wight size and cost of servodrive has led to the development of electric micromachines involving several functions.

In this connection,very perspective in the use of motosyns (combination of motors-selsyns) and motors,controlled by space shift of windings axes (DUPS) which at present are allrady used in some practical cases:in level and water head indicators electromechanical manipulators and so on.

In the report the basic theoretical principles and technical characteristics of the windings axes space shift controlled motosyns and motors worked out in NIITeplopribor are discussed.

The new induction micromachines constructed on the basis of combination of a selsyn and an electric motor with a hollow rotor,cannot be investigated using the theoretical lay out made for selsyns and for machines with a hollow rotor because the later are investigated only for fixed angle displacement (90°) between the winding axes as for the motosyns and DUPS the angular displacement is a determining factor.

This has led to the necessity of developing new theoretical principles discribed.

By realising a three-phase winding on the stator and the inner rotor,different modes of control can be received by means of change of either of the parameters:

I)voltage value (signal coefficient);

2)voltages meantime phase,supplied to the windings;

of currents in all motosyn rotor and stator winding

3)position of the three-phase windings resulting axes;

4)space shift angles of the rotor and the stator.

As a result 63 control modes /3/ can be obtained insted of the three usually used for the actuating motors:amplitude, phase and amplitude-phase control method.So,the possibilities of such a machine are made much wider.

The most important cases of motosyns and DUPS use are: selsyn-motosyn transmission (type SDS-2D,SDS-2M,SDS-IMB are used),electromechanical manipulator drive (DUPS type SDS-3M and DUPS-63M).

Table 1.

Technical characteristics motosins
(combined motor-selsyn)

№№	Type of motosins / Techical characteristics		Contact or flexible current supply device				Contactloss current supply device	
			SDS-1, (SDS-1s)	SDS-2	SDS-2D, (SDS-2n)	SDS-3	SDS-1MB	SDS-B-400
1	Exiciting voltage	V	55	110	110	220	110	115
2	Frequency	Hz	50	50	50	50	50	400
3	Power consuption	V·A	25	55	50	400	18	5
4	Torque of friction	N·cm·10^2	0,4(0,2)	1,5	1,5	2,0	0,3	0,03
5	Specific torque	$\frac{N cm}{rad}$	1,44	9,2	4,0	11,5	1,32	0,172
6	Gear ratio	–	270	1500	800; 1200 2360; 4000	7	80; 140; 240	500
7	Coupled with selsyn-transmitter	–	ND-404, SS-404	ND-501, BD-501A	BD-501A	ND-521	BD-404A	BD-160A
8	Maximum transmission error	rad	0,013÷0,026	0,013÷0,026	0,013÷0,026	0,013÷0,026	0,013÷0,026	0,0035
9	Maximum torque on a outlet rotor selsyn shaft	N·m	2,8	65	43	3	2,4	0,65
10	Temperature range	°C	+5... +50				-40...+60	-40...+60
11	Relative humidity	%		80			90	98
12	Weight	Kg	1,4	3,3	3	5,3	2,5	0,45
13	Fault intensity	$\frac{1}{h}\cdot 10^{-6}$	18	18	18(25)	20	12	15

Table 2.

Technical characteristics motors type DUPS

Motor type	Network voltage, V	Frequency, Hz	Nominal power, W	Nominal speed, rad/s	Nominal torque, N·cm	Starting torque, N·cm	Statics friction torque, N·cm	Maximum control voltage, V	Control current, A	Excitation current, A	Excitation power, W	Capacitor, μF	Idle speed, rad/s	Starting voltage, V	Rotor inertia, $g \cdot cm \cdot s^2$	Electromechanical time constant, ms	Used power at stand-still, W	Efficiency, %	Mechanical characteristics nonlinearity, %	Internal stator turn angle, rad	Weight, kg
DUPS-4	220	50	4	231	0,179	0,3	0,05	220	0,08	0,08	15	1	294	3,5	0,3	290	30	13	35	±1,6	1,3
DUPS-10	220	50	6	168	0,48	0,8	0,1	220	0,1	0,15	18	1,75	241	1,5	0,094	30	37	24	25	±1,6	2,5
DUPS-25	220	50	18	174	1,05	1,8	0,1	220	0,31	0,38	38	4	252	1,5	0,182	25,2	75	30	25	±1,6	4,2
DUPS-63	220	50	42	171	2,5	4,35	0,15	220	0,72	0,83	83	8	265	1,5	0,182	11,5	172	31	25	±1,6	6,1
DUPS-160	220	50	160	173	9,75	16,6	0,3	220	2,5	3,5	380	20	272	2,5	2,0	33	740	25	25	±2,1	25
DUPS-400	220	50	350	184	20,0	26,8	0,3	220	2,7	4,3	550	34	283	2,5	4,5	48	1300	30	40	±2,1	26

Note: Motors DUPS-63, DUPS-160 and DUPS-400 have Build-in ventilators

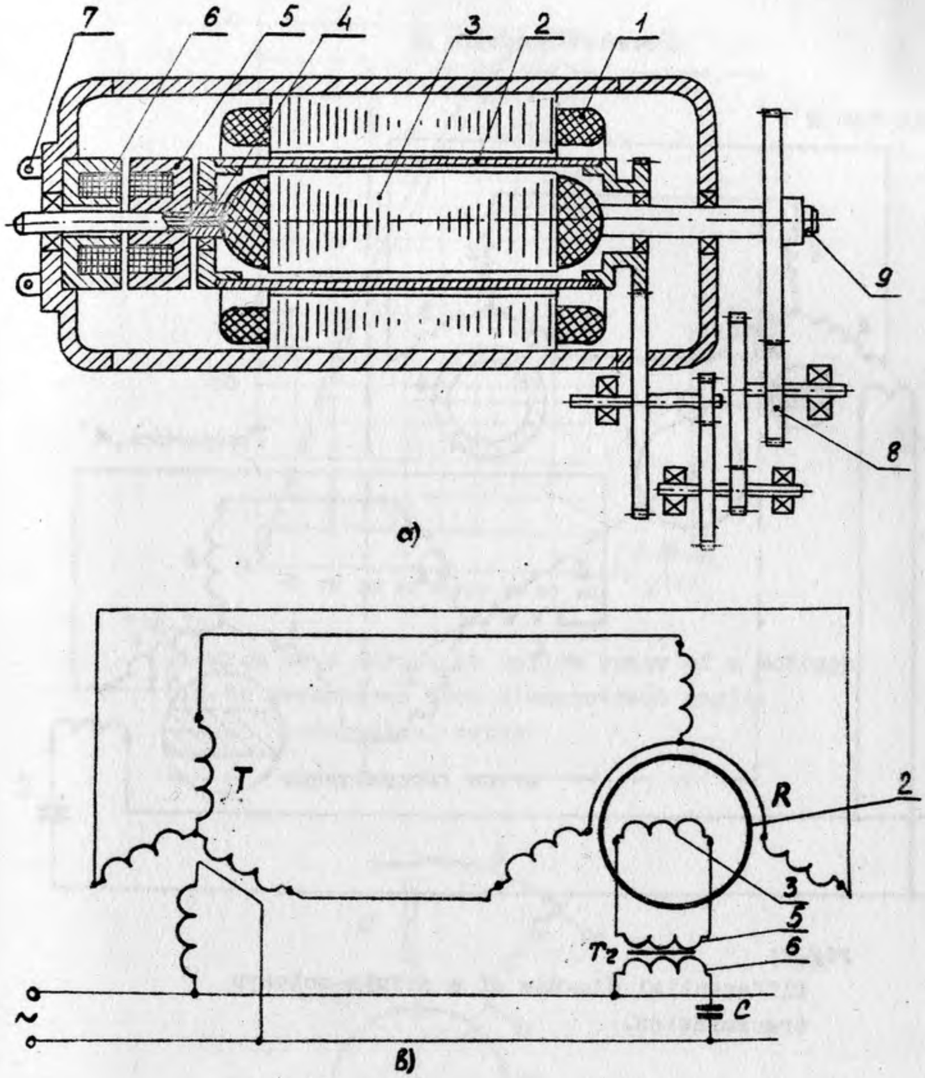


Fig.1:

a)selsyn-motosyn;

b)principle diagram of a selsyn-motosyn distance transmission.

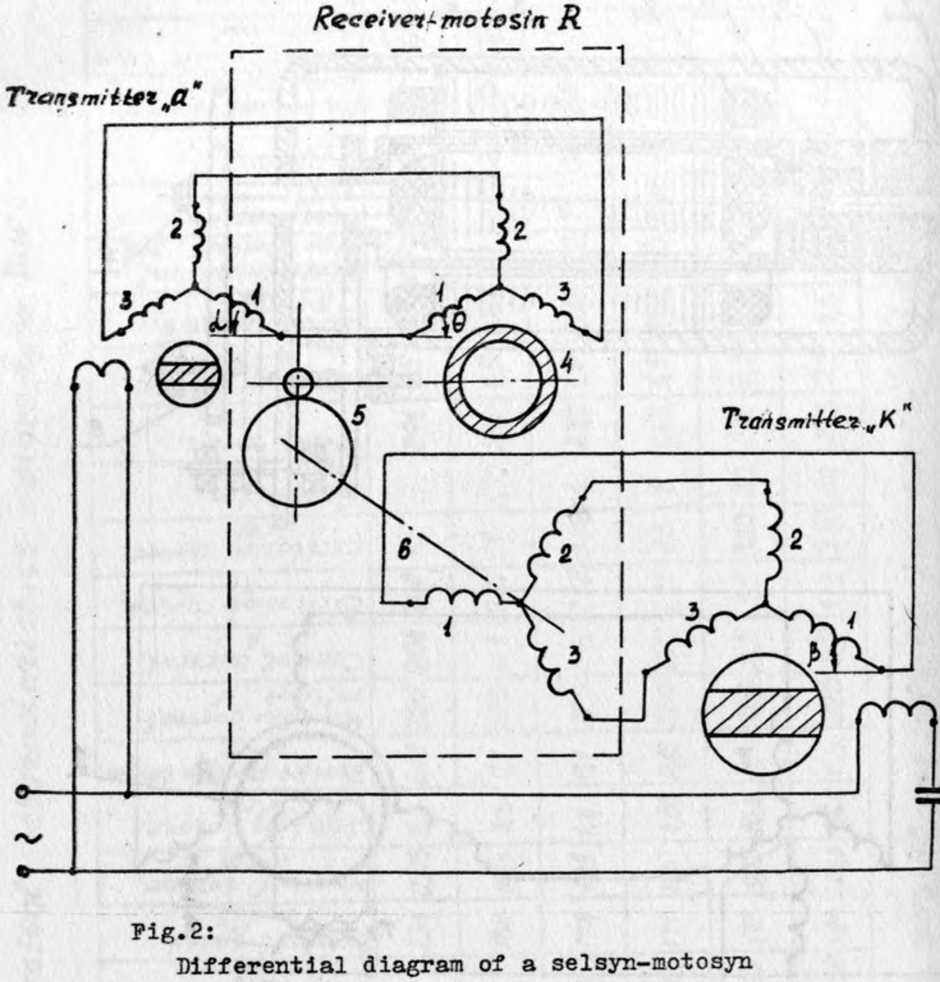


Fig.2:

Differential diagram of a selsyn-motosyn transmission.

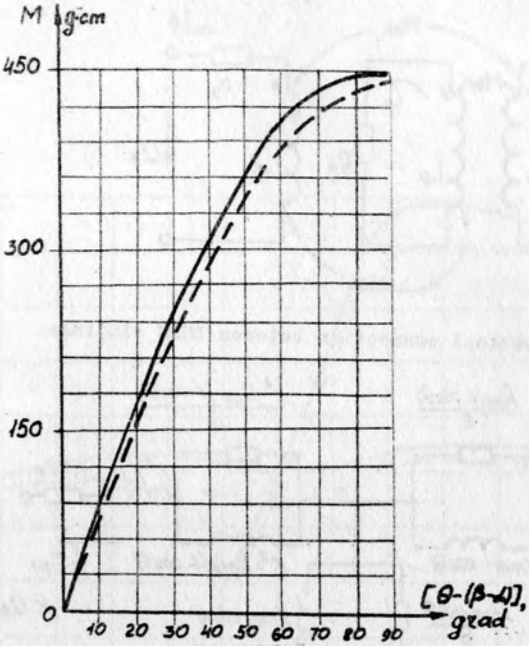


Fig.3:

Diagram of a torque on hollow rotor of a motosyn SDS-2D dependence from disagreement angle:

\- - - - theoretical curve

——— experimental curve

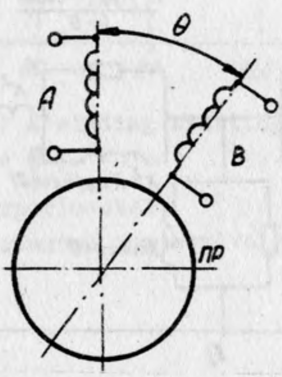


Fig.4:

DUPS Electrical diagra

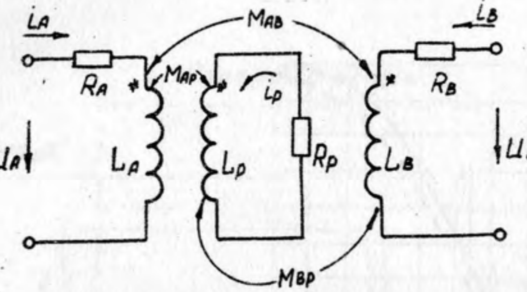
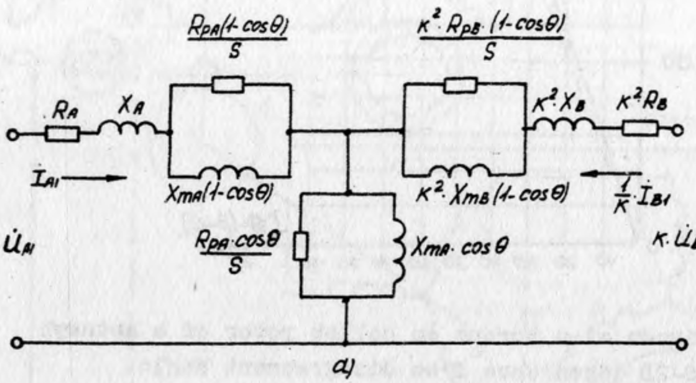


Fig.5:

Electromagnetical connection between DUPS windings.



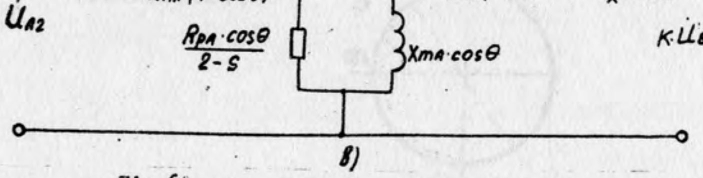


Fig.6:

Equivalent circuit diagram:

a)for direct sequence current;

b)for reverse sequence current.

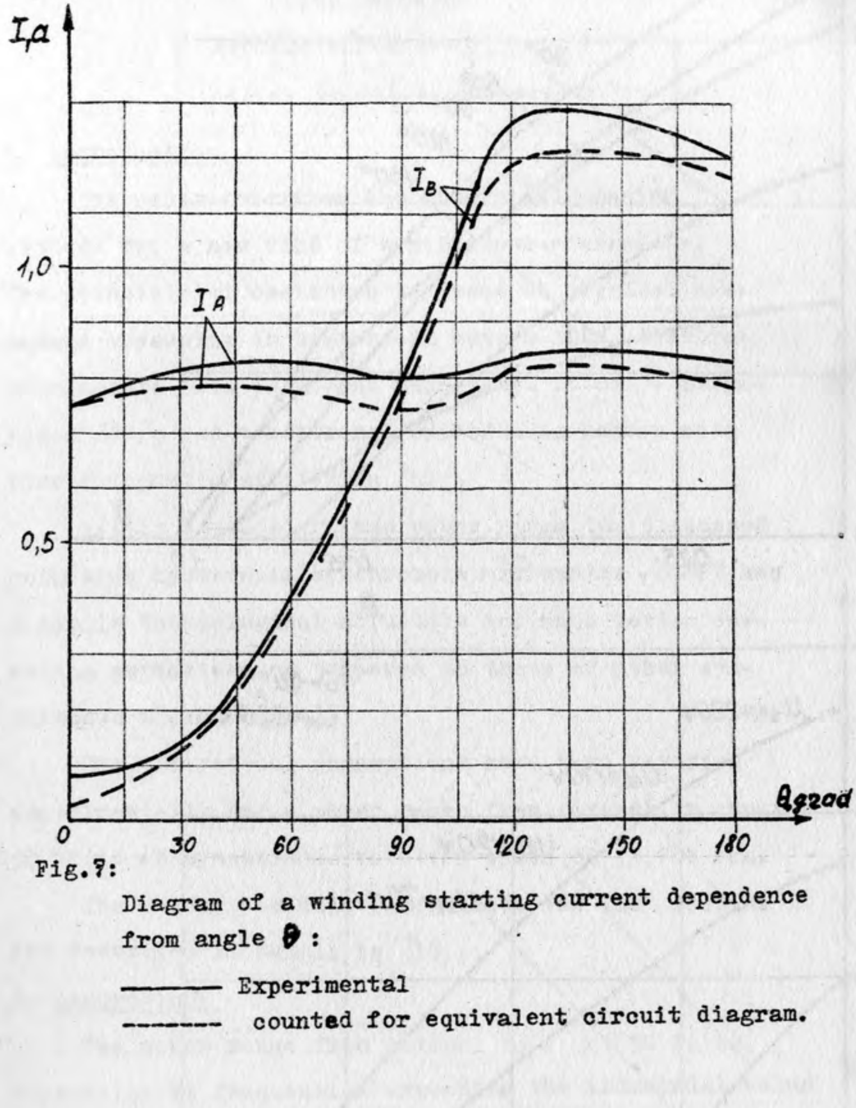


Fig.7:

Diagram of a winding starting current dependence from angle θ :

——— Experimental

------- counted for equivalent circuit diagram.

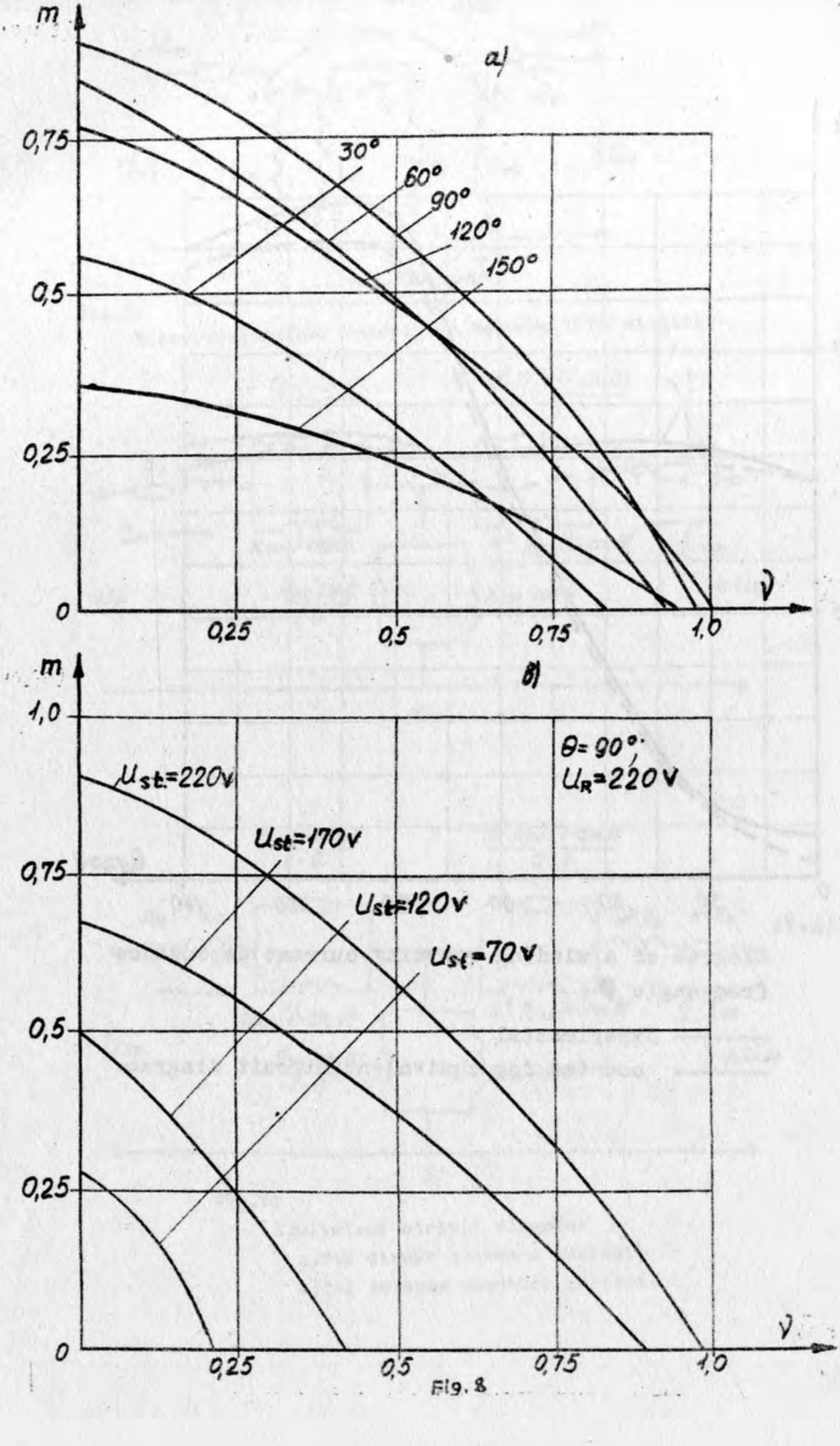

m
a)
0,75
0,5
0,25
0
30°
60°
90°
120°
150°
0,25
0,5
0,75
1,0
ν
m
b)
1,0
0,75
0,5
0,25
0
θ= 90°;
U_R=220 V
U_{st}=220v
U_{st}=170v
U_{st}=120v
U_{st}=70 V
0,25
0,5
0,75
1,0
ν

Fig. 8

15.8

NEW KIND OF SYNCHRONOUS MICROMOTOR

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1. Introduction

The paper describes the theory and design methods for a new kind of synchronous micromotor. The principle of operation is based on physical phenomena appearing in hysteresis motors /SH/, synchronous motors with permanent magnets on rotors - permasynes /SP/, and traditional synchronous motors with electromagnetic excitation /SS/.

Within a predetermined power range the discussed permasyne hysteresis synchronous micromotor /SSHP/ has a simple technological structure and much better operating parameters as compared to those of other synchronous micromotors.

The theoretical assumptions have been verified experimentally for a power range from several to about 50 Watts at synchronous rotation speed of 15,000 rpm.

The theory, calculation methods and test results are described in detail in [10].

2. Assumptions

The power range from several to about 50 Watts, especially at frequencies exceeding the industrial value is not sufficiently covered by synchronous motors of sufficiently high unit power values attained easily beyond the above power range. For instance, SH motors show good unit power values for the power of several watts and the industrial frequency, while with a rise of power and frequency the unit power drops. On the other hand, SP motors show comparatively high unit power values at the power output over 100 Watts.

An analysis of these relations leads to a conclusion that they result from the physical properties of magnetically active materials used in the above motors.

This led to an idea of incorporating in one motor the characteristic advantages of the both machine types, that is the natural self-excitation and the extremely simple construction and technology of SH motors, and the relatively high unit power values of SP motors. The difiiculty that has to be overcome consists in essentially different properties of magnetically active materials used for the both types.

Vicalloys /not available in Poland/ which present the optimum material for SH motors, do not show good properties in the regime of permanent magnetisation with flows obtainable in the micromotor stator, while in the case of extra-motor magnetisation show much worse properties than the typical materials used for SP motors. The latter, in turn, show absolutely insufficient hysteresis properties determined by non-saturated hysteresis loops corresponding to the magnetic intensity which can be obtained in the micromotor stator.

Thus the construction of a motor incorporating the advantages of both SH and SP motors depended on a selection of magnetic materials of required properties. Having determined the required properties and analysed a number of materials available in Poland the properties of which permitted to expect positive results, we concentrated on Cu Ni Fe 40 40 20.

3. Design and principle of operation

The stator constructed same as in an asynchronous motor ensures a rotating magnetic field. The rotor has the form of a smooth cylinder made of Cu Ni Fe 40 40 20 disks.

During the motor start-up a starting procedure should be followed consisting in a momentary forcing of stator flow increase and a re-magnetisation of rotor material in accordance with the extreme saturated hysteresis loop. The above forcing can be performed by several different methods which can be easily automatized, and which have been described in detail in [10].

Thus, when the motor is switched on, the rotor material is re-magnetized by the rotating field, as a result of which a hysteresis starting torque appears. The extreme hysteresis loop of the material used has no optimum properties with respect to the hysteresis torque, as its convexity factor [4] is rather small. Besides, due to almost full cylindrical shape of the rotor the effect of rotational hysteresis reduces the torque value. Nevertheless, the starting torque value is large enough, as it even slightly exceeds the rating, due to increased current during the starting procedure up to the values exceeding considerably /3 to 5 times/ the values permissible for the motor for thermal reasons. This increased current value, however, is permissible during the momentary starting procedure lasting for about 10 cycles. After that time the supply voltage is reduced to the rated value, the current value drops and the starting procedure is terminated.

After the conclusion of the starting procedure the rotor of SSHP motor remains magnetized radially. The rotor assumes the properties of a permanent magnet motor SP, while differing from it by the fact that rotor magnetization occurs following its synchronization due to which there is no effect of magnet stabilization by counter-action during the start-up, which is characteristic for SP motors. This enables a higher power output on the motor shaft.

The main physical differences between the SSHP and an over-excited SH described in detail in 10 consist in a different thickness of magnetic material layer, and thus in a different distribution of magnetic field forces and different physical properties of the material. Its basic property should be a possibility of attaining the extreme saturated hysteresis loop in the conditions of a field created in the motor during the starting procedure instead of obtaining the maximum convexity factor of the loop during the machine nominal operation of the machine.

The nominated operation range during which the motor can be loaded from the open-circuit condition up to the loss of synchronism begins after the end of the starting procedure. Within this range it can be treated as a SP motor, but cannot be described in the same way as SP motors usually are, as "a synchronous machine with excitation permanently turned on", because in this case the excitation "is turned on" only after the machine gains synchronism, which is characteristic of SS motors. with electromagnetic excitation. The design of the SSHP rotor is also similar to that used in SS motors with non-salient-pole rotors. An application of the theory of this machine to the SSHP permits to explain clearly all the phenomena occuring in it and to derive the formulas and make the calculations based on vector diagrams ensuring the required accuracy of results.

The working point on the rotor material demagnetization curve can be determined as for the SP according to the extreme hysteresis loop obtained during the starting procedure. This point determines the value of the rotor magnetic flux at no-load run and the value of no-load run emf corresponding to the value of E_o SS.

The rated supply voltage U_n should be chosen slightly above the so determined E_o value of the rotor. The difference between U_n and E_o determines the degree of under-excitation of SS motor. The geometrical difference between the two values determines, as it can be seen in the vector diagram, the current value consumed by the motor. This current value generates a stator flow Θ_t which adds to rotor "flow" Θ_m into a resultant flow Θ_w which corresponds to the resultant gap emf of SS. In case of under-excited SS at no-load run the angle between Θ_t and Θ_m is acute and Θ_t causes additional magnetization. Along with the motor load increase, the angle between Θ_t and Θ_m and at the same time between E_o and U_n called the ϑ angle of synchronous machine load increases too. At a certain moment the angle between Θ_t and Θ_m equals to $\frac{\pi}{2}$. From then on, further motor load increase is accompanied by a stator flow component which de-magnetizes the rotor. Here again we encounter an essential difference between the SSHP and the SS: the de-magnetizing action of Θ_t cannot be compensated with an increase of rotor excitation current.

The working point starts to move down the rotor material de-magnetization curve until it reaches the point determined by the maximum value of product /BH/max, and passes beyond it which makes the motor drop out from synchronism.

Thus the range of possible machine loads can be divided into a sub-range of loads which do not cause a de-magnetization of the rotor magnetized during the starting procedure, and which with a properly selected ratio of U_n and E_o covers a major part of the whole load range, and a small sub-range of loads which do cause a de-magnetization of the rotor. Within the latter sub-range a load reduction causes the working point move along the return straight line, the beginning of which corresponds to the last value of increasing load. Within this sub-range the motor is similar to an over-excited SH motor. The working point travel along the return straight line effects the flow increase at a lower rate than the rate of its former reduction, and thus does not ensure E_o emf even at a complete machine unloading.

For the same reason the value of consumed current does not drop down to the original values. For the above reasons, the motor operation within this sub-range, although possible, cannot be recommended.

The use for the working point a point located on the de-magnetization curve and not on the return straight line, as it is the case with SP motors, enables to obtain higher E_o values, and consequently higher unit power values at respectively small current consumption.

4. Vector diagrams

For the characteristic working conditions of the SSHP i.e. the ideal no-load run, the load which does not cause rotor de-magnetization and the maximum load beyond which the machine drops out from synchronism, vector diagrams can be plotted similar to those for the SS with non-salient-pole rotor acc. to 7 .

A vector diagram for the ideal no-load run is shown in Fig. 1. Parameters changing with load are marked with subscript "o". Since $\vartheta_o = 0$, supply voltage U_f complies with the direction of E_{mof} emf. The difference between them i.e. the voltage drop on impedance $I_o Z_s$ is a sum of voltage drops $I_o R$ and $I_o X_s$. Synchronous reactance X_s is composed of armature reactance X_{as} and stator winding leakage reactance X_1. Calculations made for several varieties of the SSHP proved that X_{as} which depends on the rotor material reluctance assumes values much exceeding the values of X_1. The value of X_1 is of the order of 1% of X_{as}. For this reason the vector of gap emf E_{wo} can be drawn as shown on diagram in Fig. 1. Flows Θ_{wo} and

Θ_{wo} which correspond to emf E_{mof} and E_{wo} advance then by $\frac{\pi}{2}$, while armature reaction flow Θ_{to} which joins their ends adds to Θ_{mo} and has the effect of additional magnetization. The direction of Θ_{to} determines the direction of no-load run current I_o and ideal no-load run power factor cos φ_o.

A vector diagram for the maximum load not yet causing rotor de-magnetization is shown in Fig. 2. Parameters changing with load are marked with subscript "p". Since there is no de-magnetization of the rotor the value of E_{mof} does not change with respect to Fig. 1 while the load angle $\vartheta_p = \varphi_p$, because, as it can be seen from the diagram, just then Θ_{tp} is normal to Θ_{mo}, same as their respective vectors I_pX_s and E_{mof}. Magnitudes and directions of the remaining vectors discussed above for Fig. 1 result directly from the diagram plot.

A vector diagram for the maximum load which cannot be further increased without the motor dropping out from synchronism is shown in Fig. 3. Parameters changing with load are marked with subscript "m". With an increase of load angle ϑ over the value of ϑ_p the angle between Θ_{mo} and Θ_{tm} becomes smaller than $\frac{\pi}{2}$, as a result of what component Θ_r of flow Θ_{tm} appears opposing Θ_{mo} and de-magnetizing the rotor. Since the above mentioned de-magnetization cannot be compensated with extra motor excitation, the value of rotor emf is reduced with respect to E_{mof}. At the maximum possible load for the motor synchronous run, the location of the working point on the rotor material de-magnetization curve is determined by product $/BH/_{max}$ of the curve. The values of emf E_{mm} and flow Θ_{mm} correspond to the value of flux $\phi_{\delta m}$ easily calculable at this point. The dotted lines and superscripts ' ~~xx~~ shown in the diagram refer to values which would appear if there were no rotor de-magnetization, while the continuous lines refer to values appearing in actual practice. From the diagram it results that also in this case $\vartheta_m = \varphi_m$.

From a comparison of the diagrams it can be noted that with loading the motor from the no-load run up to the maximum value, with an increase of ϑ angle increases the value of current consumed by the motor, while the power factor determined by φ angle rises until the beginning of rotor de-magnetization and then ~~fxlxxix~~ falls down again.

5. Calculation Method

The SSHP calculation method at a predetermined output power and permissible use of active materials /resulting from machine heating/ is based on the principle of operation and construction of vector diagrams described in detail in [10]. For the lack of space in this paper I shall present only the principle of composition of the calculation method.

~~Thexxxlumexxf~~ The required volume of magnetic material is calculated in accordance with radically modified method presented in [1]..Having determined it and assumed the stator geometry, the magnetic circuit is calculated and its saturation is evaluated by traditional methods [2,3], taking into account the starting procedure parameters. Parameters after the rotor magnetization are calculated according to [1], while taking into account the fact that there is no magnet stabilization. Winding structure, winding specifications, synchronous reactances and impedances can be calculated by traditional methods.

Having determined the above values we can proceed to the calculation of electrical and mechanical parameters of the SSHP.

The current value required for the rotor magnetization during the starting procedure can be determined from the flow value required for obtaining the predetermined /extreme/ hysteresis loop of the rotor material.

The starting torque which is a hystreresis torque in the course of forcing the increased flow during the starting procedure is calculated by means of the formulas for a hysteresis motor, which I had to modify because of the effect of rotational hysteresis, [4 to 6] and [10].

Load angles, power factors and currents for the characteristic working conditions of the SSHP are determined by means of original formulas derived in [10] from the vector diagrams.

The relations obtained with use of these diagrams can be useful in quantitative and not only usual in qualitative analysis of phenomena due to the fact, that simplifying assumptions made usually for such diagrams correspond in the case of the SSHP to the actual exact working conditions. For instance, the assumption concerning the non-saturation of the magnetic circuit is fulfilled here as a result of the necessity of ensuring correct reluctance during the starting procedure.

The internal and output power values are determined by well known traditional methods as for a standard SS motor.

With respect to the permissible heating of the machine the rated values are checked by my original method described in my previous publications on micromotors [9 and 10]. The constructional parameters of the machine should be selected in such a way that the parameters obtained by the test procedure correspond to the parameters of the maximum load condition which still does not cause a de-magnetization of the machine. Optimum parameter values for a given machine are then obtained.

6. Test Models

In order to check the theory and the suggested calculation method of the SSHP, 7 test models of various power values have been made.

The magnetic materials used and the electro-mechanical parameters obtained were carefully and universally studied by means of specially designed test methods [8 and 10] . Full report on test procedures and results is given in [10].

On the basis of curves of magnetization, no-load run and starting torque /hystreresis/, load after magnetization, power output and current consumption, power and efficiency factors, excited emf, no-load run and others, the electro-mechanical parameters of tested models were determined and their values compared with those obtained from calculations.

The comparison proves a high accuracy of the suggested calculation method and the correctness of the machine theory.

7. Conclusions

A characteristic feature of the discussed theory and the suggested calculation method of the SSHP is the fact that during its operation the SSHP incorporates physical phenomena peculiar to three different types of electric motors, in new combinations.

The start-up and the calculation of the starting torque should be regarded in connection with the theory of SH motors, but with consideration of rotational hysteresis which as a rule does not appear in modern correctly designed machines of this type.

Rated operation shows certain similarities to SP motors, the essential difference being the lack of rotor de-magnetization with polarity change during the starting procedure. This very difference permitted to apply in theory the best known and most reliable elements of the SP theory.

A discussion of phenomena during the motor load changes and a calculationx of most electro-mechanical parameters turned out to be most convenient if based on the theory of the SS with non-salient-pole rotor. This theory modified in an original way permitted to explain most clearly all the physical phenomena in the SSHP, and the formulas derived from it are characterized by a high accuracy of results as compared to test results.

21.1

LEARNING AUTOMATIC SYSTEMS

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Introduction

Learning automatic systems can improve their performance, behaviour and properties during their operation. This paper will discuss the principle that underlies design of learning automatic systems, their capabilities and features.

Learning automatic systems are needed when the characteristics of observable situations (which correspond to signals to be recognized or states of plants to be controlled) are not known in advance, i.e. when the a priori data on situations are scarce.

The operation of learning uses learning procedures, classification of situations observed (patterns). Therefore much attention is given to development of these procedures whereby the functional such as average risk of wrong classification is to be minimized. This method follows from the general theory of adaptation and learning[1].

With particular forms of loss functions with the known and new learning and self-learning procedures are shown to be obtainable.

The procedures obtained by us are employed to design learning control systems and a learning receiver of pulse signals.

We will present the results of experimental study of the receiver.

1. Kinds of learning

Procedures for classification of situations observed underlie the design of learning automatic systems. Situations are divided into classes by a certain rule of decision. The techniques employed to define that rule are largely dependent on   of amount of data available in advance on the situations to be classified.

With sufficient data, when the probabilistic characteristics of situations are known, the rule of decision is found by classical techniques of the theory of statistical solutions[2,3]. In that case there is no learning. Learning is needed when data are incomplete or insufficient, that is when probabilistic characteristics of situations are not known in advance. If there is additional information, i.e. data on the situation from a certain sequence belonging to some class, the rule of decision is found in that learning sequence by procedures of learning with an incentive[4,5,6].

If this additional information is also unavailable, the rule of decision has to be found only through observation of situations by procedures that can conveniently be termed self-learning procedures[7,8]. Thus we deal with two kinds learning: with incentive and self-learning.

The rules of decision for all the above cases were earlier obtained through examination of separate and at first glance non-related problems. Nevertheless all known as well as new rules of decision can be obtained in a regular way from the condition of a minimum of overall performance criterion of an average risk in classification.

We will examine now the situations classification optimality criterion and how obtain the conditions for its minimum.

2. Classification criterion

To design a decision rule the classification performance criterion should be formulated whose extremum should define the fule.

Assume that the situations x from the space X appear randomly and each appearing situation is associated with one of N classes $X_1^\circ, X_2^\circ, \dots, X_N^\circ$ unknown to us. At different appearances the same situation x can be associated with different classes.

Let us divide the space of situations X into N regions and associate the situation $x \in X_i$ that appears at any instant of time with the class X_i°. The problem of classification consists in optimal in a certain sense division of the situation space X into regions $X_1, \dots, X_N$

Let us introduce a loss function $F_{km}(x, u_1, u_2, \dots, u_N)$ $(k, m = 1, 2, \dots, N)$, where $u_1, u_2, \dots, u_N$ are certain parameters. Each function $F_{km}(x, u_1, u_2, \dots, u_N)$ estimates the losses that appear when the situation x from the class X_k^c is associated with the class X_m^c, or when the situation x from the class X_k^c appears in the region X_m.

Let also P_k be the probability that situations of the class X_k^c arise,
$p(x/k) = p_k(x)$ the conditional probability density for situations of the class X_k^c. Then the average risk of misclassification can be given by

$$R = \sum_{k=1}^{N} \sum_{m=1}^{N} \int_{X_m} F_{km}(x, u_1, u_2, \dots, u_N) P_k p_k(x)\, dx \tag{2.1}$$

The best classification corresponds to such a selection of parameters $u_1, u_2, \dots, u_N$ and the regions X_k $(k = 1, 2, \dots, N)$ at which the average risk (2.1) is minimal.

3. Conditions for a minimum of average risk

The average risk is a functional of the boundaries Λ_{km} between the neighboring regions X_k and X_m and a totality of vectors $u_1, u_2, \dots, u_N$. To find the conditions for a minimum of risk we will following Ref[8], use the classical methods of variational calculus[9] and apply them to a more complex functional (2.1). These conditions can be represented as

$$\sum_{k=1}^{N} \sum_{m=1}^{N} \int_{X_m} \nabla_{u_\ell} F_{km}(x, u_1, u_2, \dots, u_N) P_k p_k(x)\, dx = 0 \quad (\ell = 1, 2, \dots, N) \tag{3.1}$$

and for all x which belong to the boundary Λ_{sm} between the neighboring regions X_s and X_m.

$$f_{sm}(x, u_1, \dots, u_N) = \sum_{k=1}^{N} \left(F_{ks}(x, u_1, \dots, u_N) - F_{km}(x, u_1, \dots, u_N) \right) P_k p_k(x) = 0 \tag{3.2}$$

Eq. (3.2) is a plane which divides the neighboring regions X_s and X_m. Therefore it is practical to term the function $f_{sm}(x, u_1, u_2, \dots, u_N)$ as a discriminant function. The signs of the discriminant functions at $x \,\overline{\in}\, \Lambda_{sm}$ for all $s, m = 1, 2, \dots, N$ enable to distinguish the regions.

With certain assumptions[7] the decision rule can be given by

$$x \in X_m \ (m=1,2,\dots,N)\ ,\ \text{if} \quad f_{ms}(x, u_1, u_2, \dots, u_N) < 0 \quad (3.3)$$

for all $m \neq s \ \ (s=1,2,\dots,N)$

In a particular case of two classes the average risk R, conditions for its minimum and the decision rule take the form

$$R = \int_{X_1} F_{11}(x,u_1,u_2) P_1 p_1(x)\,dx + \int_{X_1} F_{21}(x,u_1,u_2) P_2 p_2(x)\,dx +$$
$$+ \int_{X_2} F_{12}(x,u_1,u_2) P_1 p_1(x)\,dx + \int_{X_2} F_{22}(x,u_1,u_2) P_2 p_2(x)\,dx \qquad (3.4)$$

$$\nabla_{u_e} R = 0 \qquad (3.5)$$

x belongs to the class X_1^0, if $f_{12}(x,u_1,u_2) < 0$
(3.6)
x belongs to the class X_2^0, if $f_{12}(x,u_1,u_2) > 0$

Then we have to finalize the finding of decision rules from these conditions.

4. Statement of the problem

Depending on the nature of the a priori and the current additional information different statements of the problems are possible where the decision rule minimizing the average risk (eq.2.1).

Problem 1. The probability P_k that situations of the classes X_k^0 will appear, and the conditional probability densities $p_k(x)$ for situations in these classes are known $(k=1,2,\dots,N)$. The loss functions $F_{km}(x,u_1,u_2,\dots,u_N) = F_{km}(x)$ are also known. In this case there is no need in learning to obtain the decision rule.

Problem 11. The probability P_k and the conditional probability density $p_k(x)$ for situations are known $(k=1,2,\dots,N)$. The loss functions are given with the accuracy to the parameters $u_1, u_2, \dots, u_N$. In this case if the parameters $u_1, u_2, \dots, u_N$ that determine the decision rule are to be estimated, learning must be used that is based on the additional current information on the situations belonging to certain classes, i.e. on incentive and penalty.

<u>Problem 11a</u>. The probability P_K and the probability density $p_K(x)$ $(K=1,2,\dots,N)$ are unknown. The loss functions $F_{Km}(x,u_1,u_2,\dots,u_N)$ are given with the accuracy to the parameters $u_1,u_2,\dots,u_N$. In this case to estimate the optimal parameters $u_1,u_2,\dots,u_N$ certain statistical characteristics of the situations have to be reproduced. For this purpose the additional current data on the appearing situations belonging to the appropriate classes have to be used as before.

In these statements the existence of a certain classification of situations was assumed and approximation to this classification was required. If there is no current information on situations this statement of the problem is naturally devoid of sense.

That is why the physical essence of the appropriate average risk (2.1) constituents in Problem 111.

<u>Problem 111</u>. The probability P_K that the situations from the regions X_K appear (rather than from the classes X_K^0 as before) and conditional probability densities $p_K(x)$ in these regions are unknown $(K=1,2,\dots,N)$

The loss functions $F_{Km}(x,u_1,u_2,\dots,u_N)=F_m(x,u_1,\dots,u_N)$ for all $K=1,2,\dots,N$, that express now the penalty for associating the situation x with the class X_m $(m=1,2,\dots,N)$ are given with the accuracy to the parameters $u_1,u_2,\dots,u_N$. Under these conditions the estimation of the decision rule must be based exclusively on the processing of the situations observed without using the current information on situations belonging to the appropriate classes.

Thus, dependin on how complete the a priori information is and whether additional current information on the situations observed is available or not the following types of recognition or classification are distinguished: without learning (Problem 1); with learning and using the additional current information, or learning with incentive (Problems 11, 11a), with learning without additional information or learning without incentive, or self-learning (Problem III).

Now we shall proceed to the finding of the decision rules. For simplicity and observability of results we shall deal chiefly with the case of two classes.

5. Finding the decision rule without learning (Problem I)

Assume that

$$F_{11}(x,u_1,u_2) = F_{22}(x,u_1,u_2) = 0$$
$$F_{12}(x,u_1,u_2) = F_1(x)$$
$$F_{21}(x,u_1,u_2) = F_2(x) \qquad (5.1)$$

This means that with proper classification of the situation the losses are equal to zero, while at errors of the first and the second kind these depend on the situation. Then the condition for a minimum of average risk of eqs (3.4) will be

$$f_{12}(x) = P_2 p_2(x) F_2(x) - P_1 p_1(x) F_1(x) = 0 \qquad (5.2)$$

for all $x \in \Lambda_{12}$. $f_{12}(x)$ is the dividing function, therefore the decision rule takes the form

$$x \in X_1 \,, \qquad \text{if} \quad f_{12}(x) < 0 \qquad (5.3)$$
$$x \in X_2 \,, \qquad \text{if} \quad f_{12}(x) > 0$$

The decision rule can be formulated through a likelihood ratio $p_1(x) / p_2(x)$, namely

$$x \in X_1 \,, \qquad \text{if} \quad \frac{p_1(x)}{p_2(x)} > h(x)$$
$$x \in X_2 \,, \qquad \text{if} \quad \frac{p_1(x)}{p_2(x)} < h(x) \qquad (5.4)$$

where $h(x) = \frac{P_2 \cdot F_2(x)}{P_1 \, F_1(x)}$ is a threshold which in this case depends on x. In a particular case when

$$F_{12}(x) = w_{12} = const \,; \quad F_{21}(x) = w_{21} = const$$

$$h(x) = h = \frac{P_2 \cdot w_{21}}{P_1 \cdot w_{12}} = const. \qquad (5.5)$$

This case is usually considered in the statistical location theory[2,3]. In any other case with incomplete apriri information the decision rule can be found only through learning procedures.

6. Procedures of learning with incentive

To obtain procedures of learning with incentive we will transform the average risk minimum conditions of eqs (3.1), (3.2) to a more convenient form

$$M\,\Phi_\ell(x, u_1, u_2) \tag{6.1}$$

$$x \in X_1 \quad \text{if} \quad f_{12}(x, u_1, u_2) < 0 \tag{6.2}$$
$$x \in X_2 \quad \text{if} \quad f_{12}(x, u_1, u_2) > 0$$

where

$$\Phi_\ell(x, u_1, u_2) = \nabla_{u_\ell} F_{km}(x, u_1, u_2)\,, \quad (k, \ell, m = 1, 2) \tag{6.3}$$

if the situation of the class X_k° belongs to the class X_m°.

Now, to find the decision rule we can use iterative learning procedures[1].

The form of learning procedures depends on how complete the apriori information is.

Problem 11. Assume that probabilistic characteristics of the situation are known. The unknowns are the parameters u_1, u_2 of the loss functions $F_{ik}(x, u_1, u_2)$ $(i; k = 1, 2;\ i \neq k)$. As earlier, we will assume $F_{mm}(x, u_1, u_2)$, $(m = 1, 2)$ equal to zero. Then the discriminant function $f_{12}(x)$ will be given by

$$f_{12}(x) = F_{12}(x, u_1, u_2)\, P_1 p_1(x) - F_{21}(x, u_1, u_2)\, P_2 p_2(x) \tag{6.4}$$

By applying probabilistic iterative procedures to (6.1) we will have

$$u_\ell[n] = u_\ell[n-1] - \gamma_\ell[n] \cdot \nabla_{u_\ell} F_{12}(x[n], u_1[n-1], u_2[n-1])\,, \quad (\ell = 1, 2)$$

if $x[n]$ is a situation of the class X_1° and

$$F_{12}(x[n], u_1[n-1], u_2[n-1]) P_1 p_1(x[n]) - F_{21}(x[n], u_1[n-1], u_2[n-1]) P_2 p_2(x[n]) < 0 \tag{6.5}$$

$$u_\ell[n] = u_\ell[n-1] - \gamma_\ell[n]\, \nabla_{u_\ell} F_{21}(x[n], u_1[n-1], u_2[n-1])\,, \quad (\ell = 1, 2)$$

if $x[n]$ is a situation of the class X_2° and

$$F_{12}(x[n], u_1[n-1], u_2[n-1]) P_1 p_1(x[n]) - F_{21}(x[n], u_1[n-1], u_2[n-1]) P_2 p_2(x[n]) > 0$$

$$u_\ell[n] = u_\ell[n-1]\,, \quad (\ell = 1, 2)$$

in other cases.

Problem 11a. Let now the probabilistic characteristics of situations are unknown. In this case we could also use the learning procedures, cited above, if in the process learning we could restore the probabilistic characteristics of situations included in the expression for the dividing function

$$f_{12}(x, u_1, u_2) = F_{12}(x, u_1, u_2) P_1 p_1(x) - F_{21}(x, u_1, u_2) P_2 p_2(x) \tag{6.6}$$

This can be done by using the known procedures of restoring the probabilities and random values distribution densities[8,10]. However; there is no need in doing so. We will describe a simpler technique actually based on the same procedures; we will use them to restore the discriminant function itself rather than its constituents.

Fix the parameters u_1, u_2, then

$$f_{12}(x, u_1, u_2) = f_{12}(x) \tag{6.7}$$

If the function $f_{12}(x)$ is approximated by a stretch of an orthonormed series

$$\hat{f}(x) = c^T \varphi(x) \tag{6.8}$$

so as the functional

$$I(c) = \int_X [f_{12}(x) - c^T\varphi(x)]^2 dx \tag{6.9}$$

be minimal.

In (6.8) and (6.9) the character T means a transposition of the vector c. By differentiating the functions $I(c)$ over c and using the orthonormality of the vector-function $\varphi(x)$ components we will find the optimal values of c:

$$c = M\Phi(x), \tag{6.10}$$

where

$$\Phi(x) = \begin{cases} \varphi(x) F_{12}(x), & \text{if } x \text{ is a situation of the class } X_1^0 \\ -\varphi(x) F_{21}(x), & \text{if } x \text{ is a situation of the class } X_2^0 \end{cases} \tag{6.11}$$

By considering eq. (6.10) as a regression equation we will obtain, through the probabilistic iterative techniques of Refs[1,11], the procedure

$$c[n] = c[n-1] - \gamma[n]\{c[n-1] - \varphi(x[n]) F_{12}(x[n])\}, \tag{6.12}$$

if $x[n]$ is a situation of the class X_1°

$$c[n] = c[n-1] - \gamma[n]\{c[n-1] - \varphi(x[n])F_{21}(x[n])\}$$

if $x[n]$ is a situation of the class X_2°. By using eq. (6.10) to solve Problem 11a at arbitrary parameters u_1, u_2 and using also eq. (6.1) we will have the necessary conditions for the parameters $u_1, u_2, c_1, c_2, \ldots, c_n$

$$M\,\Phi_e(x, u_1, u_2) = 0$$

$$M[\Phi(x, u_1, u_2) - c] = 0 \qquad (6.13)$$

$c^T\varphi(x) = 0$ for all $x \in \Lambda_{12}$ where the functions $\Phi_e(x, u_1, u_2)$ and $\Phi(x, u_1, u_2)$ are found by eqs (6.3) and (6.11) respectively.

From eqs (6.13) following learning procedures

$$u_e[n] = u_e[n-1] - \gamma_e[n]\,\nabla_{u_e} F_{12}(x[n], u_1[n-1], u_2[n-1])$$

$$c[n] = c[n-1] - \gamma[n]\cdot[c[n-1] - \varphi(x[n]\cdot F_{12}(x[n], u_1[n-1], u_2[n-1])]$$

if $x[n]$ is a situation of the class X_1° and $c^T[n-1]\varphi(x[n]) > 0$.

$$u_e[n] = u_e[n-1] - \gamma_e[n]\cdot\nabla_{u_e} F_{21}(x[n], u_1[n-1], u_2[n-1])$$

$$c[n] = c[n-1] - \gamma[n]\cdot[c[n-1] + \varphi(x[n])F_{21}(x[n], u_1[n-1], u_2[n-1])] \qquad (6.14)$$

if $x[n]$ is a situation of the class X_2° and $c^T[n-1]\varphi(x[n]) \leq 0$

$$u_e[n] = u_e[n-1]$$

$$c[n] = c[n-1] - \gamma[n]\cdot[c[n-1] - \varphi(x[n])F_{12}(x[n], u_1[n-1], u_2[n-1])$$

if $x[n]$ is a situation of the class X_2° and $c^T\varphi(x) > 0$

$$u_e[n] = u_e[n-1]$$

$$c[n] = c[n-1] - \gamma[n]\cdot[c[n-1] + \varphi(x[n])F_{21}(x[n], u_1[n-1], u_2[n-1])]$$

If $x[n]$ is a situation of the class X_1° and $c^T[n-1]\varphi(x[n]) < 0$. In the most frequent particular case which underlies the location theory[1,2]

$$F_{12}(x, u_1, u_2) = w_{12}$$

$$F_{21}(x, u_1, u_2) = w_{21} \qquad (6.15)$$

Then the procedure of estimating the coefficients c of the dividing function $\hat{f}(x) = c^T\varphi(x)$, which minimizes the average risk

$$R = \int_{X_1} w_{21} P_2 p_2(x)\,dx + \int_{X_2} w_{12} P_1 p_1(x)\,dx \qquad (6.16)$$

will have this form

$$c[n] = c[n-1] - \gamma[n]\cdot\{c[n-1] - \varphi(x[n])w_{12}\}$$

if $x[n]$ a situation of the class X_1°

$$c[n] = c[n-1] - \gamma[n]\cdot\{c[n-1] + \varphi(x[n])w_{21}\} \qquad (6.17)$$

if $x[n]$ is a situation of the class X_2^0

7. Procedures of learning without incentive (self-learning)

In this case we do not know neither probabilistic characteristics of the situations x, nor additional data on the situations belonging to classes. In other words, only those observations are used of which we do not know what are the classes they belong to. Using the notation in Section 4 the average risk R at $N=2$ will be given by

$$R = \sum_{K=1}^{2} \int_{X_K} F_K(x, u_1, u_2) P_K p_K(x)\, dx \tag{7.1}$$

The condition for the minimum is obtained from eqs (6.1), (6.2), (6.3) by replacing the conditions of the situations belonging to real classes X_K^0 with the condition $\left(x \in X_K\right)$ of the condition belonging to the region X_K:

$$M\, \Phi_e(x, u_1, u_2) = 0, \tag{7.2}$$

where

$$\Phi_e(x, u_1, u_2) = \nabla_{u_e} F_K(x, u_1, u_2) \text{ if } x \in X_K \quad (K, e = 1,2) \tag{7.3}$$

and

$$\begin{aligned} x \in X_1 &\quad \text{if } F_1(x, u_1, u_2) - F_2(x, u_1, u_2) < 0 \\ x \in X_2 &\quad \text{if } F_1(x, u_1, u_2) - F_2(x, u_1, u_2) > 0 \end{aligned} \tag{7.4}$$

To find the learning procedures which give the parameters u_1, u_2 apply to eq. (7.2) the probabilistic iterative procedures.

Then we will have

$$u_e[n] = u_e[n-1] - \gamma[n] \nabla_{u_e} F_1(x[n], u_1[n-1], u_2[n-1]),$$

if

$$F_1(x[n], u_1[n-1], u_2[n-1]) - F_2(x[n], u_1[n-1], u_2[n-1]) < 0$$

$$u_e[n] = u_e[n-1] - \gamma[n] \nabla_{u_e} F_2(x[n], u_1[n-1], u_2[n-1]), \tag{7.5}$$

if

$$F_1(x[n], u_1[n-1], u_2[n-1]) - F_2(x[n], u_1[n-1], u_2[n-1]) > 0$$

$$(e = 1,2)$$

Quite naturally, these procedures coincide with those obtained in Ref.[7].

8. Particular cases of procedures

Though in their form they do differ from situation classification procedures known earlier the procedures obtained above contain them as particular cases. We will show this for the problem of learning with incentive. Let us search for the dividing function in the form

$$\hat{y} = u^T \varphi(x) \tag{8.1}$$

where u is a vector of unknown parameters.

$\varphi(x)$ is a vector-function whose components are linearly independent. In the basic criterion select the loss functions in this way

$$F_{12}(x) = F(1 - u^T\varphi(x)) = F_{11}(x)$$

$$F_{21}(x) = F(-1 - u^T\varphi(x)) = F_{22}(x) \tag{8.2}$$

where $F(\cdot)$ is a certain convex function.

Then

$$R = \int\limits_{X_1 = \{x:\, u^T\varphi(x) < 0\}} F(1 - u^T\varphi(x)) P_1 p_1(x)\, dx + \int\limits_{X_2 = \{x:\, u^T\varphi(x) > 0\}} F(-1 - u^T\varphi(x)) P_2 p_2(x)\, dx +$$

$$+ \int\limits_{X_1} F(-1 - u^T\varphi(x)) P_2 p_2(x)\, dx + \int\limits_{X_2} F(1 - u^T\varphi(x)) P_1 p_1(x)\, dx \tag{8.3}$$

By introducing a mixed situation distribution density

$$p(x) = P_1 \cdot p_1(x) + P_2 \cdot p_2(x) \tag{8.4}$$

we can represent the average risk of eq. (8.3) as

$$R = \int\limits_{X} F(y - u^T\varphi(x)) p(x)\, dx = MF(y - u^T\varphi(x)) \tag{8.5}$$

where

$$y = \begin{cases} 1, & \text{if } x \text{ belongs to the class } X_1^0 \\ -1, & \text{if } x \text{ belongs to the class } X_2^0 \end{cases} \tag{8.6}$$

teacher's instuctions (additional information) on the classes to which the appearing situations belong. This was the form in which the functionals were treated in recognition problems (Refs[4,5,6]).

The learning procedure in this case is given by

$$u[n] = u[n-1] - \gamma[n]\nabla_u F(1 - u^T[n-1]\varphi(x[n])),$$

if $x[n]$ is a situation of the class X_1^0

$$u[n] = u[n-1] - \gamma[n]\nabla_u F(-1 - u^T[n-1]\varphi(x[n])), \qquad (8.7)$$

if $x[n]$ is a situation of the class X_2^0. The procedures of eq. (8.7) can be written in a more compact form by using eq. (8.6). In Ref.[5] such procedures were obtained directly from the functional of eq. (8.5).

9. On optimal procedures

The learning procedures cited above converge when certain conditions imposed on the quantities $\gamma[n]$ and the loss functions are met[1]. In cases where the number of situations observed is limited it is practical to select $\gamma[n]$ so as to utilize that information in the best (in a certain sense) way. In other words; optimal procedures of Ref[12,13] have to be considered. Two possibilities arise here[13]; one is multiple use of non-optimal procedures with relatively simple $\gamma[n]$ of the type $\gamma[n] = \frac{a}{n}$ with periodic use of a known sequence of situations;

the other consists in special selection of $\gamma[n]$ which minimizes a certain functional, e.g. for linear procedures such as eqs (6.12), (6.17) it is easy to prove as was done in Refs [13,14] that the optimal value of $\gamma[n]$ is

$$\gamma_{opt}[n] = \frac{1}{n + \frac{\sigma^2}{V^2[0]}}, \qquad (9.1)$$

where σ^2 is the variation of x, $V^2[0]$ is the initial variation of the estimate. This expression gives the minimal variation of estimate at each step. If there is no a priori data on the initial value of the estimate variation, then $V^2[0] = \infty$ and we obtain

$$\gamma_{opt}[n] = \frac{1}{n} \qquad (9.2)$$

As for general cases, finding the optimal values is too cumbersome to be discussed here.

10. Learning control systems

The theory of optimal control systems makes it possible to obtain the law for variation of a control action $u_y(t)$ as a time function. As for the synthesis of an optimal control law, i.e. finding the dependence of U_y on phase coordinates, this problem is far from its satisfactory solution in more or less complex cases[3,15]. For systems, optimal in speed of response

$$U_y = 1 \qquad \text{or} \qquad U_y = -1 \tag{10.1}$$

Therefore the problem of synthetizing such systems can be regarded as that of classification of a phase coordinates vector $x = (x_1, x_2, \dots, x_M)$ into the two groups (10.1). Represent the unknown to us equation of the dividing surface as

$$\hat{f}(x) = U^T \varphi(x) = 0 \tag{10.2}$$

Then

$$U_y = \begin{cases} 1 & \text{if} \quad \hat{f}(x) \geq 0 \\ -1 & \text{if} \quad \hat{f}(x) < 0 \end{cases} \tag{10.3}$$

By the results of determining the optimal control and paths of phase coordinates $X_{opt}(t)$ with the given initial conditions $X(0)$ one can compile a training sequence i.e. a relation between $X_{opt.}$ and $U_{opt.}$ The classification is performed with the aid of learning procedures. Thus in such learning control systems the so-called switching surface is restored on the basis of the training sequence.

Realisation of such systems with adaptive linear elements was discussed in Refs[16,17,18]. Assume that we have a unique controller optimal in terms of speed of response which would generate optimal control (10.1) dependent on the plant coordinates. That optimal controller can be used for adjustment of "normal" standard controllers incorporated in simple learning control systems. During the learning the adjustible parameters change so as in time the "normal" controller would function as a costly optimal controller in the best way (in terms of minimal erroneous control actions)[1,16].

Other possibilities for construction of learning control systems are based on quantization of the system input signals space and determination or investigation of the already known optimal controls fot each of the regions obtained[19,20].

11. Self-learning receivers

A learning receiver is to classify the signals received. In those cases where the signals are represented by a scalar time function we can utilize the procedures for learning of situation recognition with the simplifications usually made for a one-dimensional case. Thus with second order loss functions the quality criterion is given by

$$R = \int_{X_1} (u_1 - x)^2 p(x)\, dx + \int_{X_2} (u_2 - x)^2 p(x)\, dx \tag{11.1}$$

In this case it is more practical to use continuous procedures instead of discrete ones. The latter can give continuous procedures by the appropriate limit transition (see also Ref.[8]). In this case these self-learning procedures can be represented as

$$\begin{aligned} \frac{du_1(t)}{dt} &= -\gamma_1(t) \cdot [x(t) - u_1(t)] \cdot 1(x(t) - x^0) \\ \frac{du_2(t)}{dt} &= -\gamma_2(t)[x(t) - u_2(t)] \cdot 1(x^0 - x(t)) \end{aligned} \tag{11.2}$$

where x^0 is a threshold equal to

$$x^0 = \frac{u_1 + u_2}{2} \tag{11.3}$$

and

$$1(z) = \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{if } z < 0 \end{cases} \tag{11.4}$$

The optimal values of $\gamma_1(t)$ and $\gamma_2(t)$ can be represented in this form

$$\gamma_1(t) = \frac{1}{\int_0^t x^2(\tau)\, 1(x(\tau) - x^0)\, d\tau} \tag{11.5}$$

$$\gamma_2(t) = \frac{1}{\int_0^t x^2(\tau)\, 1(x^0 - x(\tau))\, d\tau} \tag{11.6}$$

where $t > \varepsilon > 0$

The block-diagram of a receiver is shown in Fig.1. The receiver is assembled of elemens ЭМУ-8 and specially developped blocks (semiconducting contactless relays, γ - element and a controlled potentiometer). We will describe below the results of an experimental study of a learning receiver employed in classifying the input signals (pulsed signals whose amplitude change) by different laws, and pulsed signals against a noisy background). The oscillogram of Fig.2 shows processes of learning without additional outside information. The notations are

1 input signal to the receiver,

2 output signals associated with the 1st class x/,

3 output signals associated with the 2nd class x/,

4 the process of threshold setting whose duration characterizes the learning period. For the cases under consideration the process lasts three through seven periods. The signal-to-noise ratio (in oscillograms 1 to 6) is of the order 0.85 to 0.4. The performance of this principle depends exclusively on how distinctly the signals to be classified are distinguished. Such a scheme of a learning receiver was used to establish the response parameter (threshold) in a prototype of a teletype link with noise and ensures a minimal numbers of errors. during reception of signals in case of errors statistics unknown in advance. Such receivers can be applied in numerous other fields (remote control, monitoring, etc).

Conclusion

The paper is an extension of the learning atutomatic systems theory. From the condition for minimum of average risk of situation misclassification, the probabilistic iterative procedures gave situation classifiction procedures for both complete and incomplete information on situations to be classified. In the latter case, we come to learning procedures with and with out an incentive. The learning and self-learning procedures known earlier follow

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x/ At signals of varying amplitude (Fig.2) the output signals are presumed normed.

from our procedures as particular cases. Our procedures were used to design learning control systems and learning receivers.

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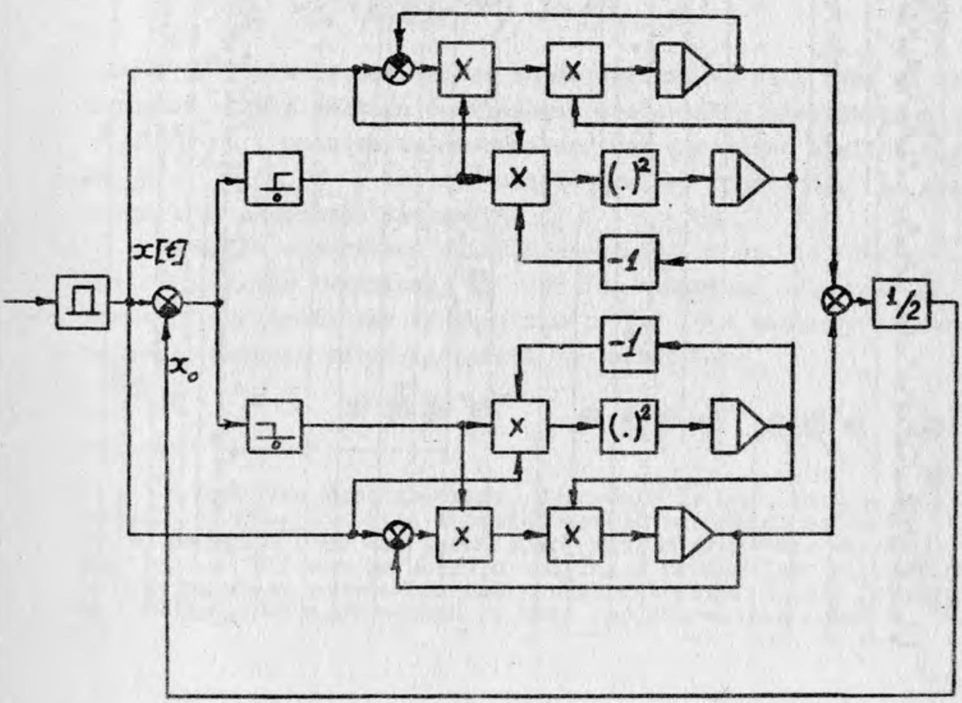
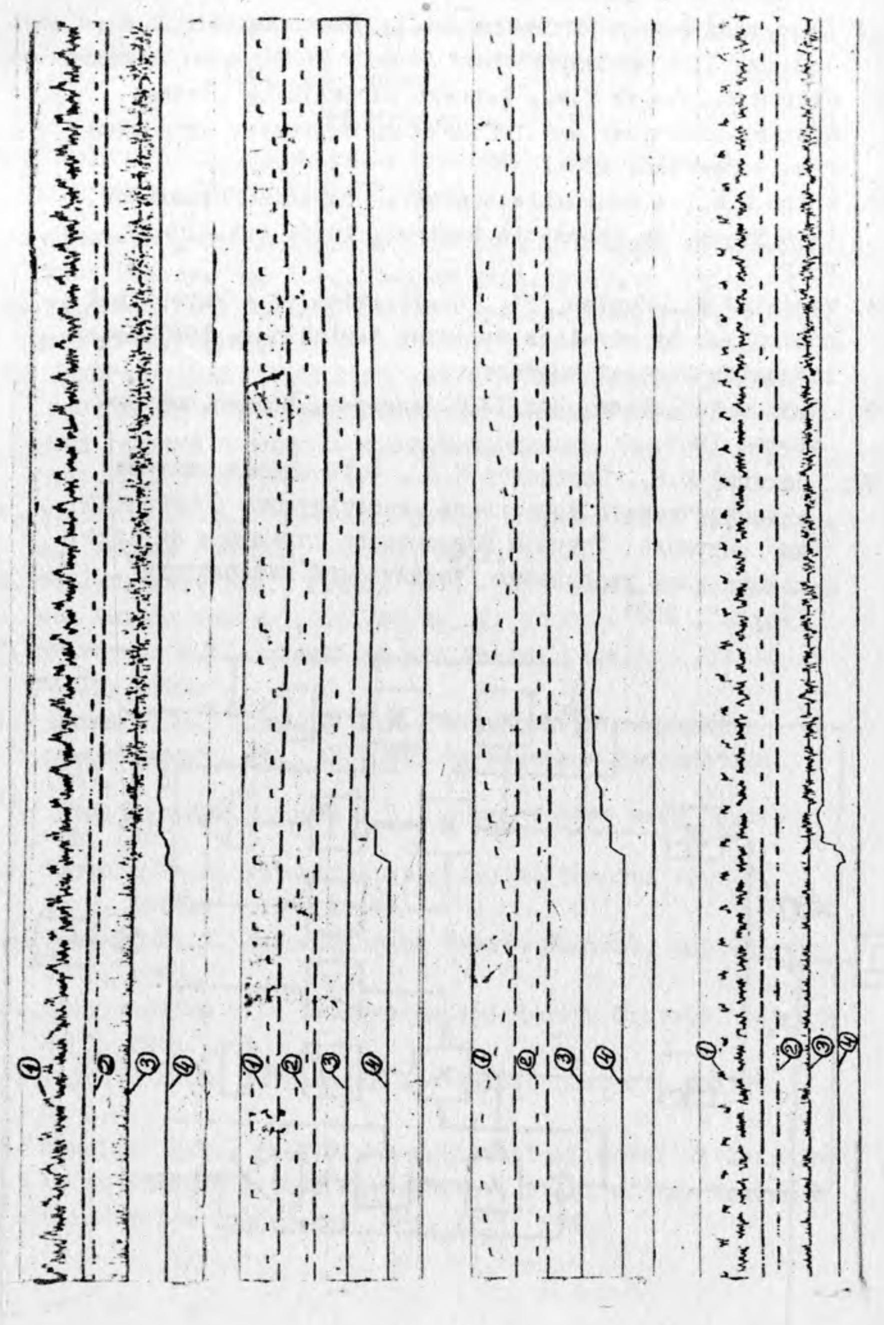


Рис. 1



21.3

ON CONVERGENCE OF RANDOM PROCESSES APPEARING IN CONSTRUCTION OF RECURRENT TRAINING AND ADAPTATION ALGORITHMS

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1. Introduction

The Robbins-Monro procedures used in the stochastic approximation method[1,2], the algorithms generated by the potential function method applied to solution of approximation problems[3-7] and some other kindred procedures make it possible to study random sequences of m -dimensional vectors $y^n=(y_1^n, y_2^n, \ldots y_m^n)$, described by recurrent relations of the form

$$y_i^{n+1}=y_i^n+\gamma_n \Phi_i(y^n, x^n), \quad i=1,2,\ldots,m \tag{1}$$

where x^n is a random vector which appears at each step in accordance with a certain conditional probability distribution $P(x^n/y^n)$, unknown in advance and not dependent explicitely on n ; $\Phi_i(y,x)$ are certain determined functions; γ_n are terms of a numerical sequence*.

Specific algorithms which are reduced to eq. (1) differ in the form of the functions Φ_i and the numerical sequence γ_n . Normally in algorithms of this kind, γ_n is a sequence of non-negative numbers which satisfies the conditions

$$\sum \gamma_n = \infty \tag{2a}$$

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*) That the Robbins-Monro procedure is akin to the procedures of the potential function method was first noted by Ya.Z. Tsypkin[8]. He has shown also[9] that a procedure of the type of eq. (1) can be used in solving a broad class of problems in the machines adaptation and training theory. Common features in the proof of convergence by both procedures was noted in [10].

$$\sum \gamma_n^2 < \infty \tag{2b}$$

In some cases, however, condition (2b) can be replaced by a weaker condition

$$\lim_{n \to \infty} \gamma_n = 0 \tag{2c}$$

(see, e.g. Theorem VIII of this paper).

Quite naturally, we come to a problem of establishing certain criteria for convergence of a random sequence, $y^1, y^2, \ldots, y^n, \ldots$ generated by procedure (1).

In Section 2 of this paper sufficient conditions are established for convergence of random processes, applicable, e.g. to procedure (1). These conditions are formulated in the following terms. A sequence of determined functions is introduced

$$U_n(y^1, y^2, \ldots, y^n) \geq 0 \quad \text{and} \quad V_n(y^1, y^2, \ldots, y^n) \geq 0 \tag{3}$$

which generally speaking are functions of the number of vectorial arguments $y^i = (y_1^i, y_2^i, \ldots, y_m^i)$ which increase with n and are realizations of the random process $y^1, y^2, \ldots, y^n, \ldots$. Also, relations are established for the functions U_n and V_n which are valid due to the properties of the random process $y^1, y^2, \ldots, y^n, \ldots$ which guarantee the trend to zero in some sense ("By probability", "almost certainly" or "average") at least for one of the random sequences $U_1, U_2, \ldots, U_n, \ldots$ or $V_1, V_2, \ldots, V_n, \ldots$

When the conditions for convergence, thus obtained, are employed in specific problems, functions of eq. (3) can be selected so that convergence of one of these functions to zero in a certain sense leads to convergence, in the same sense, of the random process $y^1, y^2, \ldots, y^n, \ldots$.

The experience accumulated in employment of the above criteria for convergence in specific problems (see e.g.[4-7]) shows that in many cases no difficulties arise in selection of such functions. In all theorems of Section 2 it is assumed that function (3), thanks to the random process $y^1, y^2, \ldots, y^n, \ldots$ meets the following condition:

<u>Condition A</u>: Mathematical expectations $M\{U_1(y^1)\}, M\{V_1(y^1)\}$ exist and

$$M\{U_{n+1}(y^1, \ldots, y^{n+1}) / y^1, \ldots, y^n\} \leq (1+\mu_n) U_n(y^1, \ldots, y^n) - \tag{4}$$

$$-\gamma_n V_n(y^1, \ldots y^n) + \zeta_n, \; n = 1, 2, \ldots$$

where $\gamma_n \geqslant 0$ and μ_n are numerical sequences such that
a) $\sum \gamma_n = \infty$
b) $\sum |\mu_n| < \infty$

and ξ_n is a certain numerical sequence. In formula (4) the symbol $M(\dots/\dots)$ denotes conditional mathematical expectation.

The properties of the random sequence ξ_n and the numerical sequence γ_n are detailed in each theorem of Section 2. When these were applied to procedure (1) $M\{\xi_n\}$ and $|\mu_n|$ were found to be proportional to γ_n^2 .

Condition A Along with detailing the properties of the sequences ξ_n and γ_n make the first condition for all theorems of Section 2. These theorems actually differ in their second conditions which establish additional requirements to the relation between the functions $U_n(y^1, \dots, y^n)$ and $V_n(y^1, \dots, y^n)$ which, together with the first conditions of the theorems make it possible to prove the convergence of the appropriate random sequences.

The second conditions of Section 2 can be weakened substantially if the limited nature of almost all realizations of the random process $y^1, \dots, y^n, \dots$ is somehow established.

<u>Definition</u>. A sequence of functions $U_n(y^1, \dots, y^n)$ is infinitely large if any sequence $y^1, \dots, y^n, \dots$ for which the limit $\lim_{n \to \infty} U_n(y^1, \dots, y^n)$ exists and is finite is limited.

For a number of theorems of Section 2 we can prove that almost all realizations of the random process $y^1, \dots, y^n, \dots$ are limited if the sequence of functions U_n not only satisfies the first conditions of the theorems but is also infinitely large. The appropriately modified theorems are denoted in this paper by the same number with the index"a". (E.g. a modification of Theorem 1 is denoted as Theorem 1a).

The approach to establishing the convergence of random processes suggested in this paper is conceptually close to the direct Lyapunoff method applied to investigation of stability of motion whereby the fact of stability is established if a certain function of phase coordinates can be selected that would meet the conditions needed to ensure its trend to zero in the process of disturbed motion. From Condition A follows that the functions U_n and $\gamma_n V_n$ play the role of "a Liapunov function" and "its derivative due to the process".

The idea itself of using certain techniques akin to the Liapunov method to study the convergence of random processes was used in quite a few papers (see e.g.[11-13]). This paper differs from them in that the conditions imposed on the "Liapunov function" reflect the specific features of random processes caused by relations (1) and (2)*. The known theorems of Blum[2] which are employed to prove the convergence inherent in procedures of the stochastic approximation method are of the same nature.

In Section 3 the general theorems of the preceding Section are used directly to establish the conditions for convengence of procedure (1). Theorem V is formulated whose conditions are analogous in their form to those of Blum's theorems. That theorem for which the results obtained by Blum and Ya.G. Gladyshev[15] are particular cases enable to establish the convergence of procedure (1) used to solve the regression equations and also in cases where the uniqueness of solutions to these equations cannot be quaranteed. Theorem VI enables to establish the convergence of procedure (1) for features of a process generated by procedure (1) other that those used in Theorem V. Section 3 presents in conclusion theorem VII which extends the results obtained by Blum to the case where the sequence γ_n incorporated in procedure (1) satisties condition (2c) instead of a stronger condition (2b).

2. Basic Theorems on Convergence

Let us consider a random process at discrete time n

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*) A continuous determined analog to the process of eqs (1), (2) is made by the equations

$$\frac{dy_i}{dt} = \gamma(t)\,\Phi_i(y_1,\dots,y_m);\quad i = 1\dots m$$

provided that $\int^{\infty}\gamma(t)\,dt = \infty$

$$\int_0^{\infty}\gamma^2(t)\,dt < \infty \qquad /\text{or } \lim_{t\to\infty}\gamma(t) = 0/$$

and described by conditional probability densities $P_{n+1}\left(y^{n+1}/y^1,\dots,y^n\right)$ for appearance of the random quantity y^{n+1} at the n+1-th time provided that at times $1,2,\dots,n$ the random quantities take the values $y^1, y^2,\dots, y^n$ respectively.

The conditions sufficient for the mathematical expectations $M\{U_n\}$ and $M\{V_n\}$ of a sequence of functions (3) in a random process, $y^1,\dots,y^n,\dots$, to go to zero are established by Theorems I and II. Because the quantities U_n and V_n are non-negative these quantities evidently go to zero in terms of probability.

Theorem 1. Let a random process $y^1,\dots,y^n,\dots$ and a sequence of scalar functions (3) which satisfy the following conditions:

1°. Condition A where

$$\lim_{n\to\infty}\gamma_n=0 \quad \sum M\{\ell_n\}<\infty$$

2°. $M\{V_{n+1}/y^1,\dots,y^n\}\le(1+A\gamma_n)V_n(y^1,\dots,y^n)+B\gamma_n+\eta_n$

where A and B are certain constants while $\eta_n \geq 0$ is a random sequence such that

$$\sum M\{\eta_n\}<\infty$$

Then

$$\lim_{n\to\infty} M\{V_n\}=0 \tag{5}$$

and a sequence of random functions V_n goes to zero in terms of probability at $n\to\infty$.

When actual processes are studied it is found that the quantities A and B of Condition 2° of Theorem 1 are certain functions of $y^1,\dots,y^n$, rather than constants:

$$A=A_n(y^1,\dots,y^n),\quad B=B_n(y^1,\dots,y^n)$$

However, if we somehow manage to establish the limited nature of the sequences A_n and B_n for almost all realizations of the random process, that V_n converges to zero in terms of probability can also be proved in this case. The limited nature of the sequences A_n and B_n can follow, in particular from the limited number of realizations of a random process. The condition which guarantees the limited nature of almost all realizations of the random process $y^1,\dots,y^n$, can as specified in Section 1, be expressed as a constraint on the form of a sequence of functions $U_n(y^1, \dots, y^n)$. It is sufficient then to require that the sequence U_n be infinitely large.

Theorem 1a. Let function (3) satisfy the following conditions

1°. Condition A where

$$\lim_{n \to \infty} \gamma_n = 0 \quad \sum M\{\ell_{n}\} < \infty$$

2°. $$M\{V_{n+1} / y^1, \dots, y^n\} \leq (1 + A_n(y^1, \dots, y^n)) V_n + B_n(y^1, \dots, y^n) \gamma_n + \eta_n$$

where $A_n(y^1, \dots, y^n)$ and $B_n(y^1, \dots, y^n)$ are functions which for $y^1, \dots, y^n$, $|y^i| \leq R$, $i = 1 \dots n$ any limited sequence are majored by the constants $A(R)$ and $B(R)$ respectively while η_k is a random sequence such that

$$\sum M(\eta_n) < \infty$$

3°. The sequence of functions $U_n(y^1, \dots, y^n)$ is infinitely large.

Then the sequence of random quantities V_n goes to zero in terms of probability at $n \to \infty$

$$V_n \xrightarrow{P} 0$$

In the formulation of Theorem 1 the requirement of convergence for the series $\sum M(\ell_{n})$ is substantial. Theorem II formulated below makes it possible to weaken that requirement by substantial strengthening of Condition 2° of Theorem I. If the requirement for convergence of the series $\sum M(\ell_{n})$ in Theorem II is jettisoned, we will manage (see Theorem VII) to abandon in some cases requirement (2b) in algorithms of stochastic approximation and use requirement (2c) instead.

Unlike Theorem I, Theorem II states that the sequence U_n rather than sequence V_n goes to zero.

Theorem II. Let functions (3) meet the conditions

1°. Condition A where

$$\lim_{n\to\infty}\gamma_n=0 \text{ and } M\{\zeta_n\}=\alpha_n\gamma_n,\ \lim_{n\to\infty}\alpha_n=0$$

2°. There is a constant, A , such that

$$V_n(y^1,\dots,y^n)\geqslant A U_n(y^1,\dots,y^n)$$

Then

$$\lim_{n\to\infty} M\{U_n\}=0$$

and a sequence of random quantities U_n goes to zero in terms of probability at $n\to\infty$.

Let us take up now the conditions sufficient for "almost certain" convergence. Theorems III, IIIa and IV formulated below make it possible to establish both the convergence in terms of probability and "almost certainly". However, the conditions of Theorem III and IIIa guarantee convergence $M\{U_n^\beta\}$ only at $\beta<1$ while those of Theorems IV and IVa do not quarantee convergence of mathematical expectations at all.

<u>Theorem III</u>. Let function (3) satisfy the following conditions

1°. Condition A where

$$\lim_{n\to\infty}\gamma_n=0 \qquad \sum_1^\infty M\{\zeta_n\}<\infty$$

2°. In almost all realizations of a random process for which $\lim_{n\to\infty} V_n=0,\ \lim_{n\to\infty} U_n=0$ is valid.

Then at $n\to\infty$ a sequence of random quantities goes to zero almost certainly $U_n\xrightarrow{a.c.}0$

and at any $\beta<1$

$$\lim_{n\to\infty} M\{U_n^\beta\}=0. \tag{6}$$

<u>Theorem IIIa</u>. Let function (3) satisfy the following condtitons

1°. Condition A where

$$\lim_{n\to\infty}\gamma_n=0,\ \sum_1^\infty M\{\zeta_n\}<\infty$$

2°. In all limited sequences for which $\lim_{n\to\infty} V_n=0$, $\lim_{n\to\infty} U_n=0$ is valid.

3°. The sequence of functions $U_n(y^1,\dots,y^n)$ is infinitely large.

Then at the sequence of random quantities goes to zero almost certainly and also at any $\beta<1$, $\lim_{n\to\infty} M\{U_n^\beta\}=0$.
Using Theorem III we can prove convergence of the known Dvo-

retsky process[14]. The establishment of the fact that functions U_n and V_n can be chosen that satisfy the conditions of Theorem III is not related to analysis of the random process in question and requires only algebraic transformations. The analysis of a random process in completely "handled" by Theorem III. The Dvoretsky theorem deals with a random process $x^1, \dots, x^n, \dots$;its convergence both almost certainly and in the mean square is proved, i.e.

$$x^n \xrightarrow{a.c.} 0, \lim_{n\to\infty} M\{|x^n|^p\} = 0 \text{ at } p \leq 2$$

If Theorem III is applied, a weaker assertion can be proved

$$x^n \xrightarrow{a.c.} 0, \lim_{n\to\infty} M\{|x^n|^p\} = 0 \quad \text{at } p < 2$$

(the case $p = 2$ corresponds to $\beta = 1$ in eq. (6)).

Theorem IV which follows is very close to Theorem I as far as their conditions are concerned but differs in that it contains the requirement for limiting the increase of the sequence V_n (Condition 2°) not in the sense of mathematical expectations but in almost each realization. This make it possible to establish the convergence of the sequence to zero V_n almost certainly.

Theorem IV. Let function (3) satisfy the following conditions 1°

1° Conditions A where

$$\lim_{n\to\infty} \gamma_n = 0 \quad \sum_1^\infty M\{\zeta_n\} < \infty$$

2°. For any $\delta > 0$ there is such a set of the random proess $y^1, \dots, y^n, \dots$ realizations whose probability is above $1 - \delta$ and there are constants $A_\delta > 0$ and $B_\delta > 0$ such that on that stretch the inequalities

$$V_{n+1} \leq (1 + A_\delta \gamma_n) V_n + B_n \gamma_n + \eta_n \tag{7}$$

are valid where η_n is a sequence of numbers such that $\sum_1^\infty \eta_n$ converge. Then at $n \to \infty$ the sequence of random quantities V_n goes to zero almost certainly.

3. Conditions of convergence for the Robbins-Monro procedure of the stochastic approximation method

In this Section the results of the preceding Section are used to find the convergence of the Robbins-Monro procedure. That procedure is known to be in constructing a sequence of vectors $y^1, \ldots, y^n, \ldots$ that would satisfy relation (1) and intended for a solution to a set of regression equations

$$M_x\{\Phi_i(x, y_1, y_2, \ldots y_m)\}=0 \quad i:=1,\ldots,m \qquad (9)$$

$P(x/y)$ V where x is a random quantity with a fixed but unknown probability distribution function V (which may depend on $y=(y_1,\ldots,y_m)$) and the symbol $M_x\{\ldots\}$ denotes mathematical expectation at each fixed vector $y=(y_1,\ldots,y_m)$. Now we have to find convergence, in some sense, of the random sequence $y^1, \ldots, y^n, \ldots$ to the roots y^* of equation set (9).

Theorems V to VII below generalize and complete the known results (see References in [17]) which establish the conditions for convergence. Theorem V covers the results obtained by Blum[2] and Gladyshev[15] and extends them, in particular, to the case where there may be more than one solution to the regression equations.

Theorem VI also establishes the conditions for convergence of procedure (1)in case where one cannot say for certain that there is just one solution to set of equations (9). The nature of this Theorem differs somewhat from that of Theorem V and cannot be derived from it.

Theorem VII modifies conditions of Theorem V for the case where the sequence γ_n of eq. (1) does not satisfy Condition (2b).

Following Blum we will introduce a non-negative twice continuously differentiable function $U(y)$ and the functions

$$V(y) \equiv -M_x\left\{\sum_1^m \frac{\partial U}{\partial y_i}\Phi_i(x,y)\right\}$$

$$W_n(y) \equiv M_x\left\{\max_{0\le\theta\le 1}\sum_{i,k=1}^m \left(\frac{\partial^2 U(z_1,\ldots,z_m)}{\partial z_i\,\partial z_k}\right)_{z=y+\theta\gamma_n\Phi(x,y)}\Phi_k(x,y)\Phi_i(x,y)\right\}$$

which are assumed existant. In all theorems of this Section it is assumed that the following condition is met.

Condition B. $V(y) \geqslant 0$ and at any $U \geqslant 1$

$$W_n(y) \leq aU(y) + bV(y) + c$$

where a, b, c are certain constants.

Note the main features in using the theorems of the preceding Section to prove Theorems VI to VIII. The rôle of Condition B in the Theorems of this Section is in that it guarantee that the random sequences

$$U_n \equiv U(y^n), \quad V_n \equiv V(y^n)$$

by virtue of recurrent procedure (1) and relations (2a),(2b) satisfy Condition A of Section 1. For us to be able to use the Theorems of the preceding Section their other conditions must be met. Besides, the fact that $U(y^n)$ (or $V(y^n)$) go to zero (established by the Theorems of the preceding Section) should guarantee that the random sequence $y^1, \dots, y^n$ converges to the solution to set of equations (9). Other conditions of Theorems V to VII serve exactly that purpose.

In order to formulate Theorem V denote a set of solutions to set of equations (9) as Y^* and say that y^n tends to Y^* at $n \to \infty$ in terms of probability (almost certainly)

$$y_n \xrightarrow{P} Y^* \quad (y_n \xrightarrow{a.c.} Y^*)$$

if $\rho(y^n; Y^*) \equiv \inf\limits_{y \in Y^*} |y^n - y|^P \to 0 \quad (\rho(y^n; Y^*) \xrightarrow{a.c.} 0)$

Theorem V. Let $y^1, \dots, y^n, \dots$ be a random process described by relations (1) and (2a), (2b). Let then the function $U(y)$ meet the following conditions:

1°. Condition B;

2°. For each sequence $y^1, \dots, y^n, \dots$ where $\lim\limits_{n\to\infty} U(y^n) = 0$ simultaneously and $\lim\limits_{n\to\infty} P(y^n; Y^*) = 0$;

3°. For each sequence $y^1, \dots, y^n, \dots$ where $\lim\limits_{n\to\infty} V(y^n) = 0$ simultaneously and $\lim\limits_{n\to\infty} U(y^n) = 0$.

Then at $n \to \infty$ the random vector y^n tends to Y^* almost certainly

$$y^n \xrightarrow{a.c.} Y^*$$

If the function $U(y)$ is infinitely large, conditions 2° and 3° can be replaced by weaker conditions

2a°. The function $U(y)$ can vanish only in the points from Y^*.

3a°. For each limited sequence where $\lim_{n\to\infty} V(y^n)=0$ simultaneoušly $\lim_{n\to\infty} U(y^n)=0$.

The proof of Theorem V is based on Theorems II and IIIa of the preceding section.

Theorem VI. Let $y^1,\dots,y^n,\dots$ be a random process described by relations (1) and (2a),(2b). Let then the function meet the following conditions:

1°. Condition B;

2°. $U(y)$ is an infinitely large function;

3°. The function $V(y)$ is continuously differentiable and can vanish only in the points from Y^*;

4°. The function

$$B_n(y)\equiv M_x\left\{\max_{0\le\theta\le1}\sum_{i=1}^{m}\left(\frac{\partial V(z_i,\dots,z_m)}{\partial z_i}\right)_{z=y+\gamma_n\theta\Phi(x,y)}\Phi_i(x,y)\right\}$$

is majored by the function $B(y)$ independent from n and bounded in any bounded area where the variable $y=(y_1,\dots,y_m)$ can change. Then at $n\to\infty$ the random vector y^n tends to Y^* in terms of probability:

$$y^n \xrightarrow{P} Y^*$$

The proof of Theorem VII is based on Theorem 1a of the preceding Section.

Theorem VII. Let $y^1,\dots,y^n,\dots$ be a random process described by relations (1) and (2a),(2b). Let then the function $U(y)$ meet the following conditions:

1°. Condition B;

2°. Condition 2° of Theorem VI*;

3°. $V(y)\ge AU(y)$ where $A>0$ is a certain constant. Then at $n\to\infty$ the random vector y^n goes to ~~zero~~ Y^* in terms of probability

$$y^n \xrightarrow{P} Y^*.$$

Condition 3° of Theorem VII is evidently substantially stronger than the condition of the same number for Theorem V. It is because that condition was strengthened that requirement (2b) could be replaced by weaker (2c). In this case, however, convergence of the random process $y^1,\dots,y^n,\dots$ is proved only in terms of probability rather then "almost certainly". The proof of Theorem VII is based on Theorem II of the Preceding Section.

Despite the fact that Condition 3° of Theorem VII is rigid enough there are quite a few applications where this Condition is met. As an example we can cite the Robbins-Monro procedure for finding the mean value of the random quantity x. In this case

$$y^{n+1} = y^n + \gamma_n (x^n - y^n)$$

and assuming that

$$U(y) = V(y) = (y - M\{x\})^2$$

we conclude from Theorem VII that $y^n \xrightarrow{P} M\{x\}$ if the sequence γ_n satisfies requirements (2a),(2b).

Condition 3° of Theorem VII is also met when the method of potential functions is used in some problems of training.

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25.7

AUTOMATIC CONTROL SYSTEM OPTIMILIZING THE BORE - HOLE BORING PROCESS

(Report theses)

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Comparative analysis of various technical and economic indices of the drilling process allows to consider the criterion of minimum cost of a boring-hole length unit to be an optimum one ta ng into account both technical and mechanical processes. The main disturbances resulting in the cost extreme drifts is the random changing of physical and mechanical properties of the rock under drilling.The simplest structure of the automatic control system is obtained by means of building of an open-loop adapting system with non-linear compensating coupling using the apriori process information.In the process of the control system operation the continuous estimation of physical and mechanic properties of the rock is performed,depending on which the o mum operation parameters ratio is set,thus providing the minimum drilling cost.

According to the analysis the control system dynamic properties depend on the physical and mechanical properties of the rock under boring,and are rather random.For example,some coefficients characterizing the automatic control system vary within the range of 40 to I60 fold.So the described control system is the system with random parameters.The dynamic properties of such a system can't be studied by common means.Special methods are suggested in this paper for analysing dynamic properties of the automatic control system.The method allows to statistically characterize the dynamic properties of the control system.Such dynamic indices of the system as stability,stability margin, limits of the transient response oscillations frequency etc. are defined by their probability.

In the report some data on industrial tests of drilling automatic control systems are also supplied.

AUTOMATIC CONTROL SYSTEM OPTIMILIZING THE BORE - HOLE BORING PROCESS

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Technical and economic indices of drilling shotholes at strip - mines are defined completely by the boring conditions parameter values.It is most expedient to assume the cost of a boring - hole length unit to be the criterion of optimum boring conditions,the boring - hole length unit reflecting both the technical and the economic sides of the process.In this case obviously the aim of the automatic control system is to provide the minimum cost of boring with random variations in physical and mechanical properties of the rocks bored

$$S_{min} = \min_{Y \in \mathcal{D}} \Phi(Y, C)$$

where $\Phi(Y,C)$ - cost function of the bore-hole length unit;

$Y=(y_1, y_2, \ldots y_n)$, y_i - independent parameters of the boring conditions;

$\mathcal{D}$ - the range of tolerable values of Y

C - constants independent of the boring conditions.

Minimization of the function S can be performed by the multi-channel extremal automatic control system,whose block-diagram is shown in figure I.In boring process the cost of the bore-holo length unit is calculated by the current $y_1 \ldots y_n$ parameter values.Under the action of disturbances $f_1(t) \ldots f_m(t)$, of which random change in physical and mechanical properties of the rock is the principle one,the cost extreme drifts at random.The controlling signals forming logical block performs the selection of the cost extreme by setting through the appropriate automatic control systems of the optimum working conditions parameters ratio,providing the minimum cost of boring in the rock encountered. In this case the limitations - H_i , put on the range of permissible parameters values Y variations must be also taken into account. These limitations are defined by the desining features of the drilling equipment or technological requirements.

For practical reasons it is expedient to control the basic parameters $y_1, \dots y_k$ alone, whose influence upon the cost is the most considerable.

The difficulties of technical realization of the minimum bore-hole length unit cost criterion are caused by the fact that the cost of boring can be calculated only after the boring has been completed, while the current boring conditions parameters must be calculated and set directly during the process itself in a manner allowing to obtain the minimum final cost of boring.

The above - mentioned circumstance is caused by the fact that the immediate realization of the minimum cost of boring criterion implies the use of operative information about the current wear and tear of the boring tool , which can't be regarded as possible, since the devices capable of supplying such an information are nowadays not available.

Taking the above-mentioned into consideration it is expedient to in advance determine the optimum values of the working conditions parameters (by means of the minimum cost criterion) for different physical and mechanical properties of rocks.

The principle working - conditions parameters to achieve the minimum cost of boring are the following: end-thrust F and rotational speed of the boring tool n must depend[I] on the hardness of the rock under boring f , as shown in fig.2. While making use of these dependences the simplest structure of the automatic boring process control system is obtained (see fig.3). The mentioned automatic control system is an open-loop-adjusting system with non-linear compensating coupling with the principle disturbance[2,3,4].

Obviously, the accuracy of obtaining the minimum cost of boring will depend on the trustworthiness of the apriori information used for tuning of functional generators and on the accuracy of estimation of physical and mechanical properties of the rock under boring.

The existing methods of defining physical and mechanical properties of rock are based on single laboratory sample tests and are not suitable for the automatic process control pur - poses,continuous estimation just on the working boring machine being necessary.

To meet these requirements,the method of estimation of physical and mechanical properties of rock can be performed this way.

In fig.4 are shown generalized dependence of drilling rate V in rocks of various hardness $f(f_1<f_2<f_3)$ on the magnitude of the end - trust F and a characteristic of the hydraulic circuit of the tool feed.

Supposing the diagram has the outline of the curve I . Points of intersection of the schematic diagram with the curves $V=\varphi(P)$ characterize the boring conditions in rocks with definite physical and mechanical properties. Obviously various end-thrust and rates of power drilling will be set in rocks of various hardness.When hardness is f_1 the end-thrust is F_{y1} and the rate is V_{y1} when $f_2 - F_{y2}$ и V_{y2} etc.

So the values of the end-thrust F_y and the rate of power drilling V_y determine the drillability of rocks and characterize physical and mechanical properties of rocks under boring.Both the end -thrust and the rate of drilling are integral characteristics of physical and mechanical properties of rocks and reflect such properties of rocks as hardness, brittleness and plasticity,liability to cracking etc.

The choice of the concrete index (F_y or V_y) in realization of the automatic control system by the boring process will be determined by additional considerations such as simplicity and reliability of the index measurement, the kind of the characteristic of the feed system of the tool etc.By adequate adjusting characteristic of the system (for instance by choice of proper resistance in the hydraulic rig system and by adjusting the relief value) optimum end-thrusts are obtained as in fig.2

The case when estimation of the rock properties is performed by the end-thrust the block - diagram is shown in figure 5.When physical and mechanical properties of the rock change there sets a new end-thrust F_y measured and converted into an electrical signal U_r by a pick - up device.The voltage U_r which is the index of physical and mechanical properties of the rock,is then supplied to the functional generator,which in its turn forms according to the programme the reference voltage U_3 of the automatic rotational speed of the tool control system.

Thus,with each new rock encountered an optimum ratio of operating parameters will be attained correspondingly insofar as the end-thrust in various rocks will be optimum by virtue of adequate adjusting characteristic of the feed system of the tool,whereas the optimum rotational speed of the tool is set by means of the control system sensitive to physical and mechanical properties of the rocks under boring.In figure 6 are supplied the oscillograms,illustrating the operation of the automatic control system by power drilling in industry. At the commencement of boring as the surface layer of the rock is passed through the resistance of the rock increases while the end-thrust magnitude increases gradually from 3 - 4 up to I3 - I4 tons.The rotational speed of the boring tool decreases accordingly from I40 - I45 r/p.m. to 65 - 70 r/p.m. (see figure 6a.). As the bore-hole becomes deeper the physical and mechanical properties of the rocks encountered are changing. The physical and mechanical properties of the rocks encountered are different at different stages of boring.Figure 6b illustrates the drilling process with the tool moving from the harder into the softer rock.In this case the end-thrust decreases from I3 - I4 tons to 7 - 8 tons,while the rotational speed of the tool raises from 60 - 70 r/p.m. up to I20 r/p.m. The drilling process with random variations in physical and mechanical properties of the rocks is shown in figure 6c.

As it is seen,every time when the properties of rock change the adjustment of operation parameters occurs in ways providing the optimum drilling process.As mentioned above,the correspondence of operation parameters to the optimum ones is defined by the trustworthness of functional dependence $n = \varphi_1(f), F_y = \varphi_2(f)$ (fig.2) and the accuracy of their realization in the system.Application of the automatic control system saves 5 - 8 thousand roubles per one rig.The drilling process control dynamic model designed for small values increm ments and corresponding to the block diagram of figure[5] is shown in figure 7,where:

ΔX - increment of the hydraulic feed system throttle plunger position;

ΔP_H - increment of the hydraulic feed system bottom hole pressure;

ΔQ_{gp} - increment of liquid supply through the hydraulic feed system throttle;

ΔF - increment of the hydraulic cylinder thrust;

$\Delta F_c(\Delta v)$ - increment of drillability of the rock owing to the change in the rate of drilling by the magnitude equal to Δv ;

$\Delta F_c(\Delta n)$ - increment of the rock drillability owing to the change in the rotation speed of the tool by the magnitude equal equal to Δn ;

ΔF_c - generalized rock drillability increment;

A - square of the piston;

V - volume of the liquid compressed;

β - modulus of compression of the liquid;

m - mass of the motion parts;

$K_1, K_2, K_3, K_{yT}, K_{Tp}$ - coefficients characterizing the hydraulic feed system;

$\hat{K}_v = \frac{\partial F_c}{\partial v}, \hat{K}_n = \frac{\partial F_c}{\partial n}$ - random coefficients,characterizing the change in the rock drillability depending on the adequate change in the drilling rate v and the rota - tion speed n of the tool.

K_F - end-thrust-sensing device transfer coefficient;

$\frac{K_\varphi}{1+pT_\varphi}$ - transfer function of the filter;

$K_{\varphi y}$ - transfer coefficient of the functional generator;

$W_{cc}(P)$ - transfer function of the automatic tool rotation speed control system;

Supposing that $\Delta X=0$, $\Delta P_H=0$,which is the same as the free system state.Let us analyse its dynamic properties by means of method[6].The transfer function of the open - loop self-adjusting automatic control system (figure 8) is :

$$W(P)=\frac{1}{\hat{K}_r}\left\{K_{rp}+pm+\frac{A^2}{K_3+K_{yr}+p\frac{V}{\beta}}\left[1+\frac{\hat{K}_n K_\varphi K_F K_{\varphi y}\,\Phi_{cc}(p)}{(1+pT_c)(1+pT_\varphi)}\right]\right\}$$

where $\Phi_{cc}(p)=\dfrac{1}{K_{cK}+\dfrac{1}{W_{cc}(p)}}=\dfrac{1}{K_{cK}+Y_{cc}(p)}$ - conversion transfer function of the closed loop servo system;

$\hat{K}_v, \hat{K}_n$ - random parameters within a wide range of dispersion (40 to 160 - fold[5]).

The characteristic equation of closed loop control system is:

$$\begin{aligned}F(\gamma)=&(K_{rp}+m\gamma)(K_3+K_{yr}+\tfrac{V}{\beta}\gamma)(1+T_c\gamma)(1+T_\varphi\gamma)[K_{cK}+Y_{cc}(\gamma)]+\\&+A^2(1+T_c\gamma)(1+T_\varphi\gamma)[K_{cK}+Y_{cc}(\gamma)]+A^2\hat{K}_n K_F K_{\varphi y}K_\varphi+\\&+\hat{K}_r(K_3+K_{yr}+\tfrac{V}{\beta}\gamma)(1+T_c\gamma)(1+T_\varphi\gamma)[K_{cK}+Y_{cc}(\gamma)]=0,\end{aligned} \quad (1)$$

where γ -are the roots of the equation.

Let us show the D-decomposition in the system of axes of the parameters $\hat{K}_n$ and $\hat{K}_v$.The condition required for the Michailov locus to pass through the null point of the coordinates is expressed as follows:

$$F(j\omega)=a(\omega)+jb(\omega)+\hat{K}_n c+[d(\omega)+je(\omega)]\hat{K}_v=0, \quad (2)$$

where

$$a(\omega)+jb(\omega)=[A^2+(K_{rp}+j\omega m)(K_3+K_{yr}+j\omega\tfrac{V}{\beta})\times$$
$$\times(1+j\omega T_c)(1+j\omega T_\varphi)[K_{cK}+Y_{cc}(j\omega)];$$
$$d(\omega)+je(\omega)=(K_3+K_{yr}+j\omega\tfrac{V}{\beta})(1+j\omega T_c)(1+j\omega T_\varphi)[K_{cK}+Y_{cc}(j\omega)];$$
$$c=A^2K_\varphi K_F K_{\varphi y}$$

The equation (2) can be divided into two ones:

$$a(\omega)+\hat{K}_n c+d(\omega)\hat{K}_n=0,\quad b(\omega)+\hat{K}_v e(\omega)=0.$$

Considering them to be simultaneous equations,let us solve it to define $\hat{K}_v$ and $\hat{K}_n$.Then we get:

$$\hat{K}_v=-\frac{b(\omega)}{e(\omega)},\quad \hat{K}_n=\frac{1}{c}\left[\frac{d(\omega)b(\omega)}{e(\omega)}-a(\omega)\right].$$

Let us now build the D-decomposition curves in accordance with the real coefficients values (figure9). After the density distribution[5] of these parameters in the system of axes has been built, the probable stability of the automatic control system can be defined:

$$P[\hat{K}_n, \hat{K}_\nu \subset (R)] = \iint\limits_{(R)} f(\hat{K}_n, \hat{K}_\nu)\, dK_n\, dK_\nu < 1, \qquad (3)$$

where (R) - the range of the control system stability. With a given density of the probability of the drilling parameters, complete probability of the control system stability can be attained by means of the adequate correction of the system. Let us illustrate that the quality of the automatic drilling control system can be determined only with a certain probability.

The equation (I) can be presented in such a form:

$$[1+\gamma^2 T_m T_\beta \alpha + \gamma(T_m + T_\beta)\alpha](1+\gamma T_c)(1+\gamma T_\varphi) + L\hat{K}_n + N\hat{K}_\nu(1+\gamma T_\beta)(1+\gamma T_c)(1+\gamma T_\varphi) = 0; \qquad (4)$$

where

$$T_m = \frac{m}{K_{TP}};\ T_\beta = \frac{V}{\beta K_3};\ K_3' = K_3 + K_{yT};\ \alpha \frac{K_{TP} K_3'}{A^2 + K_{TP} K_3'}$$

$$K_F' = K_F K_\varphi K_{\varphi y};\ L = \frac{A^2 K_F'}{(A^2 + K_{TP} K_3') K_{CK}};\ N = \frac{K_3'}{A^2 + K_{TP} K_3'}; \qquad (5)$$

Let us consider the case when $T_m, T_\beta \ll T_c, T_\varphi$. The equation (4) can be with a certain proximity substituted by

$$\gamma^2 T_c T_\varphi + \gamma(T_c + T_\varphi) + \hat{B} + 1 = 0, \qquad (6)$$

where

$$\hat{B} = \frac{L\hat{K}_n}{1+N\hat{K}_\nu} = \frac{L\sigma_{K_n}\hat{K}_{n*}}{1+N\sigma_{K_\nu}\hat{K}_{\nu*}};\ \hat{K}_{n*} = \frac{\hat{K}_n}{\sigma_{K_n}};\ \hat{K}_{\nu*} = \frac{\hat{K}_\nu}{\sigma_{K_\nu}}. \qquad (7)$$

The equation (6) gives the aproximate values of the main roots:

$$\gamma_{1,2} \cong -\frac{1}{2}\left(\frac{1}{T_\varphi} + \frac{1}{T_c}\right) \pm \sqrt{\frac{1}{4}\left(\frac{1}{T_\varphi} + \frac{1}{T_c}\right)^2 - \frac{(1+\hat{B})}{T_c T_\varphi}}. \qquad (8)$$

The following condition for the vibration process is necessary:

$$\hat{B} > \varphi\left(\frac{T_c}{T_\varphi}\right) = \frac{1}{4}\left(\frac{T_c}{T_\varphi} + \frac{T_\varphi}{T_c} - 2\right). \qquad (9)$$

After substituting for the (7) and (5) values the equation (8) can be presented in such a form:

$$\hat{K}_{n*} > K_{n_0} = \varphi\left(\frac{T_c}{T_\varphi}\right)(q + h\,\hat{K}_{v*}), \qquad (10)$$

where dimensionless coefficients:

$$q = \left(1 + \frac{K_{TP}K_3'}{A^2}\right)\frac{K_{CK}}{K_F'\,\sigma_{Kn}} = \frac{1}{L\sigma_{Kn}};\; h = \frac{\sigma_{Kv}K_3'K_{CK}}{\sigma_{Kn}A^2K_F'} = \frac{N\sigma_{Kv}}{L\sigma_{Kn}}.$$

Let us put the dependence (10) on the random parameters $\hat{K}_{v*}$ and $\hat{K}_{n*}$ distribution plane (fig.10).Obviously,the probability of the oscillating process is determined in the following way:

$$P(\Omega>0) = P(\hat{K}_{n*} > K_{n_0}) = \int_{-\infty}^{\infty}\int_{K_{n_0}} f(\hat{K}_{n*}\hat{K}_{v*})\,d\hat{K}_{v*}\,d\hat{K}_{n*} \qquad (11)$$

To determine the probability of the oscillating process with the frequency exceeding Ω_A we can use the (8):

$$\sqrt{\frac{1+\hat{B}}{T_cT_\varphi} - \frac{1}{4}\left(\frac{1}{T_\varphi} + \frac{1}{T_c}\right)^2} > \Omega_A\;,$$

which after the transformation can be presented in the following shape:

$$\hat{B} > T_cT_\varphi\,\Omega_A^2 + \varphi\left(\frac{T_c}{T_\varphi}\right).$$

Taking the (5) and the (7) into account this equation can be presented in the following form:

$$\hat{K}_{n*} > K_{n_1} = \left[T_cT_\varphi\,\Omega_A^2 + \varphi\left(\frac{T_c}{T_\varphi}\right)\right](q + h\,K_{v*}).$$

The probability of the oscullating process with the frequency exceeding Ω_A can be like in case described by (11) determined in the following way(figure 10):

$$P(\Omega > \Omega_A) = P(\hat{K}_{n*} > K_{n_1}) = \int_{-\infty}^{\infty}\int_{K_{n_1}} f(\hat{K}_{v*},\hat{K}_{n*})\,d\hat{K}_{v*}\,d\hat{K}_{n*}. \qquad (12)$$

The damping coefficient in an oscillating process depends on time constants T_c and T_φ and does not depend on the random parameters $\hat{K}_v$ and $\hat{K}_n$

$$\alpha_0 = \frac{1}{2T_\varphi} + \frac{1}{2T_c}$$

In the non-oscillating process the main absolute value of the damping coefficient is:

$$\alpha = \frac{1}{2}\left(\frac{1}{T_\varphi} + \frac{1}{T_c}\right) - \sqrt{\frac{1}{4}\left(\frac{1}{T_\varphi} + \frac{1}{T_c}\right)^2 - \frac{(1+\hat{B})}{T_cT_\varphi}}$$

Limiting this value by the maximum $\alpha > \alpha_A$ we obtain:

$$\hat{B} > \psi(T_c, T_\varphi, \alpha_A) = (T_c + T_\varphi)\alpha_A - T_o T_\varphi \alpha_A^2 - 1. \qquad (13)$$

Substituting in (I3)the value B obtained from the equation (7) we shall have:

$$\hat{K}_{n_*} > K_{n_2} = \psi(T_c, T_\varphi, \alpha_A)(q + h\hat{K}_{v_*}).$$

The probability of the non-oscillating process with the main damping coefficient less than the given value α_A can be like in case described by (II) determined in the following way(fig.IO):

$$P(\alpha < \alpha_A) = P(\hat{K}_{n_*} > K_{n_2}) \int_{-\infty}^{\infty} \int^{K_{n_2}} f(\hat{K}_{v_*}, \hat{K}_{n_*}) d\hat{K}_{v_*} d\hat{K}_{n_*}.$$

If the characteristic equation can't be expressed in the shape of a quadratic equation,the probability of a given property is determined by the dependence (3),where (R) should be regarded as the range of the given transient performance.

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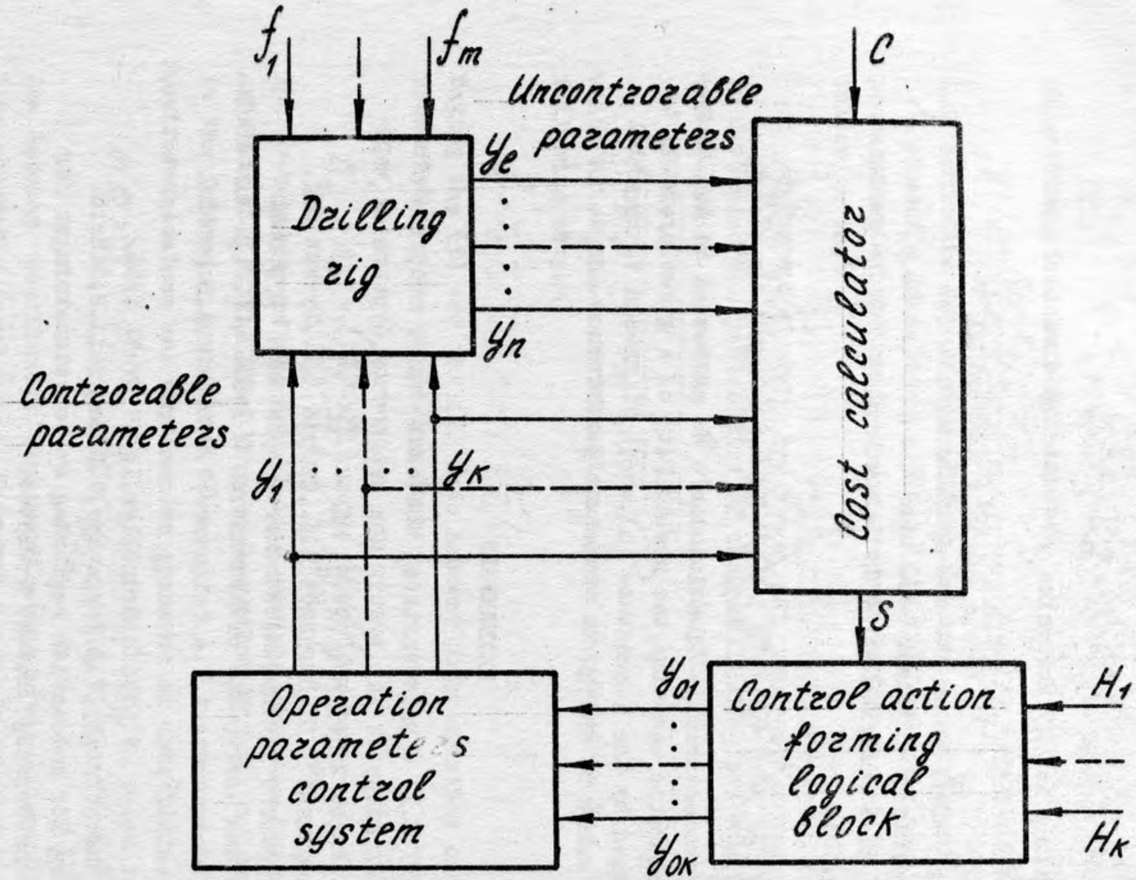

Fig.I. Block-diagram of the multichannel extremal automatic control system.

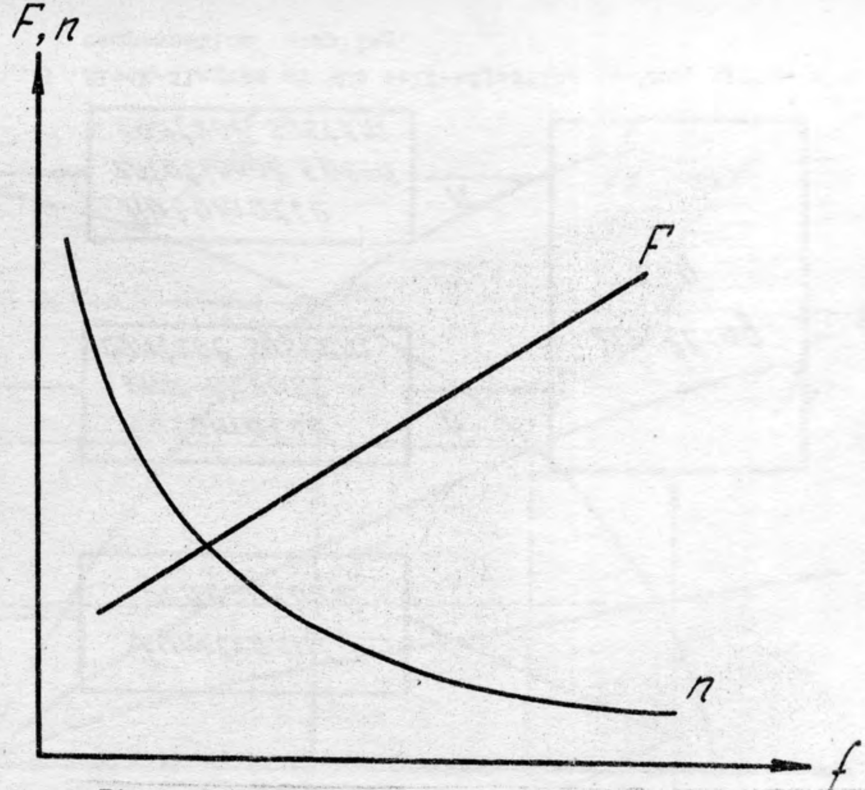

Fig.2. Coded end-thrust and rotation speed of the tool dependences on the hardness of rock.

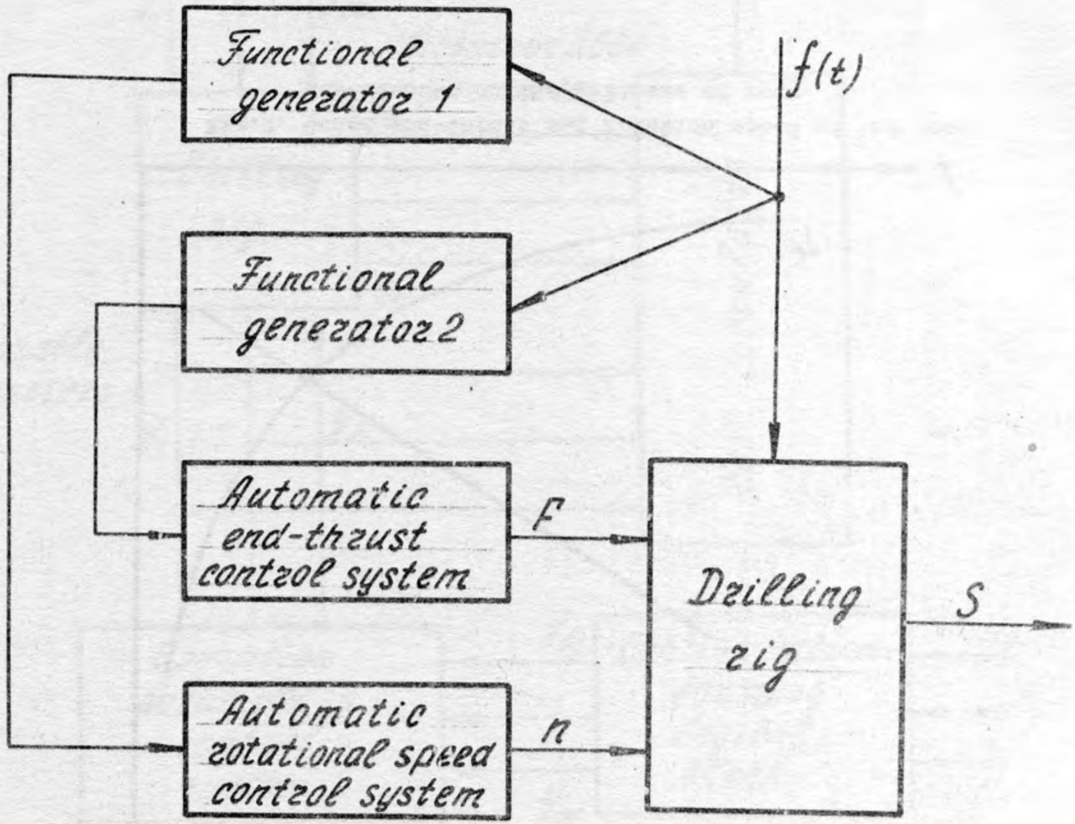

Fig.3 Block-diagram of the self-adjusting control system with compensation coupling.

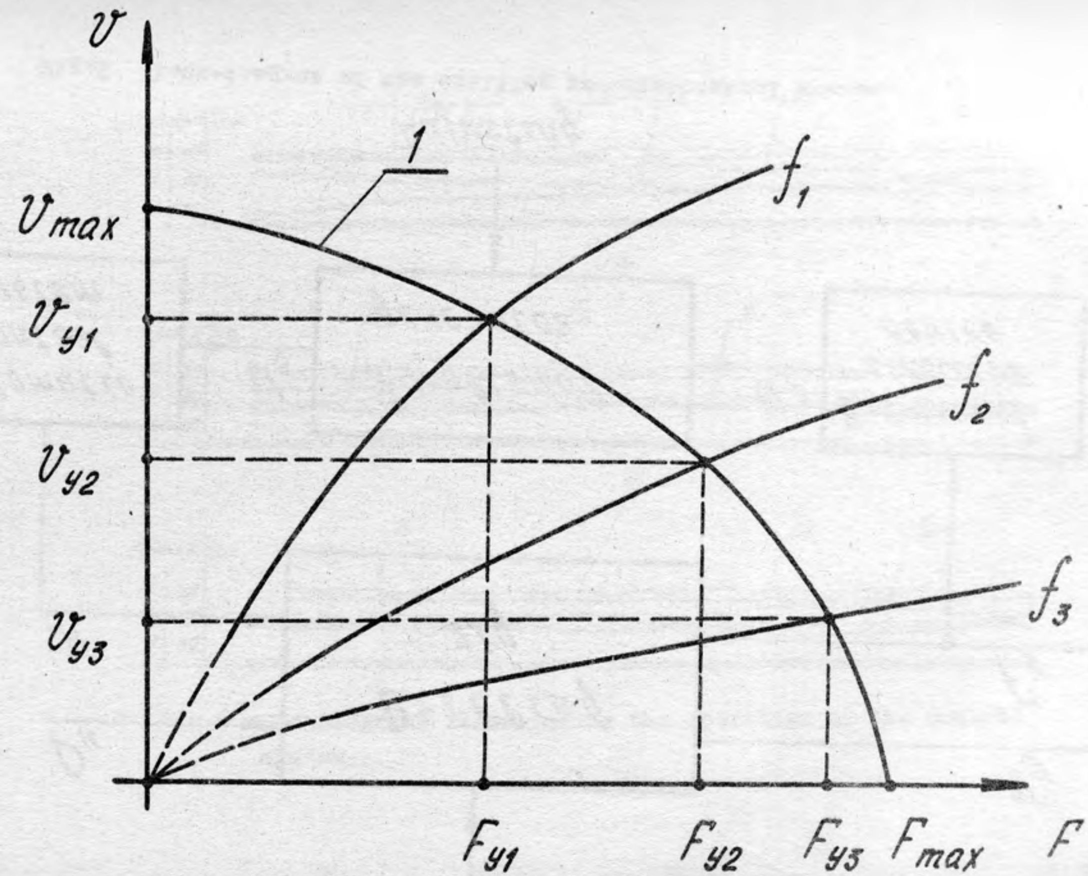

Fig.4 Generalized mechanical characteristics of rocks.

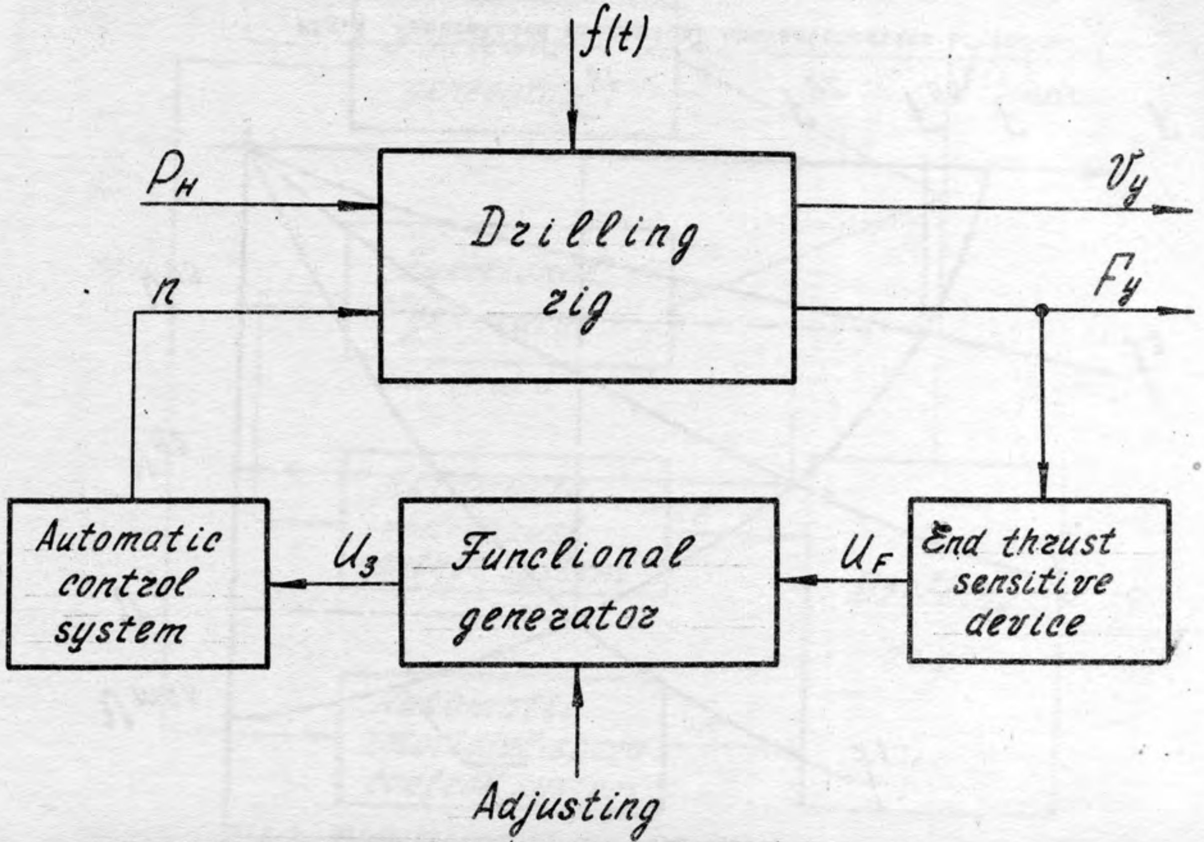

Fig.5 Block-diagram of the drilling process control system.

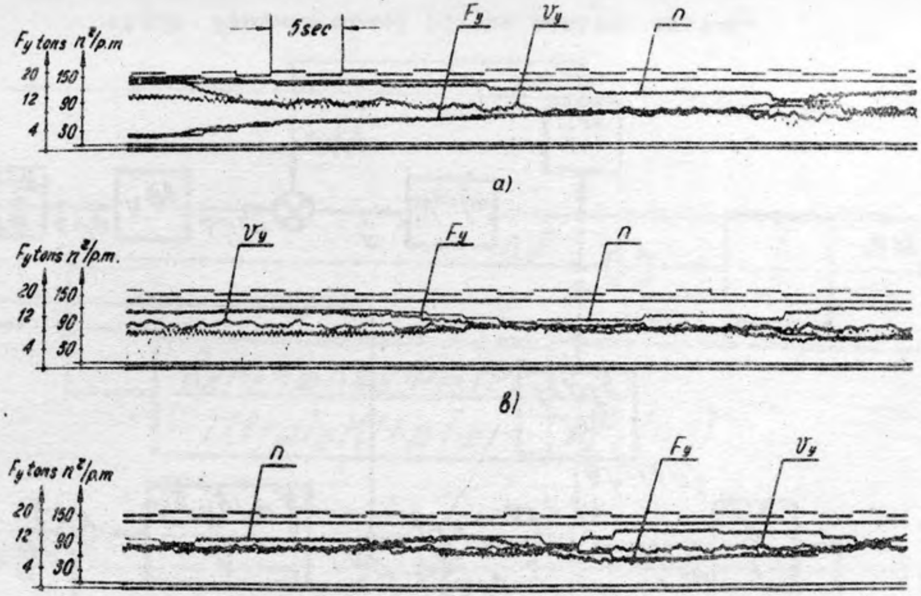

Fig.6 Oscillograms illustrating the operation of the control system.

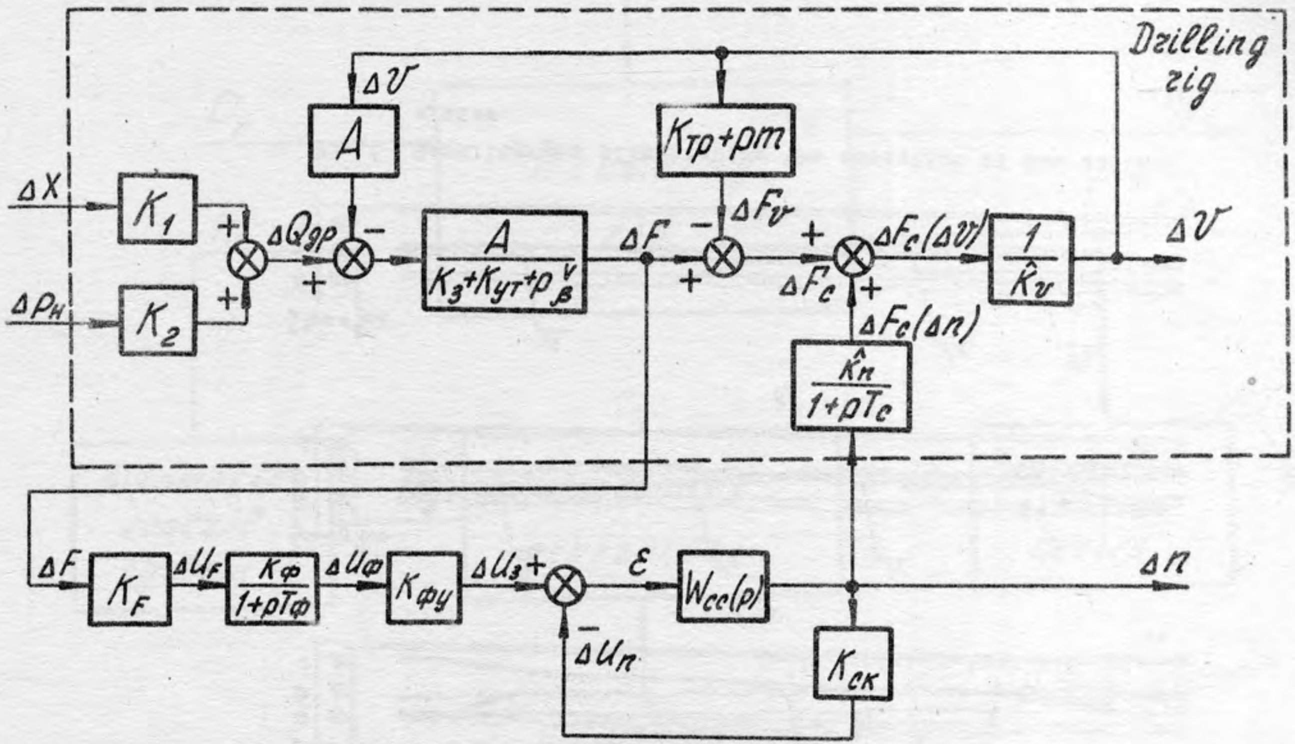

Fig.7 Dynamic model of the control system.

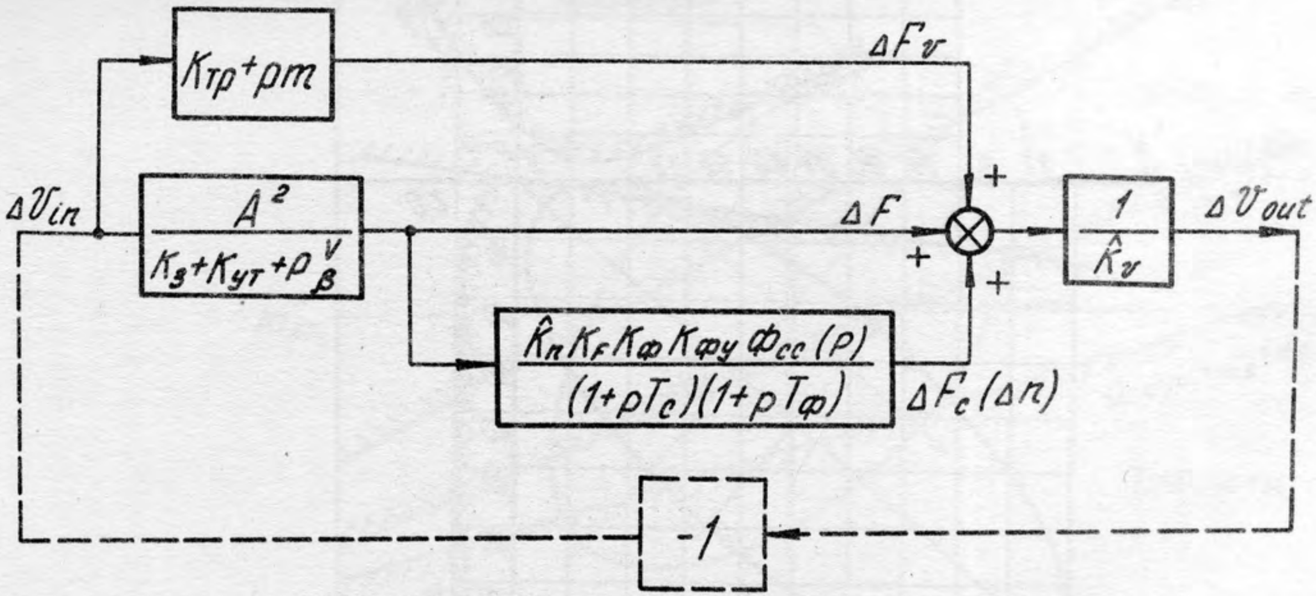

Fig.8 Dynamic model of the free system state.

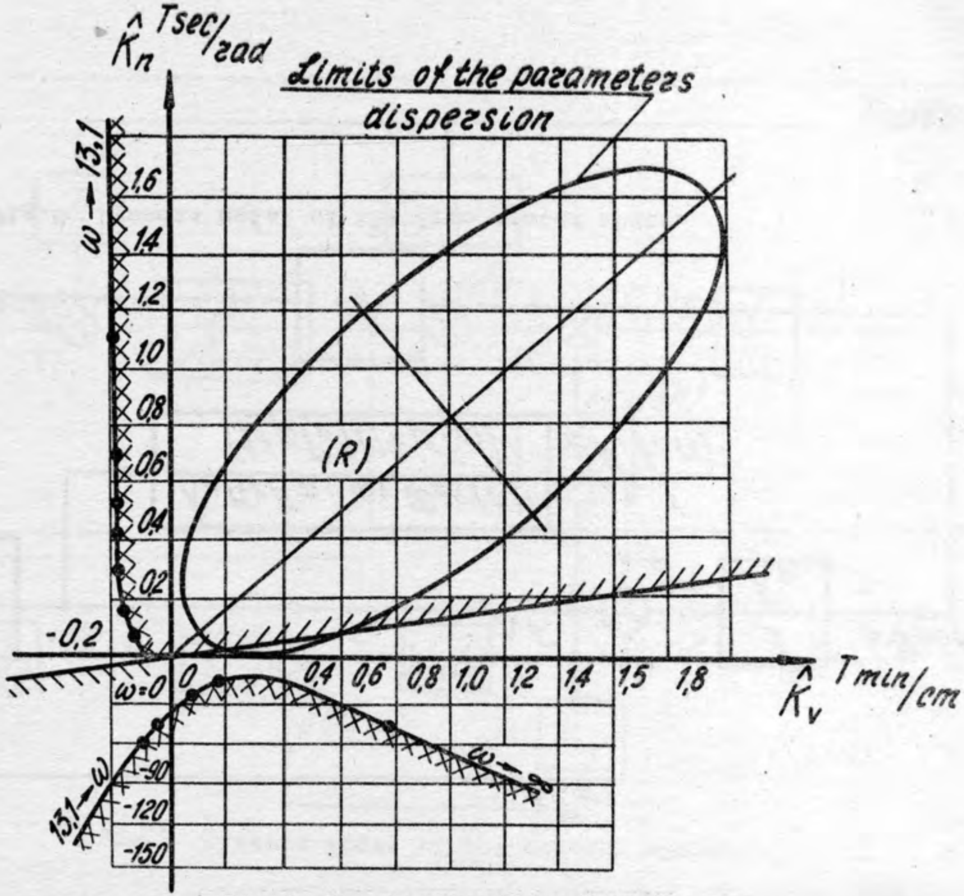

Fig.9 D - decomposition locus.

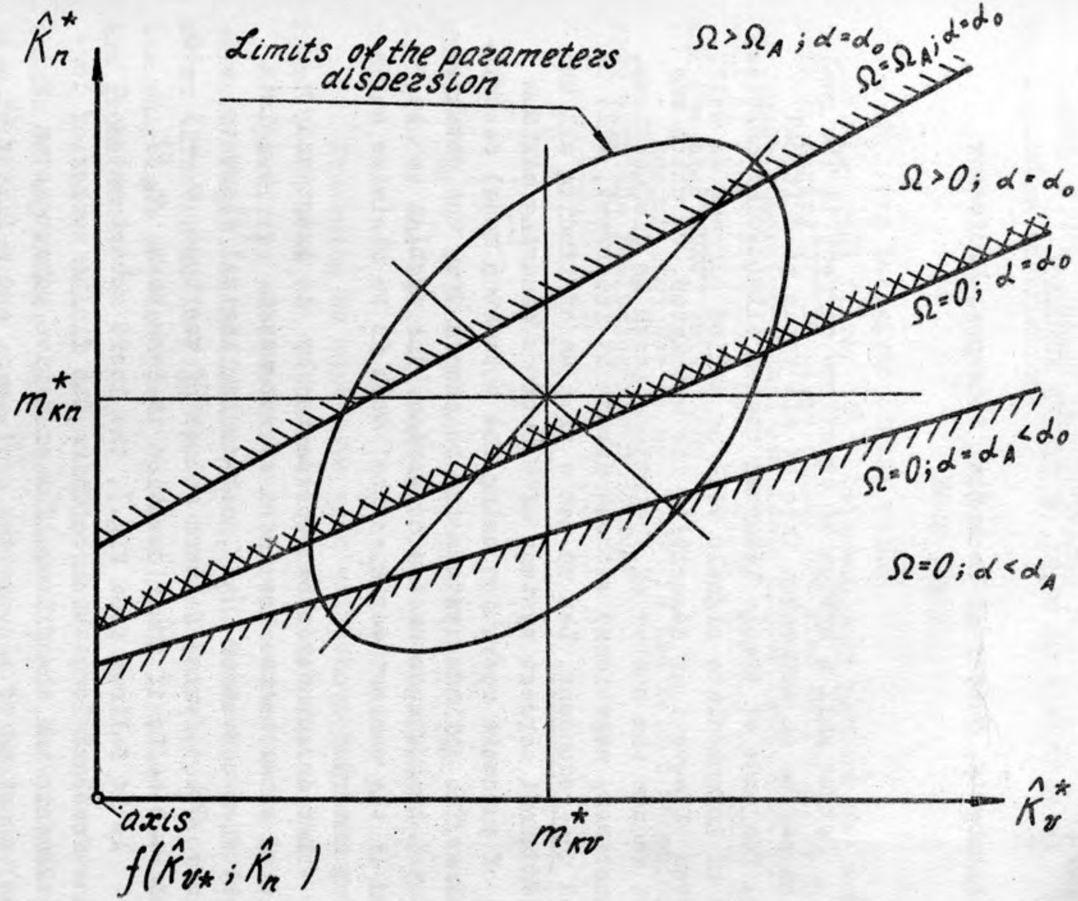

Fig.IO Statistical analysis of dynamic properties of the system.

27.3.

AN ASYNCHRONOUS MODEL OF FINITE AUTOMATA

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Let us call a system of equations correct if its realization can be an automaton free of all types of critical races. The code of stable internal states eliminating critical races of intermediate signals will be called correct as well. A system of equations describing the automaton, in which one cannot reduce the number of operations (such as disjunctions, conjunctions, negations) without making it incorrect, will be called non-redundant. Let us set a problem of finding all the non-redundant correct systems of equations ensuring maximum speed of automata operation using the automaton model described below. The optimum systems may be chosen from the obtained ones by introducing a choice criterion. The problem can be solved in the easiest way when the task is to minimize an automaton memory.

Let an automaton be represented by k inert and m primitive subautomata. The inert subautomaton q consists (Fig.1) of a converter Π_q, containing logical elements, and a filter Φ_q, which filters pulses of the type $0 \to 1 \to 0$ and $1 \to 0 \to 1$, if their duration is less than $\tau_q(t)$.

As it follows from Fig.1, the inert subautomaton q has one feedback loop which contains the finite number of logical elements and the filter. The primitive subautomaton v (Fig.2) consists of a converter Π_{k+v} and a filter Φ_{k+v}, but unlike the inert one it has no feedback loop.

The inert subautomata $1, \ldots, k$, where k is the number of feedback loops in the automaton (the number of "memory elements") generate the intermediate signals $X_1, \ldots, X_k$, and the primitive subautomata $k+1, \ldots, k+m$ produce output signals $Z_1, \ldots, Z_m$, where m is the number of these signals. $A_1, \ldots, A_n$ are input signals, t is the continuous time.

Let us assume that the subautomata converters are based on logical elements each (the j-th) having its internal delay $\tau_j(t)$ which is the random function of time with the following restrictions:

$$\tau_з \geqslant \tau_j(t) \geqslant 0, \qquad ((1))$$

where $\tau_з$ is the given finite value.

Any real filter Φ_i does not only perform the necessary filtering of the given signal but also shifts it by $\tau_i(t)$. Decompose, therefore, each available filter (see Fig.1 and 2) into an ideal filter ($И\Phi_i$), which performs filtering only, and the time delay unit $\tau_i(t)$.

Assume that the delay $\tau_i(t)$ is the random function of time with the following restrictions:

$$\tau_{\varphi} > \tau_j(t) > \tau_{пп}, \qquad ((2))$$

where τ_{φ} is the given finite value and $\tau_{пп}$ is the duration of the longest transient process occuring in the automaton.

According to what have been said above and basing on Figs. 1 and 2 an automaton model consisting of k inert and m primitive subautomata may be represented as it is shown in Fig.3. As it follows from Figs. 1 and 2, the automaton illustrated in Fig.3 has k feedback loops because the functions of the type $Y_a(t) = f(Y_a(t),...)$ called circles, are prohibited.

Assume that it is possible to measure simultaneously values of any number of intermediate signals during transition from one stable state into another, occuring in the model of automata considered.

The model shown in Fig.4 can be used as well as that shown in Fig.3. In this case (Figs. 1 and 2) signals $Y_1(t)$, ... , ... , $Y_k(t)$ are not applied to the subautomata inputs. However, the number of non-redundant correct systems of equations describing the given transformation can be reduced considerably if the model of Fig.4 is used instead of the one shown in Fig.3. There are precedents [1] when the number of systems decreases as much as hundreds of times.

It is essentially to remember that any optimum criterion of an automaton design must include expenditures for its synthesis process. The availability or absence of the programs and computers required, and what is above all, the time necessary for designing, effect the volume of the stated expenditure greatly. It is expedient, therefore, to have several ways requiring the different time for the automaton synthesis. Some of them are shown in Fig.5. They differ by the time necessary for the synthesis and, naturally, by the probability of obtaining the optimum results.

It is convenient to represent the conditions of operation of an automaton in the form of the finite automata graphs described in the paper [2]. The finite automaton graph is an oriented one the vertices of which represent the stable total states and its arcs show transitions between these states. In this paper, therefore, by the notion "vertex" we shall mean a stable total state. To obtain the systems of equations describing the automaton it is expedient to represent a graph in a form of a finite automaton matrix, as it is done in the paper [3]. The relationship between the transition table and the finite automaton matrix is simple. The number written in the left-hand side of a cell in the row β of the table gives the number of the matrix column the row β of which has the entry "one". The column ρ defines the input states and the column λ defines the output states at the vertices.

The finite automaton matrix is convenient because its column q defines the values of the intermediate signal X_q at all the vertices. Each matrix row gives the correct code of the corresponding stable state. For example, the row 1 of the matrix, shown in Table 1, has the entry 1000111, which codes the vertex 1. The code given by the matrix is always redundant and its length may be reduced if it is needed.

The elimination of critical races in the finite automaton is carried out according to the rule of inertness: an intermediate signal X_q is equal to "one" at the vertex q, at the preceding vertices and in all unstable total states occuring during direct transitions to the vertex q . At the rest vertices and in the unstable total states the signal X_q is equal to "zero".

Table 1

	00	01	10	11
1	1-00	6-11	7-00	8-01
2	2-01	5-00	7-00	8-01
3	3-10	6-11	7-00	8-01
4	4-11	6-11	7-00	8-01
5	1-00	5-00	7-00	8-01
6	2-01	6-11	7-00	8-01
7	3-10	5-00	7-00	8-01
8	4-11	5-00	7-00	8-01

Transition table

	1	2	3	4	5	6	7	8	ρ	λ
1	1	0	0	0	0	1	1	1	00	00
2	0	1	0	0	1	0	1	1	00	01
3	0	0	1	0	0	1	1	1	00	10
4	0	0	0	1	0	1	1	1	00	11
5	1	0	0	0	1	0	1	1	01	00
6	0	1	0	0	0	1	1	1	01	11
7	0	0	1	0	1	0	1	1	10	00
8	0	0	0	1	1	0	1	1	11	01

Finite automaton matrix

The correct code does not yet define the correct system. Therefore, taking into account the fact, that defining the correct code is not an end in itself, but only an intermediate stage of the automaton synthesis process, it is expedient to form the synthesis process so, that it can be possible to obtain the correct systems of equations avoiding the coding stage. The algorithms obtained in [1,2,4-6] allows to find the non-redundant correct systems of equations without performing a coding stage.

Obtaining a system of equations directly from a finite automaton matrix is carried out on the basis of the paper [2] according to the following functions (for simplification of the expression we neglect the time):

$$X_q = \bigvee_{d \in M_q} R_d X_d , \qquad ((3))$$

$$Z_v = \bigvee_{\beta \in N_v} R_\beta X_\beta , \qquad ((4))$$

where R_d is an elementary conjuction of the input signals $A_1, \dots , A_n$, equal to "one" only with the input state ρ_d ; R_β is the same only with the input state ρ_β ; X_q is an intermediate signal defined by the column q of the matrix; X_β - is the same defined by the column β ; M_q is a set of matrix rows (i.e. stable total states) in which the column q contains "ones". N_v is a set of matrix rows in which the output signal $Z_v = 1$. If the column e of the matrix contains no "zeroes", then $X_e = 1$; $e = d$ or β .

Let us call the equations of the type ((3)) or ((4)) a special disjunctive normal form of the function. For example, basing on ((3)) and ((4)) we have from the matrix of Table 1 the following:

$$
\begin{aligned}
X_1 &= \bar{A}_1\bar{A}_2X_1 + \bar{A}_1A_2X_5 \,, \\
X_2 &= \bar{A}_1\bar{A}_2X_2 + \bar{A}_1A_2X_6 \,, \\
X_3 &= \bar{A}_1\bar{A}_2X_3 + A_1\bar{A}_2 \,, \\
X_4 &= \bar{A}_1\bar{A}_2X_4 + A_1A_2 \,, \\
X_5 &= \bar{A}_1\bar{A}_2X_4 + \bar{A}_1A_2X_5 + A_1\bar{A}_2 + A_1A_2 \,, \\
X_6 &= \bar{A}_1\bar{A}_2X_1 + \bar{A}_1\bar{A}_2X_3 + \bar{A}_1\bar{A}_2X_4 + \bar{A}_1A_2X_6 \,, \\
Z_1 &= \bar{A}_1\bar{A}_2X_3 + \bar{A}_1\bar{A}_2X_4 + \bar{A}_1A_2X_6 \,, \\
Z_2 &= \bar{A}_1\bar{A}_2X_2 + \bar{A}_1\bar{A}_2X_4 + \bar{A}_1A_2X_6 + A_1A_2 .
\end{aligned}
\qquad ((5))
$$

It must be noted that obtaining a non-redundant correct system of equations by the given method allows us at once basing on the finite automaton matrix, to obtain the relatively simple results which when using the systems of perfect disjunctive normal forms of equations are obtained as a result of the rather laborous process of minimizing these functions.

The limiting complexity of the system obtained directly from the matrix is determined according to the paper [7] by the expressions:

$$D = c+m, \quad K=p, \quad O=n, \quad \Phi=c+m,$$

$$B_1 = n+2m+p+2c, \qquad ((6))$$

where D , K , O , Φ are the number of disjunctions, conjunctions, negations and filters, respectively; B_1 is the total number of operations (the number of elements each having any number of inputs and indefinite loading capability); c is the number of matrix columns (without columns ρ and λ) each having at least one "zero"; p is the total number of matrix rows.

The process of merging the columns of the submatrix i is redused according to the work [5] to their arbitrary coding by α_i signals [1]:

$$\alpha_i = \log_2 \rfloor d_i \lfloor , \qquad ((7))$$

where d_i is the number of the columns in the matrix i ; $\rfloor d_i \lfloor$ is the approximation of d_i to the nearest larger number of the row 1,2,4,8, ... After the column merging the system of equations describing the automaton is defined according to ((3)) and ((4)).

For example, for the matrix, shown in Table 1, coding is defined by Table 2. Formed by the columns a , b and c the column rows of this table may be filled in by any different numbers.

Table 2

	a	b
1	0	0
2	0	1
3	1	0
4	1	1

	c
5	0
6	1

From Table 2 it follows that the column a must be formed by disjunctions of every row of the matrix columns 3 and 4 (Table 1); the column b must be formed by disjunctions of every row of the matrix columns 2 and 4; the column c is the column 6. As a result of these transformations we obtain Table 3 from Table 1.

---

1) The submatrix is called a set of matrix columns representing the vertices with equal ρ .

Table 3

	a	b	c	ρ	λ
1	0	0	1	00	00
2	0	1	0	00	01
3	1	0	1	00	10
4	1	1	1	00	11
5	0	0	0	01	00
6	0	1	1	01	11
7	1	0	0	10	00
8	1	1	0	11	01

According to ((3)) and ((4)) using Tables 1 and 3 we write down a system of equations obtained as a result of the column merging process:

$$
\begin{aligned}
X_a &= \bar{A}_1\bar{A}_2 X_a \bar{X}_b + \bar{A}_1\bar{A}_2 X_a X_b + A_1\bar{A}_2 + A_1 A_2 ,\\
X_b &= \bar{A}_1\bar{A}_2\bar{X}_a X_b + A_1 A_2 X_a X_b + \bar{A}_1 A_2 X_c + A_1 A_2 ,\\
X_c &= \bar{A}_1\bar{A}_2\bar{X}_a\bar{X}_b + \bar{A}_1\bar{A}_2 X_a \bar{X}_b + A_1 A_2 X_a X_b + \bar{A}_1 A_2 X_c ,\\
Z_1 &= \bar{A}_1\bar{A}_2 X_a \bar{X}_b + \bar{A}_1\bar{A}_2 X_a X_b + \bar{A}_1 A_2 X_c ,\\
Z_2 &= \bar{A}_1\bar{A}_2\bar{X}_a X_b + \bar{A}_1\bar{A}_2 X_a X_b + \bar{A}_1 A_2 X_c + A_1 A_2 .
\end{aligned}
\qquad ((8))
$$

The maximum complexity of a system of equations obtained in such a way is defined according to the paper [5] by the following equalities:

$$
\begin{aligned}
&D=\sum_i \log_2]d_i[+m , \quad K=p,\\
&O=n+\sum_i \log_2]d_i[, \quad \Phi=\Sigma_i \log_2]d_i[+m,\\
&B_2=n+2m+p+3\sum_i \log_2]d_i[
\end{aligned}
\qquad ((9))
$$

The automaton in which the following unequality holds:

$$S > \frac{\prod_i Jd_i L}{2}, \tag{(10)}$$

where S is the number of stable internal states will be called limited. As it is shown in the paper [5], in the limited automaton we have after the columns having been merged a system of equations ensuring the formation of the minimum memory. After that we cannot further decrease the automaton memory by any other methods.

For the automata which are not limited memory minimization is carried out according to the paper [4] during a "special minimization" stage. This stage is rather laborous. Therefore, it is expedient to use this method only if as a result of it an automaton gets essentially simplified. After special minimization the complexity of the automaton can be evaluated by the expression:

$$B_3 = n + 2m + p + 3\log_2 \frac{JSL - 1 + \log_2 JSL}{2} \tag{(11)}$$

If $B_3 \ll B_2$, then the performance of this stage is reasonable.

The essence of the "special minimization" stage is that it is necessary to ensure differentiation of the vertices having the equal input states. According to this one can find using the finite automaton matrices the simplest elementary conjunctions which, taking into account restrictions being imposed by the races between the filters, ensure the stated differentiation. As a result, as it is shown in the paper [8], all the non-redundant correct codes of the stable internal states are obtained [1)]. It is possible to select from these codes those having the minimum length.

---

1) A code is called non-redundant if none of its digits can be omitted.

Obtaining the simplest elementary conjunctions, representing the vertices, gives one the possibility (according to the paper [1]) of obtaining all the non-redundant correct systems of equations describing the automaton. Then we shall always have the following:

$$G_4 \subseteq G_3 , \qquad ((12))$$

where G_4 is a set of systems obtained by using the simplified model, shown in Fig.4 , G_3 is the same obtained by using the general model in Fig.3.

As it is shown in the paper [1], the synthesis process can be greatly speeded up by eliminating a part of variants of the non-redundant correct systems.

We may now summarize the results which refer not only to the model considered (Fig.3), but also to the "classical" model of the finite automaton (the model in Fig.4 in which all the logical elements are non-inert) which is being intensively investigated by different authors:

1. The application to the converter inputs the signals from the delay inputs as well as from the delay outputs may lead to a considerable increase of sets of correct systems of equations.

2. For the limited finite automaton any memory minimization except the column merging operation is senseless.

3. The logical transformation of the correct systems of equations does not necessarily lead to the correct system of equations.

4. Not all the correct systems of equations are obtained as a result of minimizing the system of perfect normal forms of functions.

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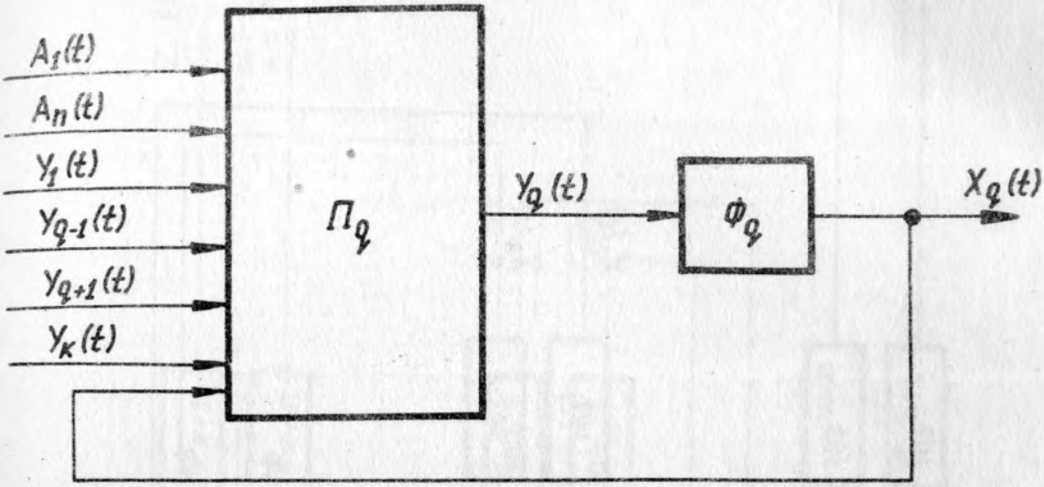

Fig.1 Inert subautomaton

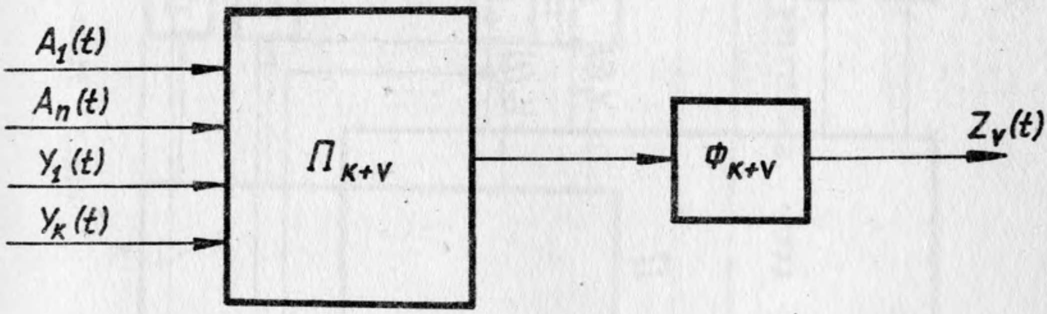

Fig.2 Primitive subautomaton

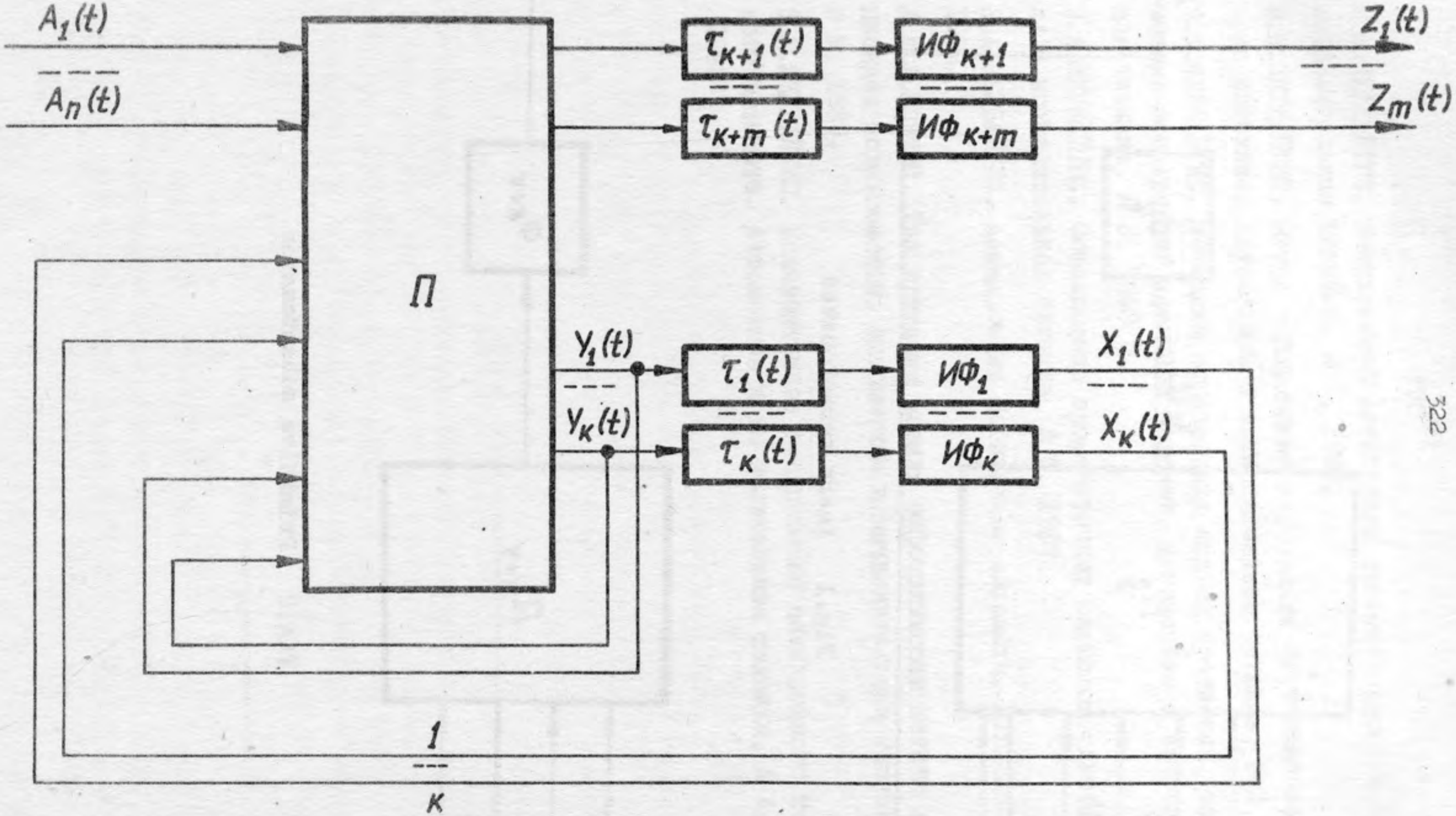

Fig.3 Finite automaton model

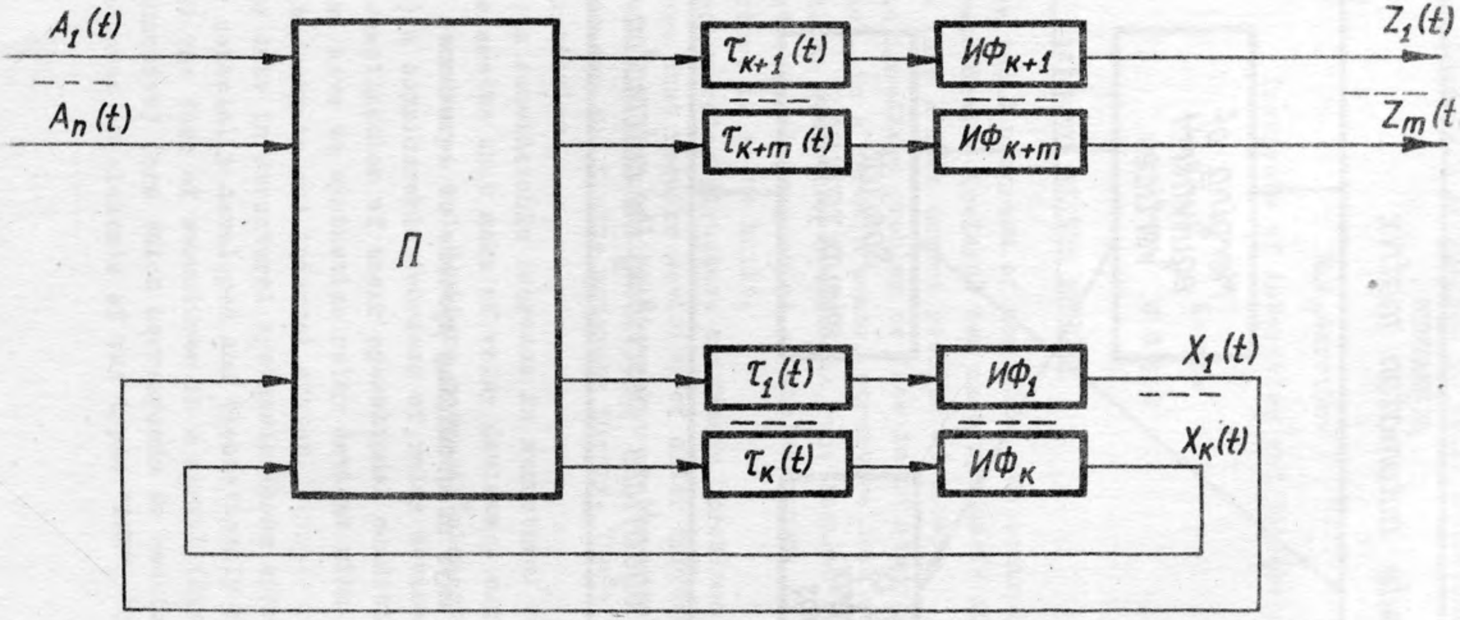

Fig.4 Simplified finite automaton model

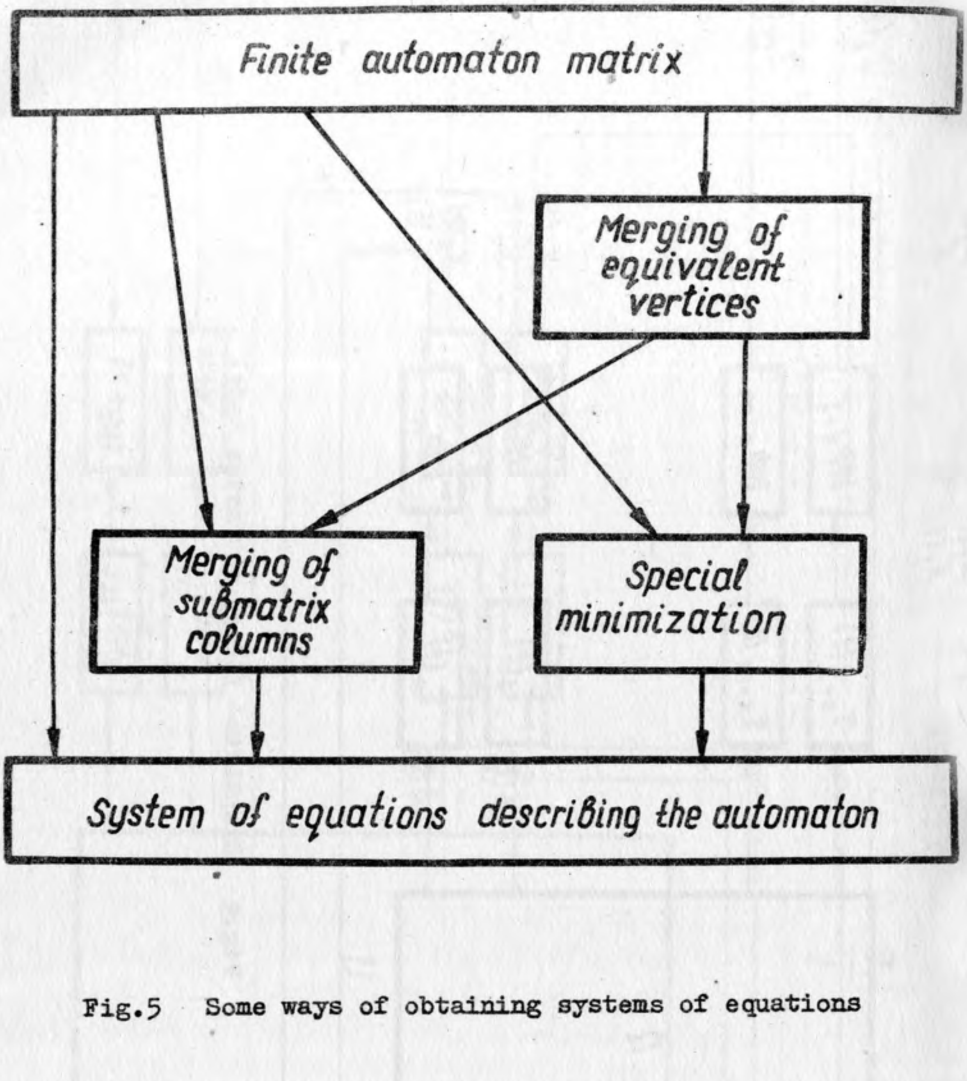

Fig.5 Some ways of obtaining systems of equations

27.5

HEURISTIC APPROACHES TO RELAY STRUCTURES SYNTHESIS

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U S S R

1. Statement of the problem

The rapid progress of computing and control technology of discrete devies (relays) has made synthesis of their structures into a most urgent problem of today.

The immediate problem we face is construction of an optimal, in a certain sense, structure that would implement the given operating conditions, the desired reliability and structural properties of the elements, of which the structure is to be built.

Modern relay structure synthesis problems have a certain feature that require revision of older approaches to synthesis and a substantial improvement of the existing relay structures and finite automata theory. These features are, in particular:

a) a considerable increase in structural complexity of the elements that make of relay devices: e.g. "micromodular" structures and "uniform networks" elements".

b) a considerable increase of relay devices capacities and complication of their operational conditions. Even now we have to synthetize relay devices with hundreds of inputs/outputs and internal states.

To date in structural synthesis those approaches have been especially developed and theoretically based that yield the form of structures in a normal (disjunctive or conjunctive) form which corresponds to realization of structures on elements of the types "AND", "OR", "NOT".

Those approaches are based chiefly on finding, from the tables of states which set the conditions for operation of relay devices, the so-called "minimal terms", i.e. those "products of letters" (conjunction of variables) where no letter can be eliminated otherwise the table becomes inconsistent: a product is obtained which is contained in a state with one at the output and in a state with zero at the output. By a search of all possible subsets in the set of minimal terms, those are selected that ensure the so-called "irredundant realizations"*); also by search all those realizations are selected that satisfy the given minimization functional, e.g. those that have the minimal number of elements (let us use the term "absolute minimal" realizations).

The search required increases very quickly. E.g., the maximal number of minterms is 32 for five variables, 4300 for ten variables, 759,488 for fifteen, etc.[1] The number of operations required to find the minimal irredundant realizations increases still quicker and to find the optimal structural realizations for more than twelve or fifteen variables is practically impossible even for digital computers. Realization of structures in factored form and with many outputs as so-called "connected" structures**) is still more cumbersome. Attempts to develop methods where incomplete

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*) Irredundancy here signifies that without obtaining an inconsistent realization one cannot a) feed 0 or 1 at the input of any element which has not received a constant before and b) no part of the structure can be eliminated by sending the signal from the output element of another part of the structure to the output of the entire structure or to the input of an element in another part.

**) Connected structures or structures of "the connected tree" type are those that have connections between structures of separate outputs.

search would suffice have not made the synthesis less cumbersome to any substantial degree,

A possible technique to eliminate these difficulties is the so-called oriented search for optimal realization of structures whereby at each stage of synthesis by appropriate estimates a version is selected from among all possible versions that realizes a near-optimal structure. Only one path is selected in the "decision tree" which makes the computations less cumbersome. The more accurate are criteria for complexity of structure realization the closer the realizations is to the optimum.

In a general form the problem of a oriented search for optimal realization of structures can be formulated as follows.

Let operational conditions of a relay device with n inputs and k outputs be given as a set of state tables, generally by partial functions, where the functions realized at each output $F_i\ (1 \le i \le k)$ are given by sets of states $M_1^i = \{\alpha_1^i, \alpha_2^i, \dots, \alpha_m^i\}$ on which the functions equal to 1 and $M_0^i = \{\beta_1^i, \beta_2^i, \dots, \beta_p^i\}$ on which they are equal to 0 (the former will be called "truth" and the latter "false" states).

Let a set of elements $\Phi = \{f_1, f_2, \dots, f_t\}$ be also given. Structural properties of each of the elements f_j, $(1 \le j \le t)$ are characterized by a completely determined state table of states with a subset of truth states $\Pi_1^j = (\psi_1^j, \psi_2^j, \dots, \psi_z^j)$ and a subset of false states $\Pi_0^j = \{\xi_1^j, \xi_2^j, \dots, \xi_s^j\}$

A relay device of these elements is to be constructed. It was to be near optimal in terms of the given optimality criterion, which might be the minimal number of elements, the minimal number of operating inputs, minimal overall "weight" of the structure if each element is estimated by a "weight" that differes from others, etc.

The Boolean function $f(y_1, y_2, y_q)$ realizable by a single element f_j with $q(f_i)$ inputs and the variables $y_1, y_2, \dots, y_q$ fed to the inputs will be termed "elementary" for a set of elements that contain this element f_i

Any Boolean function can evidently be recorded as a certain composition of elementary functions whose arguments can be either other functions (including elementary functions) or independent variables. Therefore realization of the system $F(n, k)$ with a given set of elements is performed by representing the Boolean functions which describe the operating conditions for separate outputs of the structure as compositions of element functions with respect to the given set of functions $F_i = f(f_1, f_2, \ldots, f_j, x_{jk}, \ldots, x_q)$ and selection from among all possible compositions such one that would ensure a near-optimal realization. Some of the elements which realize a relay device are output elements for the entire structure. Let us denote these as

$$f_{1k}, f_{2k}, \ldots, f_{kk}$$

Generally the algorithm of an oriented search can be assumed to consist of the following operations.

a) selection such one function (F_i) from among all such functions that are best in terms of most complete use of its parts for realization of all other output functions of the system F (n, k).

b) selection from among all possible elements at the output of the realizable functions $F_i^!$ such one that would ensure the most optimal realization of the given function a

c) selection for this element of most optimal sequence for realization of the functions $f_g \ (1 \leq g \leq q)$ at its inputs and selection of functions f_g such that would ensure a consistent, irredundant and most optimal realization of the function F_i .

After selection of the functions f_g for an output element of the function F_i realized first, the totality of these functions is considered as operational conditions for a certain multi-output relay structure and the operations are reiterated until the function F_i has been realized completely.

Then the next function F_i is realized; all outputs of the elements that comprise the structure already realized are assumed to be additional independent variables. In synthetizing other outputs of the structure these variables are considered together with the original independent variables. Thus structures of the connected tree type are obtained that in a number of cases ensure more optimal realizations.

2. Criteria for optimality of structures

As already stated, the optimality of a relay structure obtained through a oriented search depends largely on the nature and accuracy of the optimality criteria used. To obtain accurate criteria is a most complex problem in the theory of relay devices Research in this field is clearly insufficient. The papers published discuss chiefly the criteria for the numbers of elements and inputs for realization of structures in the normal form. The criteria suggested are of asymptotic nature and in a number of cases do not give the required orientation for the actual synthesis problems.

One-output structures complexity was first estimated by Shannon.[2] The asymptotic estimate obtained by him was improved by G.Povarov[3] and O.Lupanov[4]. The estimate suggested by Lupanov is of the form $\mathcal{L}(n) \sim \rho \frac{2^n}{n}$ (1) where is the number of variables and ρ is the factor which depend on the type of elements utilized and the type the structure. An asymptotic estimate was also suggested that is based on the balance between truth and false states

$$\mathcal{L}(\kappa, n) \sim \rho \frac{\log_2 C_{2^n}^{\kappa}}{\log_2 \log_2 C_{2^n}^{\kappa}} \tag{2}$$

where κ is the number of truth states.

The above estimates are valid for completely determined functions. However, actual operating conditions especially for high numbers of variables are characterizing with incompletely determined state tables. Estimates of realization complexity for relay structures was studied for this case in the Institute of Automation and Telemechanics, (Technological Cybernetics). L.Sholomov[5,6] suggested the criterion

$$\mathcal{Y} = N \log_2 N - n_1 \log_2 n_1 - n_0 \log_2 n_0 \tag{3}$$

where n_1 is the number of truth states, n_0 is the number of false states and $N = n_1 + n_0$. This criterion estimates the number of inputs in elements "AND", "OR", "NOT"

With connectivity and branches taken into consideration the estimate is of the form

$$\mathcal{L}(f) = \rho \frac{y}{\log_2 y} \tag{4}$$

The change of estimates for N and n is shown in Fig.1. A statistical verification of the estimate on a digital computer has shown that this reflects the change in complexity correctly enough also with the number of variables not going to infinity (Fig.2). P.Parkhomenko suggested that the complexity of realization of an incompletely determined state table be estimated by the number of states N. L.Sholomov has shown that this estimate is true asymptotically. However, the statistical test has proved that with the increase of complexity in structure realization decreases the accuracy of the estimate (Fig. 2).

The criterion suggested by V.Kopylenko[8] has is of a somewhat different nature. It is used to find the priorities in selection of variables fed to the inputs of elements and is of the form

$$\mathcal{S} = \max\{\mathcal{S}_1, \mathcal{S}_0\} \tag{5}$$

where

$$\mathcal{S}_1 = \frac{n_1' n_0^\circ}{n_1' + n_0^\circ} \tag{6}$$

and

$$\mathcal{S}_0 = \frac{n_1^\circ n_0'}{n_1^\circ + n_0'} \tag{7}$$

In these expressions n_1', n_1° are the numbers of truth states where the variable to be estimated takes the value one or zero respectively and n_0', n_0° is the number of false states where that variable also takes the values one or zero respectively. The values $\mathcal{S}_1$ or $\mathcal{S}_0$ describe the proximity of the function f_g to its realization by one letter (the criteria $\mathcal{S}_1$ and $\mathcal{S}_0$ are easily seen to have the highest possible value in this case). With constraints on the number of inverse values of indèpendent variables the variable ar selected in the order of highest values of $\mathcal{S}_1$.

It will be shown below that the oriented search requires estimation of realizing the multioutput structure as a whole. L.Sholomov has proved[6] that this estimate is hard to calculate. A simpler estimate is valid for the particular case where incompletely determined states are the same for all outputs. It has the form

$$\mathcal{L}(F_1 F_k) = N\log_2 N - \sum_{i=1}^{q} \ell_i \log_2 \ell_i \qquad (8)$$

where ℓ_i is the number of similar columns of the matrix whose rows are associated with the functions F_i and the columns with the states that describe the functions; in the cells ones denote truth states and zeroes, false states. An estimate which is easy to calculate is also obtained in the case where in the given system of functions the truth and false states are disjoint . Then the estimate is of the form

$$\mathcal{Y} = (N_1 + N_0)\log_2 (N_1 + N_0) - N_1 \log_2 N_1 - N_0 \log_2 N_0 \qquad (9)$$

where N_1 is the total number of truth states and N_0 the total number of false states.

In his paper L.Sholomov has also discussed the most optimal priority of functions realizations at the outputs of a multi-output structure. This problem has also been solved only for the particular case of similar don't care states for all outputs of the structure. To find that priority a) the difference is found in the magnitude of the complexity estimate realization of the entire multi-output structure and its complexity when the matrix rows which correspond to the functions $F_1, F_2, \dots, F_K$ are eliminated in turn. It has been shown asymptotically that the function whose elimination ensures the minimal difference should be the last to realized, b) the row which corresponds to this function is eliminated from the matrix completely and the procedure is repeated until the priority of realization has been found for all outputs of the structures without exception.

The above review shows that the existing research into estimates of complexity do not cover the need completely. Therefore in development of synthesis techniques especially in their programming on digital computers one has to resort to purely intuitive criteria whose practicality has to be found experimentally when the techniques are checked statistically.

3. Methods of oriented search for optimal realizations

Development of methods of oriented search for optimal realization of structures can help meet various practical needs, namely

a) the constraints on the number of inputs to elements, on decay, on the factor of input branches, on the nature of interconnections, etc;

b) necessity to eliminate hazards in the circuits of element outputs;

c) the possibility of synthetizing structures with the desired reliability;

d) the possibility of using the methods for synthesis with any kind of elements;

e) possibility of extending the technique to the case where the operational conditions of a relay device and the structural properties of elements are given instead, of a state table, in the "interval form" which is especially important for multi-output structures with a large number of input variables.

Two methods of oriented search for an optimal realization of structures are described below; they were developed by the writer in collaboration with V.M.Kopylenko. Before describing their essence I would make a few notes on classification of elements that comprise relay devices.

Let a "characteristic number" of an element, $[\tau_f]$ imply the minimal number of inputs which, when the same signals $\gamma\ (\gamma \in \{0,1\})$ are fed to them give the unique output signal $\mathcal{E}(\mathcal{E} \in \{0,1\})$. Divide relay elements into classes first by the values of the characteristic number. We will distinguish elements with $[\tau_f] = 1$ (e.g. elements "AND", "OR", *) and with $[\tau_f] > 1$ (e.g. majority elements). Further, we will distinguish elements with "symmetrical" inputs for which the output value does not change when input variables are permuted, i.e.

$$f(f_1, f_2) = f(f_2, f_1)$$

and with "nonsymmetrical" inputs for which this value changes. We will also distinguish elements with "ordered" inputs where the value $[\tau_f]$ for any totality of inputs is similar and with "unordered" inputs for which the value $[\tau_f]$ differ for different totalities of inputs.

A. The method of realization tables

Let us take an example to illustrate this method. Let the table be given by Fig. 3a. Distinguish the so-called "compulsory" letters*) which are underlined in the table of Fig. 3a. Evidently some of the variables which do not contain the compulsory letters can be eliminate. Using the criteria of (5), (6), (7) we will eliminate those variables successively starting with the variable which has the lowest value of the criterion chosen, i.e. that one which leads to the most complex realization of the function at the appropriate input of the output element. We will determine each time new compulsory letters that have appeared after the respective column of the state table has been eliminated.

---

*) A letter is termed compulsory, if the truth state where it is included has a "neighbouring" false state, i.e. that false state which differs from the truth state only in the value of that letter. If this letter is crossed-out the realization is made inconsistent; therefore it should be included into the minterm which realizes the given state.

We will do this untill all columns of the table contain the compulsory letters. The products of the compulsory letters thus obtained will form two types of minterms:

a) minimal terms of a kernel (see Ref [8]), if the products are not included in the opposite parts of the state table and b) if they are, then the so-called "incomplete" minterms which have to be made consistent, if realization is to be consistent. Let us consider for simplicity the realization of the state table as shown in Fig.3a with elements "OR". After these operations the structure will be as shown in Fig.4.

Let us construct a so-called "realization" table (Fig.5a) where the rows are associated with the minterm of a kernel and incomplete minterm and the columns are associated with truth (at the left) and false (at the right) states. The cells of the table contain crosses in each row where it intersect with columns whose states are realized by the given minimal term. The minterms of a kernel realize the state table consistently, therefore they contain crosses only in truth states. Incomplete minterms contain crosses both in truth and false states. For elements "AND" and "OR" with $[\tau_f = 1]$ it is sufficient to have just one cross in each column. The minterms of kernels should be present in any realization of the structure. Let us tick in the table of Fig.5a the states realized by them. The remaining states are realized by the incomplete minterms redundantly since there are several crosses in each corresponding column of the realization table. Therefore from their set we have to choose such a subset that would, firstly, realize all truth states irredundant and, secondly, select from among the total functions, such one that would ensure the simplest realization.

Let us present the totality of all total functions as one function $F = \bigvee_{q=1}^{k} f_q$ where k is the number of incomplete minterms which are included into some irredundant subset of them. The number of truth states to be realized by this function will be the same for all subsets. Therefore to obtain the least complex total functions by the criterion of eq.(3) (see Fig.1) a subset of incomplete minterms has to be chosen such that has crosses in a min. number of columns that correspond to the false table of states. The algorithm of this choice contains these operations: a) minimal term is chosen which contains the lowest number of crosses in the false part of the realization table, if there are several terms of the kind, then the one with the largest number of crosses in the truth part; b) in the remaining rows in both the truth and false parts of the table crosses are elimineted in the columns which contain crosses in the row chosen; c) from among the remaining terms one that satisfies the condition a) is chosen again and these operations are iterated until only those rows remain in the table that contain only the eliminated crosses. This would be sufficient evidence that all states in the truth part of the table have been realized.

In this example the table will look as shown in Fig.5b after all these operations have been completed. The optimal irredundant subset contains the minterms of the kernel $\bar{x}_9 \bar{x}_3$ and $\bar{x}_9 x_7 \overline{x_5}$ and incomplete minterms $\bar{x}_5 \bar{x}_3$, $\bar{x}_6$ and $\bar{x}_7$. From the same realization table, the state tables for total functions f_1, f_2 and f_3 are found. The truth states of the appropriate row with non-eliminated crosses are included as truth states while all states with both eliminated and non-eliminated crosses in this row of the false part of the table are included as false states (see Figs 5c, 5d and 5e). These tables are realized by the same operations as the original state table.

This synthesis method is sufficiently simple and efficient for all elements with symmetrical inputs and $[\tau_f] = 1$*)

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*) Among those elements are "AND" "OR" "NOT", "NOR", "NAND" and, conditionally, "modulo-two sum" and "equivalency".

but is much more complex for elements with $[\tau_p] > 1$ (see Ref[9])

B. Transition tables method

In this method the oriented search is performed in its form applicable for the common case of elements with $[\tau_p] > 1$ with non-symmetrical and unordered inputs. Let the operating conditions of the synthetized relay device be given by the state table of Fig 6b with the set of truth states M_1 and the set of false states M_0 while the structural properties of the element with which this device is to be realized are given by the state table of Fig 6a with the truth states set Π_1 and false states set Π_0.

Form two functions; the function h_i where Π_1 are truth and Π_0 - false states and the function g_i where Π_0 are truth and Π_1 - false states. The set of all minterms of the function h_i will be denoted as $H_i = \{v_1, v_2, ..., v_e\}$ and called the "truth characteristic" of the element and the set of the minterms of the function g_i will be denoted as $G_i = \{w_1, w_2, ..., w_e\}$ and called the "false characteristic" of the element. It is easily shown that a consistent realization of the structure at the output of an element requires that for each input state of the device $\alpha \in M_1^i$ there be at least one minterm $v_p \in H^i$ and for each of the states $\beta \in M_0^i$ at least one minterm $w_f \in G_i$

The chief operations included in the synthesis go in this order: a) the truth and false characteristics of the element are find; b) a so-called "transistional" table is constructed with the number of columns equal to the number of the element minterms and the number of rows equal to that in the state table of the realizable functions. At the left of the transitional table there is a table of the function state table, on top a table of minterms H_i and at the bottom a table of minimal terms G_i (Fig 6c); c) priorities are set for feeding the element inputs and finding the variables fed there; d) the transitional table columns are filled, the so-called "rigid" prescriptions are found and state tables for total functions are made

Initially, an element input is chosed with the least number of letters to realize it (which gives the greatest freedom of further operations) and included in the shortest minterms. This ensures the quickest realization of states. In the table of Fig.6c this input is denoted as y_1 . The variables fed to the inputs are chosen by criteria (6) and (7). The values of these, shown at bottom of the table of Fig.6b help to choose the variable x_4 by the criterion S_0
In verifying the basic state table we note those states that are realized inconsistently by this variable (digit 1 in states 3, 9, 0, 8, 12, 14) and which should thus be realized on other minterms.

The second input will be y_4 since it realizes the minterm v_2 completely. Because in G_i this input realizes two letters in terms w_1 and w_2 , in states 0, 8, 12, 14 the values of the variables fed to that input have to be rigidly prescribed: in other words a combination of those values should be valid completely for these states. That variable is $\bar{x}_4$; however when this is used, the term v_2 will not realize any of the states. Therefore the variable at the input y_4 is replaced by the function f_4
States 0, 8, 12, 14 that have rigid prescriptions must be the truth states while it is desirable to have states 2, 4, 6, 10 that can be realized by the term v_2 as false. Realization of the function f_4 is shown in the table of Fig.6d. After the structure of the appropriate element has been obtained, the states realized by the term v_2 (circled fours) and the newly appearing rigid prescriptions (digits 4 in states 5, 7, 11) are noted in the table of Fig.6c.

The realization process is over when all inputs of the element are have been filled and all states of M_1^i and M_0^i realized.

The above algorithm is sufficiently simple and can be used with any type of elements. Its modifications permit to obtain hazards-free structures and synthetize structures when these and structural properties of elements are given in the "interval form".

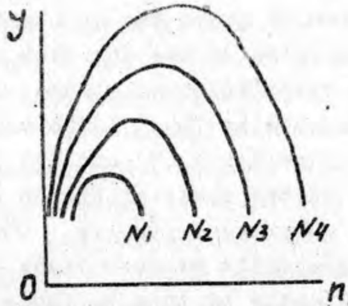


Fig.1

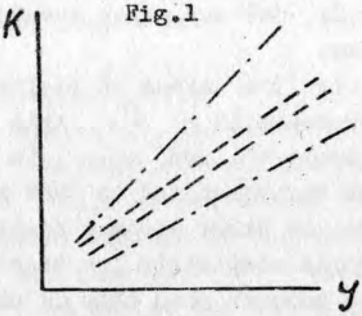


Fig.2

Avalue of complexity according to criteria, K-actual complexity,- Sholomov criteria,-·- Parphomenko criteria

	X_{10}	X_9	X_8	X_7	X_6	X_5	X_4	X_3	X_2	X_1	X_9	X_8	X_7	X_6	X_5	X_3	
65	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	
123	0	0	0	1	1	1	1	0	1	1	0	0	1	1	1	0	$\bar{X}_9\bar{X}_3$
170	0	0	1	0	1	0	1	0	1	0	0	1	0	1	0	0	
344	0	1	0	1	0	1	1	0	0	0	1	0	1	0	1	0	$\bar{X}_6$
3[illegible]0	0	1	0	1	1	0	1	0	0	0	1	0	1	1	0	0	$\bar{X}_5\bar{X}_3$
437	0	1	1	0	1	1	0	1	0	1	1	1	0	1	1	1	$\bar{X}_7$
450	0	1	1	1	0	0	0	0	1	0	1	1	1	0	0	0	
645	1	0	0	1	1	0	0	1	1	1	0	0	1	1	0	1	$\bar{X}_9 X_7 \bar{X}_5$
6[illegible]0	1	0	1	0	1	0	1	0	0	0	0	1	0	1	0	0	$\bar{X}_9$
704	1	0	1	1	0	0	0	0	0	0	0	1	1	0	0	0	
[illegible]	1	1	0	0	1	0	1	0	0	0	1	0	0	1	0	0	$\bar{X}_8$
358	0	1	0	1	1	0	0	1	1	0	1	0	1	1	0	1	X_9X_3
376	0	1	0	1	1	1	1	0	0	0	1	0	1	1	1	0	$X_9X_6X_5$
391	0	1	1	0	0	0	0	1	1	1	1	1	0	0	0	1	
424	0	1	1	0	1	0	1	0	0	0	1	1	0	1	0	0	X_9X_8
511	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	X_7
559	1	0	0	0	1	0	1	1	1	1	0	0	0	1	0	1	$\bar{X}_7$
629	1	0	0	1	1	1	1	1	0	1	0	0	1	1	1	1	X_5X_3
902	1	1	1	0	0	0	0	1	1	0	1	1	0	0	0	1	

a). б).

Fig.3

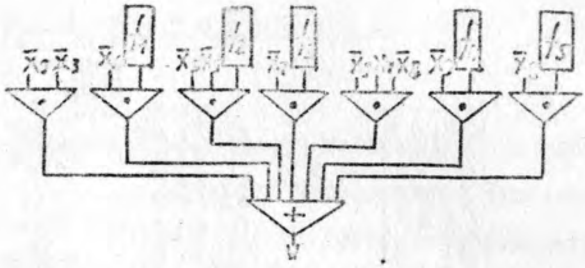
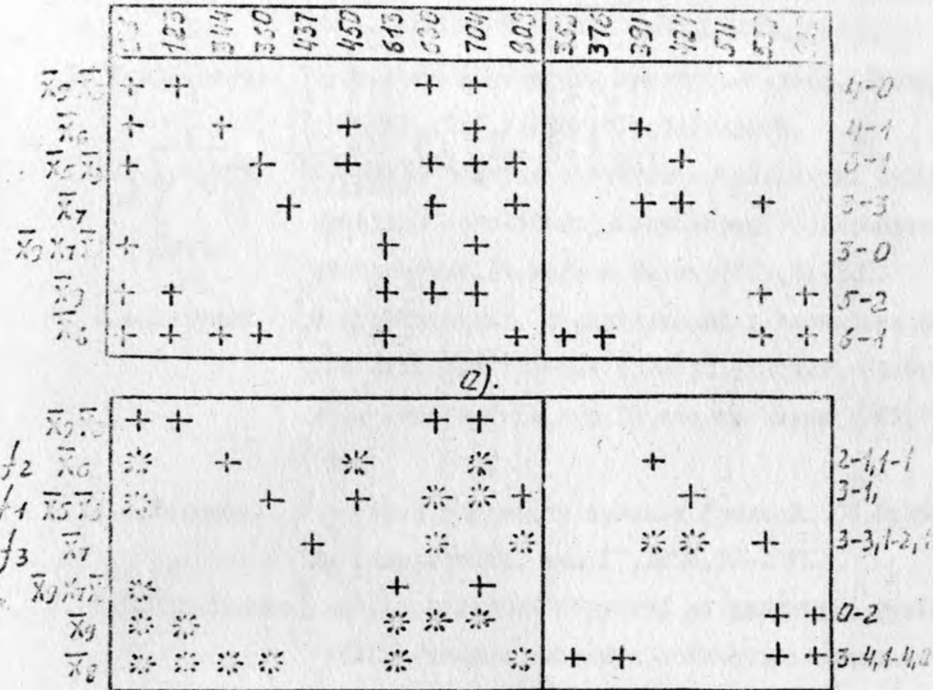


Fig.4



	x_9	x_8	x_7	x_6	x_5	x_3
437	1	1	0	1	1	1
391	1	1	0	0	0	1
427	1	1	0	1	0	0
557	0	0	0	1	0	1

$f_3 = x_5$

в).

	x_9	x_8	x_7	x_6	x_5	x_3
344	1	0	1	0	1	0
391	1	1	0	0	0	1

$f_2 = \bar{x}_8 = x_7 = x_5 = \bar{x}_3$

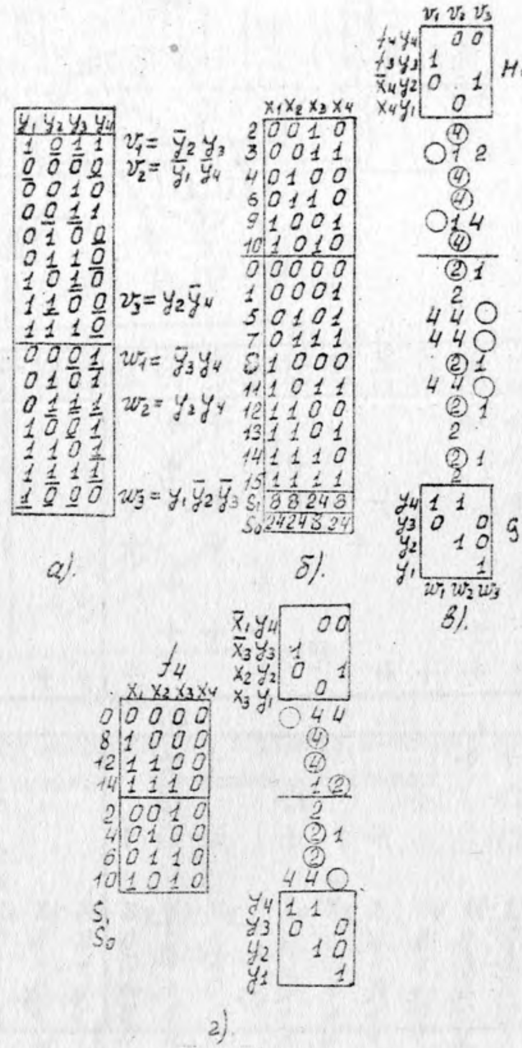
г).

	x_9	x_8	x_7	x_6	x_5	x_3
360	1	0	1	1	0	0
450	1	1	1	0	0	0
898	1	0	0	1	0	0
391	1	1	0	0	0	1

$f_1 = \bar{x}_3$

д)

Fig.5



$y_1\,y_2\,y_3\,y_4$
$v_1 = \bar{y}_2 y_3$
$v_2 = \bar{y}_1 \bar{y}_4$
$v_3 = y_2 \bar{y}_4$
$w_1 = \bar{y}_3 y_4$
$w_2 = y_2 y_4$
$w_3 = y_1 \bar{y}_2 \bar{y}_3$
a)
$x_1\,x_2\,x_3\,x_4$
S_1 8 8 2 4 8
S_0 24 24 8 24
б)
$v_1\,v_2\,v_3$
H_i
g_i
$w_1\,w_2\,w_3$
в)
f_4
$x_1\,x_2\,x_3\,x_4$
S_1
S_0
г)

Fig.6

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27.6

An Approach to Automatization of the Finite Automata Synthesis

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The automatization of synthesis of finite automata by means of digital computers is difficult due to lack of coordination between input and output languages of known algorithms which are used to solve problems on separate stages of synthesis. These algorithms do not enable to apply optimizing input signals on each stage of synthesis, with the participation of the optimal solution to problem at large.

The object of the present work is an approach to automatization of the process of the optimal synthesis of finite automata. This approach employs a class of algorithms based on the language of structure functions of excitation and outputs [1] The algorithms presented here are of an analytical character, they enable to conduct a single optimalization line in the process of synthesis. They are applicable to a sufficiently wide class of finite automata /fully and partly definable, minimum and non-minimum automata of Moore, Mealy, Moore-Mealy types/ realizable on arbitrary functionally complete set-ups of members. The efficiency of these algorithms is comparable to or higher than that known in literature for other algorithms.

To assure a general character of this investigation, we consider a model of a finite Mealy automaton defined on sets of internal states $A=(a_0, a_1, \dots, a_{n-1})$, where a_0 - initial state of the automaton, of input $/X=(x_0, x_1, \dots, x_{m-1})$, $x=\{0,1\}/$ and output signals $/Y=(y_0, y_1, \dots, y_{p-1}),\ y=\{0,1\}/$

, as well as, recurrence relations $a(t+1)=\delta[a(t), x(t)]$ and $y(t)=\lambda[a(t), x(t)]$. These last relations are determined on those sets and describe the functions of transitions and outputs of the automaton.

From the viewpoint of engineering practice, it is reasonable to interpret the automaton in the form of a block diagram composed of members of arbitrary types and complexity. This block diagram can be utilized to describe, in an abstract language, the conditions of performance of the automaton.

The following steps can be distinguished when passing from the block diagram to the language of structure functions of excitation and outputs.

1. Setting out the excitation functions $/q_i' = q_i'(x_0, x_1, \ldots, x_{m-1}, Q_1, Q_2, \ldots, Q_s)$ where Q_k - output function of k elementary automaton in the block diagram of the given automaton and $i = 1,2,\ldots, s \geq]\log_2 n[/$, and output functions $/z_j' = z_j(x_0, x_1, \ldots, x_{m-1}, Q, Q_2, \ldots, Q_s)$ where $j = 1,2,\ldots, y \geq]\log_2 p[/$.

2. From the excitation functions $q_i / i = 1,2,\ldots, s/$ follow the excitation functions $/q_i/$ of time delay members by making use of the relations given in Table 1 where the minimalization of the relations obtained is not necessary.

Table 1

Excitation function of flip-flop	Excitation function of delay element	Remarks
with countable input (q'_{si})	$q_i = \overline{q'_{si}} Q_i \vee q'_{si} \overline{Q}_i$	-
with decomposed inputs (q'_{0i}, q'_{1i})	$q_i = q'_{1i} \vee \overline{q'_{0i}} Q_i$	$q'_{0i} q'_{1i} = 0$
with doubled transitions (q'_{0i}, q'_{1i})	$q_i = \overline{q'_{0i}} Q_i \vee q'_{1i} \overline{Q}_i$	-
with three inputs $(q'_{si}, q'_{0i}, q'_{1i})$	$q_i = q'_{1i} \vee q'_{si} \overline{Q}_i \vee \overline{q'_{si}}\, \overline{q'_{0i}} Q_i$	$q'_{si} q'_{0i} = q'_{si} q'_{1i} = q'_{0i} q'_{1i} = 0$

3. Passage to the structure functions of excitation $F_k / k = 0,1,\ldots,n-1 /$ and of outputs $Y_j / j = 0,1,\ldots,p /$ is performed on the foundation of the following formulae

$$F_K = \langle \tilde{q}_1 . \tilde{q}_2 \ldots \tilde{q}_s \rangle_K , \qquad /1/$$

$$Y_j = \langle \tilde{z}_1 . \tilde{z}_2 \ldots \tilde{z}_y \rangle_j , \qquad /2/$$

where $k = 1,2,\ldots,2^s$, and $j = 1,2,\ldots,2^y$.

In /1/ and /2/ we put $\langle \tilde{Q}_1 . \tilde{Q}_2 \ldots \tilde{Q}_s \rangle_K = M_K$ and $\langle \tilde{x}_0 . \tilde{x}_1 \ldots \tilde{x}_{m-1} \rangle_j = x_j \; / \; j = 0,1,\ldots,2^m-1/$, where M_K and x_j are fundamental products corresponding to the automaton states and the structure input signal X_j .

4. Inaccessible /1.2/ states of the automaton are excluded by putting

$$M_K = 0, \quad \text{if} \quad F_K = 0 \text{ and}$$

$$M_K = 0 \quad \text{if} \quad F_K = F_K(x_0, x_1, \ldots, x_{2^m-1}, M_K)$$

If the automaton is given in terms of a structure language, it is possible, with the help of Table 2, to obtain the sought structure functions of excitation and outputs by determining according to Berge, all transitions (full states $x_j M_K$) as those corresponding to diverging branches, converging branches and reflecting branches.

The initial data for the structure synthesis are the structure functions of excitation and outputs of the automaton and the methods for coding its states given in the form of sets of binary decompositions of the type $\tau_i = \{A_0^i, A_1^i\}$ /$i=1,2,\ldots,S$/, where A_0^i and A_1^i correspond to not-intersecting sub-sets of states of the given automaton coded with states 0 and 1 of i-th elementary automaton /i-th structure input and output/.

Canonical functions of excitation and outputs of the automaton expressed in an canonical disjunctive normal form are obtained by means of relations corresponding to chosen types of elementary automata /see Table 2/. For this purpose, we substitute $M_K = \langle \tilde{Q}_1 \cdot \tilde{Q}_2 \cdots \tilde{Q}_S \rangle_K$ everywhere in the relations obtained, where M_K is determined on the basis of the chosen variant of coding /or the set of binary decompositions of the automaton states $\tau_1, \tau_2, \ldots, \tau_S$ /.

The extent of satisfying numerous requirements necessary for the automata being synthesised is determined by the choice of a proper set of binary decompositions $\tau_1, \tau_2, \ldots, \tau_S$.

The essence of efficient coding consists in the following: by applying the method of the structure synthesis described above we construct a complete set of canonical functions

Table 2

Type of elementary automaton	Structural excitation /output/ function	Transitions /states/ included in structural excitation or output function	Excitation or output function	Remarks x/
Delay element	F_K	Diverging /transient/ and reflecting /isolated/	$q_i = \bigvee_{K \in A_1^r} F_K$	-
Flip-flop with decomposed inputs	F_K	Diverging /transient/ and reflecting /isolated/	$q_{oi} = \bigvee_{K \in A_o^r} F_K \oplus \overline{\sum_{l \in A_o^i}} g\, k_j M_l$ $q_{1i} = \bigvee_{K \in A_1^r} F_K \oplus \overline{\sum_{l \in A_1^i}} g\, k_j M_l$ where $k_j M_l < F_K$	All reflecting transitions should be marked out with undefined coefficient g=0 or 1
Flip-flop with three inputs	F_K	Diverging /transient/ and reflecting /isolated/	$q_{oi} = \bigvee_{K \in A_o^i} F_K$ $q_{1i} = \bigvee_{K \in A_1^i} F_K$	All transitions should be marked out with undefined coefficient g=0 or 1. Coefficients q_{si} or q_{oi} as well as q_{si} and q_{1i} in the same implicant functions should be orthogonal
	F_{sK}	Diverging /transient/ and converging/persistent/	$q_{si} = \overline{\sum_{K \in A_1^i}} F_{sK}$	
Flip-flop with countable input	F_{sK}	Diverging /transient/ and convergent /persistent/	$q_{si} = \overline{\sum_{K \in A_1^i}} F_{sK}$	
Flip-flop with doubled transitions	F_K	Diverging /transient/ and converging /persistent/ reflecting/isolated/	$q_{oi} = \overline{\sum_{K \in A_o^i}} F_K$ $q_{1i} = \overline{\sum_{K \in A_1^i}} F_K$	All reflecting and converging transitions should be marked out with undefined coefficient g=0 or 1
Mealy	Ψ_j^i	All transitions are marked out with output signal y_j	$z_i = \bigvee_{j \in A_1^{ij}} \Psi_j^i$	-
Moore	Ψ_j^i	All states are marked out with output signal y_j	$z_i = \bigvee_{j \in A_1^i} \Psi_{j}^{i}$	-

x/ All undefined /conditional/ transients of partly definable automata should be marked out with an undefined coefficient =0 or 1

$$q_i = q_i(x_0, x_1, \dots, x_{m-1}, M_0, M_1, \dots, M_{n-1}),\ i = 1,2,\dots,2^{n-1}-1 \qquad /3/$$

$$y_j = y_i(x_0, x_1, \dots, x_{m-1}, M_0, M_1, \dots, M_{n-1}),\ j = 1,2,\dots,2^{p-1}-1 \qquad /4/$$

on a fixed set of binary decompositions characterizing all nonequivalent ways of coding the states of the automaton. Using the relation

$$\bigvee_{K \leftarrow A} M_K = M_{l_1} \vee M_{l_2} \vee \dots \vee M_{l_h} = \tilde{Q}_j, \qquad /5/$$

where $\tilde{Q}_j = Q_j$ if $(l_1, l_2, \dots, l_n) = A_1^t$, $\tilde{Q}_j = \bar{Q}_j$ if $(l_1, l_2, \dots, l_n) = A_0^t$ and $\tilde{Q}_j = 1$ if $(l_1, l_2, \dots, l_n) = A_0^t \cup A_1^t = A$,

these functions can be expressed in an absolutely minimal disjunctive form. Next, we choose such an efficient /from the viewpoint of engineering practice/ set of functions which involves the minimum number of implicit arguments. This is reduced to fulfilling the following relations

$$\sum_{i=1}^{s} b_i = B_{min}, \qquad /6/$$

$$\prod_{i=1}^{s} r_i = 0 \text{ and} \qquad /7/$$

$$\sum_{i=1}^{s} c_i = C_{min} \qquad /8/$$

where b_i - number of necessary technical elements /in adopted units/ to realize i-th function; C_i - number of implicit arguments of i-th function, B and C are certain positive integers.

With efficient coding wihtout critical race, we introduce additionally a set of characteristic decompositions $T_{K\,min} = T_{K\,min}[v_K, \delta(a, x)]$.

equivalent to the functions being analyzed. These decompositions determine the requirement of compatibility of various functions to avoid critical races. A variant of efficient coding without critical races is chosen on the basis of the relations /6/, /7/, /8/ and /9/

$$\tau_{k\,min} \leq \nu_1 \cdot \nu_2 \ldots \nu_{k-1} \cdot \nu_{k+1} \ldots \nu_s \qquad /9/$$

The described methods of coding can be easily extended for combined coding internal, input and output states of fully and partly definable, minimal and non-minimal automata [4]. In last case, the coding process show the expediency of minimizing a number of states of the automaton from the viewpoint of simplicity of structure of its realization. This consists in the following procedure.

Taking into account the compatible states of an automaton means that dimension of the set of its internal states can be reduced to a certain quantity $h < n$. Since the same codes can be assigned to compatible states, the condition of uniqueness of coding /see /7// for non-minimal automata can be expressed as

$$\prod_{i=1}^{s} \nu_i \geq 0 \qquad /10/$$

Thus, an arbitrary non-minimal automaton can be described by a set of functions consisting of $s_{min} =]\log_2 h[\leq s =]\log_2 n[$ excitation functions and $\gamma \geq]\log_2 p[$ of input functions. In order that the blocks of decomposition /10/ include only compatible states, it is necessary and sufficient, that the chosen set of excitation and output functions depend only on input signals and arguments $Q_1, Q_2, \ldots, Q_s$.

With such a statement, the problem of minimizing the number internal states is led to finding a set of excitation and output functions for which $S_{min} =]\log_2 h[$, and the problem of obtaining an efficient system of non-minima automaton - to determining such a system which is realized in the most efficient way /where $S = S_{min}$ is not obligatory/.

The fundamental criteria for the optimum variant of coding can be supplemented with other requirements fast action and restictions imposed on the number of inputs of logical elements [5] , minimum frequency of actuating secondary element, permissible loading on outputs of elementary automata, reliability and others.

The principal disadvantage of any given group of coding algorithms is their applicability to only a small class of automata. To synthesize optimally complex automata by means of these algorithms it is reasonable to employ the methods of aggregate decomposition of automaton into hierarchy and cascade structures [6] .

To avoid the mentioned disadvantage of the above described coding algorithms, a method of optimal structure synthesis is employed. It consists in applying directly the structure functions of excitation and outputs as canonical functions. This method enables to satisfy simultaneously a greater number of requirements: absence of critical races, fast operation with using two-input coincidence logical elements, minimal number of inverters and economy.

Functions of excitation and outputs for a given method are expressed according to Table 2 and minimized by relations

$$M_K = Q_K, \qquad /11/$$

$$x_j \bigvee_{i \in J} M_i = x_j \overline{\bigvee_{k \in K} M_k} \qquad /12/$$

where $\overline{\bigvee M_k}$ represents an orthogonal description of function $\bigvee_{i \in J} M_i$, K and $J^{k \in K}$ are not-intersecting sets of internal states of automaton where $J \cup K = A$.

It is seen from /11/ and /12/ that this method enables to omit the necessity of solving the coding problem, structure and combinational synthesis in their classical meaning. This fact makes the method suitable for the optimal synthesis of automata of arbitrary complexity from elements of an arbitrary functional complete technical system without applying the decomposition.

It should be noted finally that the advantages of the presented approach to the synthesis of finite automata is based, to a considerable extent, on using new structure language, that is, the language of structure functions of excitation and outputs. The algorithms of optimal synthesis of automata developed on the background of this language are characterized with following features:

1. These algorithms are of an active character /are applicable for the optimal synthesis of any types of automata of arbitrary complexity realized with arbitrary functionally complete system of elements/
2. Their efficiency is comparable or higher than that of analogous known algorithms
3. They are suitable for employing in engineering practice

/the process of synthesis does not require to construct laborious coded arrays of transitions and output of the automaton and to take into account transitions of elementary automata, they enable to perform the process of synthesis with various requirements and the selection of variants on the coding stage is significantly shortened compared to the coding algorithms commonly used; these algorithms are sufficiently formalized.

4. These algorithms are especially suitable to automatization of the synthesis by means of digital computers.

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