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A PARAMETER-ADAPTIVE CONTROL TECHNIQUE

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I. Introduction

For some years now the dual control formulation[5,6] and the dynamic programming formulation[2,3] for the so called "optimal adaptive control problem" have been available to solve control problems involving certain unknown quantities such as parameters of the system's mathematical model, parameters of the statistical descriptions for various disturbances affecting the system, or entire functional relationships involved in the mathematical representation of the control problem. Various efforts have been made to utilize these formulations and to modify them for numerous special circumstances[1,11,13]. Only limited success, however, has been achieved in dealing with the significant analytical complexities and computational burdens associated with both formulations.

This paper considers a special case of the optimal adaptive control problem for which it is possible to exploit a simple approximation technique to obtain an analytic solution of the functional equations associated with the dynamic programming formulation. The adaptive control problem itself is formulated in Section II of the paper, followed by a discussion of the approximation technique in Section III and the resulting adaptive control system in Section IV. The solution is then illustrated with a simple example in Section V.

II. A Formulation of the Adaptive Control Problem

Let the system be described by the following linear, discrete-time, stochastic model,

$$x(k+1) = A(\alpha,k)\, x(k) + B(\alpha,k)\, u(k) + \Gamma(\alpha,k)\, \xi(k) \qquad (1)$$

$$k = 0, 1, \ \ldots, \ N-1$$

$$\alpha \in \Omega_\alpha = \left\{\alpha_1, \alpha_2, \ \ldots, \alpha_s\right\}$$

with the measurement equation

$$y(k) = C(\alpha,k)\, x(k) + D(\alpha,k)\, \eta(k), \quad k = 1,2,\ldots,N \qquad (2)$$

The vector $x(k)$ is an n-vector of state variables defined at time instant t_k, $u(k)$ is an unconstrained m-vector of control inputs, and $y(k)$ is an r-vector of measured outputs. The r_1-vectors $\xi(k)$ and the r_2-vectors $\eta(k)$ form two independent sequences of independent identically distributed

Gaussian random vectors:

$$\xi(k) \sim N(0, I_{r_1}) \quad , \quad I_{r_1} = E\left\{\xi(k)\ \xi^T(k)\right\} \tag{3}$$

$$k = 0,1,\ldots,N-1$$

$$\eta(k) \sim N(0, I_{r_2}) \quad , \quad I_{r_2} = E\left\{\eta(k)\ \eta^T(k)\right\}$$

$$k = 1,2,\ldots,N$$

Similarly, the system's initial state x(0) is assumed to be a Gaussian distributed random vector:

$$x(0) \sim N\ (\mu(0), P(0)) \qquad P(0) = E\left\{[x(0)-\mu(0)][x(0)-\mu(0)]^T\right\} \tag{4}$$

The quantities $A(\alpha,k)$, $B(\alpha,k)$, $\Gamma(\alpha,k)$, $C(\alpha,k)$ and $D(\alpha,k)$ are matrices with appropriate dimensions whose elements are arbitrary but known functions of the time index k and of the ℓ-vector α. The vector α consists of unknown system parameters. It is assumed to belong to the finite set Ω_α and to be constant on the control interval $k = 0,1,\ldots,N$.

The adaptive control problem for this system consists of finding a sequence of control inputs $\left\{u(k),\ k = 0,1,2,\ldots,N-1\right\}$ as functions of the available measurements,

$$u(k) = f_k(y^k)\ ,\ y^k = \left\{y(1), y(2), \ldots, y(k)\right\},\ k = 0,1,\ldots,N-1 \tag{5}$$

such that the following average cost function is minimized:

$$J = E\left\{\ \sum_{i=1}^{N}\left[\|x(i)\|^2_{Q(\alpha,i)} + \|u(i-1)\|^2_{R(\alpha,i)}\right]\right\} \tag{6}$$

The symmetric matrices $Q(\alpha,k)$ and $R(\alpha,k)$ are again known functions of the parameters α and of the time index k.

The following additional assumptions are made:

$$\left.\begin{array}{lll} \text{(i)} & D(\alpha,k)\ D^T(\alpha,k) & > 0 \\ \text{(ii)} & Q(\alpha,k) = Q^T(\alpha,k) & \geq 0 \\ \text{(iii)} & R(\alpha,k) = R^T(\alpha,k) & > 0 \end{array}\right\} \text{ for all } \alpha \in \Omega_\alpha \text{ and } k=1,2,\ldots,N \tag{7}$$

(iv) an a-priori discrete probability distribution function q(0) for the vector α is available, where q(0) is an s-vector with components

$$0 \leq q_i(0) = \text{Prob}\ [\alpha = \alpha_i] \leq 1,\ i=1,2,\ldots,s$$

satisfying $\sum_{i=1}^{s} q_i(0) = 1$.

Since a feedback control of the form (5) is desired, the method of dynamic programming will be used to minimize criterion (6).

Define the "optimal return function":

$V(y^k, N-k) \triangleq$ cost of an N-k stage adaptive control process using the optimal control sequence $\{u^*(k), u^*(k+1), \ldots, u^*(N-1)\}$ based upon a-priori information (4) and (7 iv) and upon the measurement sequence $y^k = \{y(1), y(2), \ldots, y(k)\}$.

Applying the "Principle of Optimality"[2], the optimal return function obeys the following recursive functional equation:

$$V(y^k, N-k) = \min_{u(k)} E\left\{\|x(k+1)\|_Q^2 + \|u(k)\|_R^2 + V(y^{k+1}, N-k-1) \,|\, y^k\right\} \tag{8}$$

with $V(y^N, 0) \equiv 0$ (with probability one).

In this equation, $E\{\cdots | y^k\}$ denotes the mathematical expectation conditioned on the sequence y^k and on the a-priori data (4) and (7 iv). The dependence of Q and R upon parameters α and time index k has been suppressed. As a matter of convenience, this practice is continued in all subsequent derivations.

Equation (8) may be solved backwards, starting with a one-stage process.

$$V(y^{N-1}, 1) = \min_{u(N-1)} E\left\{\|x(N)\|_Q^2 + \|u(N-1)\|_R^2 \,|\, y^{N-1}\right\} \tag{9}$$

The conditional expectation of equation (9) can be expressed as

$$E\left\{(\cdots) | y^{N-1}\right\} = \sum_{i=1}^{s} \text{Prob}\,[\alpha = \alpha_i | y^{N-1}]\; E\left\{(\cdots) | \alpha = \alpha_i, y^{N-1}\right\} \tag{10}$$

$$= \sum_{i=1}^{s} q_i(N-1)\; E\left\{(\cdots) | \alpha = \alpha_i, y^{N-1}\right\}$$

where $q_i(N-1)$, $i=1,2,\ldots,s$, is the a-posteriori probability distribution of parameter vector α based upon measurements y^{N-1}. So (9) becomes

$$V(y^{N-1}, 1) = \min_{u(N-1)} \sum_{i=1}^{s} q_i(N-1)\; E\left\{\|x(N)\|_Q^2 + \|u(N-1)\|_R^2 \,|\, \alpha = \alpha_i, y^{N-1}\right\} \tag{11}$$

$$= \min_{u(N-1)} \sum_{i=1}^{s} q_i(N-1)\; E\Big\{ E\left[\|x(N)\|_Q^2 \,|\, \alpha = \alpha_i, y^N\right] + \|u(N-1)\|_R^2 \,|\, \alpha = \alpha_i, y^{N-1}\Big\}$$

In terms of the vectors $\mu(\alpha_i,k) \triangleq E\left\{x(k)|\alpha=\alpha_i,y^k\right\}$, define the ns-vector

$$\hat{X}^T(k) = (\mu^T(\alpha_1,k),\ \mu^T(\alpha_2,k),\ldots,\mu^T(\alpha_s,k)). \tag{12}$$

Then it is readily verified[12] that $V(y^{N-1},1)$ is quadratic in the vector $\hat{X}(N-1)$,

$$V(y^{N-1},1) = \|\hat{X}(N-1)\|^2_{S(q(N-1),1)} + T(q(N-1),1) \tag{13}$$

and the optimal control $u^*(N-1)$ is linear in $\hat{X}(N-1)$,

$$u^*(N-1) = -G(q(N-1),1)\,\hat{X}(N-1) \tag{14}$$

where matrices S and G and scalar T are nonlinear functions of the a-posteriori distribution $q_i(N-1)$, $i=1,2,\ldots,s$, which are defined in the Appendix, equations (A1), (A2), and (A3).

It is now evident that the vectors $\hat{X}(k)$ and $q(k)$ constitute a set of "sufficient coordinates"[15] for the adaptive control problem formulated in equations (1) - (7). The optimal return function can be expressed as

$$V(y^k,N-k) = V(\hat{X}(k),\ q(k),\ N-k)$$

and the functional equation (8) becomes

$$V(\hat{X}(k),\ q(k),\ N-k) = \min_{u(k)} \sum_{i=1}^{s} q_i(k)\ E\left\{\|x(k+1)\|^2_Q + \|u(k)\|^2_R + V(\hat{X}(k+1),q(k+1),N-k-1)\,|\,\alpha=\alpha_i,\hat{X}(k)\right\} \tag{15}$$

with $V(\hat{X}(N),q(N),0) \equiv 0$ (with probability one).

The existence of sufficient coordinates reduces the dependence of $V(\cdots)$ upon a growing number of variables (y^k) to the dependence upon a constant and finite number of variables $(\hat{X}(k),q(k))$.

Equation (15) can now be used to continue the dynamic programming solution, starting with the quadratic return function $V(\hat{X}(N-1),q(N-1),1)$ of (13). As defined by equation (A2), however, the matrix $S(q(N-1),1)$ is a nonlinear function of the a-posteriori distribution $q_i(N-1)$, $i=1,2,\ldots,s$. This fact prevents the successful completion of the solution in closed form. The function $V(\hat{X}(N-2),q(N-2),2)$ and all subsequent optimal return functions are no longer expressible in terms of quadratics or in terms of other similarly convenient functional forms.

The solution of (15) must therefore be obtained by numerical techniques[10] or by approximation methods. Because the computing time and memory requirements of numerical solutions are prohibitive for all but the simplest problems, the following discussion will deal with an approximation method which is based upon a very intuitive and appealing linearization technique.

III. Linearization of the Weighting Matrix of the Optimal Return Function

It has been pointed out that the optimal return function for a single stage of the adaptive control problem formulated above is quadratic in $\hat{X}(N-1)$ with a weighting matrix $S(q(N-1),1)$ which is a nonlinear function of the a-posteriori distribution $q(N-1)$. That is,

$$S(q(N-1),1) = f_1(q_1(N-1), q_2(N-1), \ldots, q_{s-1}(N-1)), \tag{16}$$

where $f_1(\cdots)$ is the matrix-valued function of $(s-1)$ independent variables defined by equations (A1) and (A2). The fact that $f_1(\cdots)$ only has $(s-1)$ arguments is a consequence of the relation

$$\sum_{i=1}^{s} q_i(k) = 1 \qquad k=0,1,\ldots,N-1 \ . \tag{17}$$

Let the matrix $\tilde{S}(q,1)$ be the matrix-valued "tangent plane" to the matrix $S(q,1)$ at the point $q(0)$. This new matrix $\tilde{S}$ can be computed by considering S itself to be a matrix "surface" on an $(s-1)$ dimensional Euclidean space.

$$f_1(q_1, q_2, \ldots, q_{s-1}) - S = 0 \tag{18}$$

Then the "tangent plane" at the point $q(0)$ is defined by

$$\sum_{i=1}^{s-1} \frac{\partial f_1}{\partial q_i}\bigg|_{q(0)} [q_i - q_i(0)] - [\tilde{S} - S(q(0),1)] = 0 \ , \tag{19}$$

Using (17), this expression can be rewritten as

$$\tilde{S}(q,1) = q_1\, U_1(1) + q_2\, U_2(1) + \cdots + q_s\, U_s(1) \tag{20}$$

where $U_i(1)$, $i=1,\ldots,s$, are $(ns \times ns)$ matrices defined by

$$U_s(1) = S(q(0),1) - \sum_{i=1}^{s-1} \frac{\partial f_1}{\partial q_i}\bigg|_{q(0)} q_i(0) \tag{21}$$

$$U_i(1) = \frac{\partial f_1}{\partial q_i}\bigg|_{q(0)} + U_s(1) \qquad i=1,2,\ldots,s-1 \ .$$

The optimal return function of the one-stage adaptive control process (13) will now be approximated by replacing the weighting matrix $S(q(N-1),1)$ by the linearized matrix $\tilde{S}(q(N-1),1)$ defined in equations (20) and (21). Using this approximation, the return function of a two-stage adaptive control process can be obtained analytically from the equation

$$\tilde{V}(\hat{X}(N-2), q(N-2), 2) = \min_{u(N-2)} \sum_{i=1}^{s} q_i(N-2)\, E\left\{\|x(N-1)\|_Q^2 + \|u(N-2\|_R^2 + \|\hat{X}(N-1)\|^2_{\tilde{S}(q(N-1),1)} + T(q(N-1),1) \,\middle|\, \alpha=\alpha_i, \hat{X}(N-2)\right\}. \tag{22}$$

The resulting return function is again quadratic

$$\tilde{V}(\hat{X}(N-2), q(N-2), 2) = \|\hat{X}(N-2)\|^2_{S(q(N-2),2)} + T(q(N-2),2) \tag{23}$$

and the corresponding control is linear[12]

$$\tilde{u}^*(N-2) = -G(q(N-2),2)\, \hat{X}(N-2). \tag{24}$$

The matrices $S(q(N-2),2)$ and $G(q(N-2),2)$ are nonlinear functions of the a-posteriori distribution $q(N-2)$ which have exactly the same functional forms as the corresponding matrices of the one-stage return function.

The symbols $\tilde{V}$ and $\tilde{u}^*$ in equations (22) - (24) are used to emphasize the fact that these quantities are no longer the optimal return function and the optimal control respectively but rather that they depend upon the approximation of $S(q(N-1),1)$ by the linearized matrix $\tilde{S}(q(N-1),1)$. Since this approximation is directly involved in the minimization indicated by equation (22), the return function $\tilde{V}$ of (23) and the control $\tilde{u}^*$ of (24) have a meaningful interpretation only if an inequality of the type

$$\|\hat{X}\|^2_{S(q,1)} \le \|\hat{X}\|^2_{\tilde{S}(q,1)} \tag{25}$$

for all $\hat{X}$ and all $q \in \left\{q \middle| 0 \le q_i \le 1, i=1,\dots,s;\ \sum_{i=1}^{s} q_i = 1\right\}$

can be established. $\tilde{V}(\hat{X}(N-2), q(N-2), 2)$ is then the minimum cost of a two-stage adaptive control process for which the "cost of the final stage is somewhat higher than the optimal cost." $\tilde{u}^*(N-2)$ is, of course, the corresponding minimizing control. The inequality (25) is indeed satisfied as a consequence of the following property.

Upper Bound Property of the $\tilde{S}$ Approximation: For any fixed but arbitrary vector $\hat{X}$, the function $\|\hat{X}\|^2_{\tilde{S}(q,1)}$, considered as a function of the s-vector q, defines a supporting hyperplane[7] of the closed convex set Ω

$$\Omega = \left\{(z,q)\,|\,0 \le z \le \|\hat{X}\|^2_{S(q,1)};\ q \in \Omega_q\right\} \tag{26}$$

at the point $\left[\|\hat{X}\|^2_{S(q(0),1)}, q(0)\right]$

where $\Omega_q = \left\{ q\,|\,0 \le q_i \le 1,\ i=1,2,\ldots,s;\ \sum_{i=1}^{s} q_i = 1\right\}$.

Proof: The proof of this property consists of two parts:

(i) A proof of convexity for the set Ω, which reduces to a proof of convexity for the function $\|\hat{X}\|^2_{S(q,1)}$ on the domain Ω_q. The details can be found in reference 12.

(ii) A proof of the fact that $\|\hat{X}\|^2_{\tilde{S}(q,1)}$ defines a supporting hyperplane of Ω at $\left[\|\hat{X}\|^2_{S(q(0),1)}, q(0)\right]$. This follows directly from the definition of $\tilde{S}$ as the matrix-valued "tangent plane" to S at $q=q(0)$, and from the convexity of Ω.

Inequality (25) is now a direct consequence of the fact that the set Ω and particularly its boundary $\|\hat{X}\|^2_{S(q,1)}$ lies in one closed half-space produced by the supporting hyperplane $\|\hat{X}\|^2_{\tilde{S}(q,1)}$.[7]

The inequality (25) is a property of the $\tilde{S}$ approximation which lends a meaningful interpretation to the two-stage return function $\tilde{V}(\hat{X}(N-2), q(N-2), 2)$. Equally important, however, is the fact that this function itself is again quadratic, with a nonlinear weighting matrix $S(q,2)$ which has exactly the same functional form as the matrix $S(q,1)$. It is therefore possible to approximate the new weighting matrix by the same linearized form

$$\tilde{S}(q,2) = \sum_{i=1}^{s} q_i\, U_i(2)\ , \tag{27}$$

where the matrices $U_i(2)$, i=1,...,s, are defined by analogy to equation (21). This approximation again satisfies an upper bound property

$$\|\hat{X}\|^2_{S(q,2)} \le \|\hat{X}\|^2_{\tilde{S}(q,2)} \quad \text{for all } \hat{X} \text{ and all } q \in \Omega_q \tag{28}$$

and further, it permits the computation of an approximate three-stage return function

$$\tilde{V}(\hat{X}(N-3), q(N-3), 3) = \|\hat{X}(N-3)\|^2_{S(q(N-3),3)} + T(q(N-3),3) \tag{29}$$

with the minimizing control

$$\tilde{u}^*(N-3) = -G(q(N-3),3)\ \hat{X}(N-3)\ . \tag{30}$$

Since this three-stage return function is again quadratic with the same nonlinear weighting matrix, it is evident that the $\tilde{S}$ approximation may be applied once more to yield an approximate four-stage return function. and that repeated applications of the same procedure can be used to obtain a solution for the entire N-stage adaptive control process. The computations required for such a solution are summarized by the following recursive equations:

Solve backwards for k=N, N-1, ..., 1

$$Q_i(k) = \left[I-\bar{K}C_i(k)\right]^{-1^T}\left[W_i(k) + U_i(N-k)\right]\left[I-\bar{K}C_i(k)\right]^{-1} \qquad i=1,2,\ldots,s \tag{31}$$

$$\bar{R}(q,k) = \sum_{i=1}^{s} q_i \, R(\alpha_i, k) \tag{32}$$

$$\bar{Q}(q,k) = \sum_{i=1}^{s} q_i \, Q_i(k) \tag{33}$$

$$G(q,N-k+1) = \left[\bar{B}^T \bar{Q}(q,k)\bar{B} + \bar{R}(q,k)\right]^{-1} \bar{B}^T \bar{Q}(q,k)\,\bar{A} \tag{34}$$

$$S(q,N-k+1) = \bar{A}^T \bar{Q}(q,k)\left[\bar{A} - \bar{B}G(q,N-k+1)\right] \tag{35}$$

$$\begin{aligned}\left.\frac{\partial f_{N-k+1}}{\partial q_i}\right|_{q(0)} = {} & \bar{A}^T\left[Q_i(k) - Q_s(k)\right]\bar{A} - \\ & - \bar{A}^T\left[Q_i(k) - Q_s(k)\right]\bar{B}G\,(q(0),N-k+1) - \\ & - G^T(q(0),N-k+1)\,\bar{B}^T\left[Q_i(k) - Q_s(k)\right]\bar{A} + \\ & + G^T(q(0),N-k+1)\left\{\bar{B}^T\left[Q_i(k) - Q_s(k)\right]\bar{B} + \right. \\ & \left. + R(\alpha_i,k) - R(\alpha_s,k)\right\} G(q(0),N-k+1) \\ & \qquad i=1,2,\ldots,s-1\end{aligned} \tag{36}$$

$$U_s(N-k+1) = S(q(0),N-k+1) - \sum_{i=1}^{s-1} q_i(0) \left.\frac{\partial f_{N-k+1}}{\partial q_i}\right|_{q(0)} \tag{37}$$

$$U_i(N-k+1) = U_s(N-k+1) + \left.\frac{\partial f_{N-k+1}}{\partial q_i}\right|_{q(0)}, \quad i=1,2,\ldots,s-1 \tag{38}$$

with initial conditions $U_i(0) = 0$, $i=1,2,\ldots,s$.

Again, detailed derivations of these recursion equations are available in reference 12. The needed definitions of known matrices $\bar{A} = \bar{A}(k-1)$, $\bar{B} = \bar{B}(k-1)$, $\bar{K} = \bar{K}(k)$, and $C_i(k)$, $W_i(k)$ are found in the Appendix, equations (A4), (A5), (A6), and (A10), (A11), respectively.

Repeated applications of the $\tilde{S}$ approximation thus yields a closed form approximate solution of the adaptive control problem formulated by equations (1) - (7). This solution can be readily interpreted in the form of a closed-loop adaptive control system.

IV. The Resulting Adaptive Control System

As shown in the derivations above, the adaptive controller must generate control signals $\tilde{u}^*(k)$ defined by

$$\tilde{u}^*(k) = -G(q(k), N-k)\, \hat{X}(k) \qquad k=0,1,\ldots,N-1 .$$

The controls are thus function of the "sufficient coordinates" $\hat{X}(k), q(k)$ and of the matrices $G(q,k)$. Expressions for the feedback matrices can, of course, be obtained entirely off-line by solving equations (31) - (37) recursively. The sufficient coordinates, on the other hand, must be computed on-line by the adaptive controller itself. The computation of $\hat{X}(k)$ may be interpreted as "state estimation", which can be performed by the simultaneous operation of s Kalman-Bucy filters[9] (equations (A13) - (A16)), and the computation of q(k) may be interpreted as "parameter identification", which can be performed by the application of Bayes theorem[8]

$$q_i(k+1) = \frac{p(y(k+1)\,|\,\alpha=\alpha_i, \hat{X}(k))\, q_i(k)}{\sum_{j=1}^{s} p(y(k+1)\,|\,\alpha=\alpha_j, \hat{X}(k))\, q_j(k)} \qquad i=1,2,\ldots,s \qquad (39)$$

where $p(y(k+1)\,|\,\alpha=\alpha_i, \hat{X}(k))$ is the probability density function of the (k+1)-th measurement conditioned on $\alpha=\alpha_i$ and $\hat{X}(k)$. With these interpretations, the resulting closed-loop adaptive control system will have the form shown in Figure 1. It is important to observe that the apparent separation of the state estimation and parameter identification functions of this controller is not a consequence of an a-priori assumption[4,14], but rather, that it is a consequence of the approximation method used to solve the recursive dynamic programming equations. A further consequence of this method is the fact that the feedback matrices G(q,k) are intimately related to both the state estimation and parameter identification schemes.

V. An Example

The parameter adaptive control technique presented above is now illustrated with the following simple example:

Let the system be described by the discretized version of a continuous-time stochastic second order system with a known natural frequency $\omega_n = 1$ radian/second and with an unknown damping ratio δ, which may assume one of two possible values δ_1 and δ_2 with probability $q_1(0)$ and $q_2(0)$ respectively. The system equations will have the following form

$$x(k+1) = \begin{bmatrix} 1 & \Delta \\ -\Delta & 1-2\delta_i\Delta \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ \Delta \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ \sigma_i \Delta^{\frac{1}{2}} \end{bmatrix} \xi(k) \qquad (40)$$

$$y(k+1) = (1, 0)\, x(k+1) + \gamma_i\, \eta(k+1) \qquad (41)$$

$$i=1,2 \qquad k=0,1,\ldots,49$$

where Δ is the sampling period of the discretization and where σ_i and γ_i, $i=1,2$, are the standard deviations of the disturbance input and the measurement noise respectively. Choosing the cost function

$$J = E\left\{ \sum_{i=1}^{50} \left[x_1^2(i) + x_2^2(i) + u^2(i-1) \right] \right\}, \qquad (42)$$

and the values

$$q(0) = \begin{bmatrix} .5 \\ .5 \end{bmatrix}, \qquad x(0) \sim N\left[\begin{bmatrix} 0 \\ -6.0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right]$$

$$\Delta = 0.1\,, \quad \sigma_1 = 1, \ \sigma_2 = 2, \quad \gamma_1 = \gamma_2 = .316\,,$$

the control gains $G(q,k)$, $k=1,\ldots,50$, defined by recursive equations (31) - (38) were computed for several sets of values of δ_1 and δ_2. Using these gains and a "true" system corresponding to the index $i=1$, the 50-stage adaptive control process was simulated on a CDC 6500 digital computer. The average cost of 100 simulation runs was then used to compare the performance of the adaptive control technique presented here with the performance of two other controllers, (1) the optimal stochastic controller computed for a plant with known parameter values corresponding to $i=1$, and (2) the optimal stochastic controller computed for a plant with known parameter values corresponding to $i=2$. Note that neither of these controllers is optimal for the adaptive control problem formulated in Section II. The true optimal solution for this problem is, of course, not available. The two controllers do, however, provide a meaningful standard of comparison. They are the controllers which will be obtained if an a-priori decision is made about the value of the parameter vector

$\alpha = (\delta, \gamma)$. Controller (1) for i=1 corresponds to the correct decision and controller (2) for i=2 corresponds to the incorrect decision.

A typical comparison between the adaptive control system and the two controllers above is given in Table I. Additional comparisons can be found in Figures 2 and 3. Figure 2 compares the per-stage costs of the three processes represented in the first row of Table I, while Figure 3 compares their phase plane trajectories. Again, both figures were obtained by averaging 100 separate simulation runs. To conserve space, the behavior of the a-posteriori probabilities q(k), k=0,1,...,50, associated with the adaptive control process is not shown. It is sufficient to state that these probabilities exhibit well-behaved convergence properties from the a-priori values $q^T(0) = (.5, .5)$ toward $q^T(\infty) = (1, 0)$.

δ_1, δ_2	Cost using optimal controller for i=1	Cost of adaptive control process	Cost using optimal controller for i=2
0.1 , 0.9	571	587	958
0.25, 0.75	447	455	536
0.40, 0.60	357	358	367

Table I. Comparison of Controllers

The comparisons of Table I and Figures 2 and 3 all indicate that the proposed parameter-adaptive control scheme represents a promising approach to the solution of appropriately formulated control problems.

VI. Conclusions

This paper has presented an approximation technique for the closed form solution of the functional equation of dynamic programming associated with a particular class of linear, parameter-adaptive control problems. The method leads to a simple and intuitively appealing adaptive controller whose performance, at least in the example presented, appears quite promising. Many questions, however, remain unresolved. For example, the control processes considered here are limited to finite duration. This restriction eliminates the need to consider the convergence question of the approximate solution of the dynamic programming equation[3]. It is clear, however, that if convergence is indeed obtained, then the storage requirement associated with the gains G(q,k), k=1,2,...,N, can be significantly reduced by storing only $G(q, \infty)$ for an infinite time adaptive control process. Another interesting question concerns the fact that the linearization procedure makes g(q,k) an implicit function of the a-priori distribution q(0). It would, therefore, seem appropriate to recompute the feedback gains occasionally as the control process evolves and as a-posteriori probabilities become available about which to re-linearize. These and related questions are subjects of further research.

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APPENDIX

Matrices for the one-stage adaptive control process are

$$G(q,1) = \left[\bar{B}^T \bar{Q}(q,N)\,\bar{B} + \bar{R}(q,N)\right]^{-1} \bar{B}^T \bar{Q}(q,N)\,\bar{A} \qquad (A1)$$

$$S(q,1) = \bar{A}^T \bar{Q}(q,N)\left[\bar{A} - \bar{B}\,G(q,1)\right] \qquad (A2)$$

$$T(q,1) = \sum_{i=1}^{s} q_i T_i(1) \qquad (A3)$$

where

$$\bar{A} = \bar{A}(N-1) = \text{diag}\left\{\left[I-K(\alpha_i,N)\,C(\alpha_i,N)\right] A(\alpha_i,N-1)\right\} \qquad (A4)$$

$$\bar{B} = \bar{B}(N-1) = \text{column}\left\{\left[I-K(\alpha_i,N)\,C(\alpha_i,N)\right] B(\alpha_i,N-1)\right\} \qquad (A5)$$

$$\bar{K} = \bar{K}(N) = \text{column}\left\{K(\alpha_i,N)\right\} \qquad (A6)$$

$$\bar{R}(q,N) = \sum_{i=1}^{s} q_i\, R(\alpha_i,N) \qquad (A7)$$

$$\bar{Q}(q,N) = \sum_{i=1}^{s} q_i\, Q_i(N) \qquad (A8)$$

$$Q_i(N) = \left[I-\bar{K}\,C_i(N)\right]^{-1^T} W_i(N)\left[I-\bar{K}\,C_i(N)\right]^{-1};\ i=1,\ldots,s \qquad (A9)$$

$$C_i(N) = (0\ \cdots\ C(\alpha_i,N)\ \cdots\ 0)\,,\ i=1,\ldots,s \qquad (A10)$$

$C(\alpha_i,N)$: ith partition

$$W_i(N) = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & Q(\alpha_i,N) & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \quad \text{ith diagonal partition,} \quad i=1,\ldots,s \qquad (A11)$$

$$T_i(N) = \text{Trace}\left(Q(\alpha_i,N)\,P(\alpha_i,N)\right) + \text{Trace}\left[W_i(N)\,\bar{K}(CMC^T + DD^T)\bar{K}^T\right],\ i=1,\ldots,s \qquad (A12)$$

and where the matrices $\bar{K}$, P, and M are obtained from the solution of s Kalman-Bucy filter equations[9]:

$$\mu(\alpha_i,k+1) = (I-KC)\left[A\,\mu(\alpha_i,k) + B\,u(k)\right] + Ky(k)$$
$$\mu(\alpha_i,0) = \mu(0) \qquad (A13)$$

$$K = P(\alpha_i, k+1)\ C(DD^T)^{-1} \qquad \text{(A14)}$$

$$P(\alpha_i, k+1) = M - MC^T \left[CMC^T + DD^T \right] CM \qquad \text{(A15)}$$

$$M = AP(\alpha_i, k)\ A^T + \Gamma\Gamma^T, \qquad P(\alpha_i, 0) = P(0) \qquad \text{(A16)}$$

$$i=1, 2, \ldots, s \ , \qquad k=0, 1, \ldots, N-1$$

In equations (A12) - (A16) the suppressed α and time dependence is given by

$$K = K(\alpha_i, k+1)\ ,\ C = C(\alpha_i, k+1),\ D = D(\alpha_i, k+1),$$

$$M = M(\alpha_i, k+1)\ ,\ A = A(\alpha_i, k),\ B = B(\alpha_i, k)\ ,\ \Gamma = \Gamma(\alpha_i, k)\ .$$

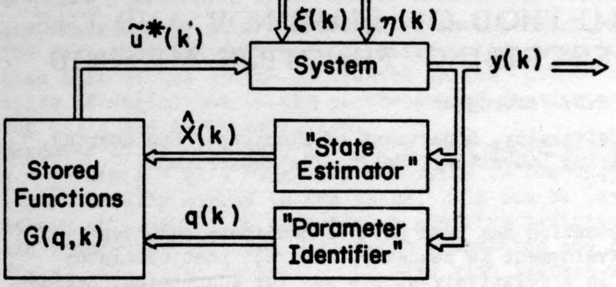

Figure 1.
The Adaptive Control System.

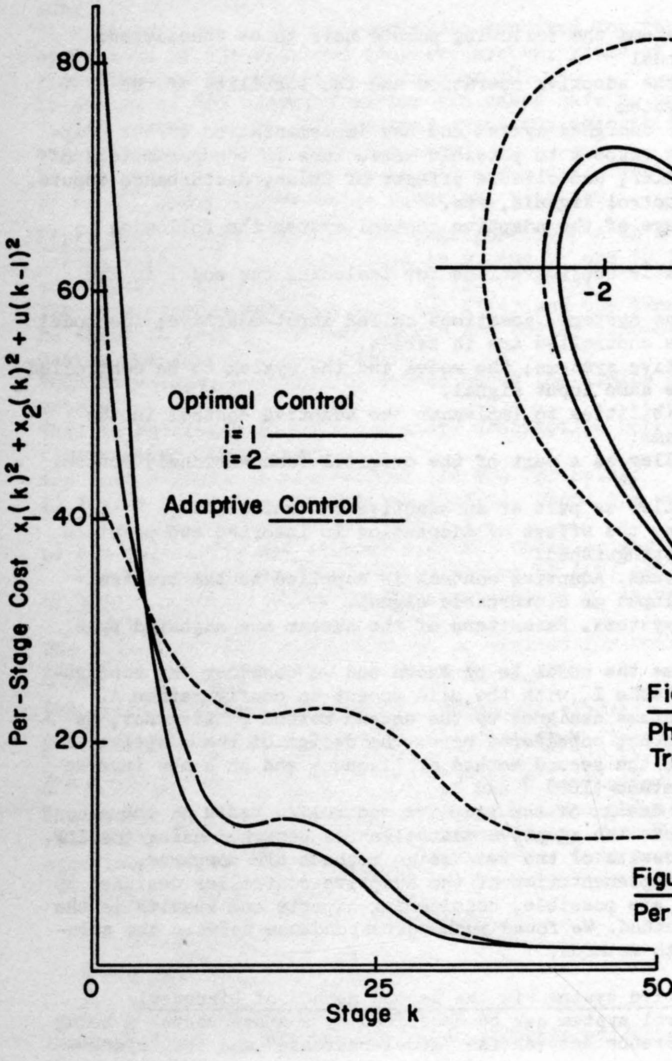

Figure 2.
Per-Stage Costs

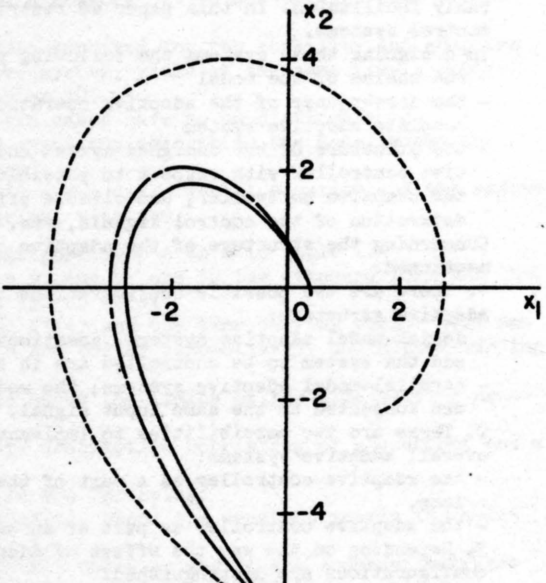

Figure 3.
Phase-Plane Trajectories

DESIGNING MODEL-ADAPTIVE CONTROL SYSTEMS USING THE METHOD OF LIAPUNOV AND THE INVERSE DESCRIBING FUNCTION METHOD

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1. Introduction.

In the last few years attention has been paid to nonlinear adaptive control systems. This development is caused by the fact that nonlinear elements can be applied in a relatively simple way for adaptation. Besides, the application of the theories for nonlinear systems has been considerably facilitated. In this paper we restrict ourselves to model-adaptive control systems.

In designing these systems the following points have to be considered:
- the choise of the model
- the convergence of the adaptive operation and the stability of the complete adaptive system
- the structure of the designed system and the implementation of the adaptive controller with respect to possible variations in the parameters of the adaptive controller, undesirable effects of noise, disturbance inputs, saturation of the control signals, etc.

Concerning the structure of the adaptive control system the following is mentioned:

1. There are two possible configurations for including the model in the adaptive structure:
- serial-model adaptive systems, sometimes called input-adaptive; the model and the system to be controlled are in series.
- parallel-model adaptive systems; the model and the system to be controlled are subjected to the same input signal.

2. There are two possibilities to implement the adaptive control in the overall adaptive systems:
- the adaptive controller as a part of the original (conventional) control loop.
- the adaptive controller as part of an adaptive control loop.

3. Depending on the way the effect of adaptation is inserted two possible configurations are distinguished:
- signal-adaptive systems. Adaptive control is supplied to the original system as an extra input or disturbance signal.
- parameter-adaptive systems. Parameters of the system are adjusted by adaptive control.

In this paper we assume the model to be known and we consider the configurations summed up in table I, with the main accent on configuration A. Parameter-adaptive systems designed by the second method of Liapunov, as treated in [1] and [2] are not considered here. The design of the adaptive controller is based on the second method of Liapunov and on a new inverse describing function method (IDF) [3] and [4].

First we consider the design of the adaptive controller based on the second method of Liapunov. Next the adaptive controller is designed using the IDF method. Finally the results of the two design methods are compared. Modifications in the implementation of the adaptive controller designed by either of the methods are possible, considering aspects and results in the design by the other method. We found much correspondence between the solutions generated by both methods.

2. Design of the adaptive system via the second method of Liapunov.

A model-adaptive control system can be described by a state vector $\underline{x}$ being a measure of the difference between the "model-response" and the "system-

response", see table I. Herein is assumed that both the model and the system are described by an n^{th} order differential equation.
The case that the order of the differential equation of the model is lower than that of the system is treated in the "second case" of this section. In spite of variations in the parameters of the system S the requirement $\underline{x} = \underline{0}$ for all t has to be satisfied.
Moreover it is necessary to have a fast convergence to the equilibrium point $\underline{x} = \underline{0}$ when $\underline{x} \neq \underline{0}$. A necessary condition is the requirement that $\underline{x} = \underline{0}$ is asymptotically stable in the large. This can be investigated by the second method of Liapunov by selecting a positive definite Liapunov function $V(\underline{x})$; and requiring that some well-known properties are fulfilled for $V(\underline{x})$ and $\dot{V}(\underline{x},t)$, the first derivative of $V(\underline{x})$ with respect to time. $V(\underline{x})$ can also be interpreted as a measure of the performance of the adaptive system, while the quotient of $\dot{V}(\underline{x},t)$ and $V(\underline{x})$ is a measure of the convergence time of the adaptive operation [5].
It will be shown that the conditions required for the stability and adaptive operations of the complete adaptive system, yield at the same time a control algorithm for the adaptive controller AC.
In designing the adaptive system two cases have to be distinguished.
1. The orders of the differential equations describing the model M and the original system S are the same (m=n).
2. The model is described by an m^{th} order differential equation and the system by an n^{th} order differential equation, with n>m.

First case (m=n). Consider the configurations A to D in table I.
Assume that the state equations in $\underline{y}_m$ and $\underline{y}_s$ are in the standard form and that the transfer function $\underline{c}^T(sI-A)^{-1}\underline{b}$ has no zeros.
The time varying matrices A(t) and A'(t) and the time varying vector $\underline{b}(t)$ can be split up into a constant matrix and vector respectively, and a time varying matrix and vector respectively.
Some knowledge, however, is needed about the limits of the time varying parameters.
This is necessary to obtain a state description consisting of a constant and a time varying linear part.
The most reasonable separations are the following:
a) $A(t) = A_o+\Delta A_o(t)$ and $A'(t)=A_o+\Delta A'_o(t)$, where the constant matrix is chosen to be equal to the model matrix A.
b) $A(t)=A_1+\Delta A_1(t)$ and $A'(t)=A'_1+\Delta A'_1(t)$, where A_1 and A'_1 are constant matrices whose elements are the average values of A(t) and A'(t) respectively.
c) $\underline{b}(t) = \underline{b}_o+\Delta\underline{b}(t)$ where the constant vector $\underline{b}_o$ is chosen to be equal to the model vector $\underline{b}_o$.
The following state equations can be derived for configuration A:

$$\dot{\underline{x}}=A_o\underline{x}-\{\Delta A'_o(t)\underline{y}_s+\underline{b}(t)r+\underline{b}(t)u\} = A_o\underline{x}-\underline{f}_{11}(\underline{y}_s,r,u) \quad (1)$$

$$\dot{\underline{x}}=A_o\underline{x}-\{\Delta A_o(t)\underline{y}_s+\underline{b}(t)(u+e)-\underline{b}_o r\} = A_o\underline{x}-\underline{f}_{12}(\underline{y}_s,e,u,r) \quad (2)$$

$$\dot{\underline{x}}=A'_1\underline{x}-\{\Delta A'_1(t)\underline{y}_s+(A'_1-A_o)\underline{y}_m+\Delta\underline{b}(t)r+\underline{b}(t)u\}=A'_1\underline{x}-\underline{f}_{13}(\underline{y}_s,\underline{y}_m,r,u) \quad (3)$$

$$\dot{\underline{x}}=A_1\underline{x}-\{\Delta A_1\underline{y}_s+(A_1-A_o)\underline{y}_m+\underline{b}(t)(u+e)-\underline{b}_o r\}=A_1\underline{x}-\underline{f}_{14}(\underline{y}_s,\underline{y}_m,r,u,e) \quad (4)$$

The vector $\underline{f}_{1j}$ includes all time varying terms and all terms not directly related to the state vector $\underline{x}$. Eliminating $\underline{y}_s$ from $\underline{f}_{1j}$ yields the state equations (5) to (8) where $\underline{f}_{1j}$ now includes time varying terms in $\underline{x}$.

$$\dot{\underline{x}}=A_o\underline{x}-\underline{f}_{15}(\underline{x},\underline{y}_m,r,u) \quad (5)$$

$$\dot{\underline{x}}=A_o\underline{x}-\underline{f}_{16}(\underline{x},\underline{y}_m,e,u,r) \quad (6)$$

$$\dot{\underline{x}}=A'_1\underline{x}-\underline{f}_{17}(\underline{x},\underline{y}_m,r,u) \quad (7)$$

$$\dot{\underline{x}}=A_1\underline{x}-f_{18}(\underline{x},\underline{y}_m,e,u,r) \quad (8)$$

Analogously it is possible to yield a number of state equations for configurations B to D.

Second case (n>m). In order to get a description of the adaptive system with an n^{th} order state vector $\underline{x}$, the differential equation describing the model has to be differentiated (n-m) times. The model is now described by the following equation:

$$\dot{\underline{y}}'_m = A'_o \underline{y}'_m + \underline{b}'_o \, r^{(n-m)}$$

where A'_o is a (nxm) matrix and b'_o a (nx1) vector of the form

$$A'_o = \left[\begin{array}{cccccc} 0 & 1 & 0 & \dots & & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \hline 0 & & & & A_o & \end{array}\right] \begin{array}{l} \left.\vphantom{\begin{array}{c}0\\0\end{array}}\right\} (n-m) \text{ rows} \\ \\ \end{array}$$

(n-m) columns

$$\underline{b}'_o = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_o \end{bmatrix} \quad (n-1) \text{ rows}$$

With the same assumptions as in the first case (m=n) the adaptive control system can be described now by the vector $\underline{x}=\underline{y}'-\underline{y}_s$.
There are still more degrees of freedom in this case for the elements in the last row of A'_o can be substituted by other elements as the following equations are also known

$$y_m^{(m+k)} + a_{n-1} y^{(m-1+k)} + \dots + a_o y_m^{(k)} = b_o r^{(k)}; \; o<k\leq n-m-1 \qquad (9)$$

Thus it is not only possible to get another configuration of matrix A'_o, but also vector $\underline{b}'_o$ is altered to a matrix which has to be multiplied by a vector consisting of elements $r, r^{(1)}, \dots r^{(n-m)}$.
It is possible to get a simplification if r is chosen to be a stepfunction. Then $\underline{b}=\underline{0}$ can be chosen. In [5] and [6] this simplification has been used. In view of the foregoing there are a large number of possibilities to describe the adaptive control system, depending on the structure (A to D), the splitting up of the time varying matrices A'(t) and A(t), the elimination of $\underline{y}_s$ and the difference in order of the differential equations describing the model and the system. This enables us, in the final design stage, to choose the solution best suited for the given condition.

2.1. The adaptive controller.

Choose the quadratic Liapunov function $V=\underline{x}^T P \underline{x}$ where P is a positive, definite and symmetric (nxn) matrix. Dependent on the splitting up of the matrices A(t) and A'(t) into a constant one and a time varying one, the time derivative of V is:

$$\dot{V}(\underline{x},t) = \underline{x}^T (A^T P + PA)\underline{x} - 2\underline{x}^T P \, \underline{f}_{ij} \qquad (10)$$

where A is the matrix A_o, A'_o, A_1 or A'_1, and $\underline{f}_{ij}$ depends on the configuration, the splitting up of the time varying matrix, etc.

$$\text{Define } -Q = A^T P + PA \qquad (11)$$

If by assumption A is stable then it is possible to choose a positive definite symmetric matrix Q. Since $\dot{V}(\underline{x},t)$ has to be negative definite, it is required that

$$2\underline{x}^T P \, \underline{f}_{ij} \geq 0 \qquad (12)$$

or with the given matrix representation

$2(p_{1n}x_1+p_{2n}x_2+\ldots+p_{nn}x_n)f_{ij}\geq 0$ (12a)

where $p_{1n}, p_{2n},\ldots, p_{nn}$ are elements of matrix P. Requirement (12) can be fulfilled by an adequate choice of $\underline{f}_{ij}$, for the free parameter u (the control signal) is included in $\underline{f}_{ij}$. A possible solution is to choose u to be a function of g_{ij} which includes the sum of all terms in $\underline{f}_{ij}$ except u, multiplied by the factor.

sign $(p_{1n}x_1+p_{2n}x_2+\ldots+p_{nn}x_n)$

Let the magnitude of each term in g_{ij} be greater than, or equal to, the magnitude of the corresponding term in $\underline{f}_{ij}$.

Therefore the control law is

$$u=g_{ij}\ \text{sign}\ \left(\sum_{i=1}^{n} p_{in}x_i\right) \qquad (13)$$

The control law (13) is not the only one fulfilling requirement (12). It is assumed that the transfer function of the system (including conventional controller) has at least one pure integration, so that in the steady state u=o when r is chosen to be a stepfunction.

The method of Liapunov can be used (the stability of motion in the neighbourhood of the equilibrium point $\underline{x}=\underline{o}$ is investigated) by the same input function to the system and the model and the same d-c gain for both. The effect of unequal d-c gain is investigated in [5].

For the third order time varying linear system and a second order constant linear model is found in [5].

$f_{ij}=c_1(t)x_1+c_2(t)x_2+c_3(t)x_3+c_4(t)y_{s2}+c_5(t)e+u$

Let $g_{ij}=c_1|x_1|+c_2|x_2|+c_3|x_3|+c_4|y_{s2}|+c_5|e|$

where $c_1,c_2,\ldots$ are the maximum values of $c_1(t),\ c_2(t),\ldots$

In this case the control law can be written as

$$u=\{c_1|x_1|+c_2|x_2|+c_3|x_3|+c_4|y_{s2}|+c_5|e|\}\ \ \text{sign}\ (p_{13}x_1+p_{23}x_2+p_{33}x_3) \qquad (14)$$

In this text the term g_{ij} will be called the modulus function, and the term sign $\left(\sum_{i=1}^{n} p_{in}x_i\right)$ the phase function, the reason for this will become clear in next section.

It can be seen from (14) that the phase function does <u>not</u> depend on the way the system is described, but is merely dependent on the choice of P and the order of the system. Furthermore, the modulus function depends on f_{ij}, i.e. on the way the system is described.

The speed of convergence of the adaptive operation can be estimated by the parameter η which is defined as

$$\eta=\min\left\{\frac{\dot{V}(\underline{x},t)}{V(\underline{x})}\right\} \quad \text{for all } \underline{x}\neq\underline{0}$$

therefore

$$V(\underline{x}) \leq V(\underline{x}_o)e^{-\eta(t-t_o)}$$

where $V(\underline{x}_o)$ is the value of $V(\underline{x})$ for the initial state $\underline{x}_o$ and $t=t_o$.

It is, however, quite difficult to calculate η and thus to get a measure of speed of convergence of the adaptive operation, for $\dot{V}(\underline{x},t)$ consists of a linear and a nonlinear term [5].

As appears from the forego..ng the control algorithm for the adaptive controller is by no means simple. As opposed to this much freedom is possible in the structure of the modulus function. The p_{in}-coefficients can be chosen within ample limits imposed by condition (12).

The choice of p_{1n} is, however, restricted by the matrix A. By a number of examples the control algorithms which arise from different $\underline{f}_{ij}$'s and configurations A to D are examined.
In section 4 one example is extensively examined. The conclusion can be drawn that all algorithms give excellent results and a difference in speed of convergence or response to a stepfunction of any other deterministic function can hardly be noticed.
The controller has generally the form as given in fig. 1 for the case where $\underline{y}_s$ is eliminated. In [5] the results of linearising the adaptive controller are mentioned. In some aspects the nonlinear controller has however advantages over the linear one (less effects of noise, saturation of the control signal already involved). In this paper a simplification of the adaptive controller is examined using the results of a similar investigation via the inverse describing function method.
The second method of Liapunov yields sufficient conditions for stability, so a certain simplification of the adaptive controller can be predicted intuitively. Possible simplifications of the modulus and phase function are examined and the following conclusions can be drawn.

1. Simplification of the modulus function. This function consists of the sum of terms in $\underline{x}$ and $\underline{w}$. Experiments on a variety of examples have shown for all configurations that the modulus function is not sensitive to the omission of terms other than those dependent on $\underline{x}$. Moreover the coefficients of the terms in $\underline{x}$ can be varied over a wide range or even be partially omitted.

2. Simplification of the phase function. Experiments have shown that the p_{in}-coefficients yielded by any given Liapunov function can be chosen rather freely. Variations in p_{in}-coefficients influence the adaptive operations slightly. Results are already shown in [5]. The influence of the p_{in}-coefficients is, however, quite significant when the adaptive control is not started with the same initial conditions of the system and the model (see fig. 7).
Though the p_{in}-coefficients can be chosen rather freely, none can be chosen as zero without worsening the adaptive operation.

3. Designing by using the I.D.F. method.
As in the foregoing a model-adaptive control system has to be designed in such a way that the output y_s of the system in which the parameters vary always equals the output y_m of the model, so $y_s = y_m \forall t$. Although there is more than one solution to this problem, we choose, as an example, the block diagram of fig. 2, which is analogous to configuration A in table I.
The output u of the adaptive controller (A.C.) must allow y_s to be equal to y_m. That means that the transfer function from y_m to y_s:

$$H = \frac{H_s(1+H_cH_m)}{H_m(1+H_s+H_cH_s)}$$

equals 1. This is the case for $H_c >> \frac{1}{H_m}$ and $H_c >> \frac{1}{H_s}+1$. However, these conditions mean that the model-adaptive system will probably be unstable (characteristic equation: $1+H_s+H_cH_s=0$). The controlled system can be stabilised if the phase shift of A.C. allows $\arg H_cH_s/(1+H_s)$ to be more than -180^o.
If A.C. is a linear filter, then it can be difficult to fulfil these requirements, because the interdependence of the phase and gain of linear filters generally is in conflict with the requirements. Therefore it would be preferable to design a filter of which the gain and phase shift can be adjusted independently of each other. The following explains how such a filter can be designed.

If $u_1=u_{10}+u_{11*}$ and $u_2=u_{20}+u_{21*}$, then: $u=u_1u_2=u_{10}u_{20}+u_{10}u_{21*}+u_{20}u_{11*}+u_{11*}u_{21*}$.

If it is assumed that u_{10} and u_{20} are dc-components, $u_{11*}=U_1\sin(\omega t+\phi_1)$ and $u_{21*}=U_2\sin(\omega t+\phi_2)$, then u consists of a dc-component, a fundamental harmonic and a second harmonic.

For $u_{11*}=0$ and $u_{20}=0$:

$u=u_{10}u_{21*}=u_{10}U_2\sin(\omega t+\phi_2)=U\sin(\omega t+\phi)$ where $U=u_{10}U_2$ and $\phi=\phi_2$.

If $x=X\sin\omega t$, then the transfer function from x to u is

$H_c=\frac{U}{X}e^{j\phi}=|H|e^{j\phi}$ in which $|H_c|=\frac{U}{X}$ and $\arg H_c=\phi$.

Now it is assumed that H_c can be split into two linear transfer functions and two describing functions in accordance with fig. 3. This block diagram is only related to the fundamental harmonics.

N_1 can be only a describing function of a nonlinearity with an even characteristic ($u_{11*}=0$). N_2 can be only a describing function of a nonlinear element with an odd characteristic ($u_{21*}\neq 0$).

Fig. 4a shows an even characteristic and fig. 4b shows an odd characteristic. In accordance with the foregoing

$$\left.\begin{aligned}&|H_c|=\frac{U}{X}=\frac{u_{10}U_2}{X}=u_{10}|N_2||H_2| \quad &\text{(a)}\\ &\text{and}\\ &\arg H_c=\phi=\arg H_2+\arg N_2 \quad &\text{(b)}\end{aligned}\right\} \qquad (15)$$

If $z_1=Z_1\sin(\omega t+\psi_1)$ and N_1 is related to fig. 4a, then $u_{10}=\frac{2cZ_1}{\pi}=\frac{2c|H_1|X}{\pi}$

or in accordance with (15a)

$$|H_c|=\frac{2c|H_1|X|N_2||H_2|}{\pi}$$

If it is required that $|H_c|=|H_1|$, then

$$\frac{2c\,X|N_2||H_2|}{\pi}=\frac{2cZ_2|N_2|}{\pi}=1$$

or

$$|N_2|=\frac{\pi}{2cZ_2} \qquad (16)$$

From this expression a nonlinear characteristic that belongs to N_2 can be determined by using the inverse describing function method [3].

Fig. 4b shows such a characteristic in which $b=\pi^2/8c$. This result is equivalent to the span filter in [7].

Other elements that can be used are shown in fig. 5.

By choosing these nonlinearities in this way we have obtained a transfer function H_c of which $|H_c|=|H_1|$ and $\arg H_c=\arg H_2+\arg N_2$.

That means the magnitude and the phase of H can be chosen independently of each other.

The path containing H_1 and N_1 in the block diagram of fig. 3 is called the modulus-path.

The path containing H_2 and N_2 is called the phase-path. The modulus-path and the modulus-function g_{ij} are equivalent, just as the phase-function sign $(\sum p_{in}x_i)$.

In spite of the fact that this blockdiagram contains nonlinear elements, the

transfer function H_c does not depend on the amplitude X and is only a function of ω. Therefore H_c can be seen as a linear transfer function.
If this block diagram is of the adaptive controller, then it is possible that y_s can be made equal to y_m and that the model-adaptive system will remain stable.
The following explains what the influence on H_c of the higher harmonics will be.
In accordance with fig. 3 in which the nonlinearities are defined by fig. 4:

$$u_1 = \frac{2cZ_1}{\pi}\left[1 + \frac{2}{3}\sin 2(\omega t+\psi_1) - \frac{2}{15}\sin 4(\omega t+\psi_1) + \frac{2}{35}\sin 6(\omega t+\psi_1)\ \ldots\right] \tag{17}$$

and

$$u_2 = \frac{4b}{\pi}\left[\sin(\omega t+\psi_2) + \frac{1}{3}\sin 3(\omega t+\psi_2) + \frac{1}{5}\sin 5\,(\omega t+\psi_2) + \ldots\right] \tag{18}$$

if $x = X\sin\omega t$, $z = Z_1\sin(\omega t+\psi_1)$ and $z_2 = Z_2\sin(\omega t+\psi_2)$.
These equations can also be written as: $u_1 = u_{10} + u_{12*} + u_{14*} + \ldots$ and $u_2 = u_{21*} + u_{23*} + u_{25*} + \ldots$
where u_{ij*} is the j^{th} harmonic in u_i (i=1 and i=2).
The output u equals:

$$u_1u_2 = \underline{u_{10}u_{21*}} + u_{10}u_{23*} + u_{10}u_{25*} + \ldots + \underline{u_{21*}\,u_{12*}} + \underline{u_{12*}u_{23*}} + u_{12*}\,u_{25*} + \ldots + u_{21*}u_{14*} + \underline{u_{23*}u_{14*}} + \underline{u_{14*}u_{25*}} + \ldots$$

The terms that have been underlined indicate the fundamental harmonic u_f in u. For this fundamental harmonic, for example, two approximations are possible:
1) $u = u_{10}u_{21*}$ and
2) $u = u_{10}u_{21*} + u_{21*}u_{12*} + u_{12*}u_{23*}$.

<u>First case</u>: $u = u_{10}u_{21*}$

In accordance with (17) and (18): $u_f \approx \frac{8bcZ_1}{\pi^2}\sin(\omega t+\psi_2)$.
The transfer function from x to u is

$$H_c = \frac{8bc}{\pi^2}\ |H_1|e^{j\psi_2}$$

or

$$H_c = |H_1|e^{j\psi_2}.$$

This expression is the same as that which was derived in the foregoing.

<u>Second case</u>: $u = u_{10}u_{21*} + u_{21*}u_{12*} + u_{12*}u_{21*}$.
In accordance with (17) and (18):

$$u_f \approx \frac{8bcZ_1}{\pi^2}\left[\sin(\omega t+\psi_2) + \frac{1}{3}\cos(\omega t+2\psi_1-\psi_2) + \frac{1}{9}\cos(\omega t+3\psi_2-2\psi_1)\right] \tag{19}$$

or

$$u_f \approx |H_1|X\,A\sin(\omega t+\delta)$$

The transfer function from x to u is $H_c = |H_1|Ae^{j\delta}$ (20)
where $A = \frac{g}{\cos\delta}$, $\mathrm{tg}\,\delta = \frac{d}{g}$, $g = \cos\psi_2 - \frac{1}{3}\sin(2\psi_1-\psi_2) - \frac{1}{9}\sin(3\psi_2-2\psi_1)$ and
$d = \sin\psi_2 + \frac{1}{3}\cos(2\psi_1-\psi_2) + \frac{1}{9}\cos(3\psi_2-2\psi_1)$.

In the tables II and III for some values of ψ_2 some values of A and δ have been given.

Table II. $\psi_1=0$

ψ_2	A	δ
0°	1,05	$18^\circ 26'$
45°	1,33	45°
90°	1,04	$71^\circ 34'$
135°	0,67	135°
180°	1,05	$198^\circ 26'$

Table III $\psi_1=90^\circ$

ψ_2	A	δ
0°	1,05	$18^\circ 26'$
45°	1,04	$63^\circ 26'$
90°	1,04	$108^\circ 26'$
135°	1,04	$153^\circ 26'$
180°	1,05	$198^\circ 26'$

In accordance with these tables and (20), $|H_c|$ is now not defined by H_1 only and arg H_c is not defined by H_2 and N_2 only. However, the influence of the higher harmonics in u_1 and u_2 on $|H_c|$ is relatively small. The phase shift of H_c is greater.
For these two cases we can conclude that themagnitude of H_c is mainly defined by H_1. The phase shift of H_c is defined by H_2 and N_2 and (to a lesser extent) by H_1.
It is possible to reduce the influence of the higher harmonics by using other nonlinearities. The extra phase shift of the modulus-path can be compensated for by taking double-valued non-linearities. This extra phase shift is favourable for the model-adaptive system in view of the stability. By application of this nonlinear filter as an adaptive controller it is necessary that the system will attenuate the higher harmonics as much as possible. Further the dc-component in x has to be zero. In the other case, there can be dc-components in z_1 and z_2. The latter means that the inputs of the nonlinearities are asymmetric instead of symmetric as has been assumed. The dc-components in z_1 and z_2 can be made zero by using differentiation circuits before the nonlinearities or by providing that x_o equals zero. The latter can occur by choosing $|H_1(0)|$ as large as possible. As has been mentioned before, the adaptive action has to be fast. That can be attained by adjusting the parameters in the phase path in the right way.
Remark. The above-mentioned nonlinear filter has been designed for configuration A in table I. From measurements it follows that similar filters can also be used in configurations B, C and D in table I.

4. Measurements.

The adaptive controllersthat have been designed by using the second method of Liapunov and the inverse describing function method have been tested in several model-adaptive systems. The responses to a stepfunction at the input of the model (second order) and of the controlled system were compared with each other. The parameters of the systems varied in a wide range. The systemsS that have been examined were of the first and third order. From these measurements it follows that these controllers have a good adaptive action.
As an example we shall give the measurement of a system in accordance with configuration A in table I. This system is defined by

$$\ddot{y}_m+b_1\dot{y}_m+b_o y_m = K'_m r \text{ and } \dddot{y}_s+a_2(t)\ddot{y}_s+a_1(t)\dot{y}_s = K'_s(t)\ (e+u)$$

where $0.2<K'_s(t)<0.9$, $0.8<a_1(t)<2$, $1.2<a_2(t)<2.4$, $b_o=K'_m=0.25$, $b_1=0.4$

and $r=3U(t)$.

The values of these parameters have been chosen experimentally.
From these equations it follows for the block diagram in fig. 2 that

$$H_m(s) = \frac{K'_m}{s^2+b_1s+b_o} \quad \text{and} \quad H_s(s) = \frac{K'_s}{s(s^2+a_2s+a_1)}$$

where the coefficients have the above-mentioned values.
Three cases have been considered:

1. $u=\{|c_1|x+c_2|\dot{x}|+c_3|\ddot{x}|+c_4|\dot{y}_s|+c_5|e|\}\text{sign } (p_{13}x+p_{23}\dot{x}+p_{33}\ddot{x})$
2. $u=|c_1x+c_2\dot{x}+c_3\ddot{x}+c_4y_s+c_5e| \text{ sign } (p_{13}x+p_{23}\dot{x}+p_{33}\ddot{x})$
3. $u=c_1|x| \text{ sign } (p_{13}x+p_{23}\dot{x}+p_{33}\ddot{x})$

where $c_1=0.75$, $c_2=1.2$, $c_3=3$, $c_4=7.15$ $c_5=1.5$, $p_{13}=0.3$, $p_{23}=1.3$, $p_{33}=1$.
The first two cases are related to the controller in accordance with the method of Liapunov. The third case is related to the controller in accordance with the inverse describing function method, by which $|H_c|=c_1$ and $\arg H_c=\arg (p_{13}+p_{23}s+p_{33}s^2)$.

First case. Fig. 6 shows the step responses:

1. y_m
2. y_s of the unstable uncontrolled system
3. y_s of the controlled system

It follows that the steady-state error is very small. This error is greater if $c_4\ \dot{y}_s$ and $c_5|e|$ are omitted.
Fig. 7 shows the step responses:

1. y_m
2. y_s of the stable uncontrolled system
3. y_s of the system of which the controller has been inserted at the arrow and $p'_{13}=30\ p_{13}$
4. as 3, but $p'_{23}=4p_{23}$
5. as 3, but $p'_{23}=2p_{23}$

The latter adjustment gives the best result. It can be explained by looking at the positions of the poles of H in the s-plane. After some calculations it seems that there are two dominating poles. The damping factor z that belongs to response 3 is less than the damping factors belonging to responses 4 and 5.

Second case. The same results were obtained as in the first case. However, this controller is not good with reference to the higher harmonics in u as the first controller.

Third case. Fig. 8 shows the step responses:

1. y_m
2. y_s of the controlled system and $c_1=200$
3. as 2, but $c_1=5$
4. as 2, but $c_1=100$.

By choosing c_1 as large as possible, the steady-state error can be made equal to zero.
Fig. 9 shows the step responses:

1. y_m
2. y_s of the controlled system and $p_{13}=0$
3. y_s of the controlled system and $p_{23}=0$
4. y_s of the controlled system and $p_{33}=0$

The controlled system will oscillate if $p_{33}=0$. If $p_{13}=0$, then there is a steady-state error that can be attenuated by choosing a larger c_1.
From these measurements it follows that these controllers can have a good adaptive action. The rate of convergence and the stability are determined by the parameters p_{13}, p_{23} and p_{33}. The steady-state error can be made equal to zero by a suitable choice of the gain in the modulus-path.

5. Concluding Remarks.

The controllers that have been designed by using the second method of Liapunov and the inverse describing function method, can be used very well in model-adaptive systems. They have been tested in several systems in which the model is of the second order and the system with time varying parameters is of the first or third order. These parameters may change simultaneously and with an unknown speed. If the signals in the system are bounded a stable and quickly converging adaptive system can be obtained also. The configuration of the controller in accordance with the inverse describing method is simpler than the other controllers. By using this method it is found that double-valued nonlinearities also may be used which will help the stability. By applying the method of Liapunov only single-valued nonlinearities are found. The controllers can be considered as elements of which the gain and phase shift can be adjusted independently of each other. The gain can be a function of the frequency. Because of this feature these controllers are suitable for use as compensation networks to obtain some ends that cannot be realized by linear networks. This research has been done for continuous linear time dependent systems.
With reference to nonlinear system measurements, these controllers can be used also to reduce nonlinear effects.

Acknowledgement.

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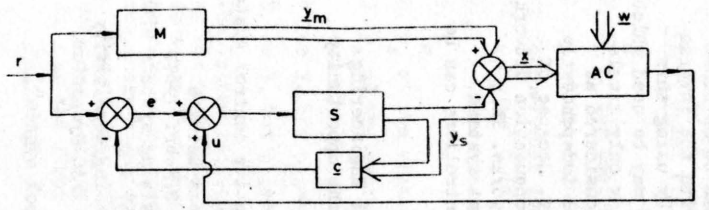
TABLE I

A



model: $\dot{\underline{y}}_m = A_o \underline{y}_m + \underline{b}_o r$

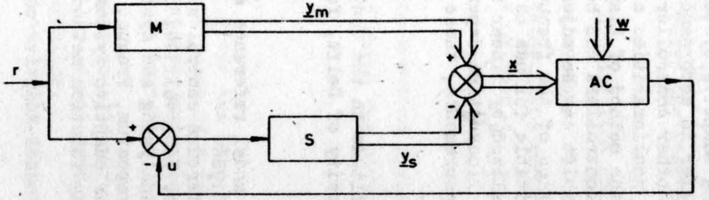
system: $\dot{\underline{y}}_s = A(t) \underline{y}_s + \underline{b}(t)(u + e)$

or $\dot{\underline{y}}_s = A'(t) \underline{y}_s + \underline{b}(t)(u + r)$

with $A'(t) = A(t) - \underline{b}(t) . \underline{c}^T$

and $e = r - \underline{c}^T . \underline{y}_s$

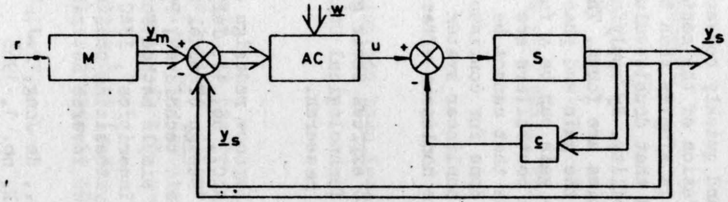
B



model: $\dot{\underline{y}}_m = A_o \underline{y}_m + \underline{b}_o r$

system: $\dot{\underline{y}}_s = A(t) \underline{y}_s + \underline{b}(t)(u + r)$

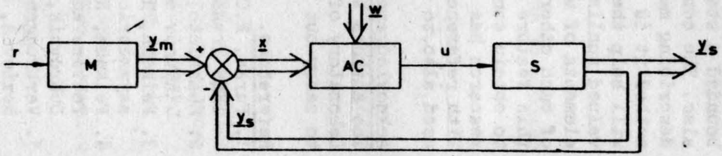
C



model: $\dot{\underline{y}}_m = A_o \underline{y}_m + \underline{b}_o r$

system: $\dot{\underline{y}}_s = A'(t) \underline{y}_s + \underline{b}(t) u$

D



model: $\dot{\underline{y}}_m = A_o \underline{y}_m + \underline{b}_o r$

system: $\dot{\underline{y}}_s = A(t) \underline{y}_s + \underline{b}(t) u$

Table I:

M is the model
S is the time varying system
AC is the adaptive controller
r is the reference input
e is the error signal between reference input and system output
u is the adaptive control signal
$\underline{y}_m$ is the (nx1) state vector of the model
$\underline{y}_s$ is the (nx1) state vector of the system

$\underline{w}$ is a vector containing all signals not directly related to $\underline{x}$

A_o is a constant (nxn) matrix

$\underline{b}_o$ is a constant (nx1) vector

A(t) and A'(t) are time varying (nxn) matrices

$\underline{b}$(t) is a time varying (nx1) vector

$\underline{c}$ is a constant (nx1) vector.

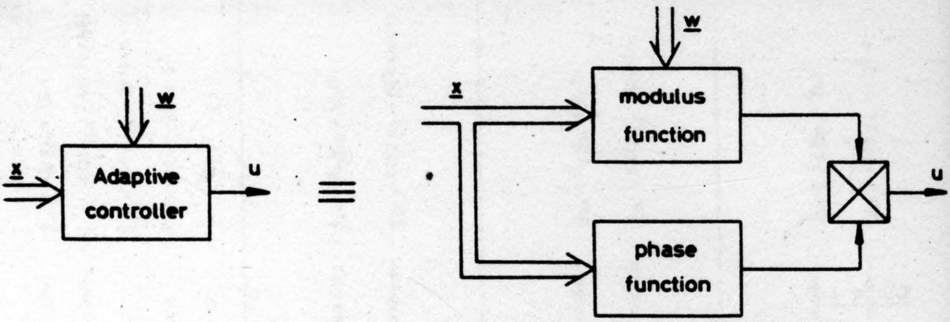

w
x
Adaptive
controller
u
w
x
modulus
function
phase
function
u

fig. 1

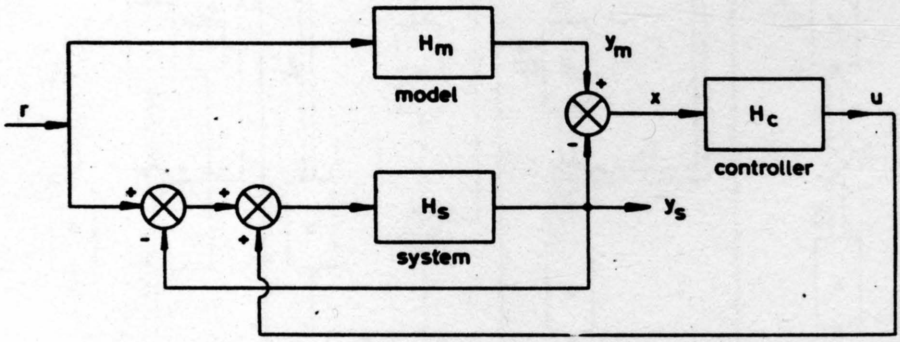

Hm
model
ym
r
x
Hc
controller
u
Hs
system
ys

fig. 2

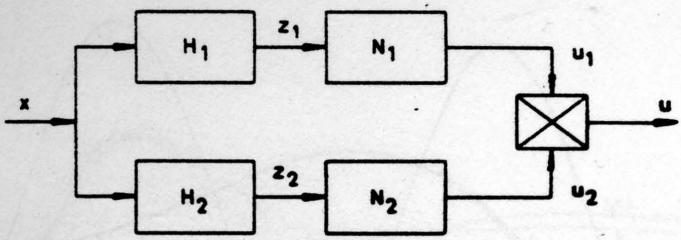


fig 3

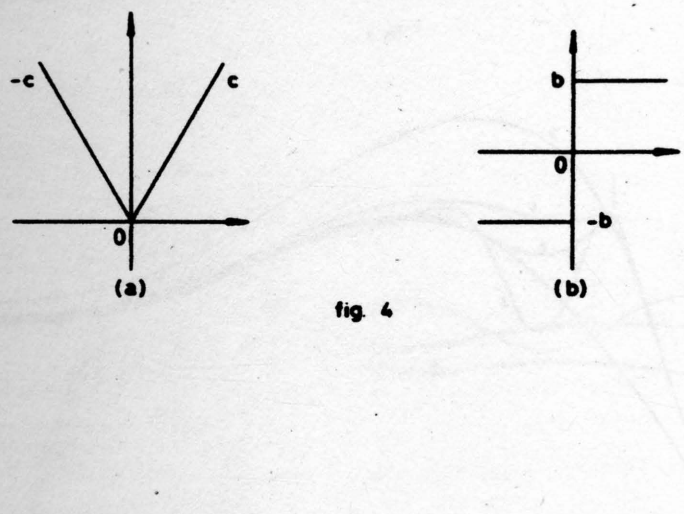


fig. 4

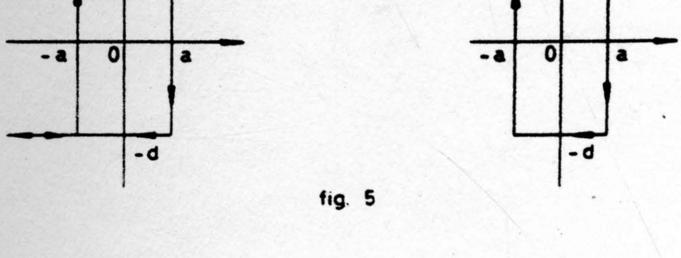


fig. 5

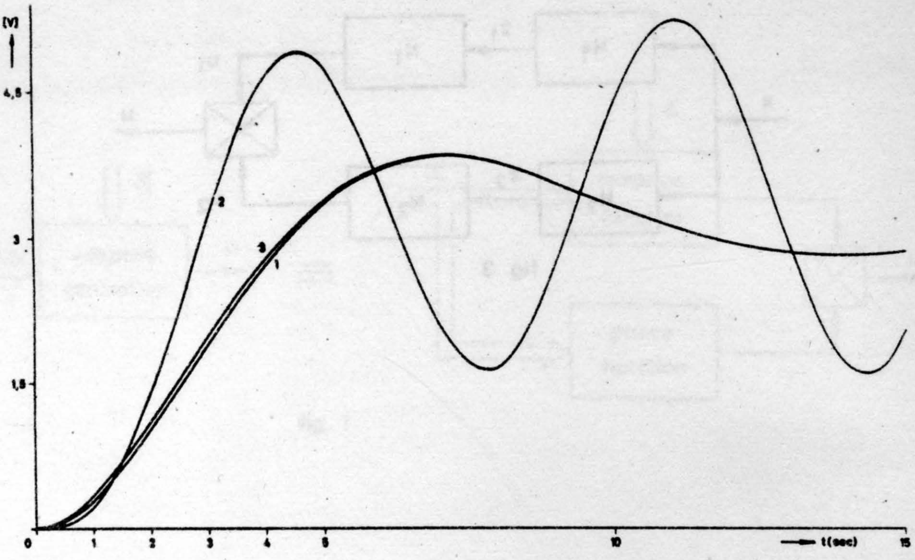


fig. 6

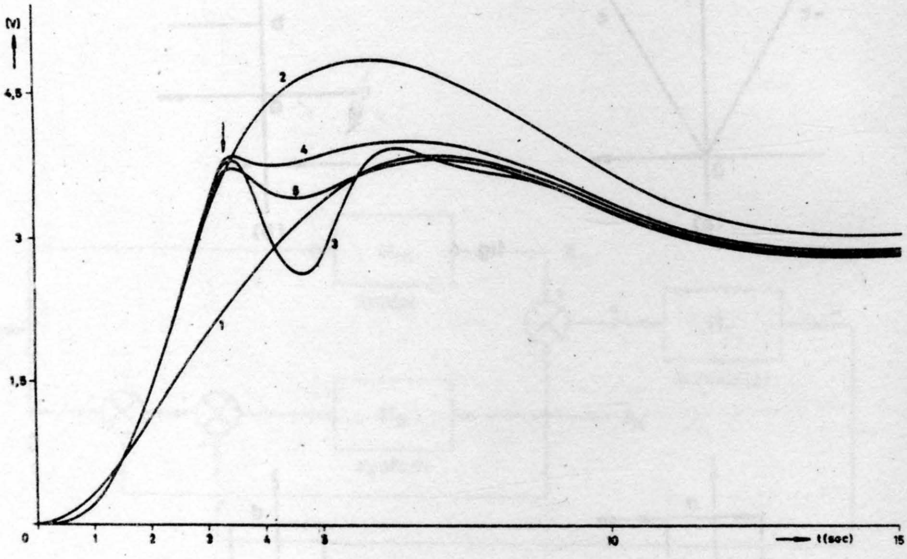


fig. 7

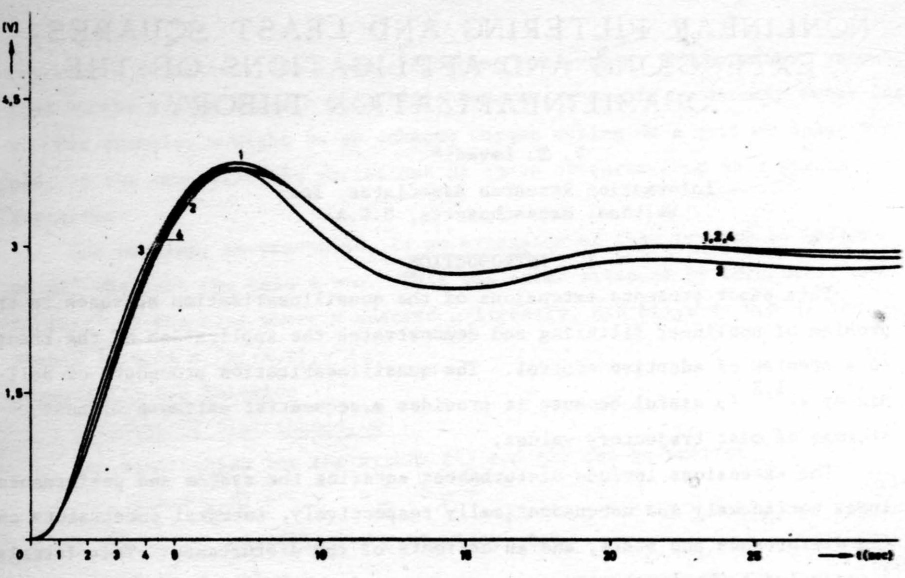

(V)
4,5
3
1,5
0
1
2
3
4
5
10
15
20
25
t(sec)
1
2
3
4
1,2,4
3

fig. 8

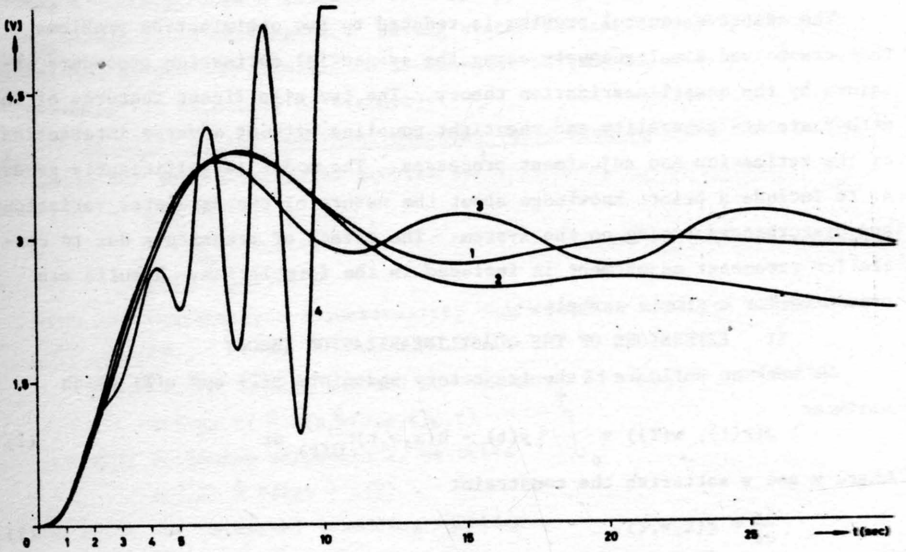

(V)
4,5
3
1,5
0
1
2
3
4
5
10
15
20
25
t(sec)
1
2
3
4

fig. 9

NONLINEAR FILTERING AND LEAST SQUARES-EXTENSIONS AND APPLICATIONS OF THE QUASILINEARIZATION THEORY

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I. INTRODUCTION

This paper presents extensions of the quasilinearization approach to the problem of nonlinear filtering and demonstrates the application of the theory to a problem of adaptive control. The quasilinearization procedure of Bellman et al[1,2] is useful because it provides a sequential estimate without storage of past trajectory values.

The extensions include disturbances entering the system and performance index nonlinearly and nonquadratically respectively, integral constraints on the disturbance and state, and an estimate of the disturbance. This formulation includes the least squares counterpart of the statistical filtering and estimation problem for "colored" noise, and the system with "randomly" varying parameter.

The adaptive control problem is reduced to two optimization problems that are solved simultaneously using the sequential estimation procedure obtained by the quasilinearization theory. The two significant features of the method are its generality and the tight coupling without adverse interaction of the estimation and adjustment processes. The model is sufficiently general to include a priori knowledge about the nature of the parameter variations and disturbances acting on the system. The effect of transients due to controller parameter adjustment is included in the formulation. Results are presented for a simple example.

II. EXTENSIONS OF THE QUASILINEARIZATION THEORY

We seek an estimate of the trajectory endpoints x(T) and w(T) which minimize

$$J(x(t), w(T)) = \int_0^T |y(t) - h(x,w,t)|^2_{Q(t)}\, dt \tag{1}$$

where x and w satisfies the constraint

$$\frac{dx}{dt} = g(x,w,t). \tag{2}$$

The estimates are designated $\hat{x}(T)$ and $\hat{w}(T)$, respectively.

Here y is a known observation or data vector; h and g are known functions and Q is a known symmetric nonsingular weighting matrix. The variable w

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represents an unknown disturbance entering the system, in the manner specified by the system dynamic model (2); however, there is no dynamic model for w. For example, w might be an unknown torque acting on a gyro or space vehicle, or the unpredictable variations or cause of variations in a system parameter.

The problem, as presented, is an extension of that treated by Bellman et al[1] who took the case $w = 0$. This was later extended by Detchmendy and Sridhar[2] to the case where w entered g linearly, and $h(x,w,t)$ was of the form $h'(x,w,t) = [h_1'(x,t), w']$.

Formulation and Solution

Estimate of Disturbance, w

The Hamiltonian for the system (1) and (2) can be written[3]

$$H(x,w,\psi,t) = -|y - h|_Q^2 + \psi' g \tag{3}$$

A necessary condition for optimality[3] is*

$$H_w(x,w,\psi,t) = 0 \tag{4}$$

From the Implicit Function Theorem[4], (4) implicitly defines the optimum $w = \hat{w}(x,\psi,t)$ as a function of x and ψ, providing that the matrix $H_{ww}(x,w,\psi,t)$ is nonsingular. Having stated the conditions under which w exists, we proceed formally to develop the estimation equations for w, assuming the existence of $w(x,\psi,t)$.

Solution of the Two Point Boundary Value Problem - The Hamiltonian system for the optimization problem is[3]

$$\dot{\psi} = -H_x(x,w,\psi,t) \tag{5}$$

$$\dot{x} = H_\psi(x,w,\psi,t) \tag{6}$$

$$\psi(0) = \psi(T) = 0 \tag{7}$$

where (7) follows from transversality conditions at the free endpoints $x(0)$ and $x(T)$.

Furthermore, having assumed the existence of $w(x,\psi,t)$, we define

$$H^*(x,\psi,t) \triangleq H(x,w(x,\psi,t)\psi,t) \tag{8}$$

Now, following Reference 2, we write

$$r(c,T) \triangleq P(T)c + x(T) \tag{9}$$

and solve the invariant inbedding equation

$$\frac{\partial r}{\partial T} = \left(\frac{\partial r}{\partial c}\right)' H_r^*(r,c,T) + H_c^*(r,c,T) \tag{10}$$

at $c = 0$.

---

*The notation convention for differentiation with respect to vectors is described in the Appendix.

Substituting (9) into (10) gives

$$\frac{dP}{dT} c + \frac{d\hat{x}}{dT} = P\, H^*_r(r,c,T) + H^*_c(r,c,T) \tag{11}$$

Differentiating (11) with respect to c gives

$$\frac{dP}{dT} = P\, H^*_{rr}(r,c,T)\, P + P\, H^*_{cr}(r,c,T) + H^*_{rc}(r,c,T)\, P + H^*_{cc}(r,c,T) \tag{12}$$

Evaluating (11) and (12) at c = 0 gives the estimation equations

$$\frac{d\hat{x}(T)}{dT} = P(T)\, H^*_x(\hat{x},0,T) + H^*_\psi(\hat{x},0,T) \tag{13}$$

$$\begin{aligned}\frac{dP(T)}{dT} = {} & P(T)\, H^*_{xx}(\hat{x},0,T)\, P(T) + P\, H^*_{\psi x}(\hat{x},0,T) \\ & + H^*_{x\psi}(x,0,T)\, P + H^*_{\psi\psi}(\hat{x},0,T)\end{aligned} \tag{14}$$

The equation for the disturbance w can be written in terms of the implicit function $\hat{w}(x,\psi,T)$, evaluated at $\psi(T) = 0$.

$$\begin{aligned}\frac{d\hat{w}(T)}{dT} &= \hat{w}_x(\hat{x},0,T)\, \frac{dx(T)}{dT} + \hat{w}_\psi(\hat{x},0,T)\, \frac{d\psi(T)}{dT} + \hat{w}_T(\hat{x},0,T) \\ &= \hat{w}_x(\hat{x},0,T)\, \frac{d\hat{x}(T)}{dT} + \hat{w}_T(\hat{x},0,T)\end{aligned} \tag{15}$$

where the last of (15) follows from the fact that $\psi(T) = 0$.

Equations (13)-(15) are the basic estimation equations. When (4) can be solved explicitly for $\hat{w}(x,\psi,T)$, $H^*(x,\psi,T)$ is an explicit function, and (13) and (14) are sufficient for the estimation. In the case where (4) cannot be solved explicitly for $\hat{w}(x,\psi,T)$, (15) must be adjoined to the system. Furthermore, in the latter case, $H^*(x,\psi,T)$ is not known explicitly, and it is necessary to express (13)-(15) in terms of $H(x,w,\psi,T)$.

<u>Estimator Results</u> - It follows from (8) that

$$H^*_a(x,\psi,t) = H_a(x,\psi,\hat{w}(x,\psi,t),t) + \hat{w}_a H_w \tag{16}$$

and

$$H^*_{ab} = H_{ab} + H_{wb}\hat{w}_a + \hat{w}_b H_{ww}\, \hat{w}_a + \hat{w}_b\, H_{ab} + \hat{w}_{ab}\, H_w \tag{17}$$

where (a,b) represent any combination of x and ψ.

The partial derivatives of w are obtained from the Implicit Function Theorem[4]

$$\hat{w}_\psi = - H_{w\psi} H^{-1}_{ww} \tag{18}$$

$$\hat{w}_x = - H_{wx} H^{-1}_{ww} \tag{19}$$

$$\hat{w}_T = - H^{-1}_{ww}\, H_{wT} \tag{20}$$

Using (3), (4) and (16)-(20) in (13)-(15) and evaluating at $\psi = 0$ gives the following estimator equations:

$$\frac{d\hat{x}}{dT} = g(x,w,T) + P\, R_x = g + 2Ph_x Q(y-h) \tag{21}$$

$$\frac{d\hat{w}}{dT} = - R_{ww}^{-1} \left(R_{xw} \frac{d\hat{x}}{dT} + R_{wT} \right) \tag{22}$$

$$\begin{aligned} \frac{dP}{dT} &= (g_x' - g_w' R_{ww}^{-1} R_{xw}) P + P(g_x - R_{wx} R_{ww}^{-1} g_w) \\ &+ P(R_{xx} - R_{wx} R_{ww}^{-1} R_{xw}) P - g_w' R_{ww}^{-1} g_w \end{aligned} \tag{23}$$

where

$$R(x,w,T) \triangleq H(x,w,0,T) = - |y - h(x,w,T)|^2_{Q(T)} \tag{24}$$

$$R_a(x,w,T) = 2h_a Q(y-h) \tag{25}$$

$$R_{ab}(x,w,T) = H_{ab}(x,w,0,T) = 2h_{ab} Q(y-h) - 2h_a Qh_b' \tag{26}$$

and a or b represent x,w, or T.

Equations (21)-(23) are the required estimator equations, although they are not necessarily expressed in the most convenient form. They form the basis for the remainder of the discussion. They are equivalent to the results of References 1 and 2 except w is included, and there is a factor of 2 which can be eliminated by substitution of Q/2 for Q.

Corollaries and Special Cases

Elimination of Matrix Inversion - The estimator equations involve computation of the matrix inverse R_{ww}^{-1}. The computation of this inverse can be replaced by a differential equation. Using the matrix identity

$$dA^{-1} = - A^{-1} dA A^{-1} \tag{27}$$

gives

$$\frac{dR_{ww}^{-1}}{dT} = - R_{ww}^{-1} \frac{dR_{ww}}{dT} R_{ww}^{-1} = - R_{ww}^{-1} [R_{wwx} \frac{dx}{dT} + R_{www} \frac{dw}{dT} + R_{wwT}] R_{ww}^{-1}. \tag{28}$$

Time Derivatives of Observation - Equation (22) for the estimate $\hat{w}$ contains the term

$$R_{wT} = 2 \frac{\partial}{\partial T} [h_w Q(y-h)] = 2[h_w Q \frac{dy}{dT} + \frac{\partial}{\partial T}(h_w Q) y - \frac{\partial}{\partial T}(h_w Qh)] \tag{29}$$

which requires the time derivative, dy/dt of the observation, y. A sufficient condition for this derivative to be unnecessary is that $h_w(x,w,T) Q \frac{dy}{dT}$ be identically zero. This is satisfied if h is independent of w.

Furthermore, if the performance index is of the form

$$J = \int \left[|y-h_1(x,t)|^2_{Q_1(t)} + |h_2(w,t)|^2_{Q_2(t)} \right] dt \tag{30}$$

Then

$$R(x,w,T) = |y-h_1(x,t)|^2_{Q_1} + |h_2(w,t)|^2_{Q_2} \tag{31}$$

and (29) becomes independent of $\frac{dy}{dT}$.

<u>Performance Index Quadratic in w</u> - Consider a performance index of the form

$$J = \int_o^T [|y(t)-h_1(x,t)|^2_{Q_1(t)} + |A(x,t)w|^2_{Q_2(t)}]dt \tag{32}$$

where A(x,t) is nonsingular. Writing

$$H(x,w,\psi,t) = - |y-h_1|^2_{Q_1} - |A_w|^2_{Q_2} + g'\psi, \tag{33}$$

equation (4) gives

$$H_w = - 2A' Q_2A\hat{w} + g_w'\psi = 0 \tag{34}$$

Since $\psi(T) = 0$, it follows that

$$\hat{w}(T) = 0. \tag{35}$$

The problem of Detchmendy and Sridhar falls into this class. It should be remembered that (35) indicates that the endpoint of the disturbance trajectory is zero, but not that the entire disturbance is zero.

<u>Disturbance Satisfying Integral Constraints</u> - Although the unknown disturbance w is not constrained by a differential equation, it may be that the average power of w is known; e.g., $\frac{1}{T}\int_o^T |w|^2 dt = a$. To treat this and similar cases we adjoin to the system (1) and (2), a constraint

$$\int_o^T f(x,w)dt = k(T), \tag{36}$$

and define the new state variable

$$x_o(t) = \int_o^T f(x,w)d\tau \tag{37}$$

Then the Hamiltonian for the system (1),(2) and (37) becomes

$$H(x,x_o,\psi,\psi_o,w,t) = - |y-h(x,w,t)|^2_Q + \psi'g + \psi_o'f \tag{38}$$

and the necessary conditions for optimality become (5),(6),(7) and

$$\dot{x}_o = H_{\psi_o} \tag{39}$$

$$\dot{\psi}_o = -H_{x_o} \tag{40}$$

$$x_o(0) = 0 \tag{41}$$

$$x_o(T) = k(T) \tag{42}$$

By making the substitution of variables

$$x_1 \triangleq \begin{bmatrix} -\psi_o \\ x \end{bmatrix} \tag{43}$$

$$\psi_1 \triangleq \begin{bmatrix} x_o - k \\ \psi \end{bmatrix} \tag{44}$$

and defining

$$H_1(x_1,\psi,w) \triangleq H(x,x_o,\psi,\psi_o,w,t) \tag{45}$$

the necessary conditions for optimality (5)-(7) and (39)-(42) reduce to

$$\dot{x}_1 = H_{1,\psi_1} \tag{46}$$

$$\dot{\psi}_1 = -H_{1,x_1} \tag{47}$$

$$\psi_1(0) = \psi_1(T) = 0 \tag{48}$$

This is the same form as (5)-(7), so that the solution (21)-(23) carries over with an appropriate change of variables.

In addition to energy constraints on the unknown disturbance, w, it may be physically consistent to model the disturbance as having certain frequency spectral properties or correlation properties, as is assumed in the statistical estimation problem. This can be done, approximately, by treating w as if it were a "white" noise, since it is unconstrained by any dynamic relationship. Then, use w to drive a "filter" with the desired spectral or time properties. The output of the filter then is used as the disturbance acting on the system.

For example, suppose the observation of the state of the system $\dot{x}_1 = g_1(x_1)$ is corrupted by noise, x_2, which is assumed to be derived from white noise, w, passing through a filter, $\dot{x}_2 = g_2(x_2, w)$, where the average power of w is 1. Figure 1 illustrates the situation. If the observation y is the noise-corrupted measurement of x_1, the criterion function becomes $J = \int_0^T |y-x_1|_Q^2 \, dt$ and it is constrained by $\dot{x}_1 = g_1(x_1)$, $\dot{x}_2 = g_2(x_2, w)$, and $\int_0^T |w|^2 \, dt = T$.

<u>Choice of Q(t)</u> - For some applications it may be desirable to weight the most recent data more heavily than past data. In such a case, the weighting matrix should be of the form Q(T,t) which tends to decrease as T-t increases. One such form is the exponential

$$Q(T,t) = Q_a \, e^{-\alpha(T-t)} \tag{49}$$

where α is a positive scalar and Q_a is a constant matrix.

Then, minimization of

$$J_a = \int_0^T |y-h|^2_{Q(T,t)} dt = e^{-\alpha T} \int_0^T |y-h|^2_{Q_a} \, e^{\alpha t} \, dt \tag{50}$$

provides the heavier weighting of the more recent data. However, since $e^{-\alpha T}$ is a constant, minimization of J_a is equivalent to minimizing

$$J = e^{\alpha T} J_a = \int_0^T |y-h|^2_{Q_a} \, e^{\alpha t} \, dt \tag{51}$$

Thus, choosing

$$Q(t) = Q_a \, e^{\alpha t} \tag{52}$$

gives the desired heavier weighting of the most recent data.

III. APPLICATION TO ADAPTIVE CONTROL

The adaptive control problem is defined herein as the on-line adjustment of controller parameters to compensate for parameter changes in the controlled system. Specifically, let the combination of controlled plant and controller be define by

$$\dot{x} = g(x,w,u,a,b,t) \qquad (53)$$

where x is the state of the combined controller and plant; u is the known system input; a is the unknown fixed system parameter; b is the known and adjustable parameter (usually belonging to the controller), and w is an unknown variation or disturbance acting on the system. Since u and t are known, they will not be explicitly expressed as independent variables for the remainder of the discussion.

The unknown parameter a represents all fixed but unknown parameters in the controlled plant. The variable w represents the unknown "white noise-like" variations driving the system. Disturbance w may be constrained by power or similar integral constraint, as discussed in Section II. Unknown variations which have known spectral properties can be derived from w by using a spectral filter, as described in Section II. These "filtered" variations are part of the state vector, x.

The variable w and resulting filtered variations are more than measurement noise; they represent all the unknown variations in the system. For example, a plant parameter, p, might be known to have a quadratic variation with time plus an additive unknown component, x_2, derived from w by a spectral filter. Then $p = a_1 + a_2 t + a_3 t^2 + x_a$ would represent the parameter with a_1, a_2, and a_3 fixed unknown parameters.

Since parameter a is constant one may adjoin to (53) the equation

$$\dot{a} = 0 \qquad (54)$$

<u>Formulation of the Adaptive Control Problem</u>

The general procedure, illustrated in Figure 2, is a two stage process consisting of plant parameter estimation and control parameter design. The controller design procedure computes the "best" value, $\hat{b}$, for the adjustable system parameter b. Thus,

$$b = \hat{b} \qquad (55)$$

for the controlled system. The controller ırameter $\hat{b}$ is chosen to make the observed system response x behave like a model response, y_m. The controller design procedure is based upon the "best" current estimate $\hat{x}$, $\hat{a}$ and $\hat{w}$ of the controlled system state, unknown parameter, and disturbance, respectively.

<u>Parameter, State, and Disturbance Estimation</u> - The input to the estimation procedure for $\hat{x}$, $\hat{w}$ and $\hat{a}$ is the observed system response y_s, the <u>known</u> value of the adjustable controller parameter $\hat{b}$, and the command input u. The estimates $\hat{x}_a$, $\hat{a}$, $\hat{w}$, of the system variables x, a, w, are chosen to minimize

$$J_a(x_a,a,w) = \int_0^T |y_s - h_a(x_a,w)|^2_{Q_a(t)}\, dt \tag{56}$$

where y_s is a specified function of the observed system responses; h_a is a corresponding function of the hypothesized system state; and Q_a is a weighting matrix.

Using the model (53), the minimization is constrained by

$$\dot{x}_a = g(x_a,w,a,\hat{b}) \tag{57}$$

and (54) (and integral constraints if appropriate).

To simplify the notation now define an augmented state and system

$$x_A \triangleq \begin{bmatrix} \dot{x}_a \\ \dot{a} \end{bmatrix} = \begin{bmatrix} g(x_a,w,a,\hat{b}) \\ 0 \end{bmatrix} \triangleq g_A(x_A,\hat{x}_B,w) \tag{58}$$

where $\hat{x}_B' = [\hat{x}_b' \mid \hat{b}']'$ is a vector, to be defined later, which contains $\hat{b}$ as one component, just as x_A contains parameter a as a component. Thus, the estimation consists of minimizing

$$J_A(x_A,w) = \int_0^T |y_s - h_A(x_A,w)|^2_{Q_a}\, dt \triangleq \int_0^T |y_s - h_a(x_a,w|^2_{Q_a} \tag{59}$$

subject to constraint (58).

<u>Control Parameter Design</u> - The controller parameter, b, will be chosen so that the controlled system response behaves like a model response, y_m. The following controller design criterion will be used: <u>The optimum value of the control parameter at time T is that fixed value that would have minimized the performance index $J_b(x,b) = \int_0^T |y_m - h_b(x)|^2_{Q_b(t)}\, dt$ assuming that the actual system state, parameters and disturbance are the same as the estimates.</u> By proper choice of the weighting matrix Q_b, the most recent performance can be most heavily weighted in the performance index as described in Section II.

According to this criterion, we choose $b = \hat{b}$ to minimize

$$J_b(x_b,b) = \int_0^T |y_m - h_b(x_b)|^2_{Q_b}\, dt \tag{60}$$

where x_b and b are constrained by

$$\dot{x}_b = g_1(x_b,\hat{w},u,\hat{a},b,t) \tag{61}$$

$$\dot{b} = 0 \tag{62}$$

Since $\hat{a}$ is a component of $\hat{x}_A$, we can define the augmented state and system

$$\dot{x}_B \triangleq \begin{bmatrix} \dot{x}_b \\ \dot{b} \end{bmatrix} = \begin{bmatrix} g_1(x_b,\hat{v},\hat{w},\hat{a},b,u,t) \\ 0 \end{bmatrix} \triangleq g_B(x_B,\hat{x}_A,\hat{w}) \tag{63}$$

and minimize

$$J_B(x_B) = \int_0^T |y_m - h_B(x_B)|^2_{Q_b} dt \triangleq \int_0^T |y_m - h_b(x_b)|^2_{Q_b} dt \tag{64}$$

subject to the differential constraint (63).

<u>Combined Estimation and Control</u> - The preceding results can be combined to give an on-line procedure for control design illustrated in Figure 2. The controller design procedure gives the parameter $\hat{b}$; for given estimates of the plant parameter $\hat{a}$ and disturbance $\hat{w}$. Similarly, the estimation procedure provides the plant parameter estimate $\hat{a}$ and disturbance estimate $\hat{w}$ given any history of the controller parameter b. The proposed method has the obvious advantage of its generality.

A more important advantage is that any adverse interaction between the estimation process and control design process is, at worst, a second order effect. The value $\hat{b}$ used in the estimation is always the current and true value of the controller parameter in the actual system. Thus, transients on the observed system output, y_s, due to adjustment of $\hat{b}$ effects are accounted for in the estimation procedure. Adjustment of $\hat{b}$ has no adverse effects on the estimation process. So, even though errors in the estimation process may result in a non-optimum value of $\hat{b}$, this non-optimum value does not introduce new errors into the estimation.

The proposed scheme requires no limitation on rate of adjustment of the parameter $\hat{b}$. Actually, transients induced due to changes in b may improve the accuracy of the estimator, particularly during periods when the system output is small for a long interval.

<u>Sequential Solution</u>

The quasilinearization procedure of Section II can be used to provide a sequential solution of these optimization problems that is well-suited to real time implementation. Applying the results of Section II to the optimization problems of (63), (67) and (74),(75), gives

$$\frac{d\hat{x}_A}{dT} = g_A(\hat{x}_A,\hat{x}_B,\hat{w}) + P_A R_{A;x_A} \tag{76}$$

$$\frac{d\hat{w}}{dT} = -R^{-1}_{A;ww} \left[R_{A;x_A w} \frac{d\hat{x}_A}{dT} + R_{A;wT} \right] \tag{77}$$

$$\frac{dP_A}{dT} = \left[g'_{A;x_A} - g'_{A;w}R^{-1}_{A;ww}R_{A;x_Aw}\right]P_A + P_A\left[g_{A;x_A} - R_{A;wx_A}R^{-1}_{A;ww}g_{A;w}\right]$$

$$+ P_A\left[-R_{A;x_Ax_A} - R_{A;wx_A}R^{-1}_{A;ww}R_{A;x_Aw}\right]P_A - g'_{A;w}R^{-1}_{A;ww}g_{A;w} \quad (78)$$

$$\frac{d\hat{x}_B}{dT} = g_B(\hat{x}_B, \hat{x}_A, \hat{w}) + P_B R_{B;x_B} \quad (79)$$

$$\frac{dP_B}{dT} = g'_{B;x_B}P_B + P_B g_{B;x_B} - P_B R_{B;x_Bx_B}P_B \quad (80)$$

where

$$R_A = - \left|y_A - h_A(x_A, w)\right|^2_{Q_A} \quad (81)$$

$$R_B = - \left|y_B - h_B(x_B)\right|^2_{Q_B} \quad (82)$$

<u>Example</u>

Illustrated in Figure 3, is a very simple adaptive control problem. A second order plant with unknown gain, a, is connected in a servo system. The plant output x_1 but not the derivative $\dot{x}_1 = x_2$ is measurable. The input, u, is known, The controller consists of an adjustable gain, b, which is to be set so that the closed loop system behaves like the model of Figure 4. The optimum fixed gain controller is to be found to satisfy the criterion

$$J_b = \int_0^T \left|y_m - x_b\right|^2_{Q_b} dt \quad (83)$$

where

$$Q_b(t) = \begin{bmatrix} e^{\beta_1 t} & 0 \\ 0 & e^{\beta_2 t} \end{bmatrix} \quad (84)$$

and y_m is the observed output of the model. The plant parameter is fixed, and the optimal estimate, using only the observable state x_1, is that which minimizes

$$J_A = \int_0^T (y_s - x_{a,1})^2 e^{\alpha t} dt \quad (85)$$

and y_s is the actual observed output of the plant, x_1. The exponential form for matrices Q_a and Q_b is used to provide heavier weighting of the most recent data, as described earlier. The terms, according to previous definitions are:

$$g_A = \begin{bmatrix} x_{a,2} \\ -x_{a,2} \end{bmatrix} + ab(u - x_{a,1}) \quad (86)$$

$$h_A = x_{a,1} \quad (87)$$

$$g_B = \begin{bmatrix} x_{b,2} \\ -x_{b,2} \end{bmatrix} + ab(u-x_{b,1}) \tag{88}$$

$$h_B = \begin{bmatrix} x_{b,1} \\ x_{b,2} \end{bmatrix} \tag{89}$$

Correct adjustment of the controlled system occurs when $a\hat{b} = 4$. Since $a = 2$, the desired value for $\hat{b}$ is 2 and the correct value of a is 2. In Figure 5, the initial values of $\hat{a}$ and $\hat{b}$ were both 1, and a step input was applied to the system. Both $\hat{a}$ and $\hat{b}$ converge to the optimal values of 2. The transient in the adjustment process is due in part to the non-optimal choice of initial conditions for P_A and P_B. The accelerated convergence of $\hat{a}$ occurring at $t = 6$ is due to the step change of the input at that instant.

IV. CONCLUSION

The quasilinearization procedure for nonlinear least squares filtering and estimation has been generalized to include unknown disturbances entering the system and the cost functional nonlinearly and nonquadratically, respectively. A sequential estimation scheme is obtained for the disturbance. In the case where the performance index is quadratic in the disturbance, the estimate of the end point of the disturbance is zero. This is true, also, for the linear quadratic case, also.

In its most general form the procedure requires the computation of derivatives of the observation; however a large number of practical cases satisfies the sufficient conditions for the derivative not to be required.

In the application to on-line adaptive control, the formulation is sufficiently general that it permits inclusion of practically all a priori knowledge about the unknown variations such as noise characteristics or random variations of parameters.

The method avoids a serious pitfall of some adaptive schemes requiring parameter identification, in which the adjustment of the controller parameter must be conducted independently of the plant parameter estimation. The method discussed in this memo provides simultaneous adjustment and estimation of parameters. There is no adverse interaction, because all adjustments of controller parameters are accounted for in the parameter estimation.

The "retrospective" definition of the optimal controller is one illustrative example of a design criterion. Predictive schemes might be used as well.

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APPENDIX

The following notation is used for differentiation with respect to vectors:

Let α be a scalar. Then $\frac{\partial \alpha}{\partial x}$ is a vector if x is a vector, having i^{th} component

$$\frac{\partial \alpha}{\partial x} = \alpha_x = \left[\frac{\partial \alpha}{\partial x_i}\right]_i$$

The derivative of a vector $b = [b_1, \ldots b_n]'$ with respect to a vector x is a matrix.

$$\frac{\partial b_x}{\partial x} = \left[\frac{\partial b_j}{\partial x_i}\right]_{ij}$$

Thus, the second derivative of a scalar is a matrix.

$$\alpha_{xb} = \frac{\partial}{\partial b}\left[\alpha_x\right] = \left[\frac{\partial^2 \alpha}{\partial x_j \partial b_i}\right]_{ij}$$

The second derivative of a vector b is a tensor of rank 3. For x, y, b, vectors,

$$\frac{\partial^2 b}{\partial x \partial y} = b_{xy} = \left[\frac{\partial^2 b_k}{\partial x_j \partial y_i}\right]_{ijk}$$

Multiplication of this tensor by a vector, a, gives a matrix

$$\left[b_{x,y}a\right] = \left[\sum_k \frac{\partial^2 b_k}{\partial x_j \partial x_i} a_k\right]_{ij} .$$

FIGURE 1 FILTER TO OBTAIN "COLORED" DISTURBANCE

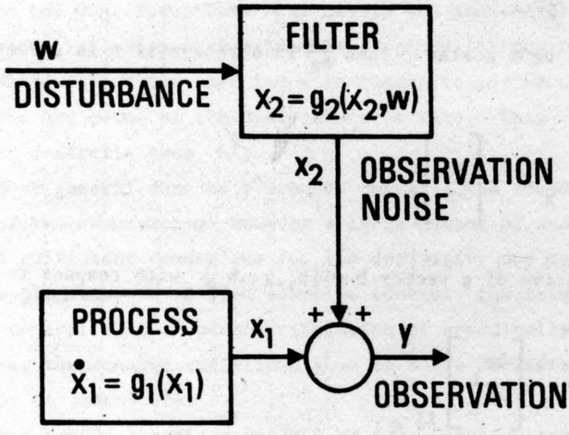


FIGURE 2 COMBINED ESTIMATION AND CONTROL DESIGN

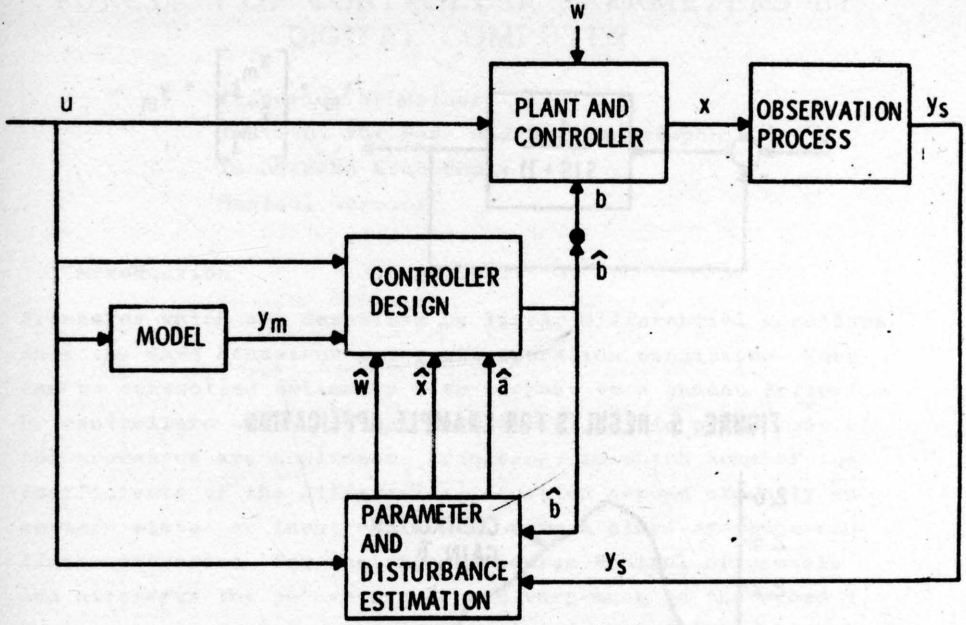


FIGURE 3 SYSTEM OF EXAMPLE PROBLEM

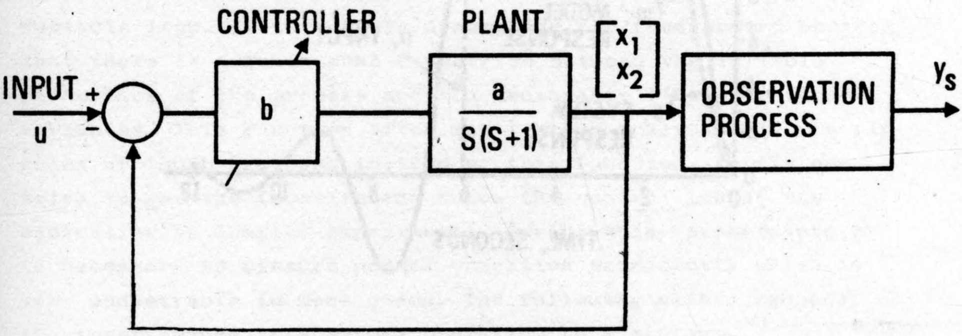


FIGURE 4 MODEL FOR DESIRED RESPONSE

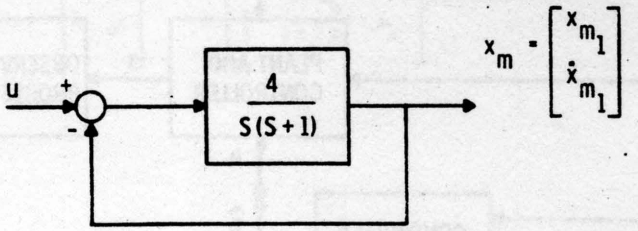
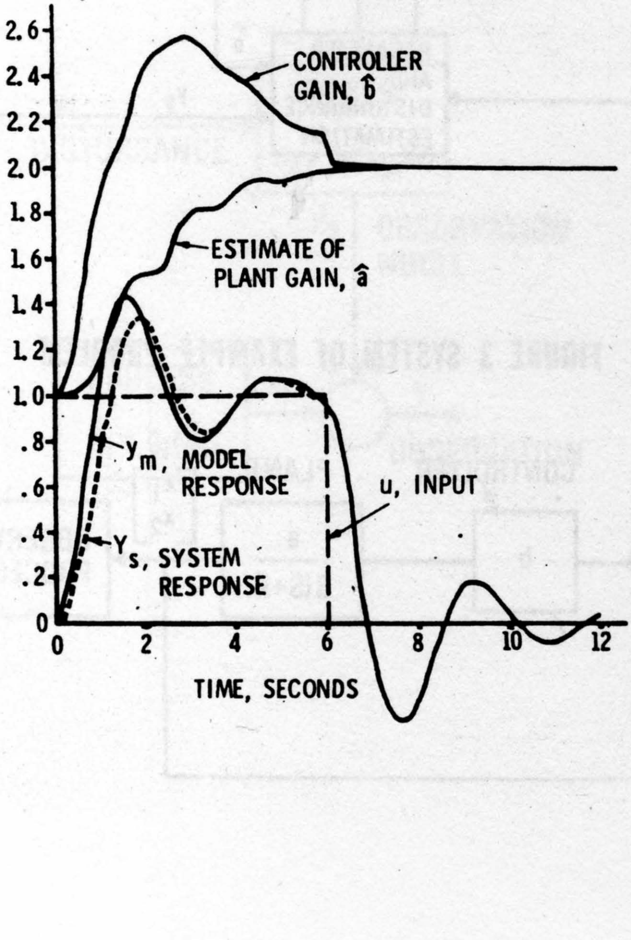


FIGURE 5 RESULTS FOR EXAMPLE APPLICATION



FINDING THE ADAPTIVE FEEDFORWARD FUNCTION OF CONTROLLER PARAMETERS BY DIGITAL COMPUTER

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1. Introduction

Processes which are described by linear differential equations show the same behaviour under all operation conditions. They can be controlled optimally with respect to a chosen criterium by controllers with constant parameters. But in praxi most of the processes are nonlinear. Processes, in which some of the coefficients of the differential equation depend strongly on certain state- or input variables, form a class of these nonlinear processes. For instance in course control of vessels and aircrafts the parameters depend very much on the speed v, or in many control loops of chemical industry (like the control of temperature, of pH-value, of analysis) on the flow of some product. These plants cannot be controlled sufficiently by controllers with constant parameters, if the operation conditions change very much. Control can be improved by adaptive feedforward control of the controller parameters. By measuring appropriate state- or input variables the parameters are fitted at once optimally to the altered process. Fig. 1 shows the simplified scheme of adaptive feedforward control in a single-variable loop. It is a basic assumption for feedforward control that there is a functional connection between the variable parameters of the process and the measurable state- or input variables. This function often isn't known analytically. Partly rules of thumb are used instead of this function, partly one tries to get the function and hence the control law of the controller by lengthy experiments. During these experiments it is necessary to disturb normal operation permanently which is very undesirable in most cases. The following method reduces the interference into normal operation to a minimum.

2. Formulation of the problem

Given a plant whose parameters depend on the measurable state- or input variables $z_1 \dots z_m$. The control of this plant shall be improved by adaptive feedforward control of controller parameters. The control law between the measurable variables $z_1 \dots z_m$ and the controller parameters $X_1 \dots X_k$, which are optimal with respect to a chosen criterium shall be learnt by means of a digital computer[2]. After a certain "starting phase" the plant shall run as a control loop with conventional adaptive feedforward control of controller parameters, the digital computer shall not be necessary any longer. Especially it is taken into consideration that permanent interference into the plant has to be reduced to a minimum. To achieve this the optimization of controller parameters for the different operating points is carried out in a model system which is simulated on a computer. Essentially there are the following problems to be solved.

1. Identification

By measuring the input- and output variables of the closed control loop a model of the plant (without controller) has to be determined for any operation point.

2. Optimization

The optimization of the controller parameters is carried out in the model system. The resulting values can be adjusted at the controller of the plant for checking.

Identification and optimization have to be repeated for as many states as necessary in the normal operation conditions. The resulting values have to be stored.

3. Adaptation

In the end the stored values have to be converted into an analog function of the measured input variables and the controller parameters. Then the control loop can work as a usual adaptive feedforward control of controller parameters later on. The realization of this analog function is usually achieved by simple function generators, as they are well known from analog computer techniques. The adjustment of controller parameters can be done by servo-multipliers.

In principle, the method is applicable for stochastic as well

as for deterministic input signals. In this paper deterministic input signals are used which seems to be admissible during the starting phase of a plant, particularly because the interferences are very seldom. By this method the control law can be learnt for continuous as well as for discrete systems with or without time delay. There will be calculated a model which is linearized about the given operation state, nonlinearities are taken into consideration by the value of controller parameters. If necessary and if the approximate form of the nonlinearity is known one can calculate even nonlinear models. In the following chapter the method is explained for continuous systems with variable parameters, in chapter 4 systems with time delay are treated. The application to discrete systems is completely analog, a thoroughly discussion therefore isn't necessary in this paper[2].
It is assumed in the following derivations that the response of the system to a change in the command variable is essential and therefore has to be optimized. It can be shown[2], that the response to a change in disturbance variables can be optimized with practical the same methods.
The controller show P, I, PI, PD or PID-behaviour. In the equations for the PD- and PID-controllers the always existing small time constants are taken into consideration. If there are used explicit controller equations in the following derivations, the equation of the PI-controller is taken (the computer programs in ALGOL of course can use all specified controllers).
The integral of time multiplied squared error ITE2 was taken as optimization criterium. This criterium on one hand results in relative small overshoots and relative small settling times, on the other hand it can be calculated analytically for linear (continuous and discrete)systems[3]. If u(t) is the error, it is

$$\mathrm{ITE2} = \int_0^\infty t \cdot u^2(t)\,dt \qquad (1)$$

Two methods for identification are explained in principle both of which do not assume any knowledge of the structure of the plant, since both don't lay stress on adjusting the structure of the plant and the model - which is impossible in general.

3. Continuous systems with variable systems.

The control loop shown in fig. 2 is given. The parameters of the plant depend much on z. It can be seen that the following explanations are valid also for more than one variable $z_1 \dots z_m$; only for the adaption the circuits for realization of the control law become expensive since in this case functions of several variables have to be realized. Even in praxi the parameter of a process depend essentially only on one variable in most cases.

3.1 Methods of identification

After a change of command variable the command variable w(t) and the output variable x(t) is recorded. The operational state of the system is known by the value of z. The transfer function of the controller $G_R(s)$ is given, but the transfer function of the plant $G_S(s)$ is not known. A model shall be found so that its output variable x_m fits as well as possible with the output x(t) (Fig.3). The controller of model and system have the same equation.

1st method

Renner[4] derived in his thesis, that the measured step response of an open system can be approximated by a model whose poles $s_i (i = 1 \dots n)$ of the transfer function are fixed. The appropriate factors R_i $(i = 0 \dots n)$ are determined by making $x(t_k)$ and $x_m(t_k)$ equal at n+1 points t_k. The step response of a system with only single poles (complex conjugate poles are also admitted, but they are omitted for sake of brevity since they don't show any important new feature) at time t_k is

$$x_m(t_k) = R_o + R_1 e^{-s_1 t_k} + \dots\dots\dots + R_n e^{-s_n t_k} \qquad (2)$$

At n+1 points this is a system of n+1 linear equations in the unknown factors R_i, which can be solved by one of the usual methods.

This method gives amazingly good results in the open loop, but isn't applicable for closed loop systems. Reinsch[5] and Unbehauen[6] have shown that even good accordance of the step functions of the open loop does not garanty good accordance of the closed loop since by closing the loop the position of the dominant poles can be very different (root locus). Moreover you can estimate only the dominant time constant of the closed loop step response but not that of the open loop. Therefore it is difficult to chose the correct time scale. On the other hand it is absolutely necessary to determine the transfer function of the open loop model, since otherwise the optimization cannot be carried out.

Therefore the following method was developped. The model consists of n terms of the first order in parallel (Fig. 4). (Complex conjugate poles are admitted, but they are for reasons of simplitity not drawn).

All poles with the exception of that with the greatest time constant, e.g. s_1, and with the exception of the common factor k are assumed fixed. The multiplication of the poles by the factor k means a change in the time scale, it is

$$s_i = k \cdot s_i' \text{ with } s_i' \text{ fixed} \tag{3}$$

Then the control loop of Fig. 4 can be described by the equations

$$\begin{aligned} u(t) &= w(t) - x_m(t) \\ r(t) &= f(u(t)) \\ \dot{\underline{q}}(t) &= \underline{A}\,\underline{q} + \underline{b}\,r(t) \\ x_m(t) &= \underline{q}^T \cdot \underline{R} \end{aligned} \tag{4}$$

in which the matrix $\underline{A}$

$$\underline{A} = \begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & & 0 \\ & & \ddots & \\ 0 & \dots & & s_n \end{bmatrix} \tag{5}$$

contains only the poles of the model. In the vectors $\underline{q}$ respectively $\underline{R}$ the values q_i resepctively R_i are summarized. $\underline{q}^T$ means the row vector of the column vector $\underline{q}$. $\underline{b}$ consists only of ones. The following method for identification is used. w(t), x(t) and by these also u(t) and r(t) of the original system are given. If we take these variables as approximate input to the model system (thus instead of $w - x_m$ $w - x$), we can calculate $\underline{q}(t)$ by eq. (4), if the time factor k and s_1 are known. If we require at least at n points $x_m = x_1$, so we can get from

$$x\,(t_k) = \underline{q}^T \cdot \underline{R} \tag{6}$$

the approximate values of the wanted R_i. The R_i can only be calculated approximately, because the input of the model wasn't $w - x_m$ but $w - x$, in principle there was the structure shown in Fig. 5. The model has to be investigated with the calculated R_i, wether x and x_m don't differ too much. If they differ too much and this happens always when the dominant poles differ markedly - the dominant poles are approached by changing the factor k or the pole s_1 in a very fast seeking process so that both output functions x and x_m agree sufficiently. By requiring $x_m(t_k)$ to be equal $x(t_k)$ at n times random inaccuracies in the values $x(t_k)$ (measuring errors) will effect the values of R_i very much. Therefore more than n values were taken for determining the R_i. If one orders these m values (m n) according to eq.(6) in a vector $\underline{X}$ one can write

$$\underline{Q} \cdot \underline{R} = \underline{X} \tag{7}$$

The m x n matrix $\underline{Q}$ consists of the values $q_i(i = 1 \dots n)$ at the times $t_k(k = 1 \dots m)$. This system of equations is overdetermined, since there are m equations for n unknowns R_i. Therefore the factors R_i are determined by the calculus of observations so that the quadratic distance between the measured and the calculated values becomes a minimum. This means the minimization of the Euclidean norm of

$$\underline{X} - \underline{Q} \cdot \underline{R} \tag{8}$$

2nd method

This method starts from the transfer function of the closed model[7,8]. It is according to Fig. 3

$$\frac{X(s)}{W(s)} = G(s) = \frac{G_R \cdot G_{SM}}{1 + G_R G_{SM}} \tag{9}$$

If the model plant is described by a rational fraction in s, of which the degree n can be chosen as one likes, it is

$$G_{SM}(s) = \frac{\sum_{i=0}^{m} b_i s^i}{\sum_{i=0}^{n} a_i s^i} \quad m \leq n \tag{10}$$

The controller shows for instance PI-behaviour

$$G_R(s) = V_P \left(1 + \frac{1}{T_n s}\right) = \frac{X_1 s + X_2}{s} \tag{11}$$

If you put eq. (10) and (11) into (9) and if you transform it into time domain, you get for the most general case m = n the following differential equation of (n+1)th order

$$(a_n + X_1 b_n) \cdot x^{(n+1)} + (a_{n-1} + X_1 b_{n-1} + X_2 b_n) \cdot x^{(n)} + \ldots$$

$$\ldots + X_2 b_o x = X_1 b_n w^{(n+1)} + \ldots X_2 b_o w \qquad (12)$$

If $w(t)$ and $x(t)$ is given and if they are differentiable n+1 times equation (12) at time t_k is an equation for the n + m + 1 unknowns $a_i (i = 1 \ldots n$, a_o can be chosen, $a_o = 1)$ and b_k $(k = 0 \ldots m)$. By the use of n + m + 1 times you get a system of equations for the n + m + 1 unknowns.
Since the differentiability, particularly of the input variable cannot be assumed (step function) and since differentiation, as it is well known, amplifies possible errors, it is impossible to apply this method in this form. Similar as in switching diagrams for the analog computer the difficulty is avoided by integrating eq. (12) n + 1-times. Then there are no longer any derivatives in eq. (12). Moreover errors are averaged by integration.
After integrating eq. (12) n+1-times and after putting the new equation in an order according to the unknowns a_i and b_i you get the following equation at time t_k

$$a_n\, x(t_k) + a_{n-1} \int_0^{t_k} x\,dt + \cdots + a_1 \underbrace{\int_0^{t_k} \cdots \int_0^{t_k}}_{n-1\,\text{mal}} x\,dt \cdots dt + b_n \cdot \Big(X_1\, x(t_k) +$$

$$+ X_2 \int_0^{t_k} x\,dt - X_1\, w(t_k) - X_2 \int_0^{t_k} w\,dt\Big) + \cdots + b_0 \cdot \Big(X_1 \underbrace{\int_0^{t_k} \cdots \int_0^{t_k}}_{n-\text{mal}} x\,dt \cdots dt +$$

$$+ X_2 \underbrace{\int_0^{t_k} \cdots \int_0^{t_k}}_{n+1\,\text{mal}} x\,dt \cdots dt - X_1 \underbrace{\int_0^{t_k} \cdots \int_0^{t_k}}_{n-\text{mal}} w\,dt \cdots dt - X_2 \underbrace{\int_0^{t_k} \cdots \int_0^{t_k}}_{n+1\,\text{mal}} w\,dt \cdots dt = - \underbrace{\int_0^{t_k} \cdots \int_0^{t_k}}_{n-\text{mal}} x\,dt \cdots dt. \qquad (13)$$

In eq. (13) the integration constants $c_o \ldots c_n$ are assumed to be zero. If the initial conditions aren't zero the integration constants c_i can be easily computed for $t = 0$ from eq. (13).
Since a computer performs the integrations very fast you easily get from eq. (13) a system of equations for determining the unknowns a_i and b_k which is analog with that of eq. (7).

$$\underline{Q}_1 \cdot \underline{B} = \underline{X} \qquad (14)$$

In eq.(14) $\underline{B}$ is the vector

$$\underline{B} = \begin{bmatrix} a_n \\ \vdots \\ a_1 \\ b_n \\ \vdots \\ b_0 \end{bmatrix} \qquad (15)$$

$\underline{Q}_1$ is a l x n+m+1 matrix which consists according to eq.(13) of the coefficients of the unknowns at times t_k. $\underline{X}$ is a 1-dimensional vector, in which the right sides of eq.(13) at times t_k are put together.

If l = n+m+1 eq.(14) is a system of equations for the n+m+1 unknowns. Its solution gives the coefficients a_i and b_k under the condition that at n+m+1 times $x(t_k)$ equals $x_m(t_k)$. In this case the resulting coefficients are quite good since in this method you use at least twice the number of measured values as in the first method. If you take l > n+m+1 you get the unknown coefficients by the calculus of observations in the same manner as with the first method. In this method it is easy to change the structure of the model automatically so that plant and model coincide as well as possible. Both methods were programmed in ALGOL and give good accordance between $x(t)$ and $x_m(t)$. As an example a system of the 4th order with a PI-controller was identified by a model of the 6th order. With the 1st respectively the 2nd method I got a numerator polynomial of degree 4 respectively 3, and a maximum error between the step response of the original and the model system of 2,5 % respectively 1,o5 % related to the stationary value of $x(t)$. In the following optimization of the controller parameters of the original and the model system, I got an error in the parameter X_1 of less than 2 ‰ respectively 4 ‰ related to X_1 of the original system. For X_2 the error was less than 2,5 ‰ respectively 4 ‰. Even the last error - which is the most unfavorable value which I got with different systems - is by far within the range in which there is an essential variation of the optimum.

3.2 Optimization

For the reasons stated in the introduction the integral of time multiplied squared error ITE2 is used as optimization criterium. This criterium shall be minimized as a function of the controller parameters X_i, which means for a PID-controller that

$$\frac{\partial ITE2}{\partial X_1} = 0; \quad \frac{\partial ITE2}{\partial X_2} = 0; \quad \frac{\partial ITE2}{\partial X_3} = 0 \tag{16}$$

have to be fulfilled. If ITE2 is given analytically as a function of X_1, X_2 and X_3, eq. (16) represents in general three nonlinear equations in X_1, X_2 and X_3. The analytical representation of ITE2 is possible in the s-domain[3], if the error u(t) can be stated as rational fraction in the s-domain.

$$U(s) = \frac{d_{n-1}s^{n-1} + \dots\dots\dots + d_0}{a_n s^n + \dots\dots\dots + a_0} \tag{17}$$

By means of Parseval's theorem it can be proved that

$$ITE2 = \int_0^\infty t\, u^2 dt = -\frac{1}{2}\cdot\frac{1}{2\pi j}\cdot\lim_{\sigma\to 0}\frac{\partial}{\partial\sigma}\int_{-j\infty}^{+j\infty} U(\sigma+s)\cdot U(\sigma-s)\, ds = \tag{18}$$

$$= -\frac{1}{4a_n}\frac{\text{Det } G}{\text{Det } H}\left(\sum_{i=2}^{n}\frac{\text{Det } G_i}{\text{Det } G} - \sum_{i=1}^{n}\frac{\text{Det } H_i}{\text{Det } H}\right)$$

The matrices $\underline{H}$, $\underline{G}$, $\underline{H_i}$ and $\underline{G_i}$ consist of the coefficients of $U(s)$[3]. Therefore they depend in general on the controller parameters X_1, X_2, X_3. This form isn't suited for calculating explicitly the derivatives (16) on a computer because of its complexity. This is achieved by the following method, which is explained for the integral of squared error IE2 for simplicity reasons. The extension to ITE2 is very easy but requires more calculations. According to[3] it is

$$IE2 = \int_0^\infty u^2\, dt = \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} U(s)\cdot U(-s)\, ds = \frac{\text{Det } G}{2a_n \text{ Det } H} \tag{19}$$

where Det H is the n-th Hurwitz-determinant of the denominator polynomial of U(s)

$$\underline{H} = \begin{bmatrix} a_{n-1} & a_n & 0 \dots 0 \\ a_{n-3} & a_{n-2} & 0 \\ & & \ddots \\ 0 \dots & \dots 0 & a_0 \end{bmatrix} \tag{20}$$

By replacing the first column of $\underline{H}$ by the vector $\underline{d}$

$$\underline{d} = \begin{bmatrix} d_{n-1}^2 \\ -(d_{n-2}^2 - 2d_{n-1}d_{n-3}) \\ \vdots \\ (-1)^{n-1} d_0{}^2 \end{bmatrix} \tag{21}$$

you get the matrix $\underline{G}$. This means you can write IE2 also as the matrix equation

$$2a_n \cdot \underline{H} \cdot \underline{w} = \underline{d} \tag{22}$$

of which only the root w_1 = IE2 is of interest. Since the coefficients a_i of the denominator polynomial of U(s) depend on the controller parameters the matrix $\underline{H}$ does also and can be separated in

$$\underline{H} = \underline{\tilde{A}} + \underline{B}\cdot X_1 + \underline{C}\cdot X_2 + \underline{D}\cdot X_3 \tag{23}$$

where $\underline{\tilde{A}}$, $\underline{B}$, $\underline{C}$ and $\underline{D}$ are matrices with constant coefficients. Neglecting in eq.(22) the factor $2a_n$ and considering changes in parameters about the operating point X_{io}, one gets from eq.(22) with $X_i = X_{io} + x_i$

$$(\underline{A} + \underline{B}\cdot x_1 + \underline{C}\cdot x_2 + \underline{D}\cdot x_3)\ \underline{w} = \underline{d} \tag{24}$$

with $\underline{A} = \underline{\tilde{A}} + \underline{B}\cdot X_{10} + \underline{C}\cdot X_{20} + \underline{D}\cdot X_{30}$ (25)

Solving the matrix equation (24) by expanding it in a series it is

$$\underline{w} = [\underline{I} - \underline{A}^{-1}\cdot\underline{B}\cdot x_1 - \underline{A}^{-1}\cdot\underline{C}\cdot x_2 - \underline{A}^{-1}\cdot\underline{D}\cdot x_3 + (\underline{A}^{-1}\cdot\underline{B}\cdot x_1 + \underline{A}^{-1}\cdot\underline{C}\cdot x_2 + \underline{A}^{-1}\cdot\underline{D}\cdot x_1)^2 + \dots]\cdot\underline{A}^{-1}\cdot\underline{d} \tag{26}$$

Now w_1 depends very clearly on x_1, x_2 and x_3 and the derivatives (16) can easily be calculated. Since these according to eq. (26) lead to three nonlinear equations in x_1, x_2, x_3 you have to use a multidimensional Newton procedure to improve the approximations X_{10}, X_{20}, X_{30}. This means in eq.(16) you take into consideration only linear terms.

$$\frac{dw_1}{dx_1} = 0 = b_1 + 2a_{11}x_1 + (a_{12} + a_{21})x_2 + (a_{13} + a_{31})x_3$$

$$\frac{dw_1}{dx_2} = 0 = b_2 + (a_{12} + a_{21})x_1 + 2a_{22}x_2 + (a_{23} + a_{32})x_3 \qquad (27)$$

$$\frac{dw_1}{dx_3} = 0 = b_3 + (a_{13} + a_{31})x_1 + (a_{23} + a_{32})x_2 + 2a_{33}x_3$$

where the b_i's respectively the a_{ik}'s are the corresponding derivatives of w_1 in the parameters x_i and x_k about the point X_{io}. It is for example

$$b_1 = \left.\frac{dw_1}{dx_1}\right|_{X_i = X_{io}} = 1.\ \text{Komponente} \left\{- \underline{A}^{-1}\ \underline{B}\ \underline{A}^{-1}\ \underline{d}\right\} \qquad (28)$$

$$a_{12} = \left.\frac{d^2w_1}{dx_1 dx_2}\right|_{X_i = X_{io}} = 1.\ \text{Komponente} \left\{\underline{A}^{-1}\ \underline{B}\ \underline{A}^{-1}\ \underline{C}\ \underline{A}^{-1}\ \underline{d}\right\}$$

By means of x_1, x_2, x_3 you get new approximate values X_{io} by the relation $X_{io} = X_{io} - x_i$. The procedure is repeated as many times as the changes in parameters x_i lie above some specified bounds. For each calculation only one matrix has to be inverted and some multiplications of matrices have to be performed. All these operations can be done very fast on a computer so that the optimum is found within very little time (even for ITE2, which takes more calculations).

3.3 Adaptation

Finally if there are determined and stored a sufficient number of combinations of the variable z and the controller parameters X_i within the operating range, of course you could approximate these relations at once by a function generator of analog computing techniques by replacing the continuous function $X_i = f(z)$ by straight lines between the calculated values.

Such a procedure wouldn't average any error. Therefore you better fit the measured values by a continuous function and approximate this by function generators. The determination of the continuous function can be done by inspection if the relation is simple. If you need an analytic solution or if the controller parameters depend on more than one variable z_i, you can get an explicit function - piecewise or for the whole range - by linear curve fitting.

When the functions $X_i = f(z)$ are implemented at the controller of the plant by means of function generators and servomultipliers the process can work as a control loop with conventional adaptive feedforward control of controller parameters, which means the controller setting is nearly optimal for all operational states. The computer isn't any longer necessary. As an example the course control of a vessel described by Oppelt[1] was taken. In this the parameter of the plant depend very much on the speed. This plant was controlled by a proportional controller with the gain V_p. With assumed values for the parameters of the plant and for the related speed v I got the step response to a change in course φ_1 shown in Fig. 6. Both curves were calculated for a speed of v = 0.2. In one case the controller parameter was optimal for v = 0.2, in the other case - as example for a fixed setting - it had the optimal value for v = 1. Fig. 7 shows the function $V_p = f(v)$ calculated by computer.

4. Systems with time delay.

In the preceding considerations it was assumed that the time delay of the system is negligible compared with the dominant time constants. This assumption isn't valid in many systems, particularly if mass transport is included. But processes with time delay are difficult to control, therefore it is very desirable to have adaption of controller parameters if the time delay of the process is variable. Subsequently the changes of the above methods for this case are outlined in short; they are discussed thouroughly in[2].

The time delay of the model loop is assumed to be in series with the plant (Fig.8). First the time delay is determined which is relatively simple since it is not important that

the time delay of the system and the model are exactly the same. For determining the time delay the following procedure was used. By the value of t, at which the output variable of the system x(t) is greater than a specified constant k (to avoid random errors), and by taking into consideration the ascent.of the output variable you get the approximate time delay (Fig. 9). Having determined the time delay one can identify the linear part of the plant by the methods of 3.1, modified for systems with time delay. The optimization cannot be performed with the very fast multidimensional Newton procedure, described in 3.2, since U(s) isn't any longer a rational fraction in s which was a necessary condition for the analytic representation of ITE2. Therefore a numerical optimization procedure had to be used. A procedure of a "direct search" was taken for which an ALGOL-program exists at the institute which is described in[9]. For each combination of parameters X_i the system of equations of the model has to be solved and the error has to be integrated according to eq.(1). Even though the seeking process is very fast, the optimization is much slower than in the linear case with the multidimensional Newton procedure, since the solution of differential equations is relatively small on a digital computer. There was a factor about 5 in the computing time. To avoid this one can replace for optimization the continuous system by a fictitious discrete system. For discrete systems you can calculate ITE2 analytically[3] even if there is time delay, since a time delay causes only a rise in the degree of the denominator and numerator polynomial of U(z). Then the Newton procedure of 3.2 - modified for discrete systems -is applicable again.
The adaptation for systems with time delay is exactly the same as for linear systems.

5. Conclusion

It was shown in this paper how to get the control law of adaptive feedforward control of controller parameters by means of a digital computer. It is an advantage of this method that the computer - on-or off-line - is only necessary during a starting phase. By simulating a model of the plant on the computer the interferences into normal operation can be reduced to a minimum.

In this method the model is determined by means of one of two described identification procedures for each operation state. The identification procedures don't need any information about the structure of the plant. The controller parameters are optimized by a multidimensional Newton procedure, respectively by a direct search for systems with time delay. A possibility was pointed out how to apply the Newton procedure for systems with time delay.
The extension of these methods to discrete systems is discussed in[2], so that it is possible by these procedures to determine the control for adaptive feedforward control of controller parameters for continuous and discrete systems with or without time delay.

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Note: [2] is a prepublication of the thesis: "Einstellung der Adaptivsteuerung von Prozessen mittels Digitalrechner", TH München

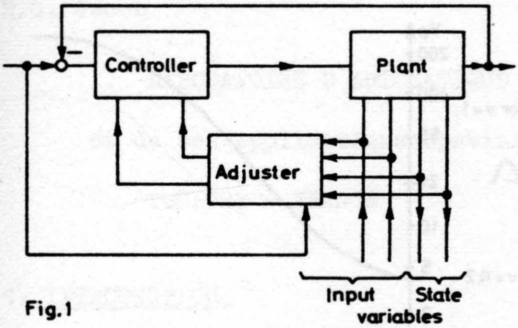

Fig. 1

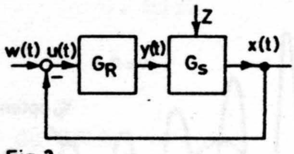

Fig. 2

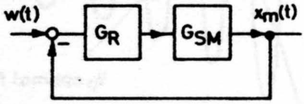

Fig. 3

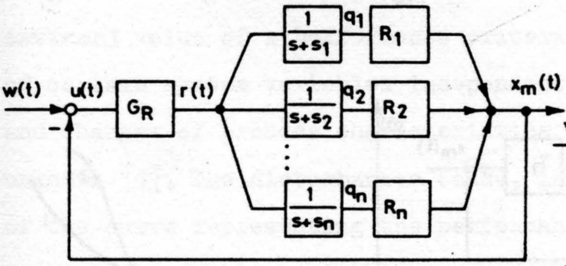

Fig. 4

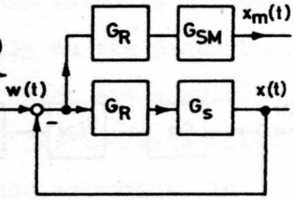

Fig. 5

Fig. 1: Adaptive Feedforward Control of Controller Parameters

Fig. 2: Continuous Control Loop without Time Delay, Parameters of Plant depend on z

Fig. 3: Model Control Loop

Fig. 4: Structure of the Model

Fig. 5: Structure of the Loop for Identification

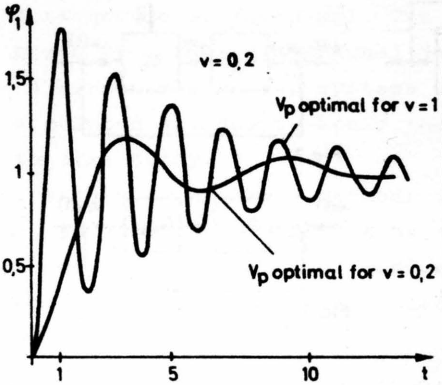

Fig.6

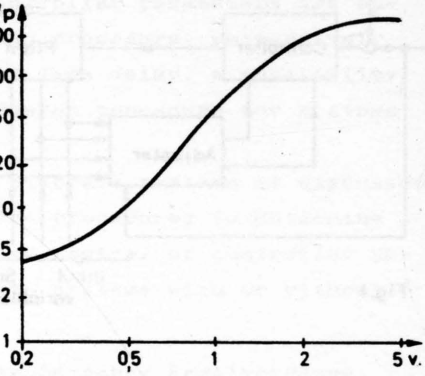

Fig.7

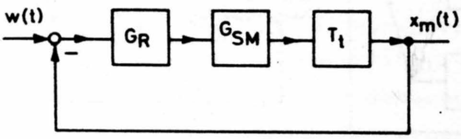

Fig. 8

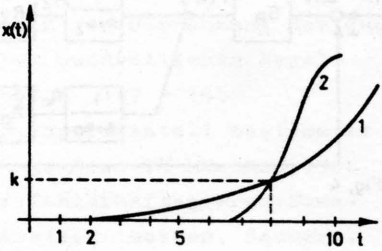

Fig. 9

Fig. 6: Step Responses

Fig. 7: Optimal Controller Parameter

Fig. 8: Model Control Loop with Time Delay

Fig. 9: Determination of the Time Delay

EXTREMUM-SEEKING CONTROLLER WITH EXTRAPOLATION

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1. INTRODUCTION

Extremum-seeking controller affects input variables of a system or a process (S) in such a way as to seek for an extremal value of a performance criterion which is a function of certain system variables independently on the disturbances and changes of process characteristics which are a priori unknown [1]. The disturbances cause, in particular, a shifting of the curve representing the performance criterion. In general, controllers of this type are applied in cases when initial information content in small [1] which results in the necessity for enlarging the amount of information by a seeking process. The presence of the seeking process lowers the quality of the system as far as speed of operation is concerned and to achieve a given accuracy the assumption is required that the changes of the performance criterion are sufficiently slow.

The extremum-seeking controller described in the paper necessitates a larger amount of initial information than a clasical extremum controller does, if a high quality is required. Owing to this factor such a controller can not be so widely applied as classical one, but on the other hand it has much better properties as far as operating speed is concerned which is the feature of all systems with large initial information.

We are considering the case when the performance criterion of the process (S) depends on a single variable only. The block diagram for the extremum-seeking control system [controller + (S)] is shown in Fig.1, where W_1 and W_2 are the parts of the process - assumed linear, situated before and behind extremal characteristic(φ). λ and λ' are the disturbances, unknown a priori, which shift(φ)horizontally and vertically. To design the controller we assume that the following initial data are at our disposal:

- the form of(φ)is known; it has single extremum /maximum or minimum/, moreover, disturbances do not affect (φ)too much;
- the step responses of $W_1(p)$ and $W_2(p)$ are known.

The extrapolation is of parabolic type and indirectly utilizes the knowledge of two points on a parabola P which are common to (P) and(φ), and its parameter which is averaged for both slopes of the curve - divided by the extremum point of(φ) - in such a way that (P) and(φ) be as close to each other as possible. An operation - comprising four intervals - which permits to obtain the extremum of (P) will be called an optimization sequence. The extremum of(φ)is achieved in course of a number of optimization sequences.

The steps of an optimization sequence are:

- drift detection step. It has a fixed time interval over which drift of the operating point(output variable φ) - caused by the disturbances λ , λ' , searching and input control signals - is detected.

- searching step of variable duration. Over this interval two points of ($\mathcal{C}$) are determined.

- control step of variable duration resulting from calculations of the position of the extremum point of (P), and taking into account the displacement of ($\mathcal{C}$) during the optimization sequence, by extrapolation of drift measured in the first interval.

- rest step of fixed duration.

The possibility of taking into account drift of ($\mathcal{C}$) during a sequence allows the controller to remain stable even in the case of rapid shift of ($\mathcal{C}$). The modulation of the searching and control steps duration allows to increase the speed of operation if the operating points is far from the extremum of ($\mathcal{C}$) and to increase accuracy if the point is close to the extremum.

After describing the main principles on which the idea of the controller is based we describe an optimization sequence. Later on, we mention about a possibility of "tachometric correction" and we carry out stability analysis for the process - controller set. The final part deals with experimental investigations.

2. PRINCIPLES OF THE EXTRAPOLATION

Let us consider the extremal characteristic ($\mathcal{C}$) described by equation $y = F(x)$, shown in Fig.2. Its deformation caused by disturbances is not significant. For two arcs forming the slopes of the curve on both sides of the extremum point we determine a parabola P having its axis parallel to Oy. The parabola is, on average, as close as possible to the arc of the characteristic $(\mathcal{C})$.

The equation of such a parabola is

$$x^2 + ax + b = 2py$$

where p - parameter of (P). For the left part of the characteristic $(\mathcal{C})$ we determine p_1 and for the right part - p_2. The sign of the change of the output value permits to find out which of these two parameters should be applied.

The purpose of the searching step is to find two points A and B of the characteristic $(\mathcal{C})$. These two points and one of the parameters p_1, p_2, uniquely determine a parabola (P). Let us denote: Δx - increase of x after a searching step, Δy - corresponding increase of y, x_A - abscissa of point A, x_p - abscissa of the extremal point of the parabola (P), d_x^* - increase which should be added to x in order to reach the extremum of (P) - Fig.2. For the introduced symbols we have

$$x_p - x_A - \Delta x = d_x^* = - p_i \frac{\Delta y}{\Delta x} - \frac{\Delta x}{2} \qquad /1/$$

$$i = 1,2$$

In most cases of extremum-seeking systems the variables x, y are not accessible, therefore y is measured after the dynamic unit W_2 and x is affected through W_1. For the further calculations we assume that gains of W_1 and W_2 are equal to unity. /If they differ from unity the form of (φ) is modified accordingly/.

During the searching step μ is increased by an increment $\Delta\mu$ of constant value, the duration time Δt_r of the step being variable. Let $\bar{x}(\Delta t_r)$ be the value of the intermediate variable x at the instant Δt_r /Fig. 3a/. It is the response for a unit step of μ. Let $\overline{\varphi}(\Delta t_r)$ be the value of φ at the instant Δt_r obtained as the response for unit gains of W_1 and W_2 and for the same step change of μ /Fig. 3b/.

We get

$$\Delta\varphi = \frac{\Delta y}{\Delta x} \quad \overline{\varphi}(\Delta t_r)\Delta\mu \, , \qquad x = \bar{x}(\Delta t_r)\Delta\mu$$

Increment of d_x^* corresponds to an increment of d_μ^* which is given by

$$d_\mu^* = -\frac{1}{\overline{\varphi}(\Delta t_r)} \quad \frac{\Delta\varphi \cdot p}{\Delta\mu} - \frac{\Delta\mu \quad \bar{x}(\Delta t_r)}{2} \qquad /2/$$

For the controller described here $\Delta\varphi$ is a constant value, equal to $\Delta\varphi_s$, determining the variable searching interval Δt_r.

3. PRINCIPLE OF THE CONTROL

In order to increase the rate of operation at the begining of the control step μ receives a step increase /called the forcing control C_E/. The magnitude of C_E is constant and as large as it is admitted by technological constraints, its sign being dependent on the change of φ after applying the step searching signal. The duration Δt_c of the control C_E is variable in such a way, that at the end of this interval the value of x corresponds to x_p. Immediately after the time Δt_c the signal is forced to assume the form of C_D /called definitive control/ such that x preserves the value x_p. The response x(t) for different searching and control signals /Fig. 4/ permits to determine Δt_c from the formula

$$x(\Delta t_c) = \frac{d_M^*}{C_E - \Delta M \bar{x}(\Delta t_r)} \qquad /3/$$

and the control C_D from

$$C_D = d_M^* - C_E + \Delta M \bar{x}(\Delta t_r)$$

Therefore

$$C_D = -\frac{1}{\overline{\varphi}(\Delta t_r)} \frac{\Delta \varphi_s \cdot p}{\Delta M} + \frac{\Delta M \bar{x}(\Delta t_r)}{2} - C_E \qquad /4/$$

In practice, the term $\Delta M \bar{x}(\Delta t_r)$ is small in comparison with the other terms of equations (2), (3) and(4). The suggested controller makes use of the simplified relations based on the following approximation:

$$C_D \simeq C - C_E \; ; \quad C \doteq -\frac{1}{\overline{\varphi}(\Delta t_r)} \frac{\Delta \varphi_s \cdot p}{\Delta M} \; ; \quad d_M^* = C \qquad /5/$$

$$x(\Delta t_c) \simeq \frac{C}{C_E} \qquad /6/$$

In case of a process (S) for which the characteristic is not followed by dynamical unit W_2, the control step of duration Δt_c is not applied and definitive control step $C_D = C$ is applied at the begining of the rest interval.

The rest step of a constant duration is chosen according to the response time of the dynamical unit W_2 which follows the characteristic $(\mathcal{C})$. This step is necessary in order that the system could reach the new stable operating point by the time the next optimization sequence starts. However, if the duration of the rest step is shorter than the response time of W_2, i.e., if the variable φ still changes at the end of the optimization sequence then, the step of detection of the operating point drift permits to take this displacement into account.

4. PRINCIPLE OF DETECTION OF THE OPERATING POINT DRIFT AND THE EXTRAPOLATION OF IT

There are two main sources of drift of the output variable φ of the process (S). They are:

a/ the action of external disturbances λ, λ' which move the characteristic $(\mathcal{C})$ horizontally and vertically.

b/ the fact that at the end of rest step duration of which is shorter than the response time of the dynamical unit W_2, the variable φ still changes.

The displacement $\Delta\varphi_p$ is detected during the first interval of an optimization sequence through memorizing values of φ at the begining $(t = t_o)$ and at the end $(t = t_1)$ of this interval. $\Delta\varphi_p$ is integrated over the searching interval.

$$\varphi_p = \varphi(t_1) + \int_{t_1}^{t_1 + t} \Delta \varphi_p \alpha_1 \, dt, \quad \alpha_1(t_1 - t_0) = 1 \qquad /7/$$

The value $\int_{t_1}^{t_1 + t} \Delta \varphi_p \alpha_1 \, dt$,

is the predicted, extrapolated change of φ in the interval Δt_r, determined without taking into consideration the influence of the searching signal. In this interval this quantity is continuously subtracted from the resultant change $\Delta \varphi_d(t)$. The change $\Delta \varphi_d(t)$ results from the change of φ caused by the searching step as well as from the drift of the operating point. If the obtained difference exceeds a threshold value $\Delta \varphi_s$, the searching step terminates. Thus, we get a modulation of the duration of this step as a function of the slope of the extremal characteristic $(\mathcal{C})$ at the operating point.

The detection step can possibly be cancelled if the changes of $(\mathcal{C})$ caused by the disturbances λ, λ' are sufficiently slow and if the duration of the rest interval is longer than the response time of the unit W_2.

5. DESCRIPTION OF AN OPTIMIZATION SEQUENCE [2]

The different steps of each optimization sequence can be generated by means of a ring counter controlled by pulses from monostable multivibrators (MS_1, MS_2) giving pulses of a fixed duration, or by comparators (C_1, C_2, C_3) giving the steps of a variable duration /Fig.5 and 6/. The diagram in Fig.6 shows the principle of operation of a controller for a case of equal parameters $p_1 = p_2$ chosen for each side of the extremum of (φ).

- <u>At the instant $t = t_o$</u> the step of detection of the operating point drift starts. The 0 stage of the ring counter /Fig.5/ is in the "binary one position" all other stages in the "zero position". The transient state of the monostable multivibrator (MS_1) determines the constant duration of the first step. The unit M_o /Fig.6/ memorizes the value $\varphi(t_o) = \varphi_o$.

- At the instant $t = t_1$, the multivibrator (MS_1) switches to its stable state and the stage 1 of the ring counter sets to "binary one position", all other stages are at "zero position". The searching step starts. The M_1 unit /Fig.6/ holds in memory $\varphi(t_1) = \varphi_1$. The difference $\varphi_1 - \varphi_o = \Delta\varphi_p$ is integrated by the integrator I_1 over the duration of this step.

The searching signal $\Delta\mu$ is imposed on the variable μ through the control block BC , the output unit of which may be for example a motor. The main time constant of the motor is small in comparizon with the time constant of W_1 and W_2 and with the average duration of the searching step. The initial condition -u is set on the integrator I_2 with input signal + V. Its output voltage S_2 linearly decreases

$$S_2 = u - \int_{t_1}^{t_1 + t_r} \alpha_2 V \, dt = u - \alpha_2 \Delta t_r$$

The voltages + V, -u and gain $_2$ are so chosen that they determine a maximum value of the time $t_{r\,max}$ which follows from the properties of the process /Fig.7/ to be optimized.

- The searching step ends at the <u>instant $t = t_o$</u> when the difference

$$\Delta\varphi_d(t) - \int_{t_1}^{t_1 + t} \Delta\varphi_p \, dt$$

assumes the value $\pm\ \Delta\varphi_s$; $(t_2 - t_1 = \Delta t_r)$. The comparator C_1 or C_2/it depends on the sign of $\Delta\varphi_s$/ makes state transition in the 2nd stage of the ring counter from "zero" to "one". The other stages remain at "zero". The control step starts.

Forcing control of a constant magnitude C_E, the sign of which corresponds to the sign of $\Delta\varphi_s$ if the extremum is a maximum point, is imposed on the variable μ through the unit (BC) simultaneously with the step change $-\Delta\mu$, which compensates the step change of the searching signal, + /the switch r_1 in Fig.6, controlled by C_1, C_2/. The voltage $S_2(t)$ is held at the value $S_2(t_2)$. The longer the time interval $\Delta\ t_r$, the smaller this value /Fig.7/.

In Fig.6 the element (GF_1) is a function generator. Its output voltage is equal to

$$C = - \frac{1}{\bar{\varphi}\ (\Delta\ t_r)} \ \frac{\Delta\varphi_s \cdot P}{\Delta\mu}$$

The potentiometer P_1 setting determines the constant term $\frac{\Delta\varphi_s \cdot P}{\Delta\mu}$.

An initial condition z for the integrator I_3 is proportional to the time interval Δt_c. Over this interval the control C_E is to be applied. The time interval Δt_c is determined from the relation (6). The voltage z is supplied by the function generator (GF_2) ; the potentiometer P_2 makes possible to set up the constant term $1/C_E$. The voltage $-V'$ is integrated by I_3. The gain of I_3 is equal to that of the integrator I_2 until the exact compensation of the output voltage S_2 takes place:

$$S_3 = - z - \int_{t_2}^{t_2 - t} (- V')\alpha_3 \, dt$$

For $S_3 = 0$, $t = t_3$ where $t_3 - t_2 = \Delta t_c$ /Fig.8/ (V' and α_3 are so chosen that the time scale for S_3 is as shown in Fig.7).

At the instant $t = t_3$, the comparator C_3 sets the stage 3 of the ring counter to "the position one". A rest step starts. The definitive control $C_D = C - C_E$ is applied to (BC). The monostable multivibrator switches on. Its transient state determines the constant duration of the rest step. When (MS_2) switches off, the next optimization sequence begins.

REMARKS

The characteristic of the generator (GF_1) is shown in Fig.9. It represents the function $\frac{1}{\varphi(\Delta t_r)}$ defined on the base of Fig.3b. For $S_2 = u$ - the value corresponding to $t = 0$ in Fig.3b - C is set to C_E.

It results in a small modification of the curve corresponding to $\frac{1}{\overline{\varphi}(\Delta t_r)}$ in the vicinity of $S_2 = u$. If the process (S) consists of a pure time delay τ, the characteristic of (GF_1) is such as shown in Fig.10.

The searching step duration is limited by the quantity $\Delta t_{r\,max}$. Large value of Δt_r indicates that the operating point is close to the extremum. If Δt_r is greater than $\Delta t_{r\,max}$ the optimization sequence is discontinued, the controls C_D and C_E are not used, only the step signal $-\Delta\mu$ is applied to compensate the step change $\Delta\mu$ of the searching signal.

- The characteristic of the generator (GF_2) is shown in Fig.11.
- In some cases /p small, $\Delta\mu$ large/ the relation (4) instead of the approximate relation (5) has to be used to determine C_D. In such cases, to determine Δt_c one has to make use of the relation (6) instead of the relation (3).

6. "TACHOMETRIC" COMPENSATION [2]

Let us assume that the process (S) consists of a dynamical unit W_1 followed by unit with an extremal characteristic (φ). In other words, time constants and delays of the unit W_2 are supposed to be negligible with regard to those of the unit W_1. Moreover, we assume that the extremal characteristic (φ) has a maximum. In the case when x_p is practically the same as the abscissa x_F of the extremal point of the characteristic (φ) and no displacements of it occur, the motion of the output variable $y \cong \varphi$ ends at $t = t_3$. It means that at this instant $dy/dt = 0$ /the controller is supposed to be ideal: t_c and C_D are precisely determined/.

If at $t = t_3 - \varepsilon$ / ε - small positive constant/ x_A and x_p are to the left of x_F, then dy/dt is positive. It means that in the course of the optimization sequence the operating point does not overrun the maximum of (φ). If x_A is all the time to the left of x_F and at $t = t_3 - \varepsilon$ x_p is to the right of this point, dy/dt is negative what indicates that the operating point has overrun the maximum of (φ) in the course of the optimization sequence. In such a manner, the value of dy/dt measured at the time when the definitive control is to be applied, gives information about the position of the operating point with regard to the maximum of (φ) . The amplitude of the definitive control can be varied as a function of the value of dy/dt which is measured at that time when the control is to be applied

$$C_{DC} = C_D + \delta_i \frac{d\varphi}{dt} \quad \text{for } x_A < x_F \quad i = 1,2$$

$$C_{DC} = C_D - \delta_i \frac{d\varphi}{dt} \quad \text{for } x_A > x_F$$

where C_{DC} is the corrected definitive control

δ_i is a coefficient which depends upon the parameter p /see paragraph 2/.

If dynamical properties of W_2 can not be neglected, the fact that $d\varphi/dt$ is negative allows to make conclusion that the maximum point of (φ) has been overrun. But under the same conditions, the fact that $d\varphi/dt$ is positive does not allow to draw a conclusion that the maximum point of (φ) has not been overrun. The correction of the definitive control can be performed but with some complications.

When drift of (φ) occurs, the tachometric compensation may be used but it has to be taken into account that for the block of $d\varphi/dt$ the variable φ is an output variable from which the extrapolated value of shift occuring during the first step of the optimization sequence is subtracted.

When the operating point moves in the disadvantageous direction, it is wise to decrease the rest step duration. It can be done by making use of information about $d\varphi/dt$ at $t = t_3 - \varepsilon$. Then, the rest step duration is no more constant and varies with the motion of the operating point.

7. CONSIDERATIONS ON CONTROLLER STABILITY

The controller is said to be stable if in steady state under the application of an optimization sequence the operating point is not moved out of the extremum of the characteristic (φ).

7.1. Stability Versus Changes of the Parameter p of the Parabola (P)

Let the extremal characteristic (φ) be a parabola. In the case when disturbances have no effect on this characteristic, the parameter p corresponding to the parabola (P) can not be the best approximation of the characteristic (φ) for all positions of the operating point, because this parameter has been chosen as an average value. As a result of it, the controller stability under changes of the parameter p of the parabola (P) has to be considered .

In order to carry out the stability analysis it is assumed that the characteristic (φ) is a parabola with a parameter p_v and that only an estimate of this parameter, equal to p, is known. The error of the variable x which arises as a result of this fact is as follows

$$\varepsilon^* = \frac{d_x^* - d_{xv}^*}{d_{xv}^*}$$

where d_x^* is a control corresponding to the estimate p, d_{xv}^* - an ideal control which brings the operating point to an extremum of the parabola (φ). To generate this control one has to know the exact value of p, namely p_v. For operating points which are distant enough from the extremum

$$\varepsilon^* \cong \frac{p}{p_v} - 1$$

Figure 12 shows ε^* against $\frac{p_v}{p}$. The stability limit is determined by $\varepsilon^* = \pm 1$. In fact, for $\varepsilon^* = +1$, the control d_x^* is two times the control d_{xv}^* ; new and previous operating points are symmetric about the extremum. For $\varepsilon^* = -1$, $d_x^* = 0$ and the operating point does not change its position.

A condition for stability has the form

$$|p| < 2 \, |p_{v\,min}| \qquad /9/$$

where $|p_{v\,min}|$ is the smallest possible value of the parameter p. It should be noted that too small values of $|p|$ ought not to be chosen, because the optimization rate decreases with $|p|$.

7.2. Stability Versus Settings of Dynamical Characteristics of the W_1 and W_2 Units

In practice, only approximations of responses $\overline{x}(\Delta t_r)$ and $\overline{\varphi}(\Delta t_r)$ are known. Moreover, the units W_1 and W_2 are nonlinear what implies that the form of the response is a function of the amplitudes of the signals μ and y. Control C, defined by (5), is determined with an error, which is

$$\varepsilon' \cong \frac{C - C_v}{C_v} = \frac{d^*_{\mu} - d^*_{\mu v}}{d^*_{\mu v}} .$$

where

$$C_v = \frac{-1}{\overline{\varphi}_v(\Delta t_r)} \cdot \frac{\Delta \varphi_s \, p}{\Delta \mu}$$

$\overline{\varphi}_v(\Delta t_r)$ is an exact response of W_1, W_2 set

$$\varepsilon' \cong \frac{\overline{\varphi}_v(\Delta t_r)}{\overline{\varphi}(\Delta t_r)} - 1$$

It can be found that in this case the stability limit is given by $\varepsilon' = \pm 1$ or $\overline{\varphi}_v(\Delta t_r) < 2\,\overline{\varphi}(\Delta t_r)$. The dotted area in Fig. 13 represents a region for which every curve $\overline{\varphi}(\Delta t_r)$ is stable providing the exact value of the parameter p is known. If we accept a curve $\overline{\varphi}$ which corresponds to a response more rapid than that of $\overline{\varphi}_v$, the amplitude of the definitive control C_D is greater than that which ought to be applied. It is due to the fact that the control C is smaller. Hence, the value of the second term of the relation (3) is smaller than the ideal one and the time interval Δt_c is smaller than the ideal time interval Δt_{cv} providing that the generator (GF_2) exactly reproduces the response $x(\Delta t_c)$. In such a manner, errors of C and Δt_c tend to compensate each other.

The generator (GF_2) determines the time interval Δt_c and has effect upon a system transient state.

Owing to the fact that changes in settings of the dynamical characteristics of W_1 and W_2 affects stability in little, the curve generated by (GF_1) can be replaced with the straight line (d) /Fig.9/ and the curve generated by (GF_2) - with the straight line (d') /Fig.11/.

7.3. Stability in the Presence of Noise

The value of the threshold $\Delta \varphi_s$ is chosen to be large enough with regard to the amplitude of noise disturbing the variable φ. However, the increase in $\Delta \varphi_s$ results in decreasing the optimization rate and accuracy. The latter is due to the increase in the amplitude of oscillations about the extremum of the characteristic (φ).

7.4. Stability Versus Drift of an Optimal Characteristic

The existence of the step of detection of an operating point drift makes possible to preserve stability of the controller under rapid changes of the extremal characteristic (φ). Experiments which are described in paragraph 8, corresponding to Fig.17 and 18, illustrate stability of the controller under the influence of sinusoidal and triangular disturbances.

8. EXPERIMENTAL INVESTIGATIONS

The process (S) has been simulated on an analog computer. Investigations which are illustrated in Fig.14a and b have been carried out in order to point out the fact that the controller is stable under large changes of the parameter p of the parabola (P). With this in mind the extremal characteristic (φ) has been chosen to be strongly dissymmetrical /the dotted curve in Fig.14/.

The transmittances of W_1 and W_2 units have been chosen as follows

$$W_1(p) = \frac{1}{1 + T_1 p} \qquad W_2(p) = \frac{1}{1 + T_2 p}$$

$$T_1 = 10s, \quad T_2 = 5s$$

The average value of the parameter p is $p_1 = p_2 = -1$.

It can be found that the tachometric compensation results in better response speed.

In order to investigate the controller stability as related to errors in settings of the dynamical characteristics of W_1 and W_2, the response $\varphi(t)$ was recorded for various values of T_1 and T_2 ; whereas settings for (GF_1) and (GF_2) were always $T_1 = 10s$, $T_2 = 5s$ /Fig.15/. The characteristic (φ) is shown in Fig.14.

Fig.16 illustrates the operation of the extremum - seeking controller in the presence of a sinusoidal disturbance $\lambda(t)$ with the amplitude equal to 3V and the period $T_p = 80s$. This disturbance results in a horizontal displacement of the parabola $\left[\text{curve } (\varphi)\right]$ with the parameter p = -2. For $|\Delta \varphi_s| = 0{,}1$ V and without the step of detection of an operating point drift, the system is unstable, but if the drift detection step exists, the system is stable /Fig.16a, 16b/. For the treshold $|\Delta \varphi_s|$ greater than that previously mentioned, namely for $|\Delta \varphi_s| = 0{,}3$ V, the system is stable even without the step of detection of an operating point drift, but when this step exists the system performance is better /Fig.16c/.

Fig.17 presents the response $\varphi(t)$ obtained for the case of disturbances of the triangular waveform. The disturbances cause a vertical displacement of the characteristic (φ) from Fig.18. It should be readily apparent that the existence of the drift detection step has stabilizing effect on the system performance.

For investigation purposes, the generators (GF_1) and (GF_2) have been replaced with linear elements [straight lines (d) and (d') in Fig.9 and 11]. It has given conclusive evidence of the fact that the controller stability is preserved despite errors in settings of the dynamical characteristics of W_1 and W_2.

9. CONCLUSIONS

The performance of the described extremum - seeking controller can be improved by eliminating the searching step. If it is the case, the control signal with the amplitude C_E following the first step of an optimization sequence is also used as a searching signal. If under the action of control the operating point moves in the wrong direction, the control C_E is terminated after Δt_r. But if as a result of control the operating point approaches the extremum, the control C_E lasts Δt_c. Exact relations (2), (3), (4), for which $\mu = C_E$, are used. The structure of such a controller is a bit more complicated despite decreasing the number of steps of the optimization sequence by one.

Principles on which the controller construction is based can be also used when the extremal characteristics is a function of two variables, $y = F(x_1, x_2)$. If it is the case, revolutionary or elliptic paraboloids are used for extrapolation instead of the parabola (P). As the base for this choice, the initial information about (φ) is used.

To conclude these considerations it should be noted that the described controller belongs to the class of model - reference adaptive systems. As a consequence of it, the generators (GF_1) and (GF_2) as well as their input and output elements can be replaced with models of W_1 and W_2.

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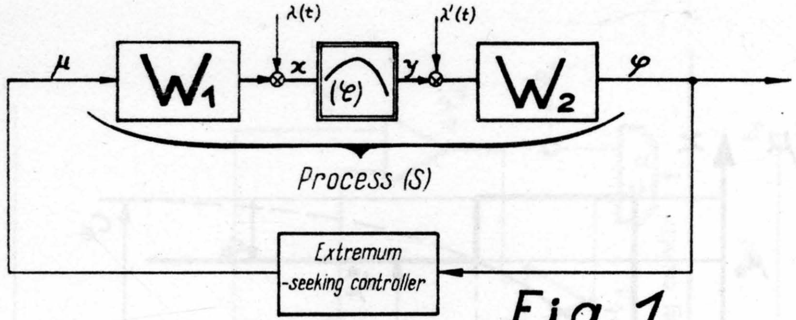


Fig. 1

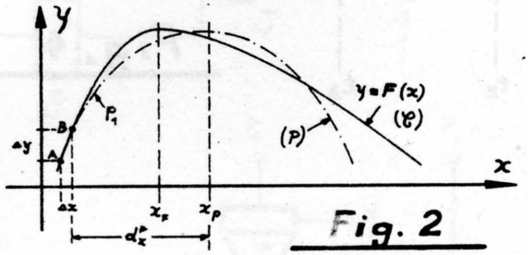


Fig. 2

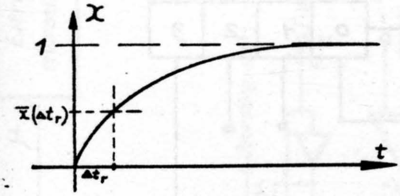


Fig.3.a

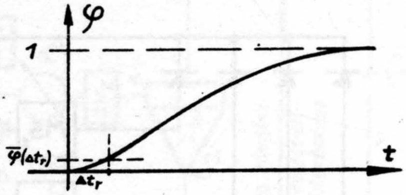


Fig .3.b

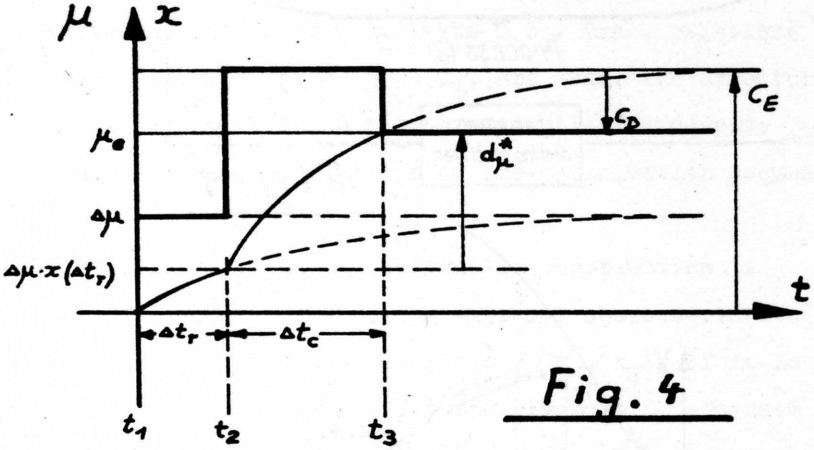

μ
x
μ_e
Δμ
Δμ·x(Δt$_r$)
C_D
C_E
d_μ^*
t
Δt$_r$
Δt$_c$
t_1
t_2
t_3

Fig. 4

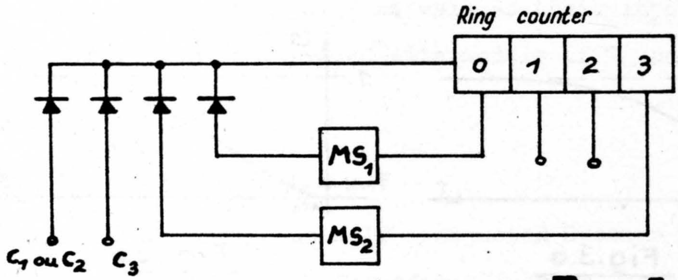

Ring counter
0
1
2
3
MS$_1$
MS$_2$
C$_1$ ou C$_2$
C$_3$

Fig. 5

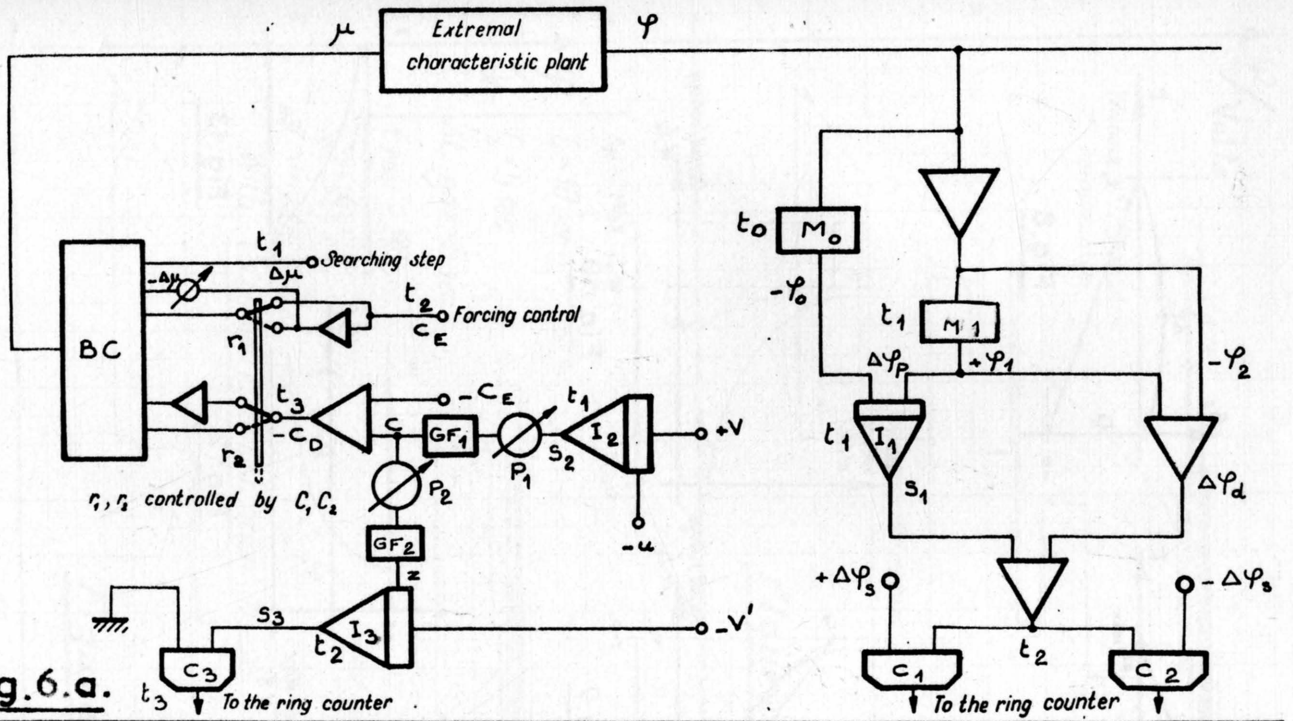

μ
Extremal characteristic plant
φ
BC
t1
Searching step
Δμ
-Δμ
t2
Forcing control
CE
r1
t3
-CE
-CD
C
r2
GF1
t1
+V
I2
S2
P1
P2
GF2
-u
z
r1, r2 controlled by C1 C2
S3
I3
t2
-V'
C3
t3
To the ring counter
t0
M0
-φ0
t1
M1
Δφp
+φ1
-φ2
t1
I1
S1
Δφd
+Δφs
-Δφs
t2
C1
C2
To the ring counter

Fig.6.a.

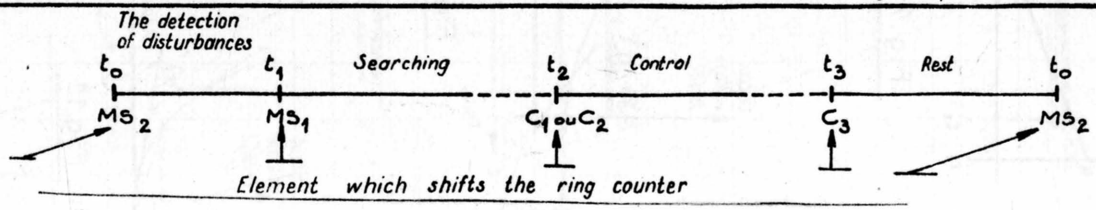

The detection of disturbances
t0
t1
Searching
t2
Control
t3
Rest
t0
MS2
MS1
C1 ou C2
C3
MS2
Element which shifts the ring counter

Fig.6.b.

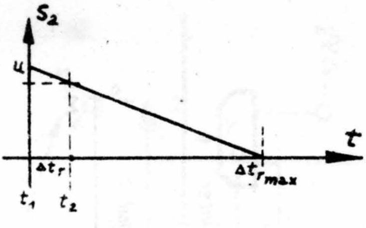


Fig.7

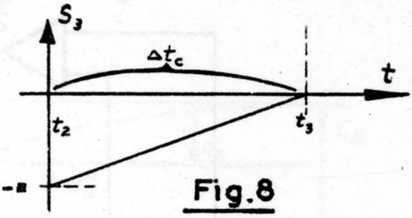


Fig.8

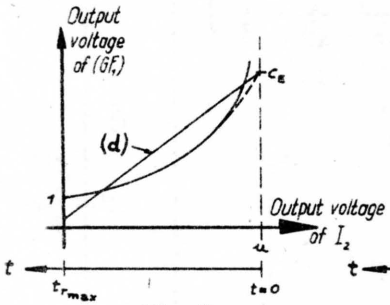


Fig.9

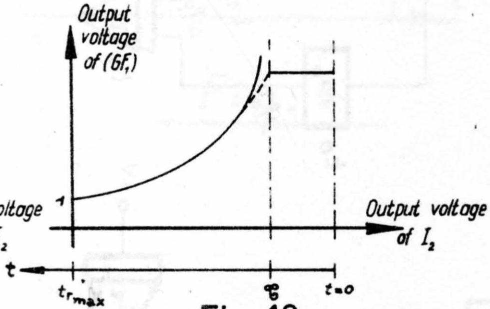


Fig.10

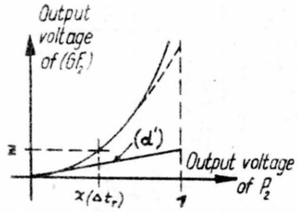


Fig.11

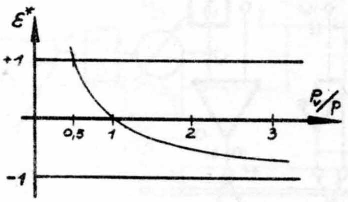


Fig.12

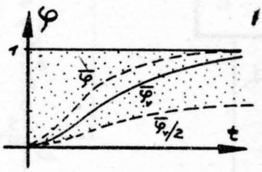
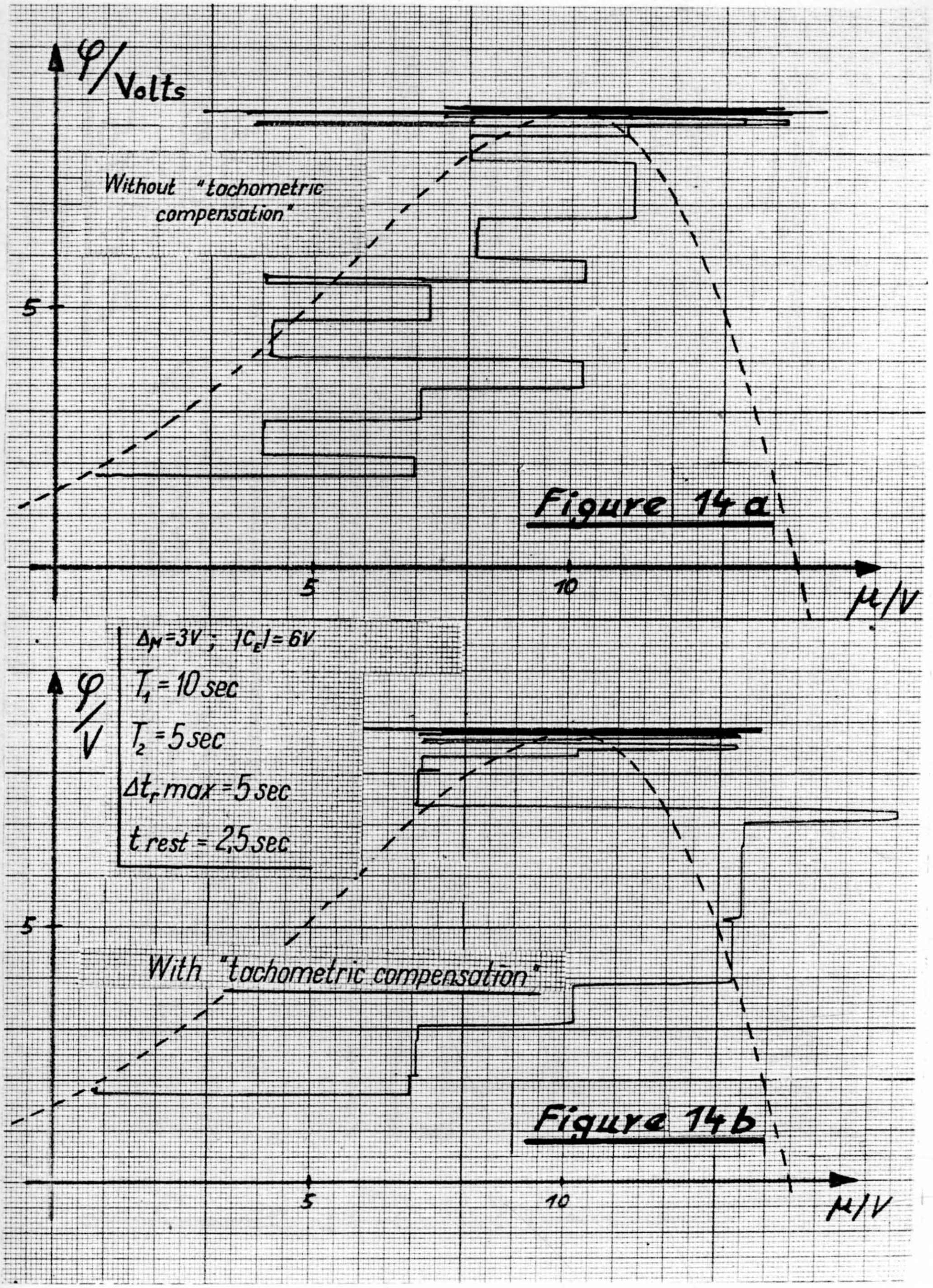


Fig.13


φ/Volts
Without "tachometric compensation"
5
Figure 14 a
5
10
μ/V
Δμ = 3V ; |C_E| = 6V
T₁ = 10 sec
T₂ = 5 sec
Δt_r max = 5 sec
t rest = 2,5 sec
φ/V
5
With "tachometric compensation"
Figure 14 b
5
10
μ/V

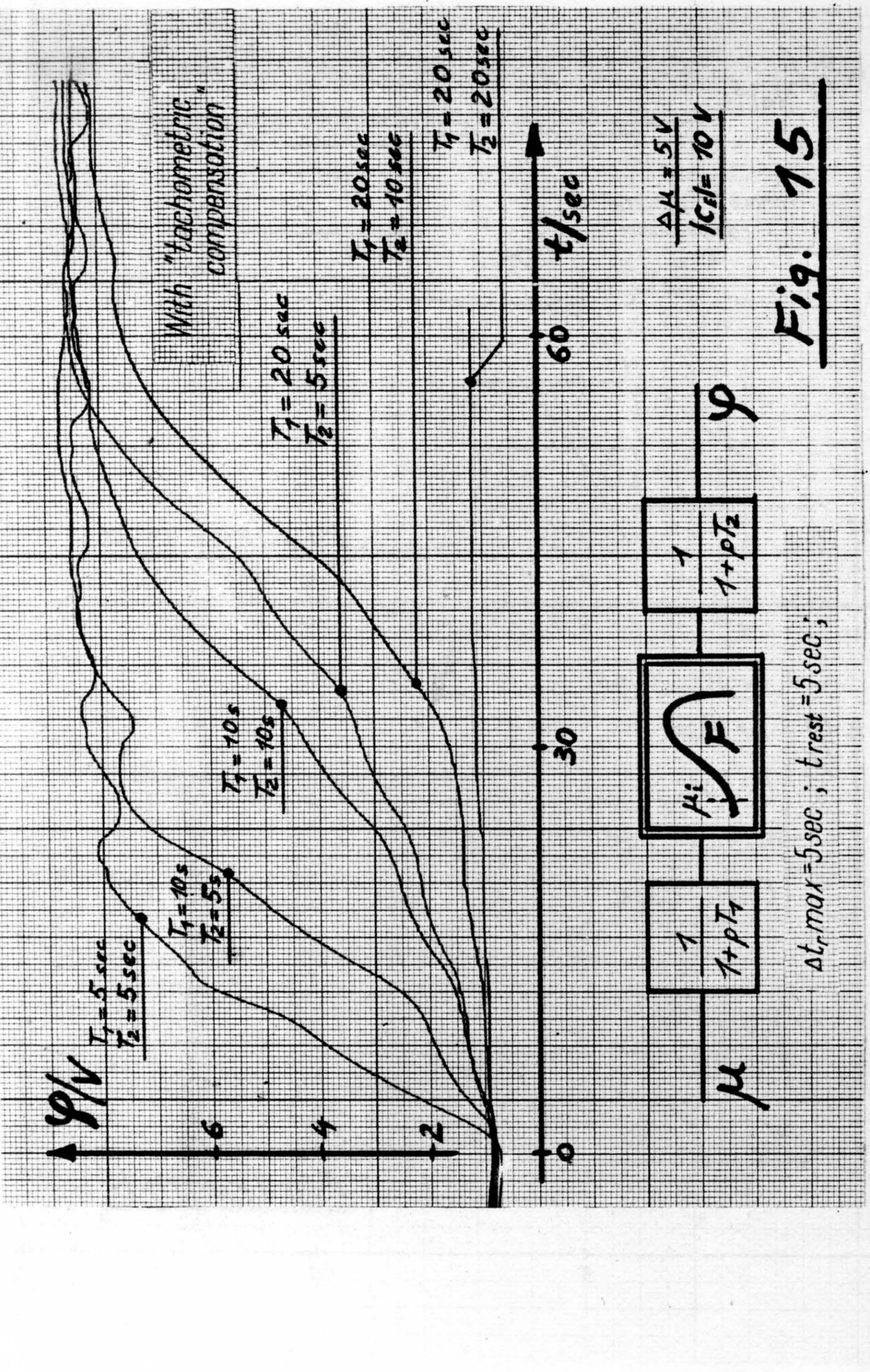

φ/V
6
4
2
0
30
60
t/sec
T1 = 5 sec
T2 = 5 sec
T1 = 10s
T2 = 5s
T1 = 10s
T2 = 10s
T1 = 20 sec
T2 = 5 sec
T1 = 20 sec
T2 = 10 sec
T1 = 20 sec
T2 = 20 sec
With "tachometric compensation"
Δμ = 5V
|C_E| = 10V
μ
1
1+pT1
μi
F
1
1+pT2
φ
Δtr max = 5 sec ; t rest = 5 sec ;

Fig. 15

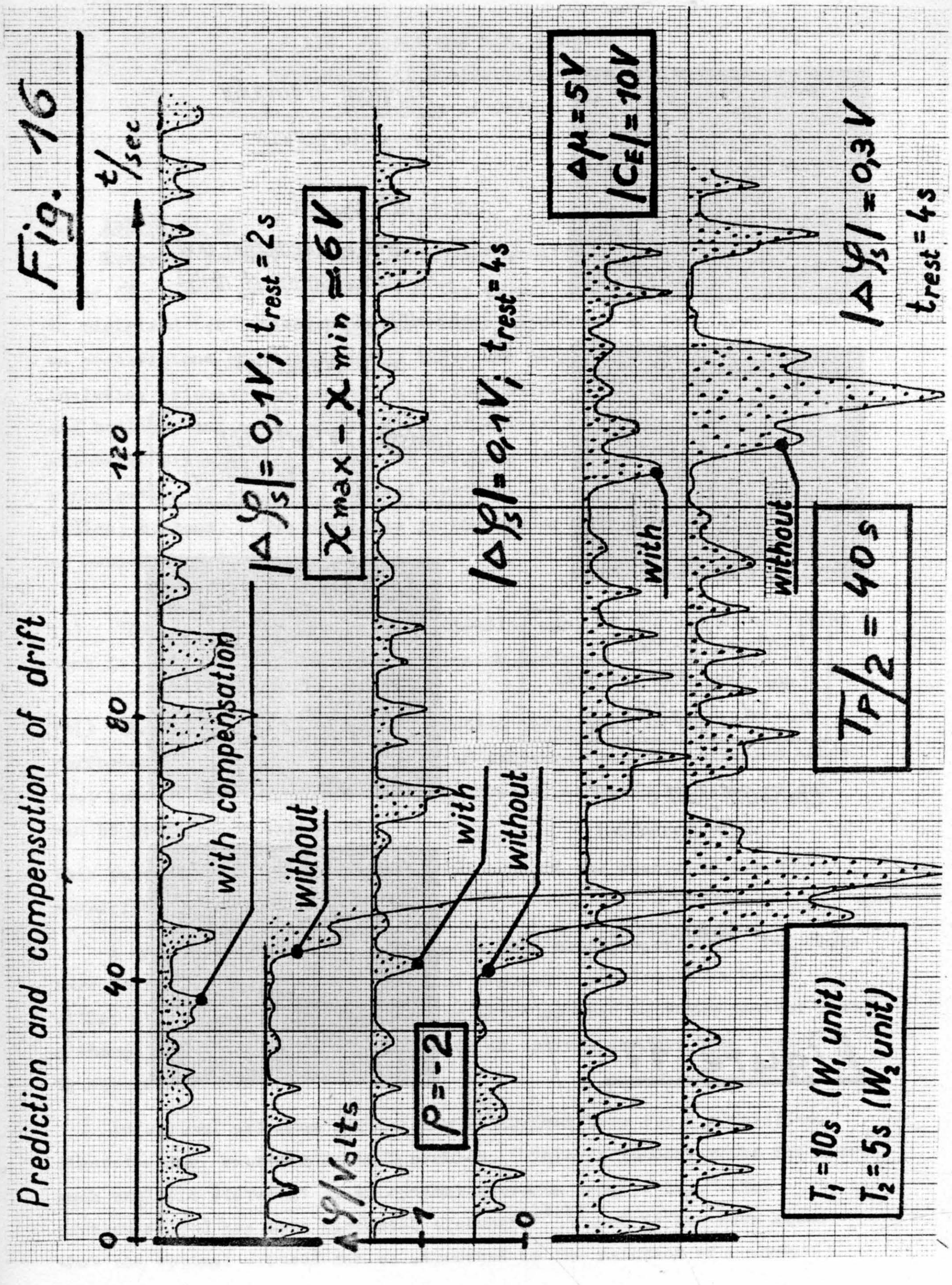
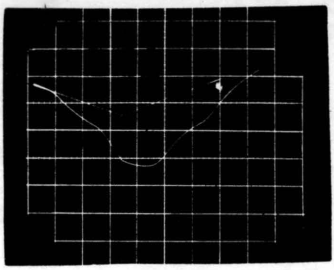


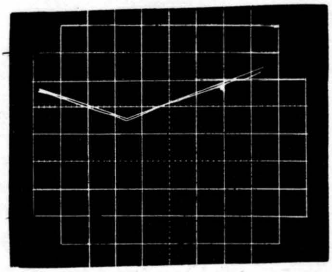
Fig. 16
Prediction and compensation of drift
t/sec
0
40
80
120
φ/Volts
with compensation
without
with
without
|Δφs| = 0,1 V; t_rest = 2s
X max − X min = 6 V
|Δφs| = 0,1 V; t_rest = 4s
ρ = −2
-1
0
Δμ = 5V
|C_E| = 10V
with
without
|Δφs| = 0,3 V
t_rest = 4s
T_P/2 = 40 s
T1 = 10s (W1 unit)
T2 = 5s (W2 unit)

Vertical triangular disturbances



a) $\lambda' = (0{,}14\ V/sec)\cdot t$

(without prediction and compensation of drift)

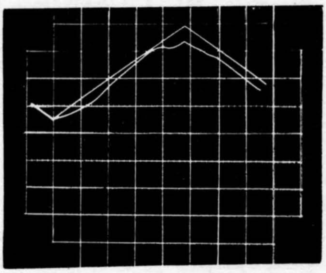


b) $\lambda' = (0{,}14\ V/sec)\cdot t$

(with prediction)

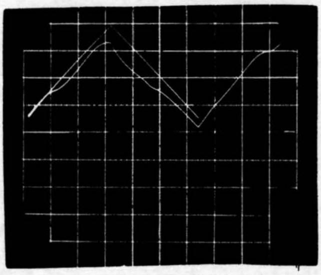
1 division = 2 V (vertically)
= 5 sec (horizontally)

$T_1 = 10\ sec$; $T_2 = 5\ sec$
$t_{rest} = 5\ sec$; $\Delta\varphi_s$ = threshold
$\Delta_\mu = 5V$; $|C_\varepsilon| = 10V$



c) $\lambda' = (0{,}28\ V/sec)\cdot t$

(with prediction)



d) $\lambda' = (0{,}42\ V/sec)\cdot t$

(with prediction)

Figure 17

AN OPTIMAL EXTREMUM CONTROL SYSTEM

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1. Introduction

Extremum control or 'hill climbing' problems are a well defined[1,2] class of control problems in which the output of a controlled process is minimised (or maximised) by automatic adjustment of the input, based on observations of the output. There are many potential applications including automatic maximisation of combustion efficiency in engines and boilers, automatic maximisation of efficiency in chemical plant, automatic minimisation of wear in drilling machinery, automatic minimisation of errors in control systems, automatic focussing of optical systems. Two obstacles in the way of widespread practical use of extremum control are:-

(i) The practical difficulty of measuring many of the output variables that it might be desirable to minimise or maximise.

(ii) The lack of theoretical information about what is the best performance that can be expected of an extremum control system and about what control law will yield that performance, assuming that the output can be measured or estimated.

This paper is a contribution to the removal of the second of the above obstacles.

A variety of extremum control laws have been proposed[1-11], but these have mostly been derived by empirical arguments and little is known either about their relative merits or about optimal extremum control laws although the question of what would be an optimal law has been raised by several authors[2,7,12-16].

An optimal extremum control law is derived here for a simplified, discrete-time, extremum control problem. This is thought to be the first such explicit statement of an optimal extremum control law. The problem solved has been discussed elsewhere[14,15] and it has been shown that the optimal control can be expressed as the solution of a dynamic programming[17,18] equation. Mozgovaya[15] showed how the problem could be simplified by separating the steady state and transient terms, but solved an incorrect equation for the steady state term; Jacobs[14] formulated a correct but impracticable equation for the transient term. The two approaches are combined here to yield the correct solution.

The resulting control law gives an indication of the structure of optimal extremum control laws and it gives a measure of the best performance that can be achieved for the particular problem investigated. This performance had previously been achieved using a simpler suboptimal policy[14] and it is concluded that the suboptimal policy should in this case be prefered for practical purposes.

2. The Simplified Extremum Control Problem

Figure 1 shows the simplified extremum control problem in which effects of measurement noise and dynamic lags are neglected. The observable output variable c is assumed to be a quadratic function of a variable x

$$c = Ax^2 \tag{1}$$

where x is the unobservable sum of a control variable u and a disturbance variable z

$$x = u + z \tag{2}$$

It is assumed that all variables are discrete-time variables, as is the case when a digital computer is used as a controller, and the integer variable i is used to represent time. The disturbance z is a stochastic variable with transformation

$$z(i+1) = z(i) + r(i) \tag{3}$$

where r is an independent random variable with zero-mean, stationary Gaussian probability density

$$p(r) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-r^2/2\sigma^2) \tag{4}$$

The problem is to make the series of control decisions u(1), u(2).... so as to minimise the expected value of the time average $\bar{c}$ of the output c defined by

$$\bar{c} = \underset{N\to\infty}{\mathrm{Lt}} \frac{1}{N} \sum_{i=1}^{N} c(i) = A\,\overline{x^2} \tag{5}$$

where the expectation is taken over the known probability densities (equation (4)) for the random variables r(1), r(2) ...

A first step towards formulating the equations of optimal control is the choice of state variables. For this purpose it is convenient to consider increments v in the control variable u

$$v(i) = u(i + 1) - u(i) \tag{6}$$

so that the system difference equation can be written (combining equations (2), (3) and (6))

$$x(i + 1) = x(i) + v(i) + r(i) \tag{7}$$

Equation (7) shows that the variable x is the state variable of the system and that its dynamics are linear. If the variable x were observable the quadratic performance criterion $\bar{c}$ of equation (5) would be minimised by a linear control law

$$v(i) = -x(i) \tag{8}$$

However x is not directly observable; what is observable is the output c and so each observation gives two possible values for x,

$$x(i) = \pm \sqrt{c(i)/A} = \pm \; w(i)$$

where w is an observable variable defined by

$$w = +\sqrt{c/A} = |x| \tag{9}$$

it being assumed that the constant A is known.

3. Sufficient Statistics

In the absence of direct observations of the state x it becomes necessary to describe the state of the system by a probability distribution for x. For each observation c the state x has two possible values ± w and so the probability distribution for x can be represented by the value of w and by a number q such that

q(i) = probability that x(i) has the positive value w

1 - q(i) = probability that x(i) has the negative value - w

This probability distribution has expected value

$$\begin{aligned} E[x(i)] &= q(i)w(i) - (1 - q(i))\, w(i) \\ &= (2q(i) - 1)\, w(i) \end{aligned} \tag{10}$$

After each stage of the process an updated probability distribution $[w(i+1), q(i+1)]$ can be derived from knowledge of the probability distribution $[w(i), q(i)]$ at the previous stage, of the control increment $v(i)$, and of the resulting output $c(i+1)$. The new value $w(i+1)$ is given by equation (9) and the new value $q(i+1)$, derived[14] by application of Bayes' rule to the system of equations (1) - (6), is

$$q(i+1) = \frac{P_1 q(i) + P_2(1 - q(i))}{(P_1 + P_3)q(i) + (P_2 + P_4)(1 - q(i))} \tag{11a}$$

where

$$\begin{aligned} P_1 &= \exp(-(w(i+1) - w(i) - v(i))^2/2\sigma^2) \\ P_2 &= \exp(-(w(i+1) + w(i) - v(i))^2/2\sigma^2) \\ P_3 &= \exp(-(w(i+1) + w(i) + v(i))^2/2\sigma^2) \\ P_4 &= \exp(-(w(i+1) - w(i) + v(i))^2/2\sigma^2) \end{aligned} \tag{11b}$$

The variables w and q completely specify the probability distribution for the state of the system at each stage; they are sufficient statistics[17,18]. They play the role of state variables in formulating the optimal control equations, and the resulting optimal optimal control law is specified by a function $v^*[w, q]$ giving the optimal control $v(i)$ as a function of the current values $w(i)$ and $q(i)$.

(4) Optimal Control Equations

The optimal control is derived by considering the possible updated values $w(i+1)$ and $q(i+1)$ that may result from a control $v(i)$. The updated value $w(i+1)$ is given by equations (1), (7) and (9)

$$w(i+1) = |x(i+1)| = |x(i) + v(i) + r(i)|$$

$$= |\pm w(i) + v(i) + r(i)|$$

so that there are two possible values depending on whether $x(i)$ is positive or negative. These are

$$\begin{aligned} w(i+1) &= |w(i) + v(i) + r(i)| \quad \text{with probability } q(i) \\ w(i+1) &= |-w(i) + v(i) + r(i)| \quad \text{with probability } 1 - q(i) \end{aligned} \tag{12}$$

It is convenient to write

$$|w(i) + v(i) + r(i)| = w^+$$

and

$$|-w(i) + v(i) + r(i)| = w^-$$

and to use the notation q^+, q^- for the two corresponding possible values of $q(i + 1)$ given by combining equations (11) and(12).

The dynamic programming approach[18] is to define an optimal expected return function

$F_N[w, q]$ = The expected value of the sum $\sum_{i=1}^{N} c(i)$ when the optimal control v^* is used for N stages starting from $w(1) = w$, $q(1) = q$.

The principle of optimality states that when the optimal policy is used and the first decision is $v(1) = v$ and the random variable $r(1)$ takes the value r the return to be expected from the remaining N - 1 stages is

$$qF_{N-1}[w^+, q^+] + (1 - q) F_{N-1}[w^-, q^-]$$

But the contribution from the first stage is Aw^2 and so, taking into account the known probability distribution for $r(1)$, the expression for total expected return can be written

$$Aw^2 + \int_{-\infty}^{\infty} (qF_{N-1}[w^+, q^+] + (1 - q) F_{N-1}[w^-, q^-])p(r)dr$$

When the optimal control law is used this expected return is minimised with respect to all decisions $v(1)\ldots$, $v(N)$, including the first decision $v(1)$, so that

$$F_N[w, q] = Aw^2 + \min_v \Big\{ \int_{-\infty}^{\infty} (q F_{N-1}[w^+, q^+] + (1 - q) F_{N-1}[w^-, q^-]) p(r)dr \Big\} \quad (13)$$

Equation (13) is the dynamic programming functional recurrence equation for the total return from a finite-time process. For a single-stage process, N = 1, the decision v cannot affect the return and the definitions give

$$F_1[w, q] = Aw^2 \quad (14)$$

Putting N = 2 in equation (13), using equation (14), performing the integration for the probability density p(r) of equation (4) and then minimising gives the optimal control for a process with only two stages

$$v_2^* [w, q] = (1 - 2q) w$$

and the optimal expected return is

$$F_2 [w, q] = A(w^2 + 4w^2 q(1 - q) + \sigma^2) \tag{15}$$

The optimal policy for a process with an infinite number of stages, as specified by the performance criterion of equation (5), could, in principle, be determined by repeated iteration of equation (13). In practice it is more convenient to regard the total return $F_N [w, q]$ as the sum of a transient term f_N that depends on the initial conditions [w, q] and a steady state term that depends only on the number of stages N

$$F_N [w, q] = f_N [w, q] + Nc_N^*$$
$$f_N [o, o] = 0 \tag{16}$$

Substituting equations (16) into equation (13) and iterating[19] from the initial solution of equation (15) it was found that for large N (N > 7) the transient term, the average steady state term and the optimal control all become independent of N

$$f_N [w, q] \rightarrow f [w, q]$$

$$c^*_N \rightarrow c^* \tag{17}$$

$$v_N^* [w; q] \rightarrow v^* [w, q]$$

Combining equations (16) and (17) gives

$$\underset{N \rightarrow \infty}{\mathrm{Lt}} \frac{1}{N} F_N [w, q] = c^*$$

and the definition of the total return function F_N shows that c* is the minimum value of the performance criterion $\bar{c}$ of equation (5).

Thus the limiting control law $v^* [w, q]$ is the optimal control for the extremum control problem specified in Section 2.

5. Optimal Control Law

The optimal control law $v^* [w, q]$ was found by writing a digital computer program to iterate a discretised form of the functional recurrence equation. Details of the computation which gives results to an accuracy of approximately $\pm$ 1% will be described elsewhere[19]. Figure 2 shows the numerical results (normalised with respect to the standard deviation σ of the random variable r) which are subject to a symmetry condition

$$v^* [w, q] = -v^* [w, (1 - q)] \tag{18}$$

and to an asymptotic approximation for large w

$$v^* [w, q] \rightarrow -(2q - 1) w \tag{19}$$

where $(2q - 1)w$ is the expected value of the state x, given by equation (10).

The computation also gave the value of the steady state performance criterion

$$c^* = \min\{\bar{c}\} = 2.2 A\sigma^2 \tag{20}$$

Another digital computer program was written[19] to simulate the performance of the system in Section 2 using the optimal control law of Figure 2. The average value of $c/A\sigma^2$ was found to be 2.2 (to accuracy approximately $\pm$2%) and this agreement with the value predicted by equation (20) is regarded as confirmation that the optimal control law has been correctly derived. (A corresponding simulation using the 'optimal' control law proposed by Mozgovaya[15] for the same system gave $\bar{c}/A\sigma^2$ the value 7, which indicates performance far from optimal).

For purposes of comparison with suboptimal policies the control v is regarded as the sum of two terms, a correction term equal in magnitude and opposite in sign to the expected value of the state x and an information-sensing term, the 'intentional error'[7].

$$v(i) = - E[x(i)] + \text{intentional error} \quad (21)$$

Equations (10) and (21) and Figure 2 were combined to give Figure 3, which shows the normalised intentional error as a function of $[w, q]$. The intentional error is subject to a symmetry condition similar to equation (18),

$$\text{Intentional error}[w, q] = - \text{Intentional error}[w, (1 - q)] \quad (22)$$

An interesting feature of the optimal control law is the discontinuity that is evident in some of the lines in Figures 2 and 3. This is due to the 'dual'[20] nature of the control which must provide both correction and information-sensing functions, as indicated by equation (21). The discontinuity arises as one of these functions becomes more important than the other.

6. Comparison with a Sub-optimal Control Law

A suboptimal control law has been described elsewhere[14] which gives performance as good as that of the optimal control law derived here. The suboptimal law uses an intentional error of constant magnitude a and with sign that can be chosen according to either of the rules

$$\text{intentional error} = (-1)^i a \quad (23a)$$

or

$$\text{intentional error} = a \text{ sign } [1 - 2q] \quad (23b)$$

It was found that when the system of Section 2 was controlled by a law based on equations (10), (21) and either of equations (23) the average value of $c/A\sigma^2$ could be reduced to the optimal value 2.2 by using an intentional error of magnitude

$$a = 0.8\sigma \quad (24)$$

This magnitude of intentional error is indicated in Figure 3 and can be seen to be typical of the magnitude of the optimal intentional errors.

7. Conclusions

It is a characteristic feature[21] of extremum control problems that the true state of the controlled process, for example the variable x in equation (1), is never directly observable. Even when measurement noise is negligible, as in the system discussed here, the non-linear extremal function introduces ambiguity between the observable variable c and the true state x. For problems like this, where there is uncertainty about the true value of the state, it is known[14, 17] that the equations of optimal control must be formulated by using as effective state variable an updated probability distribution for the true state. Such a formulation is only feasible if the updated probability distribution can be represented by sufficient statistics, and numerical solution of the resulting dynamic programming equation is not usually feasible if its dimensionality exceeds two.

The extremum control problem that has been solved here was specially simplified so that the updated probability distribution could be represented by two sufficient statistics. When additional features of practical problems are introduced, such as dynamic lags and measurement noise, the dimensionality of the problem increases and the probability distributions can no longer be represented by sufficient statistics. The optimal control equations can then be neither formulated nor solved and so optimal control theory cannot be regarded as a general procedure for the design of practical extremum controllers.

The control law derived here is therefore of particular interest as the only known example of an optimal extremum control law. The following conclusions can be drawn:-

(i) The optimal control is a 'dual control' that is the sum of a correction term and of an information-sensing term which depends on the current state. 'Dual control' (or 'adaptive') strategies have been proposed by many authors[22] but there has hitherto been some doubt about the circumstances under which such 'dual control' would be optimal[23]. It seems that it is the non-linearity of the extremum problem that forces the optimal control to be 'dual'.

(ii) The suboptimal control law described in Section 6 has the 'dual' structure of the optimal law, but is simpler to realise because its intentional error is constant in magnitude. The performance of the suboptimal law is indistinguishable from that of the optimal law and so the suboptimal law is to be prefered, on the grounds of its simplicity, for practical control of the problem specified in Section 2.

The question of how the optimal and suboptimal laws compare, and of how they might be modified, for extremum control problems where dynamic lags and measurement noise are not negligible is a matter for further research.

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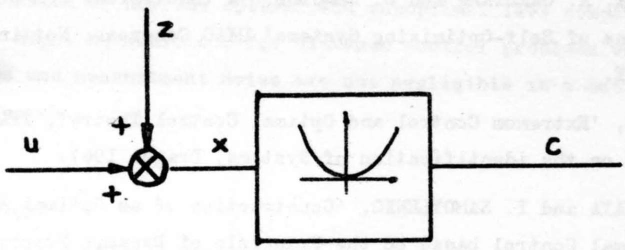

FIG I SIMPLIFIED EXTREMUM SYSTEM

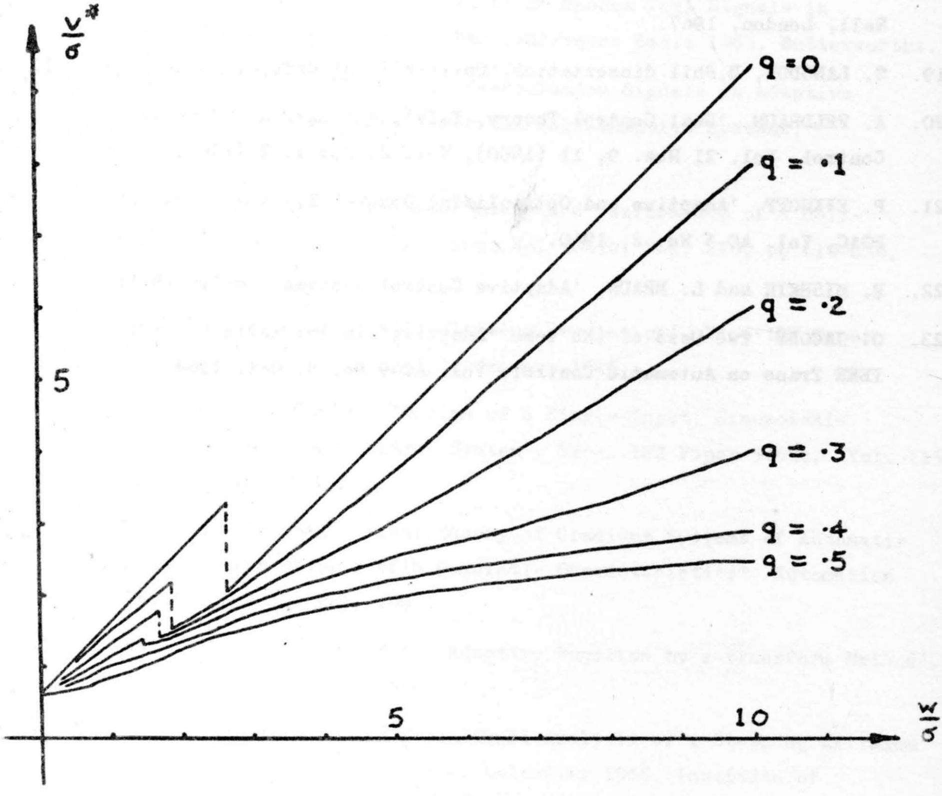

FIG 2 OPTIMAL CONTROL LAW

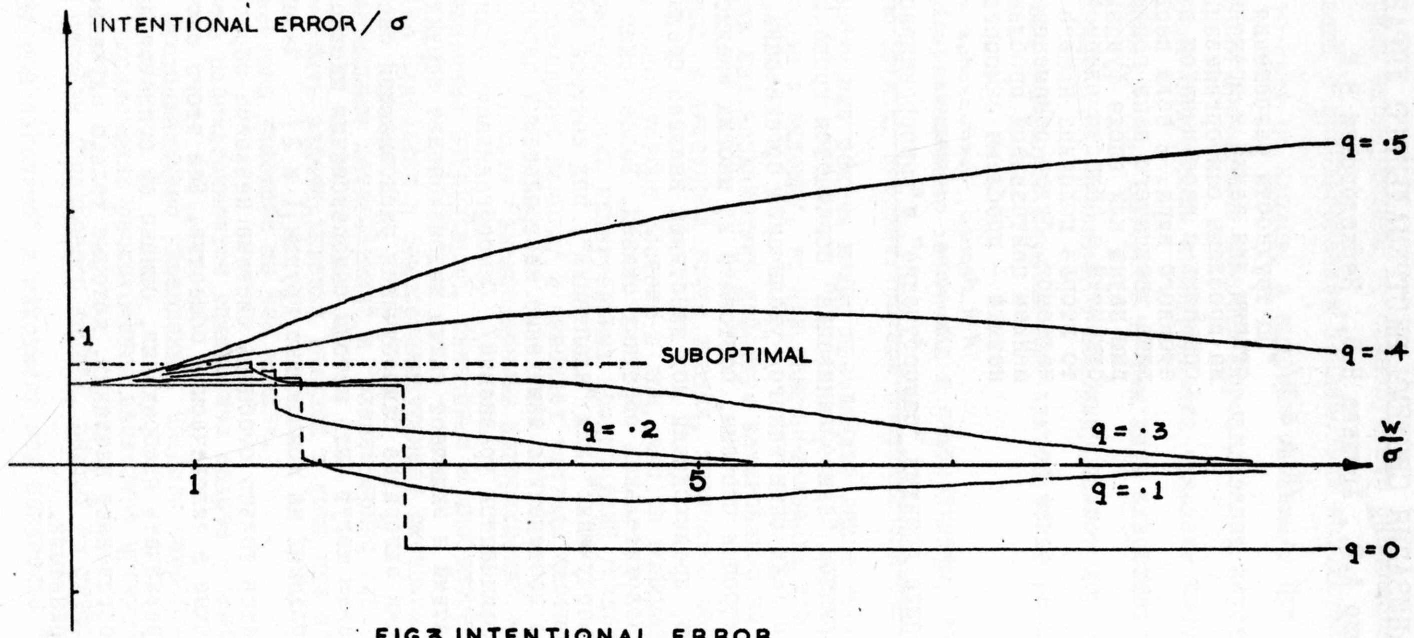

FIG.3 INTENTIONAL ERROR

САМООРГАНИЗАЦИЯ СИСТЕМЫ ЭКСТРЕМАЛЬНОГО УПРАВЛЕНИЯ

Ивахненко А.Г., Хрущева Н.В., Несходовский В.И.

/К и е в/

"По трудности разрешения и последствиям для науки и практики атаку на проблемы самоорганизации можно сравнить с наступлением на тайну атомного ядра. И если первая половина двадцатого века войдет в историю науки как эпоха фундаментальных открытий в области ядерной физики, то вторая половина нашего века, мы надеемся, будет ознаменована разрешением центральной проблемы кибернетики - проблемы самоорганизации".

А.Я. Лернер, Предисловие к книге „Принципы самоорганизации", изд. „Мир", 1966.

Определение понятия "самообучение" и "самоорганизация"

Под термином "самоорганизация" понимается процесс самопроизвольного /или спонтанного/ увеличения организации, т.е. уменьшения энтропии системы, состоящей из многих взаимосвязанных элементов, происходящий под действием внешней среды или собственных положительных обратных связей. Более узким понятием является "самообучение" или "адаптация", под которой понимается обычно только постепенное изменение настраиваемых параметров системы /коэффициентов уравнений/. Самоорганизация отличается родом воздействий и включает также видоизменение структуры.

Примерами алгоритма самообучения распознающей системы по входным сигналам могут быть методы саморазбиения множества входных изображений на компактные группы [1 и 2] . Человеку - учителю остается только сообщить системе название образа каждой группы, принятые в человеческом обществе. Без этого система также будет различать изображения, однако ей придется дать образам свои собственные названия, которые только случайно могут совпасть с принятыми.

Первые эксперименты по самообучению распознающих систем были связаны с использованием воздействий положительных обратных связей /то-есть не входов, а выходов системы/. При этом процесс самообучения прототипов подобен опрокидыванию неустойчивого тела. В случае отсутствия вмешательства человека система почти с равной вероятностью выбирает ту или иную классификацию изображений на образы /3/. Известны и комбинированное использование как входов, так и выходов системы для самообучения распознающих систем /2/.

Самоорганизация, также как и самообучение, происходит под действием внешних воздействий или обратных связей. Однако, если при самообучении всегда можно проследить линии передачи воздействий /входы и выходы/, то при самоорганизации это почти невозможно, так как воздействия действую интегрально на множество однотипных, с некоторой точки зрения, элементов системы. Роль человека-конструктора состоит в том, что он выбирает нелинейные характеристики этих интегральных воздействий /например, нелинейность подоходного налога/ и снабжает частицы системы /например, фирмы/ определенными "элементарными алгоритмами" взаимодействия.

Известные затруднения, связанные с многомерностью сложных систем, не относятся к самоорганизующимся системам, где действуют "интегральные воздействия" и "элементарные алгоритмы". Примером может быть задача укладки деталей в ящик при помощи тряски - одновременного действия на все детали.

Ниже рассматривается инженерная задача - самоорганизация системы экстремального управления. Процесс самоорганизации приводит к упорядочению расположения множества прототипов распознающей системы /"полюсного газа"/, каждый из которых имеет свой "элементарный алгоритм" и находится под контролем "интегральных воздействий".

Комбинированная система экстремального управления с корректором - распознающей системой. Ограничения задачи и определение области применения

Выше мы говорили о самообучении распознающих систем потому, что предметом данной работы является самоорганизация комбинированной системы экстремального управления, состоящей из разомкнутой части /РЧ/ и ее корректора /К/, где в качестве последнего предлагается использовать распознающую систему /рис.I/.

Выясним ограничения задачи. Рассматриваются как одномодальные, так и многомодальные экстремальные характеристики /холмы/, удовлетворяющие такому требованию: оптимальная характеристика объекта управления /"множество желаемых состояний"/ в области рабочих режимов представляет собой достаточно плавную линию $O_2^I\ O_2^{II}$, которую можно аппроксимировать кусочно-линейной функцией /рис.2/. При этом предполагается, что многомерные задачи при помощи приема, получившего название дивергенции, сводятся к одномерной задаче, решаемой в пространстве трех переменных λ, μ и ψ, где λ - обобщенное возмущающее воздействие и μ - обобщенное регулирующее воздействие, ψ - обобщенный показатель качества /учитывающий как показатель экстремума φ / так и значения возмущений $\psi = f(\varphi, \lambda)$/. Например, в простейшем случае, при линейной экстремальной характеристике будет

$$\varphi = r_0 + r_1 \mu + r_2 \lambda + r_3 \mu^2 + r_4 \lambda^2 + r_5 \mu \lambda$$

оптимальная характеристика $O_2^I\ O_2^{II}$ определяется выражением

$$\frac{\partial \varphi}{\partial \mu} = 0 \quad \text{или} \quad \mu = k_0 + k_1 \lambda \text{, где } k_0 = -\frac{r_1}{2 r_3} \text{ и } k_1 = -\frac{r_5}{2 r_3}$$

Потребуем, чтобы во всех точках этой характеристики было ψ = I. Исключая μ, получим /на линии $O_2^I\ O_2^{II}$/:

$$\psi = b_0 + b_1\lambda + b_2\lambda^2,$$

где

$$\bar{\bar{b}}_0 = \bar{\bar{r}}_0 - \frac{\bar{\bar{r}}_1^2}{4\bar{\bar{r}}_3};\ \bar{\bar{b}}_1 = \bar{\bar{r}}_2 - \frac{\bar{\bar{r}}_1 \cdot \bar{\bar{r}}_5}{2\bar{\bar{r}}_3};\ \bar{\bar{b}}_2 = \bar{\bar{r}}_4 - \frac{\bar{\bar{r}}_5^2}{4\bar{\bar{r}}_3}.$$

Очевидно, для выполнения такого требования следует применить преобразователь:

$$\psi = \frac{\varphi}{\bar{\bar{b}}_0 + \bar{\bar{b}}_1\lambda + \bar{\bar{b}}_2\lambda^2} = \bar{\bar{f}}(\varphi, \lambda).$$

При таком преобразовании в любой точке холма будет: $\psi \leq 1$ а на его гребне ψ = I. Более точно

$$\psi = \frac{\bar{\bar{r}}_0 + \bar{\bar{r}}_1\mu + \bar{\bar{r}}_2\lambda + \bar{\bar{r}}_3\mu^2 + \bar{\bar{r}}_4\lambda^2 + \bar{\bar{r}}_5\mu\lambda}{\bar{\bar{b}}_0 + \bar{\bar{b}}_1\lambda + \bar{\bar{b}}_2\lambda^2} = 1 - (-\bar{k}_0 - \bar{k}_1\lambda + \mu)^2$$

Практически такой преобразователь содержит в себе таблицы /карты/, указывающие изменение наших требований к величине показателя экстремума φ в зависимости от диапазона, в котором находится величина λ. Например, при одном сорте руды λ_I, мы можем сказать "достаточно хорошо", т.е. ψ = I, если содержание железа в отходах дбудет φ = 10%. При другом сорте λ_2 мы ту же оценку ψ = I дадим, если φ= 8% и т.п.

Процессы адаптации вызываются действием неизмеряемых аддитивных помех, вызывающих дрейф и поворот экстремального холма. В данном примере будет:

$$\psi = 1 - \{-[\bar{k}_0 + N_0(\bar{t})] - [\bar{k}_1 + N_1(\bar{t})]\lambda + \mu\}^2$$

где $N_0(\bar{t})$, $N_1(\bar{t})$ - медленно изменяющиеся помехи /смещение и поворот/

Чтобы устранить влияние переходных процессов, датчик показателя качества должен иметь некоторое усреднение или инерционность объекта должна быть компенсирована при помощи упредителей /5/, что в измерительных цепях не вызывает больших затруднений.

Следовательно, мы полагаем, что величины ψ, μ, λ поддаются измерению, причем ψ достаточно оценить только по двухбальной системе, например:

$$1 \geq \psi \geq 0{,}8 \text{ - "достаточно хорошо"}$$

$$\psi < 0{,}8 \text{ - "регулировать"}$$

Опыт, хоть и небольшой, по сравнению различных систем адаптивного экстремального управления с предлагаемой здесь системой показывает, что она оказывается конкурентно-способной именно в этом, весьма распространенном в жизни, случае. Если показатель экстремума можно измерять достаточно точно, то целесообразность применения самоорганизации и распознающих систем может вызывать сомнения. В то же время во многих сложных задачах мы можем сказать только "хорошо" или "что-то не нравится" т.е. различать два уровня качества. Именно в таких случаях рекомендуется описываемая здесь система.

Разомкнутая часть системы и "зубцы"

Разомкнутая часть системы представляет собой функциональный преобразователь с характеристикой, которую легко смещать, поворачивать или даже видоизменять. При оптимальной настройке характеристика разомкнутой части должна соответствовать оптимальной характеристике объекта управления. В примере, рассмотренном выше, требуется линейная зависимость:

$$\mu = \bar{\bar{d}}_0 + \bar{\bar{d}}_1 \lambda \quad \text{где} \quad \bar{\bar{d}}_0 = \bar{\bar{k}}_0 + N_0(\bar{\bar{t}}), \quad \bar{\bar{d}}_1 = \bar{\bar{k}}_1 + N_1(\bar{\bar{t}})$$

В более сложных случаях характеристика объекта выражается полиномом: $\mu = [\bar{\bar{k}}_0 + N_0(\bar{\bar{t}})] + [\bar{\bar{k}}_1 + N_1(\bar{\bar{t}})]\lambda + [\bar{\bar{k}}_2 + N_2(\bar{\bar{t}})]\lambda^2 + \ldots + [\bar{\bar{k}}_n + N_n(\bar{\bar{t}})]\lambda^{\bar{\bar{n}}}$,
тогда характеристика разомкнутой части будет:

$$\mu = \bar{\bar{d}}_0 + \bar{\bar{d}}_1 \lambda + \bar{\bar{d}}_2 \lambda^2 + \ldots + \bar{\bar{d}}_n \lambda^{n}.$$

Задача самообучения /адаптация/ состоит в том, чтобы коэффициенты характеристики РЧ "следили" за изменениями коэффициентов оптимальной характеристики объекта, то-есть чтобы

$$\bar{d}_{(0)} \to [\bar{\bar{k}}_{(0)} + \underline{N}_{(0)}(\bar{t})],\ \bar{\bar{d}}_{(1)} \to [\bar{\bar{k}}_{(1)} + \underline{N}_{(1)}(\bar{t})], \ldots, \bar{\bar{d}}_{(\bar{\bar{n}})} \to [\bar{\bar{k}}_{(\bar{\bar{n}})} + \underline{N}_{(\bar{\bar{n}})}(\bar{\bar{t}})]$$

Осуществление такой адаптации <u>принципиально</u> возможно только при измерении не менее двух точек поверхности экстремального холма, так как последний представляет собой четную функцию. Еще в работе /6/ было показано, что при четной характеристике поиск на объекте принципиально необходим. Описываемая здесь система все же называется беспоисковой, так как вместо периодических изменений регулирующих воздействий в функции времени в ней применено специальное наложение небольших "зубцов" на характеристику РЧ, заменяющие собой пробные шаги /6/. Указанное выше выражение $\underline{\mu}(\underline{\lambda})$ при этом относится лишь к средней линии характеристики. Амплитуда и распределение зубцов выбираются по форме холма и распределению возмущений так, чтобы обеспечить максимальное быстродействие системы. При отсутствии корректора "зубцы" не нужны.

Практически разомкнутая часть выполняется в виде матрицы ключей, открытие которых зависит от положения "значущего разряда" в унитарном коде обобщенного возмущения, а также от номера сработавшего триггера в кольцевых счетчиках импульсов [7]. Другая конструкция управляемого функционального преобразователя представляет собой логическую схему с пороговыми элементами /8/.

<u>Распознающая система-корректор /первый вариант/</u>

Как известно, распознающая система представляет собой логическое устройство, которое с целью классификации входных сигналов /обычно называемых "изображениями"/ на классы /или "образы"/ сопоставляют между собой некоторые меры близости этих сигналов к сигналам прототипов /или эталонов/, образующихся в системе в процессе ее обучения.

Ниже описываются алгоритмы действия двух вариантов распознающих систем, используемых для коррекции характеристики РЧ, причем обращается внимание на процессы самоорганизации множества прототипов. В корректоре по первому варианту в качестве входных сигналов /признаков/ используются координаты представляющей точки $\bar{\bar{X}}_{(i)}$, отвечающей "состоянию" объекта в данный момент. Как показано на рис.3 плоскость $\mu - \lambda$ можно разделить на три области или "ситуации"[x/]

I. Регулировать, уменьшить μ

П. Достаточно хорошо, так держать

Ш. Регулировать, увеличить μ.

Распознающая система является весьма гибкой и удобно настраиваемой моделью объекта управления. Например, в случае прямолинейной формы оптимальной характеристики объекта достаточно применить распознающую систему всего с тремя точечными прототипами:

$$\alpha_{(1)}(\mu_{(1)}, \lambda_{(1)}),\ \alpha_{(2)}(\mu_{(2)}, \lambda_{(2)}),\ \alpha_{(3)}(\mu_{(3)}, \lambda_{(3)})$$

Система подсчитывает меру близости входного сигнала и прототипов, например, так:

$$\Sigma_{(1)} = |\mu - \mu_{(1)}| + |\lambda - \lambda_{(1)}|,\quad \Sigma_{(2)} = |\mu - \mu_{(2)}| + |\lambda - \lambda_{(2)}|,$$

$$\Sigma_{(3)} = |\mu - \mu_{(3)}| + |\lambda - \lambda_{(3)}|.$$

Далее компаратор ИМН выбирает наименьшее из трех расстояний и тем самым указывает ситуацию. Например, представляющая точка $\bar{\bar{X}}_{(i)}$, показанная на рис.3, а будет отнесена к ситуации I.

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x/ Термины "состояние" и "ситуация" являются аналогами терминов "изображение" и "образ", применяемыми в распознавании графических изображений

Зная ситуацию, легко указать, в какую сторону следует изменять μ, чтобы попасть в ситуацию П, что является целью регулирования.

Распознающая система правильно указывает ситуацию только тогда, когда границы "области притяжения" полюсов $\Sigma_1 = \Sigma_2$ и $\Sigma_2 = \Sigma_3$ совпадают с границами ситуации П /показаны на рис.3 пунктиром/. При прямолинейных границах достаточно поставить три полюса в "перпендикулярное положение", указанное на рис.3,б. Процесс установления полюсов в нужное положение и постоянное поддержание такого положения при изменениях и перемещениях "экстремального холма" и есть процессом самоорганизации распознающей системы.

Самоорганизация трех полюсов по методу взвешенного смещения

Один из возможных алгоритмов самоорганизации трех полюсов показан на рис.3. В случае успешного окончания процесса самоорганизации полюса далжны придти в так называемое "перпендикулярное положение" /рис.3,б/.

Сначала два крайних полюса разводятся в верхний левый и нижний правый углы плоскости, и система представляется самой себе и действию внешних возмущений. Начинается самопроизвольный случайный процесс самоустановления полюсов. Датчик обратной связи, установленной на объекте, дает только два указания: "регулировать" или "достаточно хорошо, так держать". Распознающая система имеет три прототипа и, следовательно, три выхода: "регулировать, уменьшить μ", "достаточно хорошо, так держать" и "регулировать, увеличить μ". Ясно, что между указаниями датчика и распознающей системы может быть либо несоответствие, либо соответствие.

В первом случае, т.е. при несоответствии, один из крайних полюсов, который оказался ближайшим к представляющей точке /именно в этом появляется влияние зубцов характеристики разомкнутой части на выбор направления поворота полюсов/ делает шаг в направлении к ней /рис.3,а и 3,в/, а второй крайний полюс движется параллельно ему в обратном направлении. Шаг уменьшается с уменьшением расстояния по закону экстремального сглаживания и рекомендациям стохастической аппроксимации. Средний полюс с помощью экспоненциального сглаживания все время удерживается в "центре тяжести" /т.е. в средней точке/ множества состояний хорошей работы объекта.

Во втором случае, т.е. в случает соответствия выходов датчика и распознающей системы, крайние полюса не перемещаются. Система допускается к управлению /т.е. к коррекции положения или формы характеристики разомкнутой части/ только после достаточно длительного существования соответствия между выходами датчика и распознающей системы /рис.3,б/.

Предложенный нами метод самообучения прототипов был впоследствии назван заграницей "методом взвешенного смещения" /9/. Характерной чертой этого метода является то, что на приход представляющей точки реагируют ближайшие два прототипа /полюса/, причем "правильный" /по указанию учителя или выходов самой системы/ прототип движется по направлению к представляющей точке, а "неправильный" - от нее /рис.4/.

При таком методе многомерная стохастическая аппроксимация всех координат полюсов

$$\underline{\underline{K}}_N = \underline{\underline{K}}_{N-1} + \underline{\gamma}_N \left(\bar{\bar{y}} - \underline{\underline{K}}_{N-1}^{T} \right) \underline{\underline{X}},$$

где

$$\bar{y} = h^{T} X, \quad k = \begin{bmatrix} k_{(0)} \\ k_{(1)} \\ \vdots \\ k_{(n)} \end{bmatrix}, \quad X = \begin{bmatrix} 1 \\ x_{(1)} \\ x_{(2)} \\ \vdots \\ x_{(n)} \end{bmatrix}, \quad \gamma_{(N)} = \text{const},$$

заменяется одномерной

$$r_{i(k+1)} = r_{ik} + \gamma_{(k)} (X_{ik} - r_{ik}),$$

так как направление движения определено однозначно.

Теорема об устойчивости процессов самоорганизации "полюсного газа"

Множество прототипов удобно рассматривать, как некоторый "полюсный газ", частицы которого взаимодействуют одна с другой и с внешней средой. Элементарные алгоритмы взаимодействия полюсов выбираются человеком. Их можно выбрать так, чтобы процесс самоорганизации был сходящимся. Ниже формулируется теорема об устойчивости процесса самоорганизации полюсного газа.

Для самоорганизации частиц полюсного газа можно предложить ряд алгоритмов, в связи с чем будет изменяться и формулировка теоремы, В качестве примера приведем одну из известных формулировок. Эта формулировка действительна для множества прототипов /полюсного газа/ такой системы, принцип действия которой поясняется рис.5.

Полюса распознающей системы или жестко закреплены /те, которые ограничивают область возможных режимов работы системы/ или жестко связаны с вершинами зубцов или являются свободными. Как в "верхнем", так и в "нижнем" множестве полюсов отдельные полюса взаимодействуют друг с другом, с закрепленными неподвижно полюсами и с полюсами, координаты которых совмещены с координатами верхних /для "нижнего" множества - нижних/ зубцов характеристики разомкнутой части.

Закон взаимодействия двух соседних частиц газа является их "элементарным алгоритмом". Знак силы взаимодействия соседних частиц изменяется на противоположный по отношению к полюсу, координаты которого совпадают с представляющей точкой, если датчик двухбальной оценки показателя качества указывает "регулировать". Если же представляющая точка лежит в области хорошей работы объекта, то действует сила взаимодействия с тем же знаком, что и у большинства частиц.

Местоположение каждой свободной частицы определяется суммой влияний элементарных алгоритмов всех ее соседних частиц из того же множества. Изменение знака хотя бы одного элементарного алгоритма вызывает перемещение свободной частицы и, следовательно, средней линии характеристики разомкнутой части. Полюса перемещаются шагами, осторожно, а характеристика - как угодно быстро.

Теорема. Для того чтобы под действием перемещения частиц полюсного газа характеристика разомкнутой части всегда удерживалась посреди зоны хорошей работы системы, достаточно, чтобы:

I/ элементарные алгоритмы отвечали отталкиванию частиц полюсного газа между собой и притяжению по отношению к полюсу к представляющей точке, которая вышла за границы зоны хорошей работы объекта;

2/ выход представляющей точки за границы указанной зоны не должен превосходить высоту зубцов;

3/ вероятность каждого возмущения должна отличаться от нуля.x/

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x/Третье условие практически всегда выполняется, поскольку зубцы устанавливаются в рабочем диапазоне характеристики, т.е. отвечают значениям тех возмущений, которые действительно имеют место. Для ускорения процесса самоорганизации зубцы должны отвечать тем значениям возмущающих воздействий, которые наблюдаются чаще.

В случае программирования полюсного газа на вычислительных машинах взаимное отталкивание частиц удобно заменить требованием сохранения равных расстояний между ними, и таким образом, вычислять положение каждого из полюсов. При этом процесс установления всех этих частиц в определенное положение очень похож на тот процесс последовательной стохастической аппроксимации, который имеет место в аналого-цифровых преобразователях, в самообучающихся датчиках /4/ и других устройствах, которые используют последовательное во времени накопление информации.

Таким образом, в данном случае мы имеем пример реализации алгоритмов стохастической аппроксимации /I0/. Доказательство теоремы не отличается от доказательства сходимости процесса стохастической аппроксимации./II/.

Распознающая система-корректор /второй вариант/

В корректоре по второму варианту в качестве признаков используются несколько последних по времени значений показателя качества $\underline{\varphi}_{(1)}, \underline{\varphi}_{(2)}, \dots, \underline{\varphi}_{(k)}$, измеренные в отличных друг от друга вершинах зубцов характеристики разомкнутой части. Минимальное число учитываемых вершин равно трем, однако при наличии разброса показаний датчика, число учитываемых вершин следует увеличить до десяти-пятнадцати.

При этом в отличие от предыдущего "состоянием" называется взаимное расположение оптимальной характеристики объекта и характеристики РЧ. Каждому "состоянию" системы отвечает определенный код значений показателя качества, примеры которого указаны в таблице I. Множество различных состояний могут быть разделены, например, на шесть ос овных "ситуаций" указанных там же. В распознающей системе-корректоре заложено при помощи обучения /показа/

шесть кодов прототипов /эталонов/. В качестве меры близости система вычисляет скалярные произведения входных сигналов и кодов прототипов, причем, как легко убедиться, результат не зависит от того, в каких именно вершинах побывала в последнее время представляющая точка системы.

В таблице I приведены примеры точного соответствия входных состояний прототипам, записанным в распознающей системе. Все остальные состояния объекта система отнесет к одному или нескольким из этих прототипов. Алгоритм работы системы в этом случае поясняется следующим примером.

Пусть, например, состояние системы определяется таким кодом /учитываются три "текущие" вершины/:

$$\bar{\bar{z}} = \underline{0}\ {+I}\ \underline{0}\ \underline{0}\ {+I}\ {-I}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}$$

Находим пять скалярных произведений входного сигнала пятью первыми прототипами:

$$\Sigma_{(1)} = \underline{0}\ {-I}\ \underline{0}\ \underline{0}\ {+I}\ {+I}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0} = +I$$

$$\Sigma_{(2)} = \underline{0}\ {+I}\ \underline{0}\ \underline{0}\ {-I}\ {-I}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0} = -I$$

$$\Sigma_{(3)} = \underline{0}\ {+I}\ \underline{0}\ \underline{0}\ {+I}\ {-I}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0} = +I$$

$$\Sigma_{(4)} = \underline{0}\ {-I}\ \underline{0}\ \underline{0}\ {+I}\ {-I}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0} = -I$$

$$\Sigma_{(5)} = \underline{0}\ {+I}\ \underline{0}\ \underline{0}\ {+I}\ {-I}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0} = +I$$

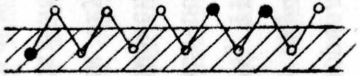
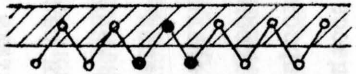
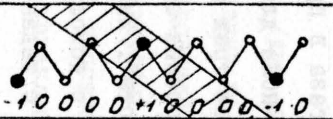
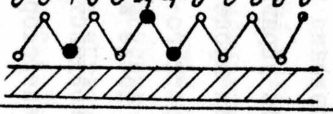
Система принимает решение по такому правилу:

Если $\Sigma_{(i)} > \underline{0}$, то данная ситуация имеет место полностью или вместе с другими ситуациями

Если $\Sigma_{(i)} \leqslant \underline{0}$, составляющей, соответствующей данной ситуации нет.

В данном примере будет принято решение: характеристика РЧ смещена вверх $/\Sigma_{(1)} = +I/$ и повернута против часовой стрелки $/\Sigma_{(3)} = +I/$, но в области рабочих режимов больших отклонений нет $/\Sigma_{(5)} = +I/$.

Таблица Шесть основных ситуации и их коды

Ситуация	Пример сигнала x	Прототип ситуации r	Скалерное произведение
„Сместить характеристики РЧ вниз"	+1 0 0 0 0 0 0 0 -1 0 -1 0 0 	+1 -1 +1 -1 +1 -1 +1 -1 +1 -1 +1 -1	$\sum_1 = -(x \cdot r) = -3$
„Сместить характеристику РЧ вверх"	 0 0 0 0 -1 +1 -1 0 0 0 0 0	-1 +1 -1 +1 -1 +1 -1 +1 -1 +1 -1 +1	$\sum_2 = +3$
„Повернуть характеристику РЧ по часовой стрелке"	 -1 0 0 0 0 +1 0 0 0 0 -1 0	-1 +1 -1 +1 -1 +1 +1 +1 -1 +1 -1 +1	$\sum_3 = +3$
„Повернуть характеристику РЧ против часовой стрелки	 0 +1 -1 0 0 0 0 0 +1 0 0 0 0	+1 -1 +1 -1 +1 +1 +1 +1 +1 -1 +1 -1	$\sum_4 = +3$
„Хорошо, так держать"	 +1 0 0 0 0 -1 +1 0 0 0 0 0 0	+1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1	$\sum_5 = +3$
„Увеличить амплитуду зубцов характеристики РЧ"	0 0 -1 0 0 -1 -1 0 0 0 0 0 	-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	$\sum_6 = +3$

Примечание: зачерненными кружками изображены зубцы, где позже других была представляющая точка

Соответствующее указание передается на устройство, корректирующее положение характеристики РЧ. Процесс корректировки выполняется медленно до момента изменения входных сигналов распознающей системы. Код, состоящий из одних сигналов "-I", означает, что характеристика РЧ вообще далеко ушла от области достаточно хорошей работы. При этом на некоторое время увеличивается амплитуда всех зубцов, пока не будет вновь найдено расположение оптимальной характеристики. В случае противоречивых рекомендаций /например, одновременно повернуть характеристику по и против часовой стрелки/ система ожидает дополнительных данных, пока конфликт не разрешится. В случае противоречивых рекомендаций /например, одновременно повернуть характеристики по и против часовой стрелки / система ожидает дополнительных данных пока конфликт не разрешится.

Моделирование процессов самоорганизации

Моделирование процессов самоорганизации - процессов самоустановления частиц полюсного газа в процэссе нормальной работы объекта управления - проведено пока для простых случаев. Моделировался процесс самоустановления трех полюсов /4, II, I2/. Путем простого графического построения определялись положения полюсов после каждого "такта работы". Имитация изменения возмущающего воздействия производилась с помощью таблицы случайных чисел или генератора случайных чисел с заданным распределением вероятности /обычно - равномерным/. Характеристика объекта аппроксимировалась полиномом второй или третьей степени. Показатель качества квантовался на 2 уровня. В начале процесса крайние полюса разводятся возможно дальше от характеристики разомкнутой части.

В процессе "работы" после каждого шага полюсов средняя линия характеристики разомкнутой части определялась как геометрическое сместо точек, равноудаленных от двух крайних полюсов. Моделирование показало, что процесс сходящийся: после некоторого числа шагов характеристика разомкнутой части оказывалась расположенной внутри зоны хорошей работы.

Аналогичное моделирование, но для девяти полюсов /что позволило аппроксимировать тремя отрезками прямой нелинейные границы зоны хорошей работы/ было проведено Т.Гергеем /I2/. Просчеты были выполнены при разных начальных положениях полюсов.

Было проведено физическое моделирование процессов самоорганизации полюсного газа. Поплавки с намагниченными стержнями в них плавали свободно на поверхности воды. По мере " работы" некоторые из них /оказавшиеся ближе остальных на данном такте к представляющей точке/ закреплялись жестко. Линия закрепленных поплавков с течением времени все точнее вырисовывала характеристику разомкнутой части. Вследствие сил взаимного отталкивания на каждом такте изменялось взаимное расположение всех остальных "полюсов".

Несмотря на то, что /как видно из изложенного/ моделирование процессов самоорганизации не было проведено в исчерпывающем объеме, проведенных исследований все же достаточно, чтоб сделать вывод о пригодности используемых алгоритмов взаимодействия полюсов для такого разделения пространства состояний на ситуации, которое делает возможным управление расположением средней линии характеристики РЧ.

<u>Работы по внедрению системы</u>

В институте кибернетики АН УССР изготовлен макет комбинированной системы экстремального управления, предназначенной для управления процессом обогащения железной руды. Секция мокрого магнитного обогащения руды представляет собой многомерный /три возмущающих и три регулирующих воздействия, один показатель качества/ объект с экстремальной характеристикой. Регулирование производится на минимум показателя качества. На первом этапе работы предусматривается использование системы в режиме "Советчика" /I4/.

Система дискретна, квантование воздействий производится на 3-4 уровня. Показатель качества φ квантуется на 2 уровня. Разомкнутая часть системы выполнена с применением матриц конвергенции и дивергенции.

Корректор состоит из селектора признаков и распознающей системы /типа "Альфа"/. Распознающая система классифицирует все возможные в ходе работы состояния объекта на 5 ситуаций. Каждой из ситуаций соответствует отдельная характеристика $\mu = \bar{f}(\lambda)$ в разомкнутой части.

Состояние характеризуется набором значений φ в трех зубцах характеристики разомкнутой части. Преобразование информации о текущих координатах объекта /возмущающих воздействиях и показателе качества/ в код состояния выполняет селектор признаков.

Система построена на полупроводниковых модулях электронной вычислительной машины "Мир". Испытания системы показали ее работоспособность. В перспективе намечается сделать систему самообучающейся /сейчас обучение распознающей системы прототипам ситуаций производится перед началом работы/ и придать большую гибкость структуре разомкнутой части.

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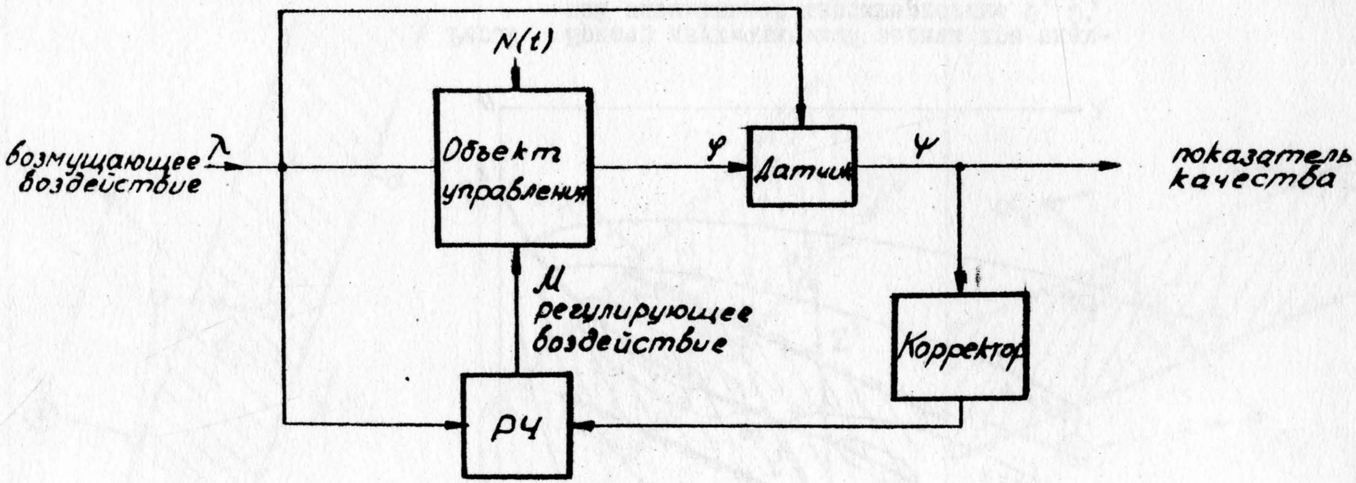

Рис.1. Схема комбинированной системы экстремального управления с двухбальным гумерением показателя качества

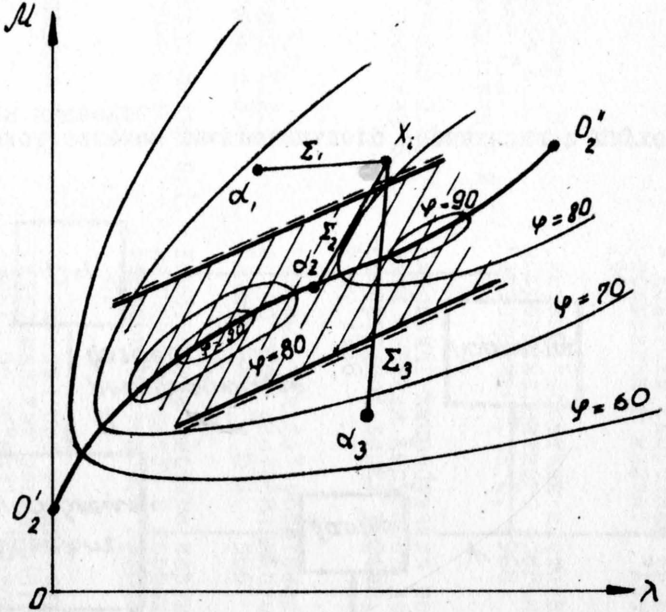

Рис.2. Пример двухмодальной задачи при плавной оптимальной характеристике O'_1 O''_2 объекта экстремального управления X – представляющая точка, $\alpha_1, \alpha_2, \alpha_3$ – прототипы /полюса/

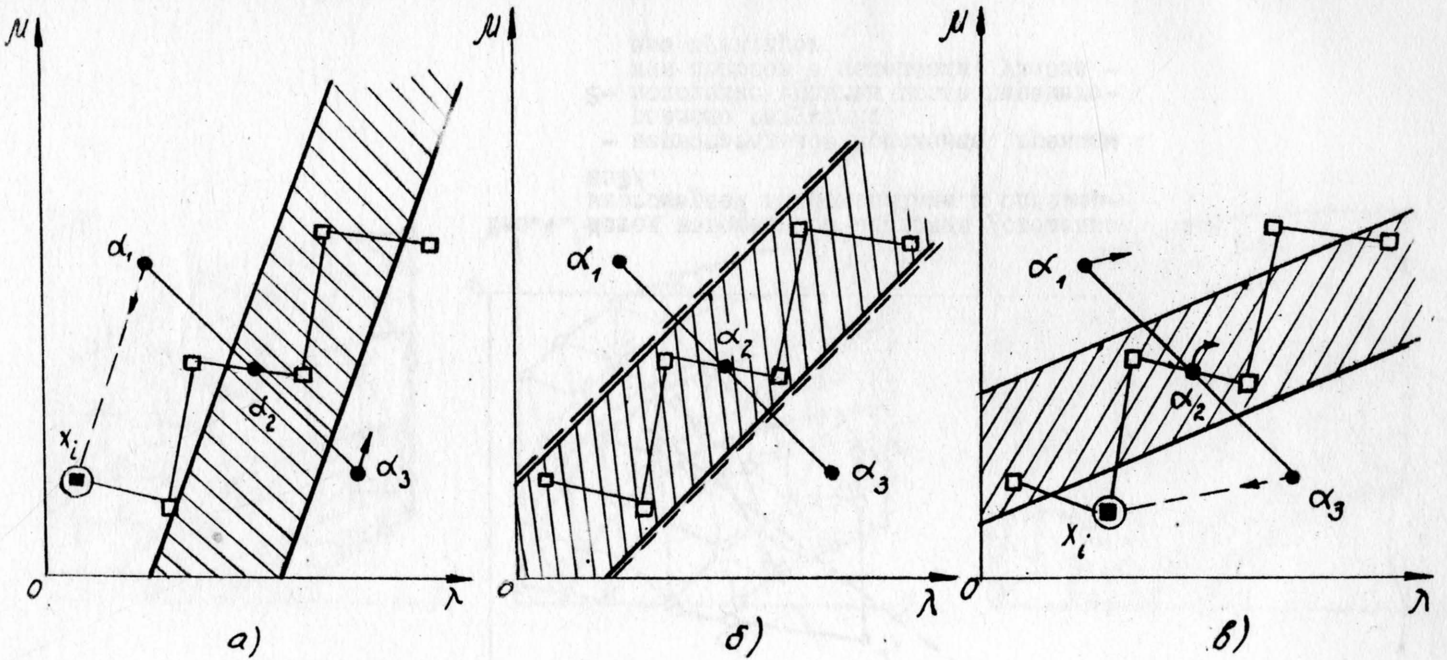

Рис.3. Алгоритм самообучения полюсов

а/ – первый полюс движется к представляющей точке X ,
б/ – полюса неподвижены – дается разрешение на цправление,
в/ – третий полюс движется к представляющей точке.

Заштрихована ситуация "достатогно хорошо" работы объекта.

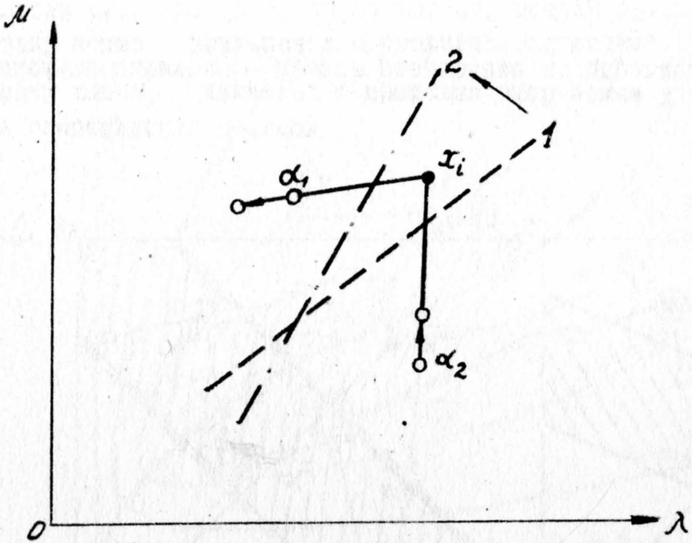

Рис.4. Метод взвешенного смещения /сведение многомерной аппроксимации к однамерной/.

- первонагальное положение границы раздела ситуаций;

2- положение границы после перемещения полюсов в положения, указан - ные пунктиром

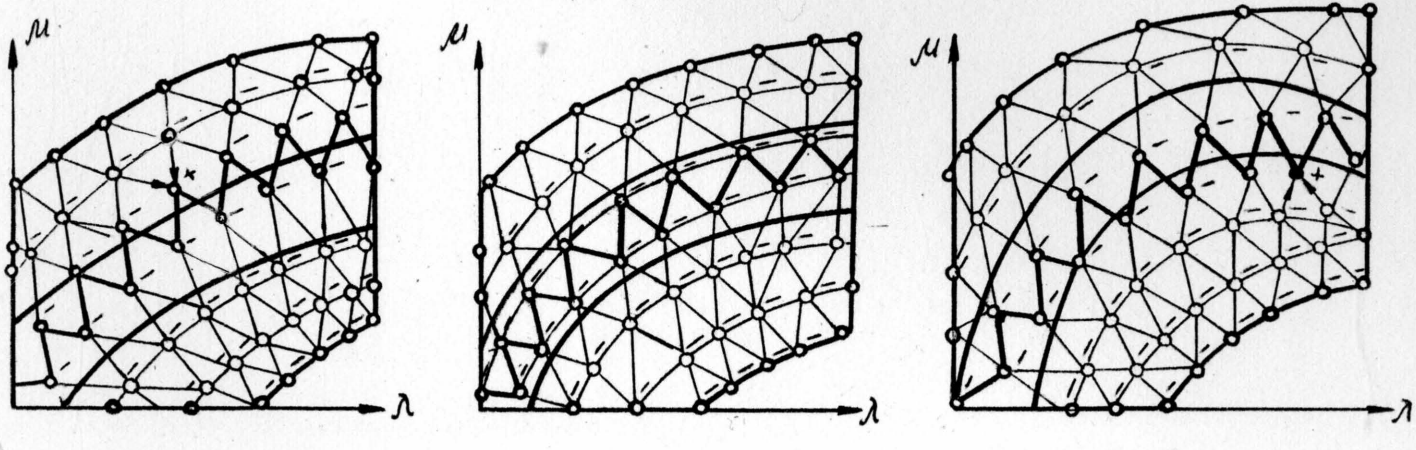

Рис.5. К формулировке теоремы об устойчивости процессов самоорганизации "полюсного газа"