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INTERNATIONAL FEDERATION
OF AUTOMATIC CONTROL

WARSZAWA 1969

Computational Methods in Optimisation

Computation and Applications Experience

Fourth Congress of the International Federation of Automatic Control
Warszawa 16–21 June 1969

TECHNICAL SESSION
25

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FEDERATION OF AUTOMATIC CONTROL
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Contents

Wydawnictwa Czasopism Technicznych NOT - Polska

Zakład Poligraficzny WCT NOT. Zam. 73/69.

A NUMERICAL PROCEDURE FOR COMPUTING OPTIMAL CONTROL BY USING PONTRYAGIN'S MAXIMUM PRINCIPLE

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Introduction

Pontryagin's maximum principle is the powerful theorem available for the solution of optimal control problems. However, when one wishes actually to compute optimal control by using the maximum principle, the initial conditions of the auxiliary differential equations are not prescribed. Hence, it is necessary to determine the initial conditions of the auxiliary variables so that the optimal trajectory satisfies given terminal conditions.

This paper presents an iterative procedure for solving such two-point boundary-value problems, when approximate initial conditions of the auxiliary variables are given. Some numerical examples are also shown.

Formulation of the problem

Let us consider control systems described by

$$\frac{dx}{dt} = f(x, u(t)), \qquad (1)$$

where x is an n-dimensional state vector in an n-dimensional state space, and $u(t)$ is an r-dimensional control vector. The control vector $u(t)$ is assumed to be constrained as

$$u(t) \in U, \qquad (2)$$

where U is a fixed bounded set in the r-dimensional Euclidean space. The initial and final states of the system (1) are given such as

$$\begin{aligned} x(0) &= x_0 \ , \quad \text{at } t = 0 \ , \\ x(T) &= x_1 \ , \quad \text{at } t = T \ , \end{aligned} \qquad (3)$$

where T represents the duration time of the control process and is not specified beforehand. The cost to be minimized is given by

$$J = \int_0^T f^0(x, u)\, dt \ , \qquad (4)$$

where $f^0(x, u)$ is a suitable function for the cost. By defining a new state variable x^0 as

$$\frac{dx^0}{dt} = f^0(x, u), \quad x^0(0) = 0, \qquad (5)$$

$x^0(T)$ represents the cost J.

According to the maximum principle, n + 1 auxiliary variables satisfying the differential equations

$$\frac{d\psi_i}{dt} = -\sum_{\alpha=0}^{n} \frac{\partial f^\alpha(x,u)}{\partial x_i} \psi_\alpha \ , \qquad (6)$$

$$i = 0, 1, \ldots\ldots, n$$

are defined. In equation (6), it is clear that $d\psi_0/dt = 0$. The condition is necessary for the optimum, hence we put

$$\psi_0(t) = -1. \qquad (7)$$

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The Hamiltonian H is defined by

$$H = \sum_{i=0}^{n} \psi_i(t) f^i(x(t), u). \qquad (8)$$

The optimal control u(t) must satisfy the following condition:

$$H(\Psi(t), x(t), u(t)) = M(\Psi(t), x(t)) = 0, \qquad (9)$$

for almost all t in the interval $[0, T]$, where

$$M(\Psi(t), x(t)) = \sup_{u \in U} H(\Psi(t), x(t), u). \qquad (10)$$

The optimal control u(t) is obtained by solving Eqs (1), (6), and (9). However the initial conditions of Eq. (6)

$$\psi_i(0) = \eta_i, \quad i = 1,2,\ldots\ldots,n \qquad (11)$$

are unknown. Eq. (11) can be written in a vector form as

$$\Psi(0) = \eta. \qquad (12)$$

If one chooses adequate initial conditions $\psi_i(0)$, the optimal control which transfers the initial state x_0 to the final state x_1 is obtained. But if one chooses inadequate initial conditions, the optimal control fails to transfer the state from x_0 to x_1. Thus, the proper initial conditions $\psi_i(0)$ (i = 1,2,....,n) must be found. Setting t = 0 in Eq. (10) yields

$$M(\Psi(0), x(0)) = M(\eta, x_0) = 0, \qquad (13)$$

where $\psi_0(0) = -1$.

Relation between the initial condition η and the final state of the system

Suppose that the initial values and time T are given. Then the final state of the system, $x(T) = \xi^T$, are determined by solving Eqs. (1), (6), and (10) according to the flow diagram given in Figure 1. The relation will be described as follows:

$$\xi_i^T = h_i(\eta_1, \ldots\ldots \eta_n, T), \qquad (14)$$

$$i = 1,2,\ldots\ldots,n.$$

In linear control systems, the functions h_i (i = 1,2,....,n) in Eq.(14) can be expressed in an analytic form since Eq. (1) is integrable. In nonlinear control system, however, the functions h_i can not be specified in an analytic form. The existense and uniqueness theorem in the theory of an ordinary differential equation ascertains that Eq. (14) exist in a nonlinear case.

Suppose that the optimal control which transfers the state from x_0 to x_1 exists. This optimal control is designated as $\hat{u}(t)$ and the optimal trajectory corresponding to $\hat{u}(t)$ as $\hat{x}(t)$. The initial conditions for Eq. (6) which give $\hat{u}(t)$ and $\hat{x}(t)$ are called the optimal initial conditions, which are denoted as $\hat{\eta}$.

The time when the optimal trajectory $\hat{x}(t)$ reaches x_1 is designated as $\hat{T}$, then

$$x_1^i = h_i(\hat{\eta}_1, \ldots, \hat{\eta}_n, \hat{T}), \quad (15)$$

$$i = 1,2, \ldots, n.$$

Equation (15) means that the optimal trajectory which is obtained by setting $\eta = \hat{\eta}$ and solving Eqs. (1), (6), and (10) reaches x_1 in time $\hat{T}$.

Now put

$$\eta_i = \hat{\eta}_i + \delta\eta_i, \quad i = 1,2,\ldots,n, \quad (16)$$

$$T = \hat{T} + \delta T.$$

Substituting Eq. (16) into (14), we define the deviation, $\delta\xi_i$, of ${\xi_i}^T$ from x_1^i,

$$\delta\xi_i = {\xi_i}^T - x_1^i, \quad i = 1,2,\ldots,n. \quad (17)$$

We call $\delta\xi = (\delta\xi_1, \ldots, \delta\xi_n)$ the terminal deviation. Assuming that the functions h_i ($i = 1,2,\ldots,n$) and M are differentiable with respect to η_i and T at $\eta_i = \hat{\eta}_i$ and $T = \hat{T}$, we obtain

$$\delta\xi_i = \sum_{k=1}^{n} \frac{\partial h_i}{\partial \hat{\eta}_k}\delta\eta_k + \frac{\partial h_i}{\partial \hat{T}}\delta T + \cdots\cdots, \quad (18)$$

$$i = 1,2, \ldots, n,$$

$$\sum_{k=1}^{n} \frac{\partial M}{\partial \hat{\eta}_k}\delta\eta_k + \cdots\cdots = 0.$$

We assume that $\delta\eta_i$ ($i = 1,2,\ldots,n$) and δT are so small that the higher order terms in Eq. (18) can be neglected. If the Jacobian defined by

$$J = \begin{vmatrix} \partial h_1/\partial\hat{\eta}_1 & \cdots & \partial h_1/\partial\hat{\eta}_n & \partial h_1/\partial\hat{T} \\ \vdots & & \vdots & \vdots \\ \partial h_n/\partial\hat{\eta}_1 & \cdots & \partial h_n/\partial\hat{\eta}_n & \partial h_n/\partial\hat{T} \\ \partial M/\partial\hat{\eta}_1 & \cdots & \partial M/\partial\hat{\eta}_n & 0 \end{vmatrix} \quad (19)$$

is not zero in a neighborhood of $\hat{\eta}_i$ and $\hat{T}$, then we can solve Eq. (18) with respect to $\delta\eta$ and δT:

$$\delta\eta_i = \sum_{j=1}^{n} A_{ij}\delta\xi_j + \cdots\cdots,$$

$$i = 1,2, \ldots, n, \quad (20)$$

$$\delta T = \sum_{j=1}^{n} A_{0j}\delta\xi_j + \cdots\cdots,$$

where A_{ij} ($i = 0,1,\ldots,n$; $j = 1,2,\ldots,n$) is some constants determined by $\hat{\eta}$ and $\hat{T}$.

Iteration Method

It is difficult in general to calculate A_{ij} and the derivatives $\partial h_i/\partial\hat{\eta}_j$, $\partial h_i/\partial\hat{T}$, and $\partial M/\partial\hat{\eta}_j$ in a nonlinear case. Even in a linear case it is not easy. Therefore, we presents much more simple iteration method for obtaining the optimal initial conditions $\hat{\eta}$. This method is essentially based on Kahne's method to solve a two-point boundary-value problem of differential equations. By neglecting the higher order terms in Eq. (20), we define

$$\bar{\eta}_i = \eta_i - \sum_{j=1}^{n} A_{ij}\delta\xi_j, \quad i = 1,2,\ldots,n,$$

$$\bar{T} = T - \sum_{j=1}^{n} A_{0j}\delta\xi_j. \quad (21)$$

where $\bar{\eta}_i$ ($i = 1,2,\ldots,n$) and $\bar{T}$ are modified values, respectively, and $\delta\xi_i = {\xi_i}^T - x_1^i$ ($i=1,2,\ldots,n$).

Now let $(\eta^{(1)}, T^{(1)}), \ldots\ldots, (\eta^{(n+1)}, T^{(n+1)})$ be n+1 different pairs of initial guesses for the initial condition η and time T. From Eq. (21), we obtain

$$\bar{\eta}_i = \eta_i^{(k)} - \sum_{j=1}^{n} A_{ij}\,\delta\xi_j^{(k)}, \quad i = 1,2,\ldots,n;\ k = 1,2,\ldots,n+1,$$
$$\bar{T} = T^{(k)} - \sum_{j=1}^{n} A_{oj}\,\delta\xi_j^{(k)}, \quad k = 1,2,\ldots,n+1. \tag{22}$$

Eq. (22) can be written in the following matrix form:

$$\begin{bmatrix} 1 & \delta\xi_1^{(1)} & \cdots & \delta\xi_n^{(1)} \\ \vdots & \vdots & & \vdots \\ 1 & \delta\xi_1^{(n+1)} & \cdots & \delta\xi_n^{(n+1)} \end{bmatrix} \begin{bmatrix} \bar{\eta}_1 & \cdots & \bar{\eta}_n & \bar{T} \\ A_{1n} & \cdots & A_{nn} & A_{on} \end{bmatrix} = \begin{bmatrix} \eta_1^{(1)} & \cdots & \eta_n^{(1)} & T^{(1)} \\ \vdots & & \vdots & \vdots \\ \eta_1^{(n+1)} & \cdots & \eta_n^{(n+1)} & T^{(n+1)} \end{bmatrix}. \tag{23}$$

Eq. (23) can be rewritten as

$$\begin{bmatrix} \bar{\eta}_1 & \cdots & \bar{\eta}_n & \bar{T} \\ A_{1n} & \cdots & A_{nn} & A_{on} \end{bmatrix} = \begin{bmatrix} 1 & \delta\xi_1^{(1)} & \cdots & \delta\xi_n^{(1)} \\ \vdots & \vdots & & \vdots \\ 1 & \delta\xi_1^{(n+1)} & \cdots & \delta\xi_n^{(n+1)} \end{bmatrix}^{-1} \begin{bmatrix} \eta_1^{(1)} & \cdots & \eta_n^{(1)} & T^{(1)} \\ \vdots & & \vdots & \vdots \\ \eta_1^{(n+1)} & \cdots & \eta_n^{(n+1)} & T^{(n+1)} \end{bmatrix}. \tag{24}$$

The first row of the matrix in the left hand side of Eq. (24) represents the modified values of η and T.

For each guess $\eta^{(k)}$ and $T^{(k)}$ $(k = 1,2,\ldots,n+1)$ we define

$$E_k = \sum_{i=1}^{n} \left| \delta\xi_i^{(k)} \right|, \quad k = 1,2,\ldots,n+1. \tag{25}$$

Eq. (25) represents the deviation of the trajectory from the given final state. We call Eq. (25) a terminal error. We can say that the smaller E_k, the nearer $\xi_i^{(k)}$ to x_1. The terminal error, $\bar{E}$, corresponding to the solution of Eq. (24), $\bar{\eta}$ and $\bar{T}$, is expected to be smaller than any of E_k's $(k = 1,2,\ldots,n+1)$. If n+1 pairs of guesses $\eta^{(k)}$ and $T^{(k)}$ $(k = 1,2,\ldots,n+1)$ are good approximations for $\hat{\eta}$ and $\hat{T}$, the solutions of Eq. (24), $\bar{\eta}$ and $\bar{T}$, are better approximations for $\hat{\eta}$ and $\hat{T}$. However if some pair, $\eta^{(k')}$ and $T^{(k')}$, among all pairs of $\eta^{(k)}$ and $T^{(k)}$ $(k = 1,2,\ldots,n+1)$ is a bad approximation for $\hat{\eta}$ and $\hat{T}$, it is unreasonable to assume that Eq. (24) holds good as the approximation equation. And so we can not always assert that the solution of Eq. (24), $\bar{\eta}$ and $\bar{T}$, is better than any pair of guesses $\eta^{(k)}$ and $T^{(k)}$ $(k = 1,2,\ldots,n+1)$. Thus n+1 pairs of guesses $\eta^{(k)}$ and $T^{(k)}$ must be selected carefully.

Now let $E_m = \max_i (E_1, E_2, \ldots, E_{n+1})$. We substitute $\bar{\eta}$ and $\bar{T}$ into the m^{th} row of the second matrix of the left hand side in Eq. (23) and the final deviation, $\delta\xi_i$, corresponding to $\bar{\eta}$ and $\bar{T}$ into the m^{th} row of the first matrix of the left hand side. Then the new matrix equation can be established:

$$\begin{bmatrix} 1 & \delta\xi_1^{(1)} & \cdots & \delta\xi_n^{(1)} \\ \vdots & \vdots & & \vdots \\ 1 & \delta\xi_1 & \cdots & \delta\xi_n \\ \vdots & \vdots & & \vdots \\ 1 & \delta\xi_1^{(n+1)} & \cdots & \delta\xi_n^{(n+1)} \end{bmatrix} \begin{bmatrix} \bar{\eta}_1 & \cdots & \bar{\eta}_n & \bar{T} \\ A_{1m} & \cdots & A_{nm} & A_{0m} \\ \vdots & & \vdots & \vdots \\ A_{1n} & \cdots & A_{nn} & A_{0n} \end{bmatrix}$$

$$= \begin{bmatrix} \eta_1^{(1)} & \cdots & \eta_n^{(1)} & T^{(1)} \\ \vdots & & \vdots & \vdots \\ \bar{\eta}_1 & \cdots & \bar{\eta}_n & \bar{T} \\ \vdots & & \vdots & \vdots \\ \eta_1^{(n+1)} & \cdots & \eta_n^{(n+1)} & T^{(n+1)} \end{bmatrix} . \quad (26)$$

The solution of Eq. (26), $\bar{\eta}$ and $\bar{T}$, is expected to be better approximation for $\bar{\eta}$ and $\bar{T}$ than any row of the matrix in the right hand side of Eq. (26).

The operation as stated above is iterated until the terminal error is smaller than a given positive number ε. The positive number, ε, is called an admissible error.

H. Knudsen also presents the iterative procedure to obtain the time optimal control of linear systems based on the assumption that the unknown initial conditions of the auxiliary differential equation and the duration time of the control process is a function of the terminal state. In his procedure, however, it is necessary to calculate a matrix $(\partial h_i / \partial \hat{\eta}_j)$ and its inversion. It is difficult to calculate such a matrix in a large linear system. Compared with his procedure, our procedure needs only an inversion calculation of a matrix and is applicable to nonlinear systems. Our procedure has such a shortcomings that it does not give security for convergency in the case where n+1 initial guesses deviate from the optimal values $\hat{\eta}$ and $\hat{T}$.

Numerical Example

We give several numerical examples of linear and nonlinear systems. Figure 2 shows a detailed flow chart for the procedure presented above.

First we consider a linear control system described by

$$\frac{dx^1}{dt} = x^2$$
$$\frac{dx^2}{dt} = -x^2 + u \quad (27)$$

The constraint for the control variable u is given as follows:

$$|u(t)| \leq 1, \quad 0 \leq t \leq T. \quad (28)$$

The initial and final states of the system (27) are given as follows:

$$x_0 = (-2.95, -2.95), \text{ at } t = 0,$$
$$x_1 = (0, 0), \text{ at } t = T. \quad (29)$$

The problem is to obtain a time optimal control which transfers the state of the system from x_0 to x_1.

Let three pairs of the initial guesses for $\hat{\eta}$ and $\hat{T}$ be as follows:

$$\eta^{(1)} = (1.0, -0.4), \quad T^{(1)} = 2.0,$$
$$\eta^{(2)} = (1.0, -0.35), \quad T^{(2)} = 2.0,$$
$$\eta^{(3)} = (1.0, -0.3), \quad T^{(3)} = 2.0. \tag{30}$$

The terminal error for these guessed values are $E_1 = 0.7057$, $E_2 = 0.4558$, and $E_3 = 2.0524$, respectively. The terminal error at the fifth iteration results in $E = 0.0034$, and the initial condition η and time T are as follows:

$$\eta = (0.9992, -0.3677), \quad T = 1.998.$$

Figure 3 shows the convergency of the iteration step.

Second we consider Van der Pol equation:

$$\frac{dx^1}{dt} = x^2 \tag{31}$$
$$\frac{dx^2}{dt} = -\varepsilon\left\{(x^1)^2 - 1\right\}x^2 - x^1 + u.$$

The constraint for the control u is given as

$$|u(t)| \leq 0.5. \tag{32}$$

We set $\varepsilon = 0.1$. The initial and final states of the system (31) are given as follows:

$$x_0 = (1.14, 1.35), \quad x_1 = (0, 0).$$

Let three pairs of the initial guesses for η and T be as follows:

$$\eta^{(1)} = (-0.8023, -1.0737), \quad T^{(1)} = 6.0,$$
$$\eta^{(2)} = (-0.7023, -1.2737), \quad T^{(2)} = 6.1,$$
$$\eta^{(3)} = (-0.9023, -0.9737), \quad T^{(3)} = 5.9.$$

The terminal error for these guessed values are $E_1 = 0.0291$, $E_2 = 0.4796$, and $E_3 = 0.304$, respectively.
The calculation is stopped at the sixth iteration. The values of η and T and the terminal error at the fifth iteration are as follows:

$$\eta = (-0.7995, -1.0507), \quad T = 6.0,$$

and $E = 0.0001$.
Figure 4 shows the convergence of the iteration step.

Third we consider the same example as above. We set the initial and final states as follows:

$$x_0 = (0.0, -2.0), \quad x_1 = (0, 0).$$

Let three pairs of the initial guessed values for η and T be as follows:

$$\eta^{(1)} = (0.1, 0.9654), \quad T^{(1)} = 6.65,$$
$$\eta^{(2)} = (0.072, 1.063), \quad T^{(2)} = 6.7,$$
$$\eta^{(3)} = (0.01, 0.924), \quad T^{(3)} = 6.75.$$

The terminal errors for these assumed values are $E_1 = 0.3572$, $E_2 = 0.2189$, and $E_3 = 0.0812$, respectively.
The calculation is stopped at the tenth iteration. The values of η and T and the terminal error at the ninth iteration are found:

$$\eta = (-0.0043, 0.2496), \quad T = 6.65,$$

and $E = 0.0026$.
Figure 5 shows the convergence of the iteration step.

Conclusion

We treat the optimal control problems with unknown time, but the iterative method stated in this paper is also applicable to the problems with fixed time. In such a case, guessing the value of T is not necessary and the algorithm changes slightly. This procedure is a local method in nature. That is, we can not obtain a good result unless the guessed values for η and T are in a neighborhood of the optimal values.

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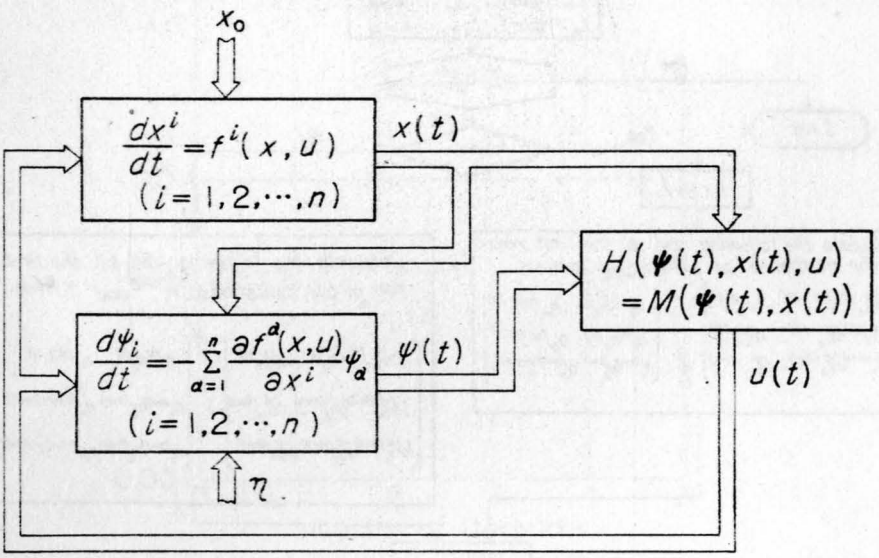

Fig. 1 From diagram for Computing the optimal Control and Optimal trajectory

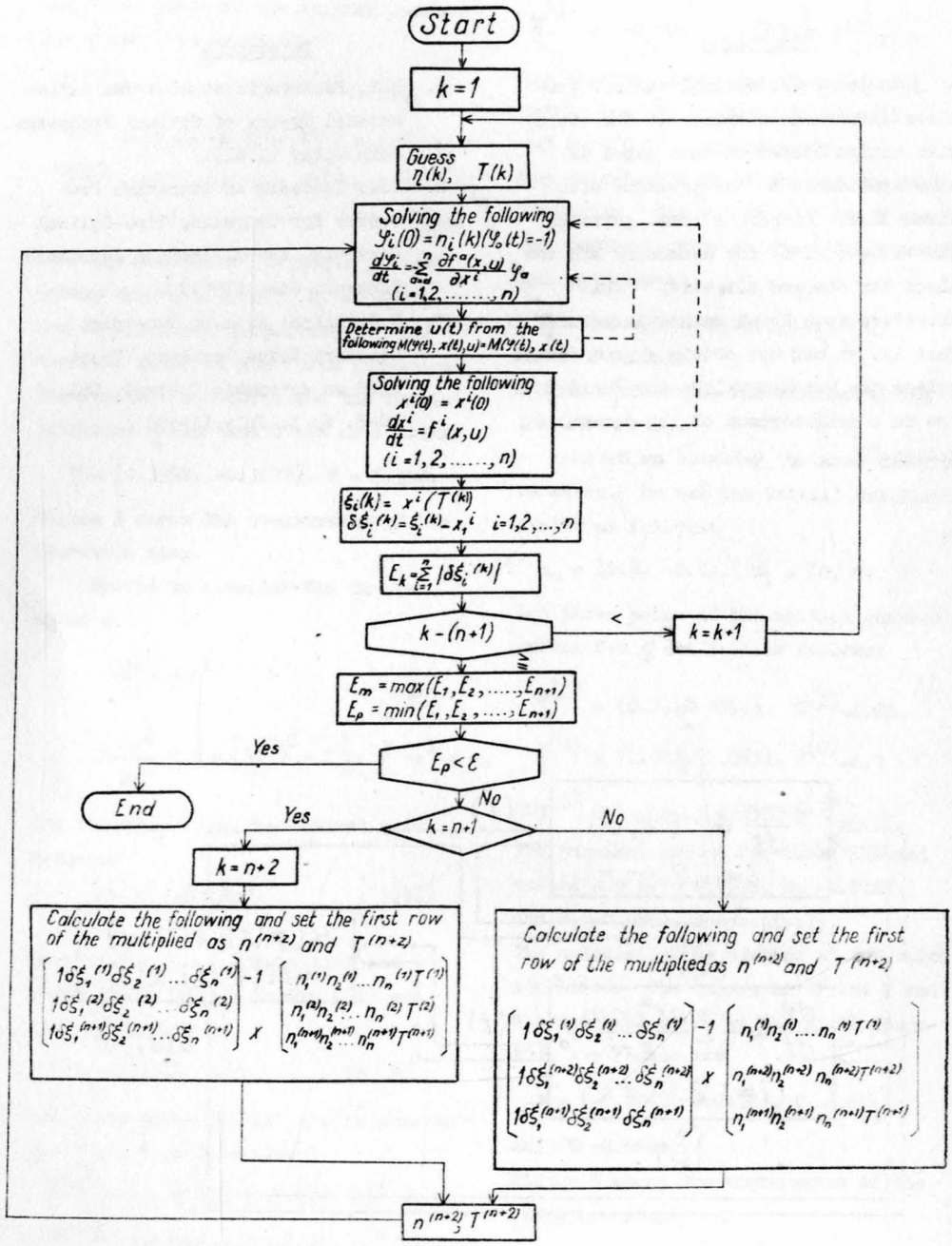

Fig 2 Detailed flow chart

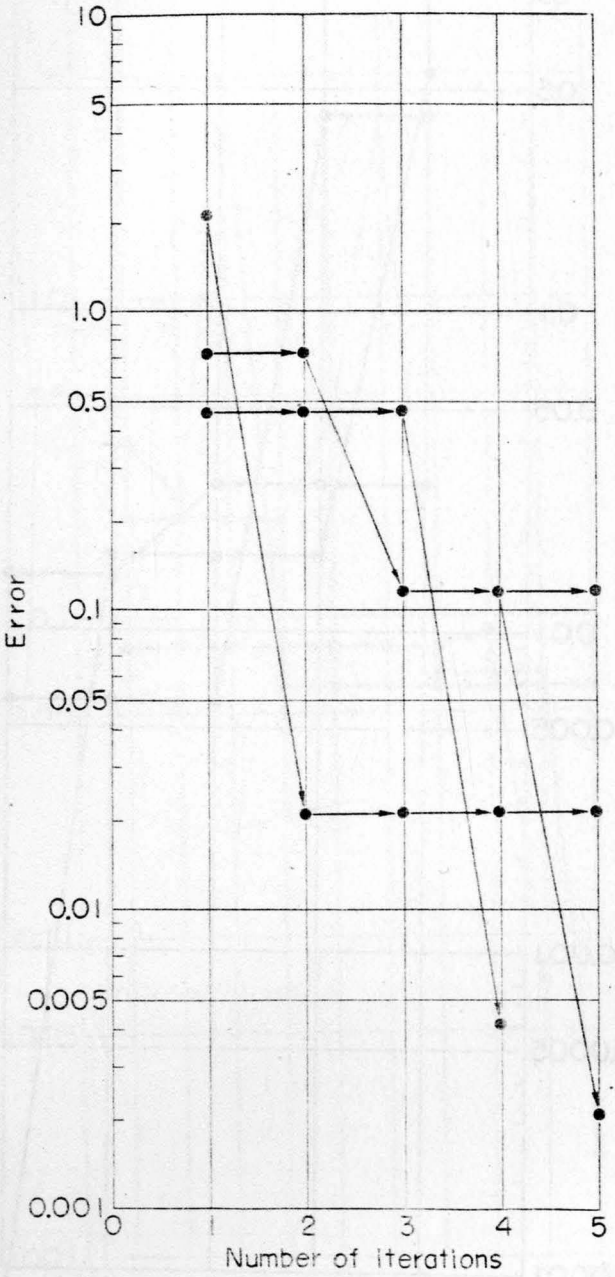

Fig. 3 Computational result for Eq. (28)

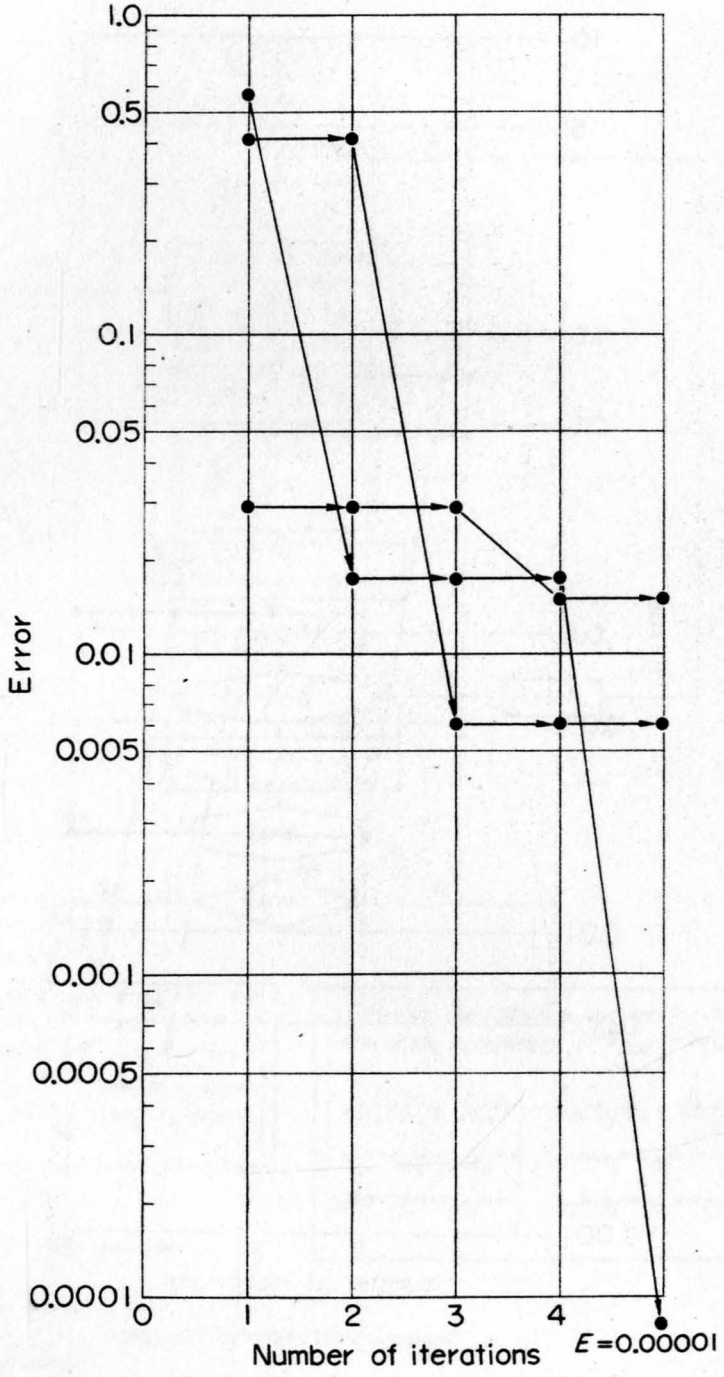

Fig. 4 Computational result for Eq. (32)

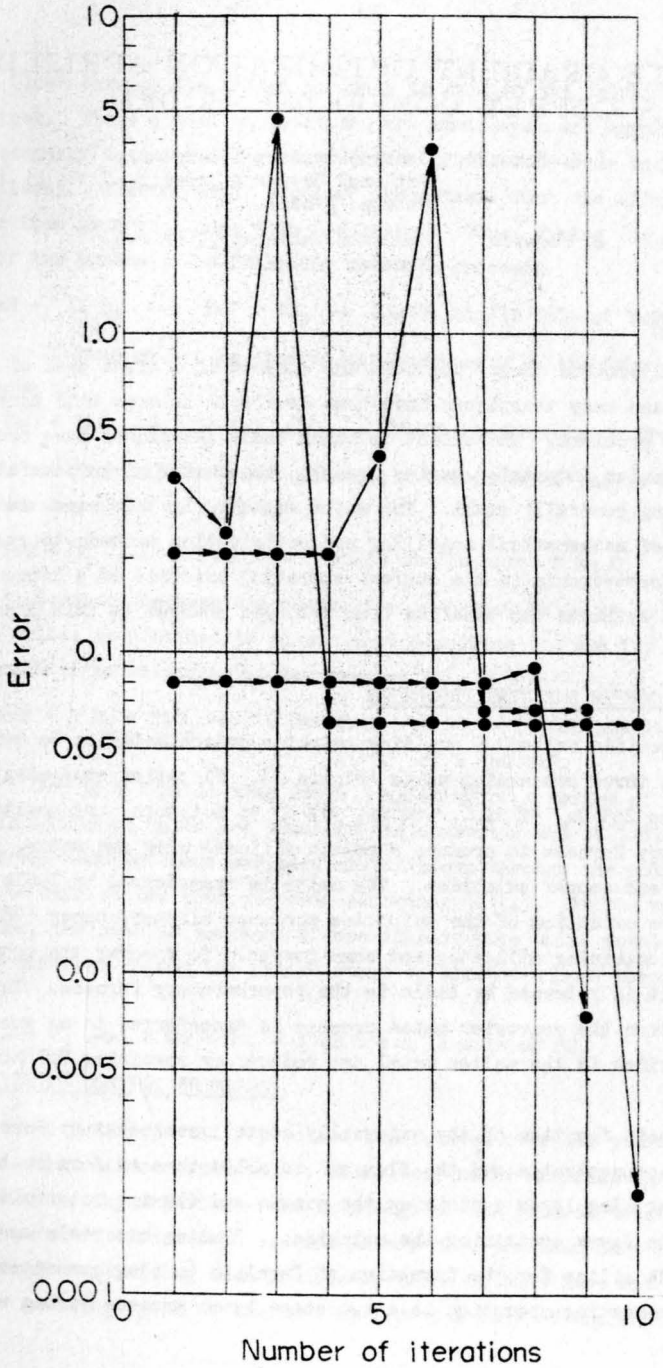

Fig. 5 Computational result for Eq. (32)

CONJUGATE GRADIENT OPTIMIZATION APPLIED TO A COPPER CONVERTER MODEL

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1.0 INTRODUCTION

Optimal control theory has been the subject of a great deal of research effort and many techniques have been developed to cope with many general types of problems. The extent to which these techniques have been applied in particular industries varies greatly, the number of successful applications being generally small. The major emphasis in this paper is on the application of mathematical modelling and optimization methods to evaluate possible improvements in the current operating practice of a copper converter and to estimate the benefits from proposed changes to this practice.

1.1 OUTLINE OF COPPER SMELTING PROCESSES

The specific method of smelting sulphide concentrates to be considered involves three processing units (Figure 1). Flotation concentrates (typical analyses 25% Cu, 5% SiO_2, 31% Fe, 31% S, 9% moisture) are smelted in a reverberatory furnace to produce a molten silicate slag and matte, a mixture of iron and copper sulphides. The matte is transferred by ladle to a converter where oxidation of the sulphides produces blister copper (98% Cu) and a slag containing silicates and some copper. To recover the copper from the slag, it is returned by ladle to the reverberatory furnace. The blister copper from the converter batch process is transferred to an anode furnace where oxides in the molten metal are reduced or remaining sulphides are oxidized.

The basic function of the externally heated reverberatory furnace is to smelt the concentrates and the flux and to allow them to form in two layers, a lighter slag layer containing the gangue and fluxing materials and the heavier matte layer containing the sulphides. Fluxing materials are required to provide silica for the formation of fayalite (a slag component).

The converting operation is a two stage batch process during which

air is blown through the molten contents to oxidize the sulphides of iron and copper. These oxidation reactions are exothermic and supply all of the heat necessary to maintain the temperature of the converter bath at the required level. Thermodynamic studies[8] demonstrate that the affinity of oxygen for iron is much higher than for copper. Thus, during the initial slag stage of the process, the following reaction proceeds

$$FeS + {}^{3}/_{2}\, O_2 \longrightarrow FeO + SO_2 - 200000 \text{ btu/lb mole of FeO} \qquad (1)$$

If free silica is present, it combines with the iron oxide (FeO) as follows

$$2\, FeO + SiO_2 \longrightarrow 2\, FeO.SiO_2 - 39600 \text{ btu/lb mole of } 2FeO.SiO_2 \qquad (2)$$

If free silica is not present, the iron oxide reacts with the air to produce magnetite as follows

$$6\, FeO + O_2 \longrightarrow 2\, Fe_3O_4 - 155250 \text{ btu/lb mole of } Fe_3O_4 \qquad (3)$$

However, because the production of magnetite is to be avoided, sufficient silica is provided to ensure that reactions (1) and (2) proceed, their combined behaviour being represented by

$$2\, FeS + 3\, O_2 + SiO_2 \longrightarrow 2\, FeO.SiO_2 + 2\, SO_2 - 439600 \text{ btu/lb mole of } 2\, FeO.SiO_2 \qquad (4)$$

During a typical slag stage consisting of a series of blowing periods, 13 ladles of matte are added to the converter and 17 ladles of slag are removed. During these periods, air is blown through the molten bath via tuyeres to oxidize the iron sulphide according to (4). At the end of the slag stage, the bath is composed of essentially pure white metal (Cu_2S) which is oxidized during the finish stage to blister copper according to the reaction

$$Cu_2S + O_2 \longrightarrow 2\, Cu + SO_2 - 114300 \text{ btu/lb mole of } SO_2 \qquad (5)$$

1.2 CONVERTER CONTROL PROBLEMS

Converter operating practice has evolved over the years from early trial-and-error efforts followed by relatively minor modifications based on increased operating experience and knowledge of the process behaviour. Although the recent advances in control theory have provided many systematic techniques for optimizing such processes, these methods have not been applied to copper smelting. One of the reasons for this lack of application involves

the current state of process knowledge. Although a great deal of operating experience exists concerning the converter process, basic research concerning reaction kinetics and other fundamental issues has not been emphasized[9]. The program to be described considers the application of modern optimization techniques to a current converter control problem and provides some results concerning the possible effects of the use of oxygen enriched air on the converter operation.

During the slag stage of the two stage converter batch process, iron sulphide is oxidized (eq. 1) and silica flux is added to combine with the iron oxide to form a silicate slag (eq. 2). The converter operator exercises control over the rate of flux addition, attempting to control the bath temperature while ensuring an adequate supply of silica to satisfy the process chemistry. Silica flux is also required in the externally heated reverberatory furnace. A limited amount of excess silica flux can be added to the converter to absorb some of the heat from the exothermic reactions and, as this excess silica would be transferred to the reverberatory furnace with the skimmed converter slag, a saving in fuel in that unit would be realized. From these observations, an optimal control problem can be formulated based on the flux addition rate as a control variable.

In addition to considering the optimization of current converter flux addition practice involving the use of standard air, the effects of the use of oxygen enriched air on the smelting power of the unit will be considered.

Following early experiments with various levels of oxygen enriched air for converter operation[10], the Hitachi Smelter in Japan introduced the concept of oxygen smelting on a commercial scale[11]. At the Hitachi Smelter, the converter is used to smelt flotation concentrates, the necessary heat being provided by the use of oxygen enriched air with resulting reductions in flue gas heat losses and increased rate of heat generation. The significant advantages to the use of enriched air include increased production rates, minimized reverberatory furnace (or blast furnace) smelting, generation of flue gases richer in SO_2 for improved production of sulphuric acid, and the additional flexibility of control to cope with variable production requirements and input materials. In another program[1] concerning the use of oxygen enriched converter air, the motivation for the study involved the possibilities of converting high grade mattes and of increasing production rates such

that three converters with enriched air could replace four units operating with standard air. In this study, the use of oxygen enriched air to increase production rates and to smelt excess silica flux or concentrates will be considered for various levels of oxygen enrichment.

2.0 FORMULATION OF THE OPTIMAL CONTROL PROBLEM

To determine the optimum flux addition rate for a complete slag stage, it is necessary to determine the optimal rate for each blowing interval. Therefore, a mathematical model must be developed which will represent converter behaviour during such intervals and which will accommodate differing initial and terminal conditions.

The performance index is specified in terms of the deviations of the bath temperature from a best value, the heat contents of the output materials, and the weights of excess silica in the slag. If oxygen enriched converter air is used, additional factors involving oxygen costs, savings in processing time, and the smelting of flotation concentrates must be considered.

2.1 DEVELOPMENT OF THE MATHEMATICAL MODEL

Since relatively little is known of the kinetics of the copper conversion process, the mathematical model of the thermal behaviour of the slag stage of the process is derived from material and heat balances based on the physical chemistry. Noting that the slag stage chemistry depends on the presence or absence of free silica in the converter bath and that operation without free silica is undesirable, the mathematical model is based on the assurance that free silica is always available. Thus, the control variable u is chosen to be the rate of addition of excess silica flux, the total rate being the sum of this excess rate u and the stoichiometric rate defined by the process chemistry. From equation (4), the stoichiometric rate of flux addition (lb.moles/min) is one third of the rate at which oxygen from the input air (lb.moles/min) reacts with iron sulphide. The stoichiometric rate depends not only on the oxygen flow rate but also on a parameter known as the oxygen efficiency. Oxygen efficiency defines the proportion of input oxygen which participates in reaction (4) and is an empirically determined factor approximating for the effects of variations in depth of blowing, bath composition and bath temperature. A more complete model development program is underway but, because only preliminary results were available[4], a simplified

mathematical model was developed for this study based on the use of the oxygen efficiency factor.

The state equations forming the basis for the mathematical model are

$$\dot{x}_1 = \frac{\Sigma \text{ rates of heat addition} - \Sigma \text{ rates of heat loss}}{\Sigma_s C_{ms} M_s} \quad (5)$$

$$\dot{x}_2 = u \quad (6)$$

where $\dot{x}_1$ is the rate of change of bath temperature, $^{o}R/min$

$\dot{x}_2$ is the rate of change of the amount of free silica in the bath, lb.moles/min.

u is the rate of addition of excess silica, lb.moles/min.

C_{ms}, M_s are the molar heat capacities (btu./lb.mole^{o}R) and molar weights (lb.moles) of the bath constituents (including x_2)

By combining the expressions for the various heat addition and loss terms and the rates of change of bath composition as the reaction proceeds, the form of the first state equation becomes

$$\dot{x}_1 = \frac{C_o - C_1 x_1 - C_2 x_1^2 - C_3/x_1 - C_4 x_1^4 + P_5 u - u x_1}{d_o + d_1 t + x_2} \quad (7)$$

where the coefficients C_o, C_1, C_2, C_3, C_4, P_5, d_o, and d_1 are expressed in terms of specific converter operating parameters. Since the heat capacity of the converter bath is time dependent, time appears explicitly in equation (7). Of the converter parameters, the conduction and radiation heat loss coefficients and the oxygen efficiency were adjusted during a series of model calibration runs to obtain a calibrated mathematical model for the specific converter employed. A typical model calibration run is shown in Figure 2.

The model calibration runs were based on data obtained during a series of converter charge cycles at the Gaspe Copper Mines Smelter in Murdochville, Que. Although model accuracy is limited by the fact that equipment was not available to measure the weights of material added or removed with the crane operated ladles, estimates of these weights were available based on average converter throughput.

2.2 SPECIFICATION OF AN INDEX OF PERFORMANCE

To evaluate current converter flux addition practice with standard air, the following factors must be considered in the formulation of a performance index which is to be maximized:

(1) Because high bath temperatures damage the refractory lining of the converter and low bath temperatures reduce the oxygen efficiency, a best operating temperature can be specified for the converter bath.

(2) Some of the heat from the exothermic converter reactions is absorbed by the converter slag which is returned to the externally heated reverberatory furnace. If the slag temperature at skimming is as high as possible, the heat requirements at the reverberatory furnace and the fuel costs would be minimized.

(3) By adding as much silica as possible to the converter, equivalent reductions in the amount of cold flux required at the reverberatory furnace could occur resulting in a fuel saving.

Based on these factors, a performance index can be specified as follows:

$$V = C_F x_1(T)x_2(T) + C_s M_s x_1(T) - C_R \int_o^T (x_1(t)-TOP)^2 dt \qquad (8)$$

where V is the value of the performance index at the end of the blowing interval

$C_F x_1(T)x_2(T)$ accounts for the value of the heat content of the excess silica at t = T

$C_s M_s x_1(T)$ accounts for the heat content of the converter slag. M_s is the equivalent heat capacity of the fayalite in terms of lb.moles of silica flux

$C_R \int_o^T (x_1(t)-TOP)^2 dt$ accounts for the deviations in bath temperature from the best value TOP^oR

To solve the optimization problem, it is convenient to introduce another state variable

$$\dot{x}_3 = (x_1(t)-TOP)^2 \qquad (9)$$

Such that the performance index becomes

$$G(\underline{x}(T)) = C_F x_1(T)\, x_2(T) + C_s M_s\, x_1(T) - C_R x_3(T) \qquad (10)$$

The relative importance of each term in the index should be based

on economic data but, because such data was not readily available, the weighting factors C_F, C_s and C_R were based on certain specific assumptions and were later varied to determine their effects on the final optimal solutions.

The performance index (10) involves only the thermal behaviour of the process and stresses the recovery of as much heat as possible. To evaluate the optimal solutions with oxygen enriched air, factors such as oxygen costs and savings in processing time should be included in an overall study. However, as oxygen smelting is commercially feasible[11], the emphasis will remain on the optimization of the thermal behaviour of the process for the oxygen enrichment runs and the index (10) will be employed.

3.0 SELECTION OF AN OPTIMIZATION TECHNIQUE

In general terms, the optimization problem to be solved is the following:

GIVEN System Equations $\dot{x}_i(t) = f_i(\underline{x}(t), u(t), t)$, $i = 1, \ldots n$ (11)

Initial values of the state variables $x_i(to) = x_{io}$ $i = 1, \ldots n$ (12)

Control variable constraints $o \leq u \leq U_{max}$ (13)

Fixed Process Time T

Index of Performance $G(\underline{x}(T))$ (14)

PROBLEM Determine the control trajectory $u^*(t)$ in the interval $t_o \leq t \leq T$ which maximizes the index of performance $G(\underline{x}(T))$ subject to the control variable constraints (13).

To determine the optimal control u^*, one could try to satisfy the necessary conditions provided by the Maximum Principle of Pontryagin. More explicitly, after determining $u^* = u^*(t, x, p)$ by maximizing the Hamiltonian

$$H = H[\underline{x}(t), \underline{p}(t), u(t)] = \sum_{j=1}^{n} f_j [\underline{x}(t), u(t), t] p_j(t) \quad (15)$$

one could try to solve the canonical equations

$$\dot{x}_i = \frac{\partial H}{\partial p_i} (\underline{x}(t) \underline{p}(t), u^*(t)) \quad i = 1, \ldots n \quad (16)$$

$$\dot{p}_i = -\frac{\partial H}{\partial x_i} (\underline{x}(t), \underline{p}(t), u^*(t)) \quad i = 1, \ldots n \quad (17)$$

with the boundary conditions

$$x_i(t_o) = x_{io}, \quad i = 1, \ldots n$$

$$p_i(T) = \left(\frac{\partial G}{\partial x_i}\right)_T \quad , \quad i = 1, \ldots n \tag{18}$$

using a boundary value iteration procedure to solve the two point boundary value problem. In this particular application, the state equations are linear functions of the control variable u and the Hamiltonian function is of the form

$$H(\underline{x}(t), \underline{p}(t), u(t)) = I(\underline{x}(t), \underline{p}(t)) + u\, F(\underline{x}(t), p(t)) \tag{19}$$

For such problems, the Maximum Principle seems to indicate a bang-bang solution of the form

$$u = \begin{cases} U_{max} & \text{for } F > o \\ o & \text{for } F < o \end{cases} \tag{20}$$

However, if $F(\underline{x}(t), \underline{p}(t))$ should turn out to be identically equal to zero over a finite interval of time (as in this application), the optimal control cannot be determined by maximizing the Hamiltonian. Such a region is called a region of singular control. In general, one cannot predict that such singular regions of control will form part of the optimal control trajectory for a given problem. If singular regions of control do exist, boundary value iteration methods for solving the two point boundary value problem fail.

Solution of the problem was achieved by applying a gradient method in function space involving the selection of a nominal control $\bar{u}(t)$ and successive adjustments to this trajectory to maximize the performance index $G(\underline{x}(T))$.

Several gradient methods are available, differing in the directions in which control perturbations are made. The steepest ascent method of Bryson and Denham generates these perturbations in the gradient direction $g_i = \frac{\partial H}{\partial u_i}$. A conjugate gradient method has been developed by Lasdon, Mitter, and Waren[6] and has been extended to bounded control problems by Pagurek and Woodside[7]. The significant advantage of the conjugate gradient method relative to the steepest ascent approach is a more rapid rate of convergence with a substantial saving in computer time. Because of its computational simplicity and rapid convergence properties, the extended conjugate gradient method for bounded control problems of Pagurek and Woodside was selected for this ap-

plication. In general terms, the method proceeds as follows:

To maximize the quantity $V(\underline{x}(t))$, the change in V resulting from a small change Δu is given to a first order approximation by the inner product

$$\Delta V = < H_u, \Delta u > = \int_{to}^{T} H_u \, \Delta u \, dt \tag{21}$$

Consider the situation at the ith iteration in the procedure where u_i has been obtained

(1) Evaluate the gradient trajectory $g(u_i) = H_u$ (22)

(2) Evaluate $\beta_{i-1} = \dfrac{< g(u_i), g(u_i) >}{< g(u_{i-1}), g(u_{i-1}) >} = \dfrac{\int g^2(u_i)dt}{\int g^2(u_{i-1})dt}$ (23)

(3) Determine the new direction of search $S_i = g(u_i) + \beta_{i-1} S_{i-1}$ (24)

(4) Determine a new control trajectory $u_{i+1} = u_i + \alpha_i S_i$ (25)

where α_i is found by a search procedure which selects the α_i yielding the largest value for $V(\underline{x}(t))$.

For bounded control problems in which saturation constraints of the type $0 \leq u \leq U_{max}$ are imposed on $u_{i+1}(t)$, β_{i-1} is computed by omitting the saturation regions of the previous control $u_i(t)$ from the integrals forming the numerator and denominator of β_{i-1}. Then $S_i(t)$ is computed for all t as described in step (3) and finally after $u_{i+1}(t)$ is computed in step (4), it is truncated at the upper bound U_{max} and the lower bound zero. No attempt has been made here to explain the reasons behind the modifications made to allow for the bounded control variable. The interested reader may refer to reference (7) for a complete development of the procedure with several simple examples. Note that if $\beta_{i-1} = 0$ in (24), the direction of search S_i becomes the gradient direction $g(u_i)$ and the method of steepest ascent is obtained. In fact, the conjugate gradient procedure is always started with one steepest ascent step and sometimes it is convenient to start with two or three such steps.

4.0 APPLICATION OF THE CONJUGATE GRADIENT METHOD

Previous comparisons of the convergence properties of the conjugate gradient and steepest ascent procedures demonstrated the superiority of the former approach. For this application, a similar comparison can be made.

The results are shown in Table 1 and Figure 6. In addition, it was found that by taking periodic steepest ascent steps (i.e. setting $\beta = 0$ periodically) during the conjugate gradient procedure a slight increase in the convergence rate occurred.

PERCENTAGE OF MAXIMUM INDEX OF PERFORMANCE ACHIEVED 100% = 3204.8	NO. OF EVALUATIONS OF $g(u_i) = H_u$ i		NO. OF EVALUATIONS OF STATE EQUATIONS		APPROXIMATE COMPUTER TIME IBM 360/50 MINUTES	
	CG	SA	CG	SA	CG	SA
90.00	7	12	25	38	2.7	4.2
99.00	14	34	50	108	5.4	11.8
99.90	30	70	102	252	11.0	26.8
99.99	60	147	198	605	21.5	62.6
100.00	100	232	320	964	36.0	99.7

TABLE 1: CONVERGENCE PROPERTIES OF CONJUGATE GRADIENT (CG) AND STEEPEST ASCENT (SA) PROCEDURES

$C_F = .0058$ $C_S = .000272$ $C_R = 1.0$

INCREASING RAMP (0 to 23 lb.moles/min in 30 min) AS NOMINAL CONTROL

Convergence rates depend on the choice of nominal control and from a practical viewpoint, the optimal solution was reached in a maximum of forty iterations for any nominal control. After forty iterations, the gradient values and step sizes were very small such that the control perturbations were practically zero (less than $\pm$ .005 lb.moles/min) over the singular region. In addition to a more rapid convergence rate based on the number of iterations, the search procedure was more efficient for the conjugate gradient method resulting in a substantial reduction in computation time (60%).

Optimization runs with radically different initial nominal control inputs yielded the same optimal solutions suggesting that the global optimal solutions had been found.

5.0 DISCUSSION OF RESULTS

Since the weighting factors for each term in the performance index

were determined from limited economic data and since, in current practice, little apparent emphasis is placed on the addition of excess silica (i.e. $C_F = 0$), optimization runs were carried out with different sets of weighting factors. In addition, the effects of variations in converter parameters on the optimal trajectories were determined. Referring to Figure 3, with heavy weighting on the temperature deviation term ($C_R = 1.0$), the optimal flux addition rate maintained the bath temperature x_1 at the best value TOP ($2600^\circ R$) over the majority of the blowing interval. For a smaller weighting ($C_R = .01$) on the temperature deviation term, the optimal bath temperature x_1 was lower than the best value in order to permit the addition of more excess silica flux. For each of these cases, the bath temperature was allowed to decrease over the final few minutes of the interval in order to maximize the addition of excess silica according to the weighting ($C_F = .0058$) of its heat content in the performance index. When the weighting factor on the heat content of the excess silica (C_F) is zero, the temperature deviation term predominates and the corresponding optimal flux addition rates resemble those currently employed (actual u, Figure 3). In fact, as temperature control is an important function of current practice and little emphasis is placed on the smelting of excess silica, optimization runs with $C_F = 0$ suggest guidelines for current practice. However, it appears that the capacity of the converter to smelt excess silica with standard air is limited and only marginal improvements to current practice appear possible.

PARAMETER VARIATIONS FROM NOMINAL VALUES	VARIATIONS IN PERFORMANCE INDEX DUE TO PARAMETER VARIATIONS AS PERCENTAGES OF NOMINAL VALUES		
	RADIATION LOSS COEFFICIENT C_{RL}	CONDUCTION LOSS COEFFICIENT C_C	OXYGEN EFFICIENCY P_2
+ 5%	- 13.8	- 55.6	+ 67.3
- 5%	+ 17.3	+ 24.2	- 540

TABLE II. SENSITIVITY OF PERFORMANCE INDEX TO PARAMETER VARIATIONS

$C_F = .0058$ $C_S = .000272$ $C_R = 0.1$

The current study is based on a simplified mathematical model of the process, its calibration involving adjustments to certain converter parameters. Since the results of Table II indicate that the optimal solutions

are very sensitive to such variation, the accuracy of the model is particularly important. The optimal trajectories resulting from these parameter variations are similar in form to those shown in Figure 3 but with differing steady levels and turn-on times.

Using standard air, the smelting power of the converter is limited and the flux addition policy is dictated by the process chemistry. The smelting power can be increased by the use of oxygen enriched air to increase the rates of reaction and heat generation and to reduce flue gas heat losses.

Optimization runs were carried out with various levels of oxygen enriched air and the results of Figure 4 indicate the increases in the optimal rates of flux addition with increases in the level of enrichment. Since sufficient excess silica to satisfy the needs of the reverberatory furnace can be added to the converter during operation at very low levels of oxygen enrichment, the excess smelting power at higher enrichment levels could be employed to smelt flotation concentrates in the converter. From Figure 5, it is seen that at an oxygen enrichment level of 42%, concentrates could be added at a rate sufficient for continuous operation without further reverberatory matte heat additions. It is interesting to note that the oxygen enrichment levels at the Hitachi Smelter are in the range 35 to 39%. The slightly higher value predicted by this preliminary study can probably be attributed to the approximations in the calibrated mathematical model and to differences in the physical plant and the compositions of the concentrates at Hitachi and at Gaspe.

6.0 SUMMARY

This preliminary study has demonstrated that practical results can be obtained from the application of mathematical modelling and optimization techniques to industrial processes. It has been found that for the particular copper converter operation considered, current flux addition practice closely resembles the optimum recommended practice, the major constraint being the limited smelting power of the unit. However, optimization of converter operation with various levels of enriched oxygen has provided an estimate of the benefits to be expected and an estimate of the oxygen enrichment level at which converter operation could proceed without reverberatory matte additions. An extension of this preliminary study is proposed involving two

control variables, the rate of concentrate addition and the level of oxygen enrichment. Additional complications arise due to the variable processing time which results and state variable constraints may be imposed to satisfy certain other chemical characteristics of the conversion process.

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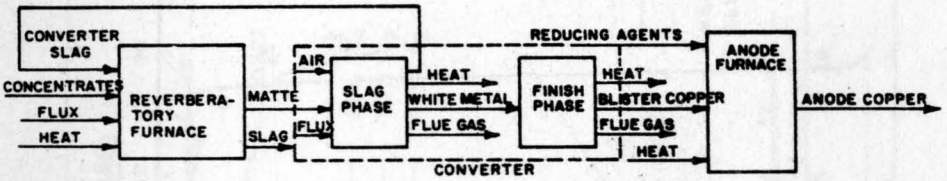

FIG.I SCHEMATIC FLOW DIAGRAM OF THE COPPER SMELTING PROCESS

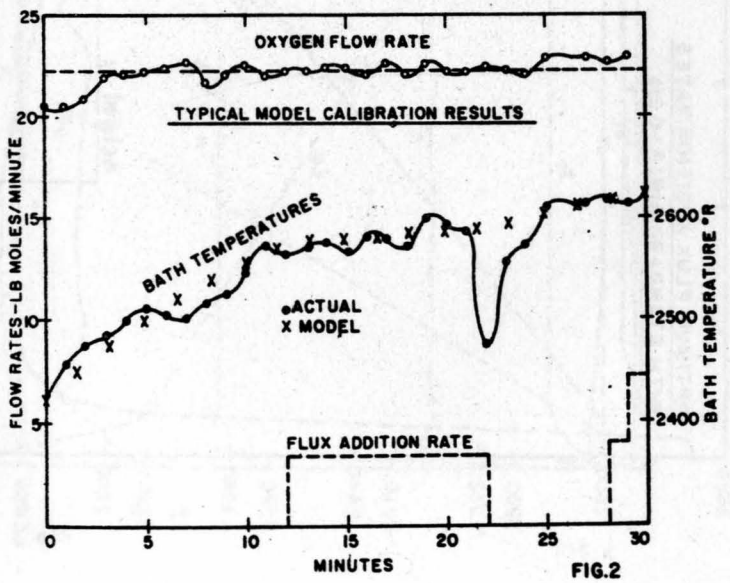

FIG.2

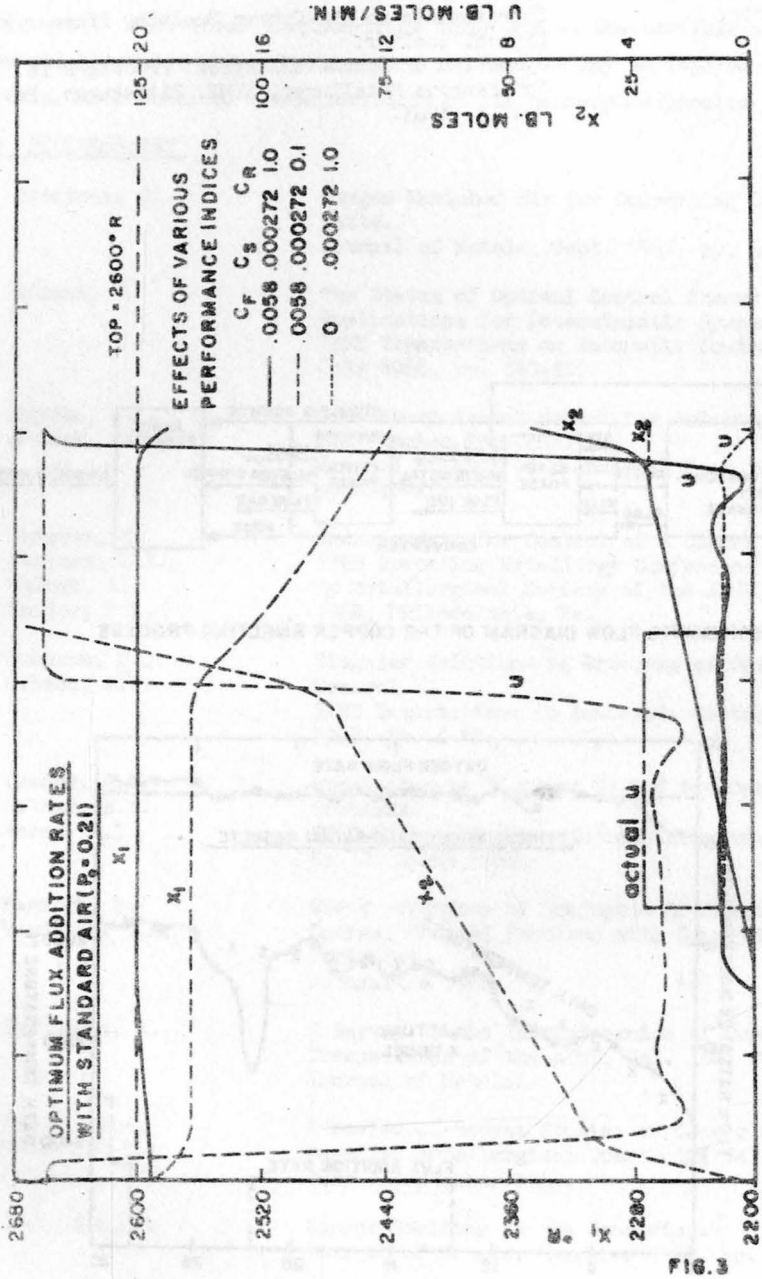
OPTIMUM FLUX ADDITION RATES
WITH STANDARD AIR (P_0 = 0.21)
TOP = 2600° R
EFFECTS OF VARIOUS
PERFORMANCE INDICES
C_F C_S C_R
.0058 .000272 1.0
.0058 .000272 0.1
0 .000272 1.0
x_1
x_2
u
actual u
x_2 LB. MOLES
u LB. MOLES/MIN.
θ_x
TIME - MINUTES
2680
2600
2520
2440
2360
2280
2200
125
100
75
50
25
20
16
12
8
4
0
0
5
10
15
20
25
30

FIG. 3

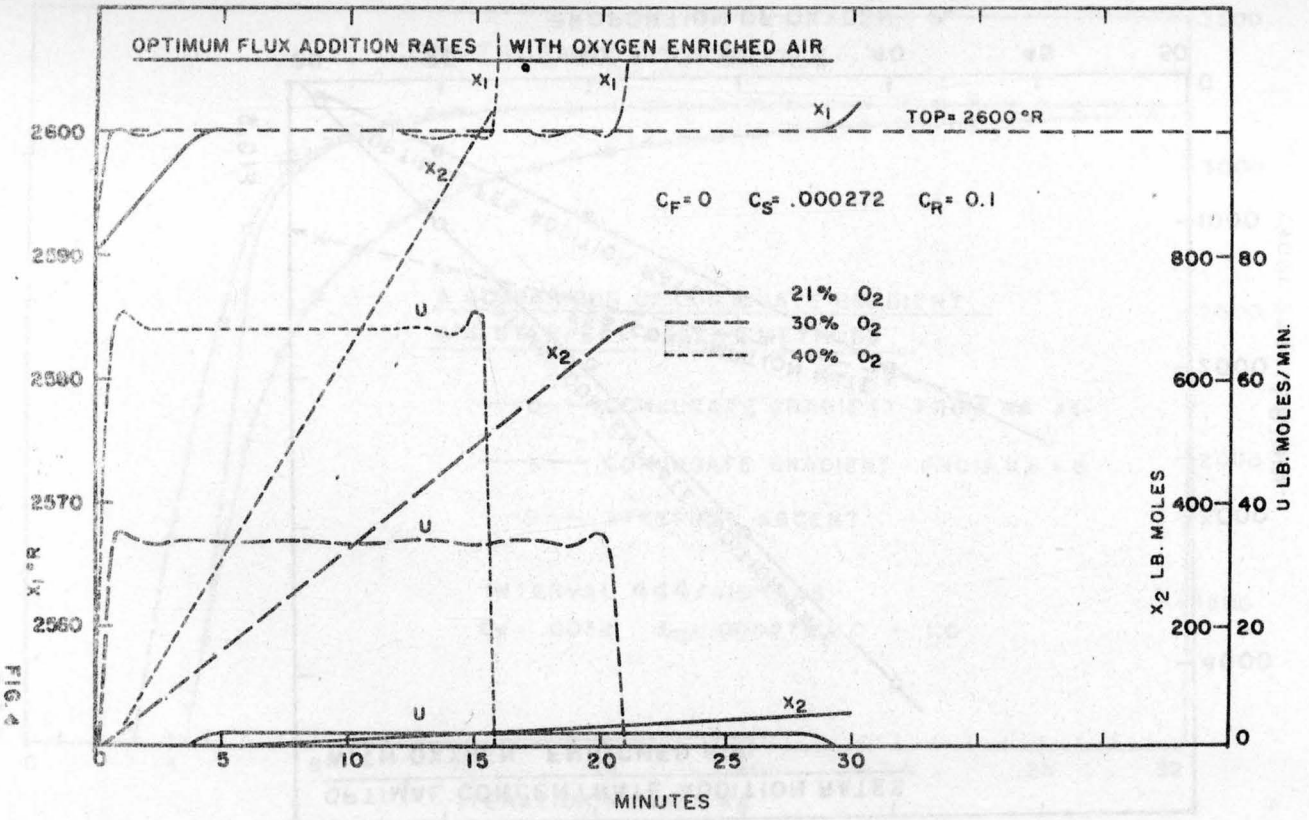
OPTIMUM FLUX ADDITION RATES | WITH OXYGEN ENRICHED AIR
TOP= 2600 °R
C_F= 0 C_S= .000272 C_R= 0.1
21% O_2
30% O_2
40% O_2
X_1
X_2
U
2600
2590
2580
2570
2560
X_1 °R
800
600
400
200
0
X_2 LB. MOLES
80
60
40
20
U LB. MOLES/MIN.
0
5
10
15
20
25
30
MINUTES

FIG. 4

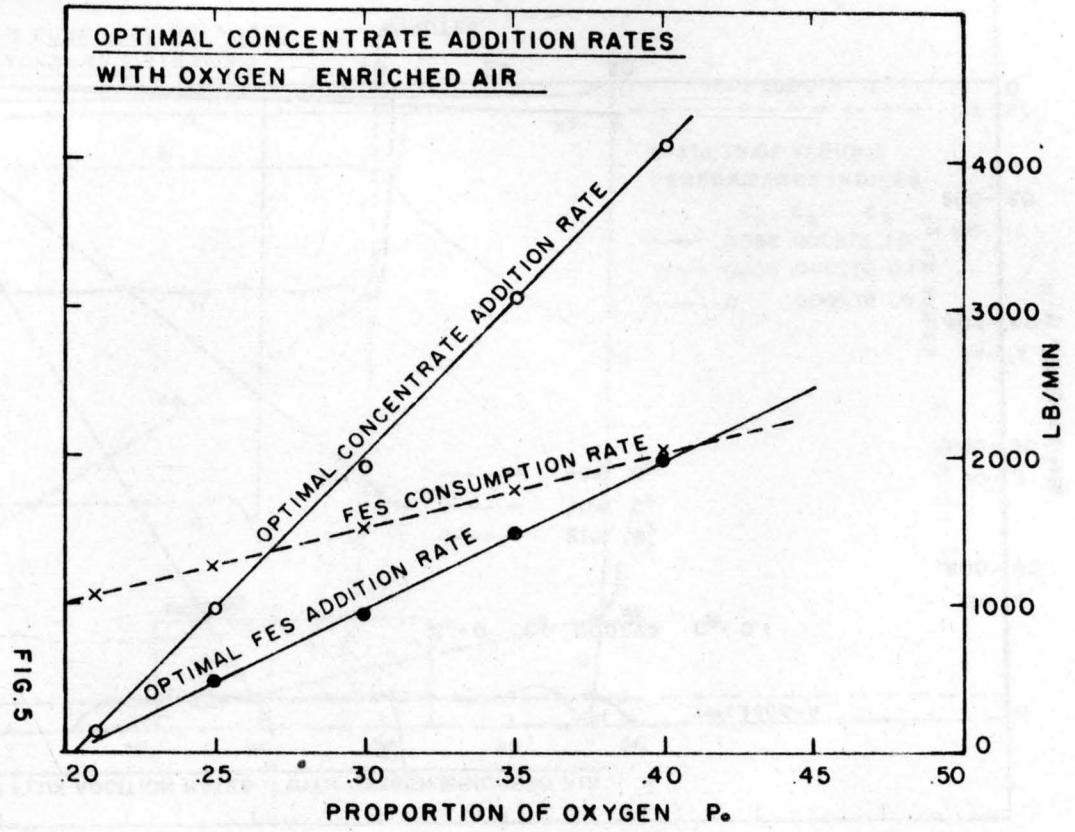
OPTIMAL CONCENTRATE ADDITION RATES
WITH OXYGEN ENRICHED AIR
OPTIMAL CONCENTRATE ADDITION RATE
FES CONSUMPTION RATE
OPTIMAL FES ADDITION RATE
4000
3000
2000
1000
0
LB/MIN
.20
.25
.30
.35
.40
.45
.50
PROPORTION OF OXYGEN Po

FIG. 5

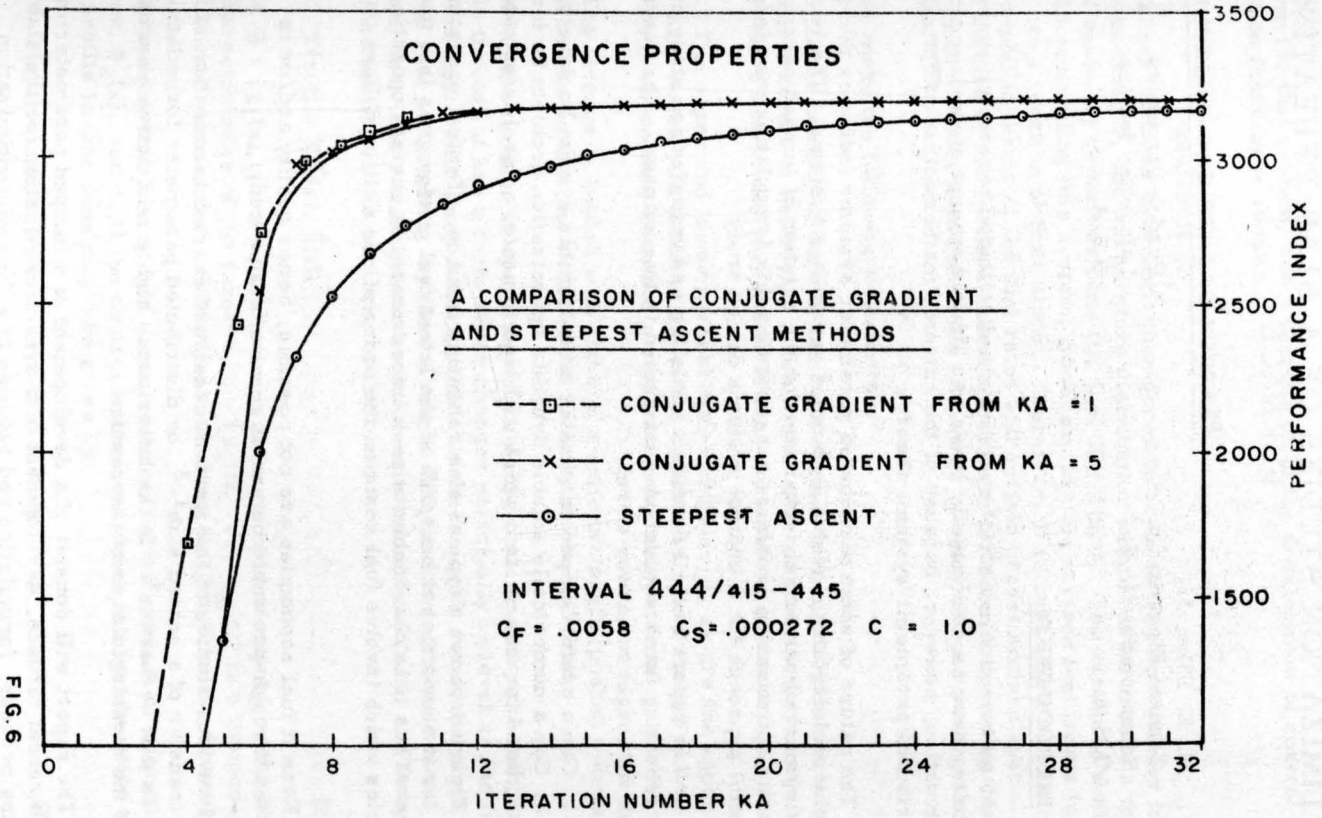
CONVERGENCE PROPERTIES
A COMPARISON OF CONJUGATE GRADIENT
AND STEEPEST ASCENT METHODS
CONJUGATE GRADIENT FROM KA = 1
CONJUGATE GRADIENT FROM KA = 5
STEEPEST ASCENT
INTERVAL 444/415-445
C_F = .0058 C_S = .000272 C = 1.0
3500
3000
2500
2000
1500
PERFORMANCE INDEX
0
4
8
12
16
20
24
28
32
ITERATION NUMBER KA

FIG. 6

OPTIMIZATION STUDIES OF A SLAB REHEATING FURNACE

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1. INTRODUCTION

Many complex industrial systems are distributed parameter systems. Modeling them as such usually leads to a limited amount of useful control information, however, because of the extreme mathematical complexity of distributed parameter systems theory.

The nature of some distributed parameter systems indicates that they may be meaningfully modeled as lumped parameter systems. The recent development of numerical techniques for the solution of lumped parameter dynamic optimization problems suggests that this formulation may indeed b a useful approach for a control systems design study.

In this report we will discuss a modeling and optimization study of the slab reheating furnace which was conducted in this manner. The questions which we sought to answer were:

1. Can a useful lumped parameter model of this system be developed?
2. Can a numerically soluble dynamic optimization problem be formulated for this system which will lead to useful control information?

Figure 1 shows a typical slab reheating furnace. Typical operating data for it shows that at best,35% of the latent fuel energy goes into the slabs. Thus it is reasonable to seek improvement in current operating policies which involve fuel costs on the order of one million dollars per year.

Even if fuel economies are not possible, better quality and/or increased through-put would be useful results of this study.

Previous studies of this system consist of exact determination of the heat transfer of a heated slab[1, 2], or distributed parameter formulations such as that by Lerner[4]. In the latter case, highly restrictive assumptions limit the meaningfulness of the results.

The report will consist of a development of a lumped parameter model of the reheat furnace, development of a dynamics equation for the slab heating process, formulation and solution of a dynamic optimization

problem for this system, and a discussion of the development of control information from these results.

2. MODELING THE REHEAT FURNACE

Reheat furnaces have been considered to be distributed parameter in nature because T, the temperature distribution within the furnace is a function of space (x) as well as time (t), i. e. $T = T(x, t)$. The variables available for controlling this system, however, are the various fuel inputs (u(t)), which are a function of time alone. Therefore, it is reasonable to search for a model in terms of the fuel rates and certain predetermined spatial terms defined by the furnace characteristics under normal operating conditions; such a model would have the advantage of being lumped parameter in nature.

We make the following assumptions:

Little heating is done in the soak zone, so we may formulate this problem in terms of delivering slabs to this zone at some required average temperature and model only the first four zones of the furnace.

The upper and lower preheat zones are slaved, as are the upper and lower heat zones. In addition the effect of skids is small enough to be neglected, so we may consider a symmetric heating process.

The furnace is loaded with slabs of a uniform length (pushed sideways) so variations across the width of the furnace may be ignored.

When the furnace is operating in a steady-state manner, the temperature profile through it has a form which changes principally in level as the firing rates change. This temperature profile is determined, for some given pushing rate and loading, by the fuel input rates for the various zones. Considering this it is reasonable to seek an expression for T(x, t), the temperature distribution in the furnace, of the form

$$T(x, t) = T(\vec{g}(x), \vec{u}(t)) \tag{2.1}$$

where $\vec{g} = \{g_1(x), \ldots, g_r(x)\}$ is a set of functions determined by the physical characteristics of the furnace, $u = \{u_1(t), \ldots, u_m(t)\}$ is a lumped parameter control vector, and x is the position along the length of the furnace.

Assuming that T(x, t) may be represented by the product of a function of x alone, $\eta_2(x)$, and $u_2(t)$, the control in the heat zone, the steady-state temperature profile in this zone might be given by:

$$T(x, t) = u_2(t)\eta_2(x) \qquad 45 \leq x \leq 79 \tag{2.2}$$

where $\eta_2(x)$ accounts for the spatial variation in the temperature profile, and $u_2(t)$ is the normalized fuel rate for this zone.

The steady-state temperature profile in the preheat zone may be written the same way, with an additional term due to the routing of the exhaust gas from the heat zone through the preheat zone, where u_1 is the fuel rate for the preheat zone.

$$T = u_1(t)\,\eta_1(x) + u_2(t)\,\eta_2(x) \qquad 0 \leq x \leq 45 \qquad (2.3)$$

The term $u_2(t)\,\eta_2(x)$ accounts for the effect of the carryover from the heat zone.

What we have called a steady-state operating regime corresponds to the pushing of slabs of a constant thickness and length into the furnace at a constant rate. If we hold $\vec{u}$ constant while increasing the thickness of the slabs pushed into the furnace the equilibrium temperature profile decreases, because the increased load in each zone absorbs more heat. Similarly, increasing the push rate while maintaining the same slab thickness increases the rate at which thermal energy is carried from the furnace, thus decreasing the temperature profile. Since the load in any zone (in tons) is directly proportional to the average thickness of the slabs in that zone we define

w_o = the nominal slab thickness

$\bar{w}_P$ = the average thickness of the slabs in the preheat zone

$\bar{w}_H$ = the average thickness of the slabs in the heat zone

ω_P = the normalized deviation from the nominal load in the preheat zone

ω_H = the normalized deviation from the nominal load in the heat zone

ω_P and ω_H are given by

$$\omega_P = \frac{\bar{w}_P - w_o}{w_o} \qquad \omega_H = \frac{\bar{w}_H - w_o}{w_o} \qquad (2.4)$$

Parameters B_P and B_H, which will determine the general extent of the coupling effect due to loading, are introduced by writing

$$\Omega_P = 1 - B_P \omega_P$$

$$\Omega_H = 1 - B_H \omega_H \qquad (2.5)$$

For slabs in the heat zone, the coupling effect is introduced by writing the temperature as

$$T(x, t) = u_2(t)\eta_2(x)\Omega_H \qquad 45 \leq x \leq 79 \qquad (2.6)$$

In the preheat zone the temperature profile is influenced by both control variables and the loading of both zones. In writing equation (2.3) this temperature profile is assumed to be the sum of the effects of the two controls. Since the portion of the preheat zone temperature profile which is affected by the heat zone control u_2 (say T_2) is influenced by the preheat zone loading as well as the heat zone loading, T_2 is written

$$T_2(x, t) = u_2(t)\eta_2(x)\Omega_H\Omega_P \qquad (2.7)$$

Adding to this the effect of the preheat control modified by the preheat loading coefficient one obtains

$$T(x, t) = [u_1(t)\eta_1(x) + u_2(t)\eta_2(x)\Omega_H]\Omega_P \qquad 0 \leq x \leq 45 \qquad (2.8)$$

To introduce the effect of varying v, the push rate, we define:

v_o = the nominal push rate

B_v = the push rate weighting factor

and

$$\xi = 1 - B_v(v - v_o)/v_o \qquad (2.9)$$

Then following the same arguments as discussed above the effect of varying v is introduced by modifying equation (2.8) to yield

$$T = u_2\eta_2(x)\Omega_H\xi \qquad 45 \leq x \leq 79 . \qquad (2.10)$$

$$T = [u_1\eta_1(x) + u_2\eta_2(x)\Omega_H\xi]\xi\Omega_P \qquad 0 \leq x \leq 45 .$$

Equation (2.10) will be used to represent our two-zone reheat furnace. It is a lumped parameter model with linearized factors accounting for the following effects: a) heat carry-through from heat to preheat zones, b) zone load interactions with the temperature distribution, and c) push rate interactions with the temperature distribution. When $B_P = B_H = B_v = 0$, the coupling effects do not appear, and equation (2.10) reduces to equation (2.3).

It now remains to select the functions $\eta_2(x)$ and $\eta_1(x)$ and the parameters w_o, v_o, B_P, B_H, and B_v which might yield a realistic model of our furnace, and to examine the properties of this model.

η_1 and η_2 have been selected to be of the form:

$$\eta_1(x) = \sum_{i=0}^{11} a_i x^i$$

$$\eta_2(x) = \sum_{i=0}^{7} b_i x^i \qquad (2.11)$$

where the values found for a_i and b_i were selected by constructing graphical the desired curves from temperature profile data for nominal operating con tions and then using least squares polynomial fitting techniques to find the values of a_i and b_i which best describe these curves.

This data was taken in 1964 and early 1965 from a five zone reheat furnace at the Spencer works of the Richard, Thomas, and Baldwin Steel Company during a study conducted by the International General Electric Co.

The exact values of B_P, B_H, and B_v and more exact forms for $\eta_1(x)$ an $\eta_2(x)$ might be determined by a carefully designed experimental program. For example, the nature of the temperature profile at the junction of the pre heat and heat zones might be more closely determined. Such a program, while refining the accuracy of the result, does not affect the validity of this feasibility study.

For the computational work which follows the following values were assumed:

$$B_v = .25$$

$$B_P = B_H = .75$$

$$w_o = 6.0$$

$$v_o = 1.0$$

3. DERIVATION OF DYNAMICS EQUATION

We will see below (Section 4) that in a lumped parameter dynamic optim zation problem the systems dynamic equation is of the form

$$\frac{d\vec{y}}{dt} = \vec{f}(\vec{y}, \vec{u}, t)$$

where $\vec{y}$ is the state of the system, and $\vec{u}$ is the control. If we select $\bar{\tau}$, th average temperature of a slab, to be the state variable, we then seek a

dynamic equation of the form

$$\frac{d\bar{\tau}}{dt} = f(\bar{\tau}, \vec{u}, t)$$

In Section 2 it was shown that it is possible to describe the temperature in the furnace by equation (2.10), which, for any given point x, yields T = T(u, t). If we can derive a differential equation for the heating process such that

$$\frac{d\bar{\tau}}{dt} = g(T(\vec{u}, t), \bar{\tau}, t)$$

the desired form will be obtained, i. e.

$$\frac{d\bar{\tau}}{dt} = g(T(\vec{u}(t), t)\ \bar{\tau}, t) = f(\bar{\tau}, \vec{u}, t) \qquad (3.1)$$

The general nature of the process to be described is that of radiative heat transfer. A concise dynamic equation of the form of (3.1) is difficult to develop for this process because of the non-linear radiation boundary condition. Scharbrough and Nichols[1], and El-Wazari[2] present typical differential-differencing solutions of this heat conduction problem. The work accurately describes the particular heat transfer process they consider but does not provide a dynamic equation of the required form.

If the temperature distribution through the slab is considered to be uniform ($\tau = \tau(t)$), and the specific heat and thermal conductivity of the steel are assumed to be constant, the one dimensional heat conduction equation may be integrated over its spatial variable and combined with the radiation boundary condition to yield

$$\frac{d\tau}{dt} = K(T^4 - \tau^4) \qquad (3.2)$$

which may be used to describe the dynamics of the slab heating process.

Lerner[3] linearizes this equation in his discussion of the reheat furnace. Sakawa[4] also limits his discussion to the linearized case. Experience has shown that this approach is inconsistent with the wide temperature range of the slab heating process.

Roe[5] has developed a method of treating a one-dimensional radiative heat transfer problem which may be used to develop a dynamics equation. Consider a slab to be in the moving frame x', y', z' as shown in Figure 1. Since it is assumed that local cold spots will be removed in the soak zone, the slab temperature may be considered independent of y'. It will also be assumed that there is no heat transfer in the x' direction in this frame. The

latter assumption is equivalent to neglecting the heat transferred between neighboring slabs. Neglecting this heat transfer simplifies the modeling problem, and is consistent with the goals of this study because this is at most a secondary effect, which will be neglected because heat transfer between adjacent slabs tends to draw their temperatures closer together. The control problem based upon ignoring this effect is more difficult to solve than the problem which includes this heat transfer. If the former can be solved numerically, it is reasonable to assume the latter may also be solved.

The differential equation governing the slab-heating process, and an expression for the surface temperature are derived in Reference 10 in terms of:

$\bar{\tau}$ = average slab temp ($^{\circ}$F + 459)

W = slab thickness (in)

C = slab heat capacity (BTU/LB $^{\circ}$F)

K = slab thermal conductivity

T = furnace temperature ($^{\circ}$F + 459)

τ_S = slab surface temp ($^{\circ}$F + 459)

These equations are:

$$\frac{d\bar{\tau}}{dt} = \frac{.99 \times 10^{-12}}{W\,C(\bar{\tau})} \left[\left| T^4(t) - \bar{\tau}^4(t) \right| \left(1 - .673 \times 10^{-10} \frac{W\,\bar{\tau}^3}{K(\bar{\tau})} \right) \right] \qquad (3.3)$$

$$\tau_s = \bar{\tau} + 17\,w^2 \frac{C(\bar{\tau})}{K(\bar{\tau})} \frac{d\bar{\tau}}{dt} \qquad (3.4)$$

Functions have been determined which express the material properties as segmented polynomials in τ. The result for the specific heat is

$$C(\tau) = \begin{cases} .1 - .7 \times 10^{-4}\bar{\tau} + .141 \times 10^{-6}\bar{\tau}^2 & 0 \le \bar{\tau} < 1382 \\ 1.006 - .527 \times 10^{-3}\bar{\tau} + .919 \times 10^{-8}\bar{\tau}^2 & 1382 \le \bar{\tau} < 1652 \\ .082 + .4 \times 10^{-4}\bar{\tau} & 1652 \le \bar{\tau} \end{cases} \qquad (3.5)$$

$K(\bar{\tau})$, the thermal conductivity, is approximated by the expression

$$K(\bar{\tau}) = \begin{cases} 33.5 - .0097\,\bar{\tau} & \bar{\tau} < 1600 \\ 10. + .005\,\bar{\tau} & 1600 \le \bar{\tau} \end{cases} \qquad (3.6)$$

The reheat furnace may now be simulated by integrating equation (3. 3) with T(t) as specified by equation (2. 10). The results of a typical simulation for the model are shown in Figure 2.

4. THE OPTIMIZATION PROBLEM

It will be shown in this chapter that it is possible to formulate the problem of controlling a reheat furnace as a lumped parameter dynamic optimization problem. Such a problem is of the form:

Given a system described by the state variables

$\vec{y} = \{y_1, \ldots, y_n\}$ and control variables $\vec{u} = \{u_1, \ldots, u_m\}$ select $\vec{u}(t)$ on $[t_0, t_f]$ such that

$$I = \phi(\vec{y}(t_f), t_f) + \int_{t_0}^{t_f} F(\vec{y}, \vec{u}, t)\, dt$$

is minimized, subject to the differential constraints

$$\frac{d\vec{y}}{dt} = f(\vec{y}, \vec{u}, t)$$

and end conditions which might be of the form

$$\vec{y}(t_0) = \vec{y}_0$$

$$\vec{y}(t_f) = \vec{y}_f$$

In addition one might have inequality constraints of the form

$$\vec{c}(\vec{y}, \vec{u}, t) \leq 0$$

$$\vec{s}(\vec{y}, t) \leq 0$$

To put the problem of optimum control of the reheat furnace into this form, consider a sequence of N slabs passing through the furnace at some velocity (or push rate) v(t). Associated with the i^{th} slab is its thickness w_i, and $x_i(t_0)$, its initial position. The position of the i^{th} slab at time t, $x_i(t)$, is given by

$$x_i(t) = x_i(t_0) + \int_{t_0}^{t} v(t')\, dt' \qquad (4.1)$$

The temperature which the i^{th} slab sees as it passes through the furnace, $T_i(t)$, is determined by equations (4. 1) and (2. 10) for any given control $\{u_1(t), u_2(t)\}$ as

$$T_i(t) = \begin{cases} u_2(t)\eta_2(x_i(t))\Omega_H \xi & 45 \le x_i(t) \le 79 \\ \left[u_1(t)\eta_1(x_i(t)) + u_2(t)\eta_2(x_i(t))\Omega_H \xi\right] \Omega_P \xi & 0 \le x_i(t) < 45 \end{cases} \tag{4.2}$$

Equation (3.3) is the dynamic equation governing the average temperature of the i^{th} slab. (For convenience the notation $\bar{\tau}$ will be dropped. Henceforth when we speak of slab temperature we will mean average slab temperatures.) It is

$$\frac{d\tau_i}{dt} = \begin{cases} \dfrac{.99 \times 10^{-12}}{w_i\, c(\tau_i)} \left[(T_i^4 - \tau_i^4)\left(1 - .673 \times 10^{-10} \dfrac{w_i \tau_i^3}{K(\tau_i)}\right)\right] & 0 \le x_i \le 79 \\ 0 & \text{otherwise} \end{cases} \tag{4.3}$$

where $c(\tau_i)$ and $K(\tau_i)$ are the material properties specified in Chapter 3.

The dynamic optimization problem is to minimize

$$I = \int_{t_0}^{t_f} F(\vec{u}(t))\, dt \tag{4.4}$$

where $F(\vec{u}(t))$ is the function which relates the normalized fuel rates $\{u_1(t), u_2(t)\}$ to the actual heating cost. The minimization is to be carried out subject to the dynamic equation (4.3) with the initial and final conditions on $\tau_i(t)$ specified. In addition the surface melting constraint must be met, with τ_s specified by equation (3.4).

$$\tau_{s_i} - \tau_{wash} \le 0 \tag{4.5}$$

$$\tau_{s_i} = \tau_i + 17\, w_i^2 \frac{c(\tau_i)}{K(\tau_i)} \frac{d\tau_i}{dt} \tag{4.6}$$

When slabs of a constant thickness, w, are pushed into the furnace at a constant rate, v, the furnace is defined to be operating in the steady state, and the optimum control $\{u_1^*, u_2^*\}$ is constant. Each slab is assumed to enter the furnace at the same initial temperature $\tau(t_i)$ and is required to leave the furnace at the desired final temperature τ_{DES}. The steady-state optimum control problem is to select $\{u_1, u_2\}$ so as to minimize $F(\vec{u})$ while satisfying the conditions $\tau(t_f) = \tau_{DES}$ and $\tau_{s_{max}} < \tau_{wash}$. ($\tau_{s_{max}}$ is the maximum surface temperature.)

5. SOLUTION OF THE STEADY-STATE OPTIMIZATION PROBLEM

In order to gain insight into the nature of the steady-state optimum control problem we will examine the general nature of curves of constant $\tau(t_f)$, $\tau_{s_{max}}$ and F(u) in the u_1 - u_2 plane. These curves may be generated as follows:

Since the furnace is operating in the steady state, it is only necessary to follow one slab through it. If the slab is at the entrance of the furnace at $t_i = 0$, its position in the furnace is given by

$$x = vt$$

Then T, the furnace temperature seen by slab, is a function of time alone, and is given by equation (2.10) with x = vt. The average slab temperature $\tau(t)$ and the surface temperature τ_s are determined by equations (3.3) and (3.4).

The form selected for $F(\vec{u})$ was $F(\vec{u}) = u_1^2 + u_2^2$. This expression was used (rather than a linear form) because it penalizes more heavily extreme operating conditions in either zone and it does not allow negative fuel rates to improve the index of performance. Thus it accounts for, in a limited manner, the fact that high firing rates are more expensive to implement as well as leading to higher fuel costs. However, it should be noted for future generalization that the method developed for solving the steady-state optimum control problem is independent of the exact form selected for $F(\vec{u})$.

The contours of constant $\tau(t_f)$, $\tau_{s_{max}}$ and $F(\vec{u})$ may now be generated for given values of v and w by selecting some point $\{u_1, u_2\}$ in the control plane, integrating equation (3.3) for the furnace temperature profile then specified by equation (2.10) and recording $\{u_1, u_2\}$, $\tau(t_f)$, $\tau_{s_{max}}$ and $u_1^2 + u_2^2$.

Figure 3 shows the general features of the control plane. Curve AC consists of the points such that the final slab temperature is the constant τ_{DES}, and curve DE consists of the points which lead to the constant maximum allowable surface temperature. Contours of constant $F = u_1^2 + u_2^2$ are also shown, as is the direction of decrease of F. The area above DE is a forbidden operating zone. As a result, if the curves AC and DE intersect at some point $B = \{u_{1B}, u_{2B}\}$, then the optimum operating point $\{u_1^*, u_2^*\}$ must lie on the segment BC. The steady-state optimum control problem may now be divided into two cases: That in which curves AC and DE intersect, and that in which they do not.

The steady-state optimization problem is now reduced to the simple one of conducting a one-dimensional search along AC for the operating point which minimizes F(u) while remaining in the allowable operating zone. Table 1 shows the results of several such searches.

Bozarth[6] gives the melting point of iron as 2800°F. Typical specifications for various steels give melting points of 2550°F to 2750°F. Thus typical current steady-state operating procedures for the furnace modeled, which are represented by cases 1 and 2 in Table 1, are well removed from the constraint boundary.

6. SOLUTION OF THE DYNAMIC OPTIMIZATION PROBLEM

It was shown in Section 4 that the problem of optimal control of the reheat furnace operating under non-steady-state conditions could be formulated as a lumped parameter dynamic optimization problem.

A number of computational techniques are available for the solution of dynamic optimization problems of the type stated. A general program, DYNOP [DYNamic Optimization Program], utilizing algorithms discussed in References 8 and 10 has been prepared at the General Electric Research and Development Center and was used to solve the dynamic control problem for the reheat furnace.

DYNOP is an iterative computational technique. In such techniques computations are started by guessing an initial form of u(t). After the dynami equations are integrated using this "nominal control," expressions are developed which determine a modification of u(t) which improves the index of performance. This is then repeated until some degree of convergence to the optimal control is achieved. The computational process is assumed to have converged when successive iterations do not appreciably change the index of performance.

Dynop itself will not handle as general a problem as was stated in Section 4 because it does not provide explicitly for specification of final conditions ($y(t_f)$) or inequality constraints ($c(y, u, t) \leq 0$). If either of these are present, they must be handled by the penalty function method.

Since typical steady-state operating conditions involve operating away from the surface melting constraint boundary, we will not include the constraint in the optimization problem. Instead, the results will be verified by examining the maximum surface temperatures during simulation of the final control law.

The penalty functions to provide the desired final average slab temperature are introduced into the index of performance as follows:

$$\hat{I} = I + \phi(\vec{\tau}(t_f))$$

$$= \phi(\vec{\tau}(t_f)) + \int_{t_0}^{t_f} F(\vec{u})\, dt \qquad (6.1)$$

with

$$\phi = \sum_{i=1}^{n} c_i \left[\frac{(\tau_i(t_f) - \tau_{i\,DES})}{\Delta\tau_i} \right]^{2\gamma_i}$$

$$F(\vec{u}) = u_1^2 + u_2^2 \qquad (6.2)$$

where

$\tau_i(t_f)$ = the final average temperature of the i^{th} slab

$\tau_{i\,DES}$ = the desired final average temperature of the i^{th} slab

$\Delta\tau_i$ = the permissible final temperature deviation for the i^{th} slab

c_i = an arbitrary weighting factor

γ_i = an integer which determines the "sharpness" of the penalty function

The same considerations as those discussed for the steady-state case lead to the form selected for $F(\vec{u})$. In addition, the quadratic form lends itself well to the computational technique used. Typical computational procedure is to start with $\Delta\tau_i$ fairly large and γ_i small, increasing γ_i and decreasing $\Delta\tau_i$ as computations proceed.

For computational purposes $c_i = 1$, $\gamma_i = 5$, $i = 1, \ldots, n$ were used. The value $\Delta\tau_i = 50$ was used for the first six iterations, and $\Delta\tau_i = 20$ for the last six. (Convergence was achieved in 12 iterations.) Figure 4 shows the resulting control law for the case of four inch slabs followed by six inch slabs. The maximum surface temperature attained by any of the slabs was 2435°F by the seventh slab.

Pertinent events on the time scale of Figure 4 are:

$t < t_A$	the entire furnace is full of four inch slabs
$t = t_A$	the first six inch slab enters the preheat zone
$t_A \leq t \leq t_B$	the thickness discontinuity is in the preheat zone
$t = t_B$	the first six inch slab enters the heat zone
$t_B < t < t_C$	the discontinuity is in the heat zone
$t = t_C$	the first six inch slab leaves the furnace
$t > t_C$	the furnace is full of six inch slabs.

Figure 5 shows the Dynop-generated optimal control laws for cases of 6 inch slabs followed by 4 inch slabs. In these cases v = 1.

The difficult part of the control problem is to heat two adjacent slabs of different thicknesses to the same final temperature range. This is evidenced by the 200° temperature difference between these slabs for the initial control. Considering this, the following physical significance may be seen in the resultant Dynop control law for the case of small slabs followed by thicker slabs.

In region 1 $u_1 < 0$. (The regions referred to are those labeled in Figure 4.) During this time the preheat zone contains mostly smaller slabs. To reduce a later tendency of these slabs to reach a final temperature $\pi(t_f) > \tau_{DES}$ we reduce their early heating in the preheat zone.

In region 2 the solution yields $u_1 > 0$. This control provides extra heat input to the large slabs which are entering the preheat zone at this time.

In region 3 the heat zone contains only smaller (four inch) slabs. In this region u_2 is equal to the optimum steady-state u_2 for the four inch slabs.

Region 4 includes the exit of the last small slab from the furnace. To make up for heat which could not be put into the large slabs without overheating the smaller ones, as the last small slab exits the heat zone u_2 is increased up to, then beyond the optimum steady-state value for the thicker slabs.

In region 5, u_2 settles down to its optimum steady-state value, which it assumes in region 6.

Figure 5 shows the Dynop-generated optimal control laws for thick slabs followed by thinner slabs. The physical interpretation of these control laws is similar to that for the previous case. In the preheat zone one increases the heating of the thick slabs and retards the heating of the thin slabs by changing u_1 appropriately. In the heat zone the value of u_2 goes from the initial optimal steady-state value to the final optimal steady-state value in a manner which provides the necessary fine temperature control for the slabs as they leave the heat zone.

7. CONTROLLER DESIGN

When empirical modeling has been done in a control study such as this, any proposed control scheme should be developed in an evolutionary manner. In this section comments will be made on how this might be accomplished, and the possibility of generalizing the Dynop results.

A controller for the reheat furnace might be developed in three stages. The first stage could be a simple feedforward controller which generates a

control law based upon the given process variables: push rate, slab thickness, and discontinuity location.

As a second step in the process of controller design one might use measurements of the final slab temperatures to modify the algorithm by which the feedforward controller generates control laws. This device might be called a terminal feedback controller.

The third and final step of the controller design process would consist of the development of a full state-variable feedback controller.

It is consistent with the level of our detailed process identification to restrict specific discussion to the possibility of developing a feedforward controller which will be based upon a generalization of the characteristics of the Dynop results discussed in Section 6. The information used to generate the control laws will be the pushing rate v, the magnitude and position of the thickness discontinuity in the input stream, the dimensions of the slabs on either side of this discontinuity, and the optimum steady-state values of u_2 found in Section 5.

There are many possible approaches to the problem of generating control laws which resemble the Dynop results. We desire a simple scheme which might be readily implemented in an on-line control computer. The method chosen is to generate the control law in segments. The general area of transition from one segment to the next is determined by the Dynop results, and the exact point of each transition determined by trial and error. The general approach for each segment is to represent the control as some perturbation about its optimum steady-state value. These perturbations were selected as a polynomial in t. Their magnitude was determined by trial and error as a function of the size of the thickness discontinuity, the slab widths, and the pushing rate.

Figure 6 shows the final form selected for the control law for small slabs followed by large slabs and Figure 7 the control for large slabs followed by small ones.

The exact form of the control expression used is shown in Reference 10. The values of I_{DYNOP}, the index of performance for the Dynop results, are compared with I, the index of performance for the synthesized feedforward control, for several cases:

w_I	w_F	I_{DYNOP}	$I_{FEEDFORWARD}$
6	5	174	180
6	4	153	164
7	5	214	230
7	4	197	213
5	6	171	179
4	6	155	164
5	7	222	227

The case of $w_I = 5$, $w_F = 7$ leads to high maximum surface temperatures; in practice discontinuities this extreme would be heated at a much lower push rate.

It is not consistent with the aims of this study to discuss in detail any further development of a controller without doing further process identification through experiments on the furnace. It is pertinent, however, to discuss how the results of such a study might be used to refine the controller.

It is useful for our purposes to categorize the process variables in terms of known variables (such as w and v) and unknown, but identifiable variables (such as the coupling parameters B_H, B_P, and B_v). A terminal feedback controller might examine the final slab temperatures and the furnace temperature, and, by inferring changes in the identifiable process parameters, appropriately adjust the parameters in the feedforward controller.

The present state of instrumentation technology for reheat furnace makes it unrealistic to propose a specific on-line, state-variable feedback control scheme for this system. The final result of an extension of this study might be an investigation of the benefits possible from such a scheme, however, thus motivating development of new measurement techniques. Such a controller might be developed in the manner suggested by Breakwell et al[7].

8. DISCUSSION OF RESULTS

It is useful to think of the results of this study as falling into two categories: information about useful tools for systems studies and information about the reheat furnace system.

The successful use of Dynop to solve the dynamic optimization problem suggests that there may be other cases in which it is reasonable to formulate large systems studies in terms of dynamic optimization problems of equivalent complexity. An ability to solve optimization problems with a fairly large number of state variables indicates that it may be feasible to model other distributed parameter systems or continuous processes in a manner similar to the modeling of the reheat furnace.

In those cases where state variable feedback is available for control purposes, computational techniques based upon the second variation may also lead directly to information useful for feedback control design in the manner suggested by Merriam[8], McReynolds and Bryson[9], and Breakwell et al[7].

The detailed computational results of this study cannot be viewed as particularly significant. They were produced for two purposes: to show that a physically significant optimization problem concerning the reheat

furnace could be solved numerically and to investigate the possible applications of such results.

Roe's work[5] on the heat transfer process has been shown to lead to highly accurate expressions for the average slab temperature and slab surface temperature. These expressions are of sufficient accuracy to be used in a predictive model-reference controller.

The steady-state results indicate that current steady-state operating points are well removed from the constraint boundary. Thus it is reasonable to conclude that steady-state through-put rates might be increased without the danger of melting slabs.

Only the $u_1 < 0$ portion of the optimal control law as determined by Dynop represents an unusual operating state. This may be due to an overstatement of the heat carry-through from the heat zone. The possible usefulness of the preheat fuel rate to influence final individual slab temperatures is an interesting possibility.

During a further study a more exact determination of the model parameters might be made. For example, the exact shape of the temperature profile near the preheat - heat zone junction might be determined. The model is thought to be representative of the fairly sharp temperature transitions which can occur there, however.

An orderly approach to the problem of control design has been developed. The initial results (the feedforward control scheme) might be applied on line immediately. The next step, the terminal feedback controller, would require development of some instrumentation, but only that consistent with current technology. This controller might then be developed into a feedback control system if it could be proved worthwhile to develop the necessary instrumentation.

ACKNOWLEDGEMENTS

The authors acknowledge the assistance of Professor T. J. Williams in this work. Primary support from a Purdue Research Foundation David Ross Fellowship and computing funds from NASA Institutional Grant NGR-15-005-021 provided the necessary financial support for the study.

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w	v	τ_{DES}	τ_{WASH}	u_1	u_2	$\tau(t_f)$ (°F)	$\tau_{S\,MAX}$ (°F)
5.0	1.0	2350	2500	0.0	0.8935	2350	2357
7.0	1.0	2350	2500	0.0	1.2774	2349	2421
7.0	1.0	2350	2400	1.0711	1.2420	2350	2399
8.0	1.2	2350	2500	0.5117	1.7148	2349	2501
8.0	1.4	2350	2500	2.3914	1.8187	2349	2500

TABLE I OPTIMUM STEADY-STATE OPERATING POINTS.

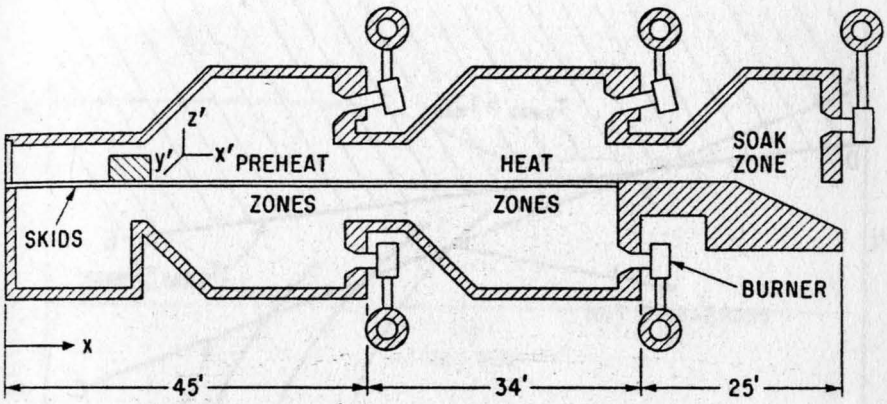

FIGURE 1 A FIVE-ZONE REHEAT FURNACE.

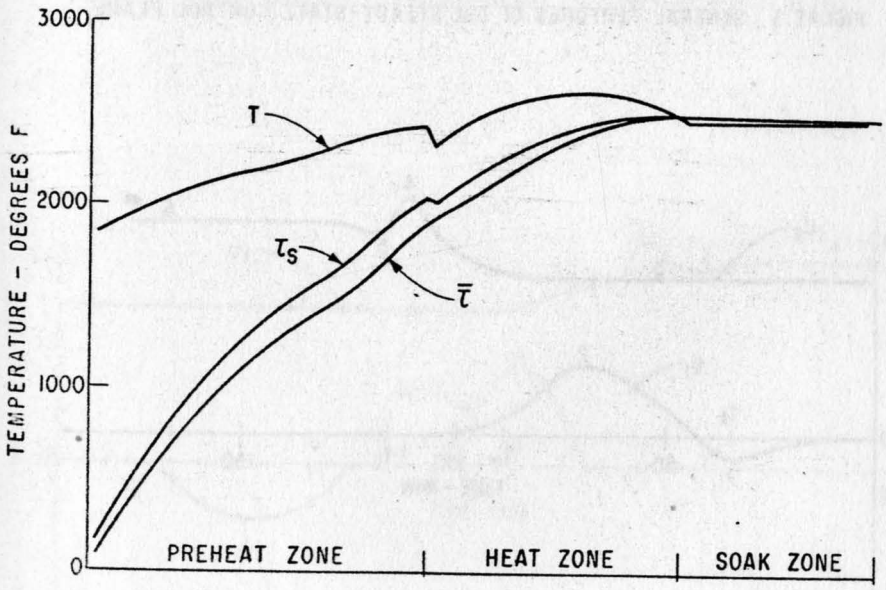

FIGURE 2 FIVE-ZONE SIMULATION RESULTS FOR $w = 5.5$, $v = 1.0$, $u_1 = 0.5$, $u_2 = 1.0$.

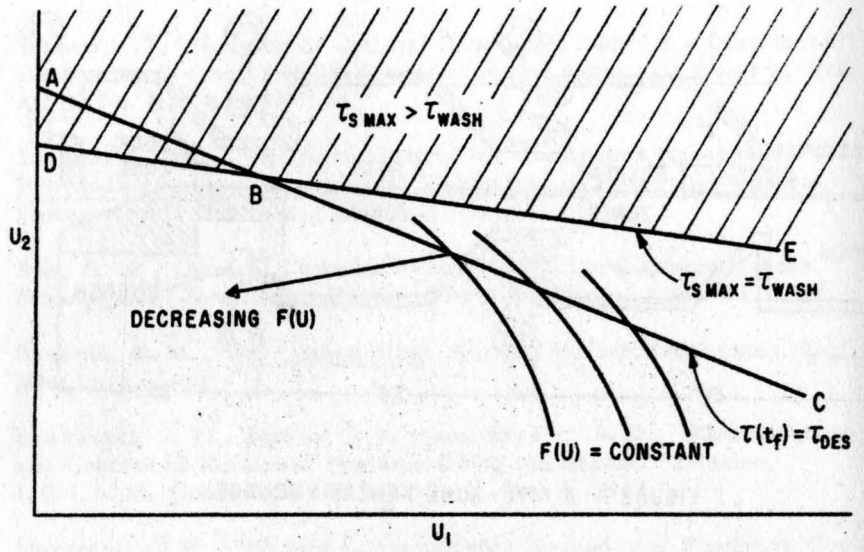

FIGURE 3 GENERAL FEATURES OF THE STEADY-STATE CONTROL PLANE.

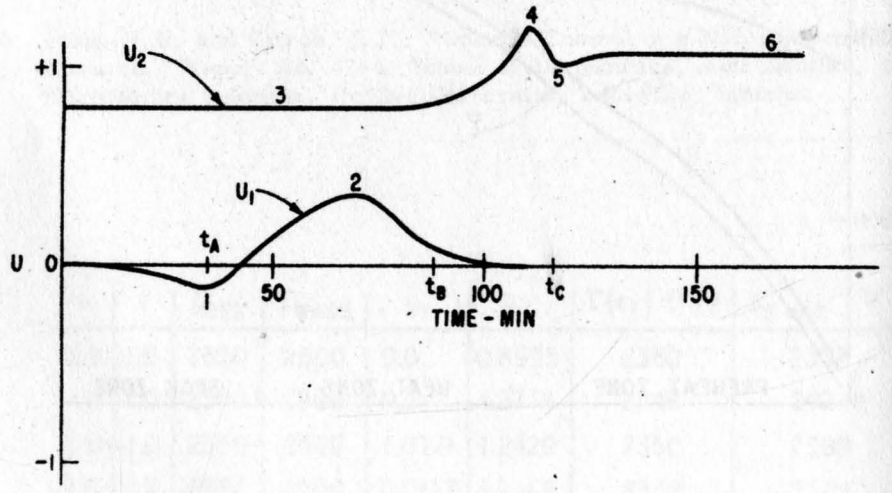

FIGURE 4 DYNOP CONTROL FOR 4 INCH SLABS FOLLOWED BY 6 INCH SLABS.

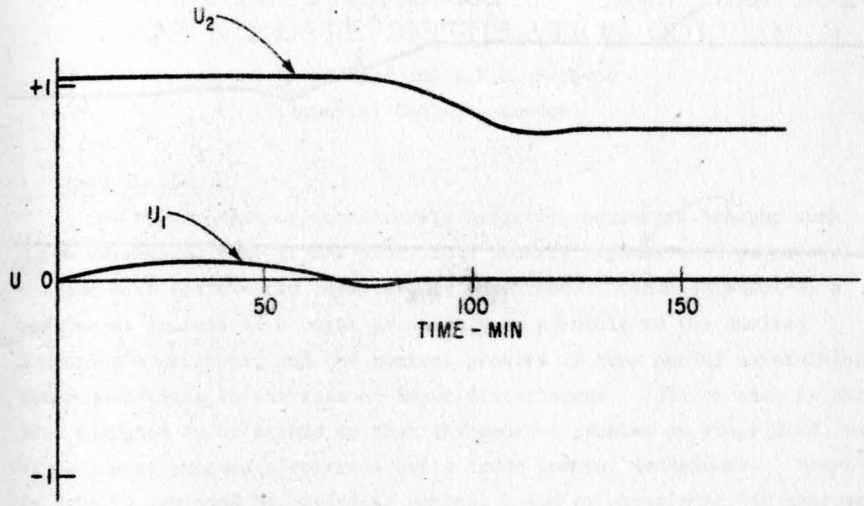

FIGURE 5 DYNOP CONTROL FOR 6 INCH SLABS FOLLOWED BY 4 INCH SLABS.

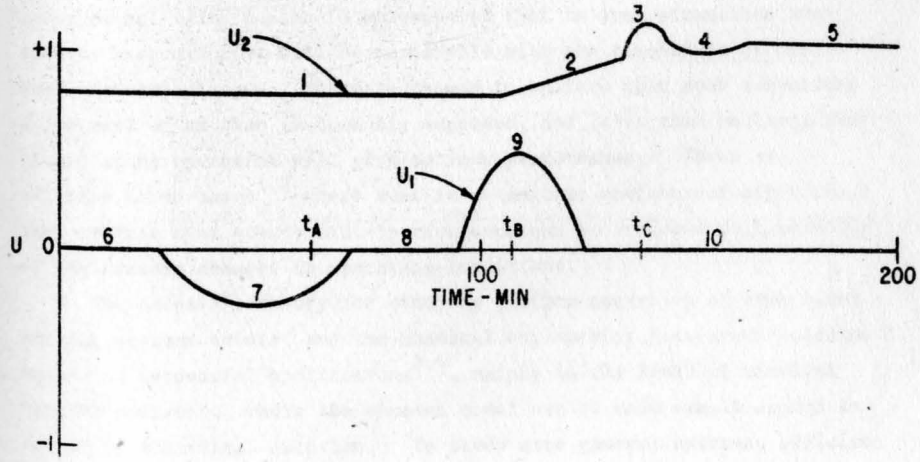

FIGURE 6 FEEDFORWARD CONTROL FOR $W_I = 4$, $W_F = 6$, $V = 1.0$.

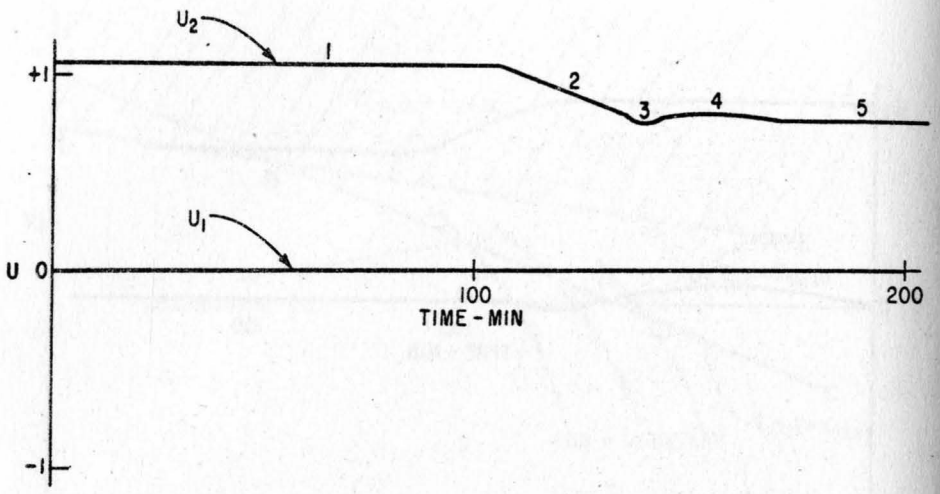

FIGURE 7 FEEDFORWARD CONTROL FOR $W_I = 6, W_F = 4, V = 1.0$.

OFF LINE COMPUTATION OF OPTIMUM CONTROLS FOR A PLATE DISTILLATION COLUMN

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1. Introduction

The development of continuously operating processes brought such large advantages that it has since been tacitly assumed that process plants operate most efficiently under steady conditions. Thus on start-up a continuous process is brought as quickly as possible to the desired operating conditions, and the control problem is then one of maintaining these conditions in the face of input disturbances. The process is therefore designed to be stable so that the control problem is simplified, and in fact most processes tolerate quite crude control techniques. Start-up is usually achieved by empirical methods based on experience, an approach which is again made possible by the natural stability of the process.

Some environmental conditions change much faster than the natural response time of the process so that they appear as fluctuations which are damped out either by the process itself or by the action of the control system. Other changes, such as in economic conditions, often take place on a time-scale much longer than the process response time, and if necessary these can be followed by changing the steady operating conditions of the process. It is also to be expected that in some situations the process response time will be comparable with the time-scale of the environmental changes; there is reason to believe that such situations arise more often than is commonly supposed, and it is then unlikely that steady state operation will give optimum performance. There are in addition known cases[1,2] where even in a constant environment significant improvements over steady state performance can be achieved with periodic or oscillatory changes in operating conditions.

The necessary theory for studying optimum operation of such time-varying systems exists, and the chemical engineering literature contains a number of successful applications[3,4], mainly in the field of chemical reactor operation, where the process model can be made simple enough to obtain an analytical solution. To study more general systems, efficient numerical techniques must be developed, and the work reported here was undertaken with this in view.

It will be noted that the methods generate feed-forward control policies based on accurate predictions. Additional problems must

therefore be solved if they are to be used on-line, but at present the computational demands make this out of the question. It is nevertheless useful to have a standard against which empirical or sub-optimal on-line methods may be compared.

2. Some Optimum Control Problems in the Operation of Distillation Columns

One general problem which arises in the operation of continuous distillation columns is the determination of the control policy which will take the system as quickly as possible from a given state to a new desired operating state. In practice it is not necessary to attain the new state exactly, since it is usually sufficient to achieve the new specifications for the products and for the rest approach the new state sufficiently closely for the normal control system to take over. A suitable formulation of the problem is then to find the control policy which minimizes the time (t_L) to take the system from the given initial state ($\underline{x}_I$) to a final state, $\underline{x}(t_L)$, which satisfies the condition:

$$\sum_{i=1}^{n} a_i \{x_i(t_L) - x_{Ri}\}^2 \leq e_s \qquad (1)$$

where e_s is some specified error and the a_i, $i = 1,2...n$, are suitable weights. The optimum final state will be on the boundary of the region in the space of the state variables defined by equation (1) since it will not take less time to reach an interior point; this follows from the continuity of the state variables in time.

The start-up problem itself is a special case of this problem, and in general the process will undergo large changes between the initial and final states. For this reason the problem cannot be solved by linearization about a suitable operating state and application of linear methods, since no linearization holds over the whole operation. On the other hand, small random fluctuations do not have a significant effect on the performance over this period so that deterministic methods may be used, and if the process model is accurate enough a feed-forward control policy can be found which solves the problem.

It is difficult to solve time-optimum control problems directly, and in any case the target region defined by equation (1) is arbitrary so that it will be necessary to explore the solution for various values of e_s and the a_i. It is therefore more efficient to find the control policy which minimizes the terminal error (e), given by:

$$e = \sum_{i=1}^{n} a_i \{x_i(t_1) - x_{Ri}\}^2 \qquad (2)$$

for a fixed final time (t_1), and repeat this for several values of t_1. The smallest t_1 for which the corresponding minimum e is equal to e_s is the solution (t_L) to the first problem, assuming that such a time-optimum control exists.

In these formulations of the start-up or change-over problem no account is taken of costs or product values during the period in question, and a more realistic requirement would be to minimize the nett cost of the operation, rather than simply the change-over time. However, unless the required new state represents steady state operation which is optimum under the new conditions, a difficulty arises in defining the length of the change-over period and it is then necessary to consider the most profitable policy for the whole period of operation of the process from start-up to shut-down. If this operating period is sufficiently long and there is indeed an optimum steady state operating policy, the complete optimum control policy will reveal minimum cost start-up and shut-down procedures with an intermediate period of operation at the optimum steady state. If no such policy exists, the policy obtained will of course be the correct time-varying policy which is optimum for the assumed conditions.

Unfortunately, examination of such complete policies over realistic operation periods makes exhorbitant demands on computing time. It therefore still seems useful to study the more restricted problem of minimum-cost change-over from a given initial state to a new steady state which does indeed represent the most profitable operation under the new conditions. Assuming that this required state is in fact attainable in finite time, then so long as the period considered is long enough to be certain of satisfying equation (1) at the end-point, we now have the problem of finding the control policy which achieves this and also minimizes a cost integral taken over the period. The integrand of this cost integral is a function of unit costs, the controls, and the properties of the product streams only, but special difficulties arise if the products must satisfy certain specifications. A practical way of dealing with this situation is to give off-specification product a reduced value, and to avoid the intractable problem which arises if the integrand of the cost integral is discontinuous in one of the state variables the reduction must be made continuously over a narrow range about the specified purity.

The more general problem discussed above of optimum operation over the

complete operating period is a problem of the same type, except that the specified state at the end-point is a suitable safe state for the column.

3. Some Methods of Solving the Optimum Control Problems Considered

3.1 Mathematical Formulation of the Problem

It is assumed that the chemical process under consideration is completely described for all instants of time t in the interval $t_0 \leqslant t \leqslant t_1$ by an n-vector of state variables $\underline{x}(t) \in E^n$. The process is under the external influence of an r-vector of controls $\underline{u}(t) \in E^r$ whose values can be set accurately on the process. The class of admissible controls is the set of piecewise continuous functions of t which assume values in a bounded set $U \subseteq E^r$, where U is known in advance and does not depend on $\underline{x}(t)$ or t.

In practice there will also be some input variables whose values cannot be adjusted and in fact there may only be information of a stochastic nature available about the values of these variables, or even no information at all. It will be assumed however for the reasons given earlier that all the non-adjustable input variables are given in advance as known functions of time and can therefore be absorbed into the model of the process.

The structure of the process is therefore such that the state variables are constrained to be solutions of the differential equations:

$$\frac{d\underline{x}}{dt} = \underline{f}(\underline{x}, t, \underline{u}) \tag{3}$$

with initial conditions $\underline{x}(t_0) = \underline{x}_I$

If the problem involves minimization of the cost integral $\int_{t_0}^{t_1} f_0(\underline{x}, t, \underline{u})dt$ this can be written in differential form by introducing a new variable $x_0(t)$ which is a solution of the differential equation:

$$\frac{dx_0}{dt} = f_0(\underline{x}, t, \underline{u}) \tag{4}$$

with initial conditions $x_0(t_0) = 0$, so that $x_0(t_1) = \int_{t_0}^{t_1} f_0(\underline{x}, t, \underline{u})dt$.

The functions $f_i(\underline{x}, t, \underline{u})$, $i = 0,1\ldots,n$ are assumed to be absolutely continuous in $\underline{x}$, t and $\underline{u}$ on $E^n \times [t_0, t_1] \times U$ and satisfy the Lipschitz condition

$$||\underline{f}(\underline{x}', t, \underline{u}) - \underline{f}(\underline{x}, t, \underline{u})|| \leqslant L(t)||\underline{x}' - \underline{x}|| \tag{5}$$

where L(t) is Lebesgue integrable on $t_0 \leqslant t \leqslant t_1$. If these conditions[5] are satisfied there is a unique, uniformly bounded, absolutely continuous

vector function $\underline{x}(t)$ of time which satisfies the differential equations (3) and initial conditions for almost all t for which $t_0 \leqslant t \leqslant t_1$ and these solutions depend continuously on $\underline{u}$.

All the optimum control problems which have been considered above may therefore be reduced to one or more solutions of the general problem of choosing the control trajectories $\underline{u}(t)$, $t_0 \leqslant t \leqslant t_1$, where $\underline{u} \in U$ so as to minimize a function of the final state variables, possibly subject to constraints on these.

Application of Pontryagin's Maximum Principle[6] to this problem results in a two-point boundary value problem in the differential equations (3) and (4) and an associated set of adjoint equations. This approach has the advantage that it deals directly with constraints on the final state variables, but after some preliminary work[7] it was concluded , in common with others, that it is very difficult to solve this boundary value problem because of the extreme sensitivity of the final values to the estimates of the initial adjoint variables. In fact this caused complete failure of the numerical methods in many cases and this approach was therefore abandoned.

If the control trajectories are approximated by piecewise constant functions as follows:

$$\underline{u}(t) = \underline{u}^{(j)} \text{ for } t_0+(j-1)\delta T<t<t_0+ j\delta T \text{ for all } j = 1,\ldots,q \qquad (6)$$

where $t_1 - t_0 = q\delta T$, then the problem becomes one of finding the control values $\underline{u}^{(j)}$ for all j which minimize a function of the final state variables, possibly with constraints on these, and subject to the condition that $\underline{u}^{(j)} \in U$.

The final state variables are of course evaluated from the controls by integrating the differential equations (3) and (4) and it has previously been remarked that these variables will be continuous functions of the control values. This is just a non-linear programming problem and it can be solved by means of any one of the standard methods available. A difficulty arises however from the fact that q must be large in order to get a reasonable approximation to the optimum control trajectory and this evidently results in a large number of independent variables $\underline{u}^{(j)}$ in the minimization. This tends to result in slow convergence of the minimization methods and an impossibly large number of evaluations of the final state from the controls.

This difficulty can be overcome to a certain extent by calculating the derivatives, since methods which make use of these tend to work better

on problems with a large number of variables. The derivatives of some function of the final state variables, $F(\underline{x}(t), x_0(t_1))$ say, with respect to the controls can be calculated at the expense of finding the first partial derivatives of $\underline{f}(\underline{x}, t, \underline{u})$, $f_0(\underline{x}, t, \underline{u})$ with respect to $\underline{x}$ and $\underline{u}$ and the integration of a set of adjoint equations associated with the differential equations (3) and (4) as follows.

Consider the following variation $\underline{u}'(t)$; $t_0 \leqslant t \leqslant t_1$ of some admissible control $\underline{u}(t)$; $t_0 \leqslant t \leqslant t_1$ for which $\underline{u}(t)$ is continuous on $T \leqslant t < T+\delta T$ and $\underline{u}(t) + \delta\underline{u} \in U$ for t such that $T \leqslant t < T+\delta T$:-

$$\underline{u}'(t) = \begin{cases} \underline{u}(t), & t_0 \leqslant t < T \\ \underline{u}(t) + \underline{u}, & \delta T \leqslant t < T + \delta T \\ u(t), & T + \delta T \leqslant t \leqslant t_1 \end{cases} \tag{7}$$

This control is also an admissible control.

$$\text{Set:} \quad x_i(t) = x_i'(t) - x_i(t) \qquad i = 0,1,\ldots,n \tag{8}$$

where $x_i'(t)$ results from $\underline{u}'(t)$ then[5] :-

$$\delta x_i(t) = O(\delta T).O(||\delta \underline{u}||) \qquad i = 0,1,\ldots,n \tag{9}$$

Differentiating (8) gives:-

$$\begin{aligned} \frac{d}{dt}\delta x_i(t) &= f_i(\underline{x}'(t), t, \underline{u}'(t)) - f_i(\underline{x}(t), t, \underline{u}(t)) \\ &= f_i(\underline{x}(t), t, \underline{u}'(t)) - f_i(\underline{x}(t), t, \underline{u}(t)) \\ &\quad + \sum_{j=0}^{n} \frac{\partial f_i(\underline{x}(t), t, \underline{u}(t))}{\partial x_j} \delta x_j + O(||\delta \underline{x}(t)||^2) \end{aligned} \tag{10}$$

$$\text{Define:} \quad \frac{dz_i}{dt} = -\sum_{j=0}^{n} \frac{\partial f_j(\underline{x}(t), t, \underline{u}(t))}{\partial x_i} z_j \qquad ,1,\ldots,n \tag{11}$$

with

$$z_i(t_1) = \frac{\partial F(\underline{x}(t_1), x_0(t_1))}{\partial x_i} \qquad ,i,\ldots,n$$

Then the expression $\sum_{i=1}^{n} z_i(t)\, \delta x_i(t)$ is absolutely continuous since $z_i(t)$ and $\delta x_i(t)$, $i = 0,1,\ldots,n$ are absolutely continuous functions of t for $t_0 \leqslant t \leqslant t_1$[5]. It therefore has a derivative almost everywhere on this interval and is recovered by integration of the derivative, so that by using equations (8), (9), (10) and (11) and the fact that $\sum_{i=0}^{n} z_i(T)\, \delta x_i(T) = 0$ it can easily be shown that the change in the function F

of the final state variables due to $\delta\underline{u}$ is given by:-

$$\delta F(\underline{x}(t_1), \underline{x}_0(t_1)) = \sum_{i=0}^{n} z_i(t_1)\, \delta x_i(t_1) + O(||\delta x(t_1)||^2)$$

$$= \int_{T}^{T+\delta T} \sum_{i=1}^{n} z_i(t)\{f_i(\underline{x}(t),t,\underline{u}'(t)-f_i(\underline{x}(t),t,\underline{u}(t))\}dt$$

$$+ O(\delta T^2).O(||\delta \underline{u}||^2) \qquad (12)$$

Hence the derivative of F with respect to $u_k^{(j)}$ can be found by letting $\delta u_k^{(j)}$ tend to zero in equation (12):

$$\frac{\partial F}{\partial u_k^{(j)}} = \tfrac{1}{2} \sum_{i=0}^{n} \left[z_i(T) \frac{\partial f_i(\underline{x}(T),T,\underline{u}(T))}{\partial u_k} + z_i(T+\delta T) \frac{f_i(\underline{x}(T+\delta T),T+\delta T,\underline{u}(T+\delta T))}{\partial u_k} \right] \delta T$$

$$+ O(\delta T^2) \qquad (13)$$

where $T = t_0 + \{j-1\}\delta T$. Thus the derivatives are approximated by the first term in equation (13) for a small enough δT.

In the examples which follow the set of points U in E^r is such that:

$$\underline{u}^{(min)} \leq \underline{u} \leq \underline{u}^{(max)} \qquad (14)$$

where $\underline{u}^{(min)}$ and $\underline{u}^{(max)}$ are vectors of given constants.

3.2 Numerical Methods of Solution

Direct use of the derivatives is made in the method of steepest descent, which can be used to improve a given set of control values by putting:

$$u_k^{(j)}\{improved\} = u_k^{(j)} - \varepsilon \frac{\partial F}{\partial u_k^{(j)}} \quad , \text{ all j and k} \qquad (15)$$

where ε is a scalar, chosen to give a reasonable reduction in the value of F at each step. The method of choosing ε in the present work is given in the Appendix.

Control constraints are dealt with simply by setting any values of $u_k^{(j)}\{improved\}$ which violate equation (14) back to the appropriate limiting values. Constraints involving functions of the final state variables, which are non-linear functions of the controls, can be dealt with by Rosen's Gradient Projection Method[8], Zoutendijk's Method of Feasible Directions[9], penalty functions[10], or Lagrange multipliers[11]. Most of these methods require derivatives of the constraint functions with respect to the controls, and these can be evaluated using equations (11) and (13). However each constraint gives rise to n adjoint differential equations, so that the computation becomes prohibitive for a large number of state variables.

It is well known that second order methods of the type given by Fletcher and Powell[12] give faster convergence than the method of steepest descent. However, for a large number of control values the advantage is greatly reduced because of the large number of function and derivative evaluations required to generate a reasonable approximation to the inverse of the Hessian matrix, which in turn is so large that it creates storage problems in the computer. The method of Fletcher and Reeves[13] does not require storage of the inverse Hessian matrix, but it is known to be less effective than methods which do generate this matrix, and the number of function and derivative evaluations is greatly increased by the necessity for fairly accurate determination of the minimum along each search direction.

With these methods, constraints may be dealt with by the use of penalty functions[10] or the more recently developed methods using accelerated penalty functions[14] or the projection method of Murtagh and Sargent[15]. It should be noted that the use of penalty functions gives rise to a steep-sided, curved valley system which constitutes a particularly difficult minimization problem. The conjugate gradient methods[12,13] require minimization along each search direction, and the initial estimate of each minimum obtained from the inverse Hessian matrix is very poor in this situation, resulting in an excessive number of function and derivative evaluations. It may therefore be preferable to use one of the methods of Murtagh and Sargent[15] for generating the inverse of the Hessian matrix, which do not rely on successive minimizations, in conjunction with the method described in the Appendix for choice of step length along each direction. Unfortunately these methods were developed too recently for use in the present work.

The second order methods given by Merriam[16] and Jacobson[17] use direct evaluation of the second partial derivatives of the state differential equations with respect to the state and control variables to generate the Hessian matrix, and hence achieve very fast convergence. However one is again faced with storage problems, and some of the computational effort tends to increase as the square of the number of state variables. It will also usually be very difficult, or even impossible, to compute the required derivatives for large complex models.

If the state differential equations are linear in $\underline{u}$ and equation (14) applies, it is possible to show from Pontryagin's maximum principle[6] that the controls will take limiting values only, unless there is a singular

control trajectory. It has further been shown[18] that for these conditions it is possible to get arbitrarily close to a final state resulting from any admissible control by means of a policy in which the controls take limiting values only. Such a control policy is completely defined by the times at which the controls switch from one extreme value to the other, and it can be hoped that a reasonably good approximation to the optimum performance will be obtained by using a relatively small number of switches.

The derivative F with respect to such a switching time can be found from equation (12) by letting δT tend to zero:

$$\frac{\partial F}{\partial T} = \sum_{i=0}^{n} z_i(T) \cdot \frac{\partial f_i(\underline{x}(T), T, \underline{u}(T))}{\partial u_k^j} \{u_k^{(max)} - u_k^{(min)}\} \qquad (16)$$

where T is the time at which u_k^j switches from $u_k^{(max)}$ to $u_k^{(min)}$.

If the number of switching times is not too large, it becomes feasible to use the various second order methods discussed above to determine them, subject if necessary to final state constraints.

If the model is such that it is impracticable to compute even first derivatives of $f(\underline{x}, t, \underline{u})$ and $f_0(\underline{x}, t, \underline{u})$ with respect to $\underline{x}$ and $\underline{u}$, a small number of switching times may make it possible to use a second order method with numerical evaluation of the required derivatives of F, as proposed by Stewart[19], or a method which does not require derivatives such as that of Powell[20].

The disadvantage of the switching time approach is that the number and relative locations of the switches is not known in advance. It is therefore necessary to apply a minimization technique to the control values in the first place, and hope that by the time progress becomes slow it will be possible to make a reasonable estimate of the optimum switching policy.

When the equations are non-linear in $\underline{u}$ and a switching policy is not appropriate, it may still be possible to approximate the optimum policy by simple functions involving relatively few adjustable parameters; the values of these can then be obtained by second-order methods, with or without determination of derivatives. However, the choice of suitable functions will again require insight and judgement on the part of the engineer.

4. Description of the Distillation Column Model used in the Optimum Control Studies

A very simple model of a plate distillation column separating a

binary mixture was used in order to test the numerical methods for optimum controls discussed above. The mass balance over plate p for the more volatile component gives:

$$\frac{d}{dt}\{\bar{x}_p C_p\} = V_{p-1} y_{p-1} - \{V_p + P_{Vp}\}y_p - \{L_p + P_{Lp}\}x_p + L_{p+1} x_{p+1} + F_p x_{Fp}$$

$$p = 1,\ldots,n \qquad (17)$$

where:

x_p = mole fraction of more volatile component in liquid leaving plate p.
y_p = mole fraction of more volatile component in vapour leaving plate p.
x_{Fp} = mole fraction of more volatile component in feed to plate p.
$\bar{x}_p$ = average mole fraction of more volatile component in hold up on plate p.
C_p = total molar hold up on plate p.
P_{Lp} = molar flow rate of liquid product leaving plate p.
P_{Vp} = molar flow rate of vapour product leaving plate p.
L_p = molar flow rate of liquid leaving plate p.
V_p = molar flow rate of vapour leaving plate p.
F_p = molar flow rate of feed to plate p.

The reboiler was taken as plate 1 and the condenser as plate n.

The following simplifying assumptions were made:

1. All the material hold up occurs in the liquid phase which is perfectly mixed so that $\bar{x}_p = x_p$.
2. The column consists of theoretical plates and the relative volatility α, is assumed constant so that the vapour mole fractions are given in terms of the liquid mole fractions by the equation:
$$y_p = x_p/\{x_p(1 - \alpha) + \alpha\} \qquad (18)$$
3. Thermal effects and the plate hydrodynamics are neglected and the hold ups are assumed to be constant.
4. There is a single liquid feed of constant composition x_F at its boiling point (introduced on plate p_F).
5. Liquid products only are taken from the reboiler and condenser only.
6. The column has three control variables, the bottom product rate P, the reflux rate R, and the feed rate F, which are constrained as follows:
$$0 \leq P \leq F$$
$$c_1 \leq R \leq \{c_2 + P\}/c_3 - F \qquad (19)$$
$$0 \leq F \leq c_4$$
where c_1, c_2, c_3 and c_4 are given positive constants and $c_1 < c_2/c_3 - c_4$ The first condition is imposed in order to prevent negative product

flow rates, and the second in order to ensure reasonable hydrodynamic operation of the column, namely prevention of flooding or dumping conditions.

7. The state variables are the mole fractions of the more volatile component in the liquid on each plate.

It follows from the above assumptions that the liquid and vapour flow rates and consequently the differential equations (17) describing the column are linear functions of the control variables as follows:

$$\begin{aligned}
&F_{pF} = F \;;\; F_p = 0 \quad p = 1,\ldots p_F-1,\ p_F+1,\ldots,n \\
&P_{L1} = P \;;\; P_{Ln} = F - P \;;\; P_{Lp} = 0 \quad p = 2,..,n-1 \\
&P_{Vp} = 0 \qquad\qquad p = 1,\ldots,n \\
&V_0 = L_1 = V_n = L_{n+1} = 0 \\
&V_p = R + F - P \qquad p = 1,\ldots,n-1 \\
&L_p = R + F \qquad\quad p = 2,\ldots,p_F \\
&L_p = R \qquad\quad p = p_F+1,\ldots,n
\end{aligned} \tag{20}$$

It is convenient to transform the control variables as follows:

$$\begin{aligned}
F &= u_3\, c_4 \\
P &= u_1\, F \\
R &= c_1 + u_2\left\{\frac{c_2 + P}{c_3} - F - c_1\right\}
\end{aligned} \tag{21}$$

so that (19) is satisfied for:

$$0 \leq u_k \leq 1 \qquad k = 1,2,3 \tag{22}$$

which has the same form as (14).

This transformation is well behaved except at the point $u_1 = u_3 = 0$ where it is easy to see that the method of steepest descent on the control values may stop prematurely in some cases. This difficulty can be overcome by setting u_1 equal to unity whenever a steepest descent step results in a value of u_3 of zero. The vertices of the polyhedron in the transformed control space given by (22) map into the vertices of the polyhedron in the original control space given by (19), so that the optimum control can be represented by the transformed control switching times.

A typical cost integral for a distillation column might be as follows:

$$x_0(t_1) = \int_{t_0}^{t_1} \{d_3F + d_4(R+F-P) - d_1x_n(F-P).S(x_n,X_n) - d_2(1-x_1)P.S(1-x_1,X_1)\}dt \tag{23}$$

where:

d_1 = value of one mole of more volatile component in top product.

d_2 = value of one mole of less volatile component in bottom product.

d_3 = cost of one mole of feed material.

d_4 = cost of evaporating one mole of liquid in the reboiler.

X_n = minimum specified value of x_n for which top product has any value.

X_1 = minimum specified value of $1-x_1$ for which bottom product has any value.

$$S(x, X) = \left(1 + \left[\frac{1 - x}{1 - X}\right]^{2m}\right)^{-1} \tag{24}$$

so that:

$$\lim_{m\to\infty} S(x, X) = \begin{cases} 0, & x < X \\ 1, & x > X \end{cases} \tag{25}$$

and $S(x, X)$ is therefore a continuous approximation to the required step function for large enough m. Note however that the larger the value of m the more severe the resulting minimization problem becomes. In practice it should be possible to estimate a reasonable value of m based on the tolerances on the product specifications. The method used here was to find the optimum steady control (i.e. the control which minimizes the integrand of (23)) for a given value of m and then to increase m until no further appreciable change occurred in the resulting optimum control.

The relevant data for the distillation column conditions used in the examples are given in Table 1 of the results.

5. Discussion of Results and Conclusions

5.1 Preliminary Remarks

Before attempting to solve any of the problems it is useful to know in qualitative terms what effect the control variables have on the process. It is well known in the case of a distillation column, and it can easily be demonstrated by a few integrations of the differential equations, that total reflux ($u_2 = 1$, $u_3 = 0$) causes maximum separation of the components in the column, that full top product rate ($u_1 = 0$, $u_3 = 1$) depletes the column of the more volatile component and that full bottom product rate ($u_1 = 1$, $u_3 = 1$) depletes the column of the less volatile component.

If it is assumed that the column is initially full of the feed material, the major start up effort is expended in establishing the compositions of the liquid in the reboiler and condenser. This is because these units contain the bulk of the material in the column and the initial compositions are the furthest away from the required steady state values. It is in fact much more difficult to establish the reboiler composition

for the example considered because the feed material is rich in the more volatile component and because of the feed plate location.

In agreement with these considerations, it was found that the minimum time ($t_1 = 45$) to establish the top product composition was achieved by applying total reflux over the whole period, whilst the minimum time ($t_1 = 63$) to establish the bottom product composition was achieved by applying full reflux rate and top product rate ($u_1 = 0$, $u_2 = 1$, $u_3 = 1$) over the whole period.

5.2 Minimum Start-up Time

For the start-up problem the final state must satisfy equation (1), which leaves it rather arbitrary unless e_s is chosen to be zero. The general intention is to choose e_s and the a_i so that satisfactory steady state control can continue from the state reached, and a number of simple special cases arise.

If only the top product composition is specified it is sufficient to require that $\underline{x}(t_L) \geq \underline{x}_R$, since the compositions will not then fall below the steady state values if the steady state controls are subsequently applied. This follows from equation (17), since y_p is a monotonically increasing function of x_p for all p. A similar remark applies if only the bottom product has any value.

In fact, for the example considered here it turns out that it is sufficient simply to establish the top product composition alone if only the top product is of interest, for the steady state controls then ensure that this will not subsequently fall below specification. The simple total-reflux policy given at the end of Section 5.1 is thus the optimum policy for this case, giving a minimum start-up time of 45. A similar situation occurs if only the bottom product is of interest, giving a minimum start up-time of 63.

The minimum time to establish both product compositions was $t_L = 66$, achieved by applying full top product rate until $t = 39$ and total reflux for the remainder of the period. The minimization methods converged rapidly for this problem, and the resulting control was used as an initial estimate for the following problem .

For establishing the complete composition profile (with $e_s = 0$), it was found to be necessary to weight the errors in equation (2) in favour of the top and bottom products, as otherwise convergence is very slow. This is logical because these compositions are the most difficult to establish, as discussed in Section 5.1, and because the column itself tends to damp

out errors in intermediate plate compositions in the subsequent operation. The minimum start-up time was found to be about 75, and the detailed results are given in Table 2.

It can be seen that the steepest descent method applied to the control values gives a rapid decrease in the terminal error over the first few steps, and progress then becomes very slow indeed. This is partly due to the "curved valley" form of equation (2), but there is a further difficulty, noted by Horn[21], that because a stable process tends to damp out the effect of earlier control action the final state tends to be very insensitive to the control policy near the start of the period; the space of the control variables is therefore naturally badly scaled.

To determine a switching policy, a minimum start-up time and policy were estimated from the steepest descent results and the latter then improved using Fletcher and Powell's[12] method. It was found to be advisable to underestimate the start-up time and use the smallest possible number of switches, as convergence is otherwise very slow. In addition, a larger number of switches increases the number of iterations required to generate the inverse of the Hessian matrix and a poor initial guess may even converge to an artificial local minimum. Once convergence is obtained from the initial guess, extra switches are added and the minimum redetermined. This procedure is repeated until no further reduction in the minimum is obtained. The whole procedure is then repeated for different values of t_1 in the region of the initial estimate, starting from the optimum switching policy found for this. The results are given in Table 3.

5.3 Minimum Start-up Cost

In order to explore start-up costs, two problems were considered. In both cases operating and feed costs were ignored ($d_3 = d_4 = 0$); in one case only top product was given a value ($d_2 = 0$) and in the other only bottom product ($d_1 = 0$). Following Jackson[3] in his work on reactor start-up, we can take the start-up cost as the difference in revenue which would have been obtained if the optimum steady state were achieved instantaneously this definition has the advantage that the cost is independent of the actual period considered if there is a true optimum steady state operating policy.

It was orginally hoped that a complete operating policy over a fixed period, with no constraints on the final state, would yield the minimum cost start-up procedure without having to consider very long periods. The longest period studied was limited to 135 by the amount of computation

involved and it turns out that this was not sufficient to provide evidence for or against the existence of an intermediate steady state policy. The results are given in Tables 4 to 7 under the columns marked $\lambda = 0$, and in each case there is clear evidence of an optimum shut-down policy, which removes as much of the valuable product as possible from the column before the end of the period and so leaves the column in an unsuitable state for further operation.

As a result a constraint on the final state was applied to force it to remain close to the optimum steady operating state, and this was dealt with by using a penalty function without acceleration. The procedure was to minimize the expression

$$x_0(t_1) + \lambda \sum_{i=1}^{n} a_i \{x_i(t_1) - x_{Ri}\}^2$$

starting with $\lambda = 0$ and successively increasing this value.

A suitable initial estimate for these policies is to use the simple policy which establishes the specified composition of the valuable product as quickly as possible, and then switch to the optimum steady state control values. As noted in Section 5.2, this produces no off-specification product. The cost, as defined above, of these simple policies is 422 when the top product is valuable, and 549 when the bottom product is valuable. The corresponding optimum policies yielded costs of 275 and 484 respectively, which show an appreciable saving in each case.

For the unconstrained problem ($\lambda = 0$), the steepest descent method worked better than for terminal error minimization, but convergence was still slow near the minimum. Fletcher and Powell's method applied to the switching times converged rapidly even from a poor initial guess, but the closely spaced switches occurring over the last part of the period tended to slow down the integrations, due to the small time-step required to deal with the transients after a switch. Fortunately it is possible to get reasonably close to the minimum with a relatively small number of switches.

For the constrained problem, the steepest descent method worked surprisingly well when it is considered that first order methods are not usually very successful in minimizing sums of squares. Convergence was rather slow however, so Fletcher and Powell's method was used to determine switching times. It was found to be necessary to increase λ slowly or the method failed to converge. A significant improvement over the steepest descent result was obtained for top product optimization, but none at all for bottom product optimization.

Details of the results are given in Tables 4 to 7.

6. Conclusions and Comment

The steepest descent method applied to the control values tends to converge very slowly after an initial rapid decrease, especially from a poor initial guess. Fletcher and Powell's[12] method cannot be applied directly to the control values because of storage and computational problems; when used to determine switching times it requires a good initial guess in most cases in order to converge at all, but if it does converge it converges rapidly. The performance of the method deteriorates for a large number of switches, partly because of the large number of function evaluations required to generate the inverse of the Hessian matrix , and partly because of the increase in the number of integration steps to deal with the transients following the switches. Fletcher and Powell's method also wastes computation time in searching for a minimum along each direction when used on strongly asymmetric functions. It would therefore be worth while exploring newer methods[15], possibly coupled with the trial and error method used for steepest descent.

On the basis of the results obtained, and assuming that derivatives can be evaluated, a three stage approach is recommended. Effort on a preliminary investigation to obtain a good initial estimate of the optimum policy is well repaid, and the steepest descent method can then be used to improve this estimate. If a switching policy is appropriate, it should then be possible to make an estimate of this, and Fletcher and Powell's method, coupled with judicious introduction of extra switches, will usually result in a further significant improvement. This approach suffers from the disadvantage that it cannot be made automatic and requires considerable judgement on the part of the user.

For minimizing the terminal error for a period of approximately 1.5 process time-constants it required 3 minutes on an I.B.M.7094 to carry out 60 steepest descent steps, and a further 1.5 minutes for 20 iterations of Fletcher and Powell's method. For minimizing the start-up cost, over a period of approximately 3 process time-constants, it took 2 minutes for 15 steepest descent steps and 18 minutes more for 30 iterations of Fletcher and Powell's method.

It is evident that it is quite impracticable to compute optimum control trajectories on-line by these methods, even for the simplified model used. However off-line computation of the optimum policy can provide a standard against which on-line sub-optimum schemes can be judged, and it also provides a nominal trajectory for the generation of a linearized process model which in turn could form the basis for a linear feed-back

controller. Off-line computation at the design stage could also be used to improve the dynamic performance of the process under the expected conditions of operation.

The examples show that appreciable gains in start-up costs for distillation columns are possible, but the results must not be taken too literally in view of the drastic simplifications in the model. The most unrealistic assumption is that the plate molar hold-up remains constant, for it is well known that it varies widely with vapour and liquid rates and with the liquid composition, and this may cause serious changes in the control policies, particularly where closely spaced switches are predicted. Vapour hold-up is nearly always negligible as assumed here, but the assumption of perfect liquid mixing will not be reasonable for large columns and the effect will be enhanced if variable hold-up is taken into account. Some of the assumptions, such as the neglect of thermal effects and the simple form of the equilibrium relation, will not cause greater errors than the natural variation between different physical mixtures. It is not expected that realistic representation of mass transfer performance or inclusion of entrainment and weeping will greatly change the character of the optimum policy, and if these were taken into account there would strictly be no need for flooding and dumping limits on the reflux rate. For the problems considered here operation is almost always at the flooding limit so that accurate representation of it would be important in real situations.

Needless to say, the use of a more realistic model or application to multicomponent mixtures would greatly increase the computing time required, but this is absorbed mainly in the integration of the equations. A detailed study[7] has been made of this latter aspect of the problem, and there is no reason to suppose that there would be repercussions on the convergence of the optimization problems discussed here. The real barrier to further progress is the almost complete lack of the requisite quantitative information on the dynamic behaviour of real columns, and further extensive computations will not be justified until such information is forthcoming.

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APPENDIX

The Method of Steepest Descent

In the algorithm given below a "step" consists of the evaluation of the derivatives $\partial F/\partial u_k^{(j)}$ for the current control policy, followed by adjustment of this policy according to equation (15), then computation of the corresponding state trajectory and value of the objective function, F. Initially one has an estimated control policy, u_0 say, the corresponding value of the objective function, F_0, and a step-length, ε. The basis of the algorithm is as follows:

1. Make a step from u_0 with the current ε to give u_1 and F_1.
2. If $F_1 > F_0$, replace ε by $\varepsilon/4$ and return to 1.
3. Replace ε by $\varepsilon/2$ and make a new step from u_1 to give u_2 and F_2.
4. If $F_2 > F_1$, replace ε by $\varepsilon/4$ and u_0 by u_1, then return to 1.
5. If $(F_1 - F_2) > (F_0 - F_1)$, go to 8.
6. Replace ε by 2ε.
7. If $(F_0 - F_1)/r < 2(F_2 - F_1) < r(F_0 - F_1)$, replace ε by 2ε.
8. Replace u_0 by u_2, then return to 1.

Steps 2 and 4 ensure that the step-length is small enough to prevent an increase in the function. The nett result of steps 3 and 5 is to halve the step-length if it is suspected that there is oscillation from side to side of a valley. Step 7 doubles the step-length if the derivatives are not changing much, and the allowable change is governed by the parameter r. Too small a value of r keeps the step-length small, whilst too large a value results in frequent increases followed by reductions due to increase of the function (step 2 or 4); the value of r was 1.5 for the results presented.

This method was arrived at by trial and error, and although it is rather crude it requires much less computer time than searching for a minimum along each direction, especially for strongly asymmetric functions and when the function evaluation requires about the same time as the derivative evaluations.

LIST OF SYMBOLS USED IN THE TABLES OF RESULTS

I number of iterations for Fletcher and Powell's method.

N number of successful steepest descent steps.

S time at which a control switches.

t_1 final time.

T time at start of constant control interval. (Control flow rates are given for a few representative constant control intervals only for the steepest descent results.)

u_j $j=1,2,3$ control values.

x_i $i=1,2,\ldots,9$ plate compositions at time t_1.

λ penalty function multiplier.

See also pp. 10-12.

TABLE 1

Distillation Column Data

$n = 9$; $p_F = 5$

$\alpha = 0.35$

$C_1 = 625.0$; $C_n = 320.0$; $C_p = 64.0$; $p=2,\ldots,n-1$

$c_1 = 5.0$; $c_2 = 40.0$; $c_3 = 1.25$; $c_4 = 20.0$

$x_{pF} = 0.55$; $x_p(t_0) = x_{pF}$ $p=1,\ldots,n$

$e_s = 0$

$a_1 = a_n = 1.0$

$a_p = 0.1$ $p=2,\ldots,n-1$

$m = 100$; $X_1 = 0.05$; $X_2 = 0.95$

$d_1 = 1.0$, $d_2 = 0.0$ for top product optimization

$d_1 = 0.0$, $d_2 = 1.0$ for bottom product optimization

$d_3 = d_4 = 0.0$

TABLE 2

Minimum terminal error for the complete composition profile using steepest descent on the control values

N \ t_1	60	65	70	75	80
0	7.95, -3	8.81, -4	1.11, -2	5.66, -4	8.55, -3
1	1.61, -3	8.45, -4	1.65, -3	1.17, -4	4.45, -3
3	1.02, -3	3.02, -4	1.25, -3	8.88, -5	3.38, -3
10	8.31, -4	2.76, -4	2.40, -4	3.00, -5	4.72, -4
20	6.60, -4	1.86, -4	1.50, -4	1.34, -5	1.01, -4
30	6.51, -4	1.63, -4	7.84, -5	1.14, -5	3.26, -5
60	6.25, -4	1.47, -4	3.07, -5	9.76, -6	1.19, -5

Resulting final states:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
Specified	0.0636	0.140	0.258	0.398	0.518	0.641	0.769	0.878	0.9534
$t_1 = 60$	0.0847	0.144	0.247	0.384	0.510	0.650	0.782	0.875	0.9429
$t_1 = 70$	0.0662	0.144	0.258	0.393	0.514	0.643	0.779	0.883	0.9509
$t_1 = 75$	0.0633	0.144	0.260	0.396	0.513	0.638	0.773	0.882	0.9531

Optimum control trajectories:

$t_1 = 60$	T	0	10	20	30	34	35	40	50	59		
	u_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
	u_2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		
	u_3	1.00	0.96	0.87	0.81	0.93	0.15	0.28	0.09	1.00		
$t_1 = 70$	T	0	10	20	30	35	40	50	60	63	66	69
	u_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.09	0.26	0.64
	u_2	1.00	1.00	1.00	1.00	0.98	0.98	0.98	0.95	0.90	0.80	0.60
	u_3	1.00	1.00	0.99	0.98	0.13	0.14	0.18	0.38	0.57	0.68	1.00
$t_1 = 75$	T	0	10	20	30	35	40	50	60	65	68	71	74
	u_1	0.01	0.03	0.04	0.06	0.07	0.01	0.02	0.03	0.07	0.15	0.41	0.66
	u_2	0.98	0.98	0.98	0.98	0.98	0.93	0.93	0.92	0.89	0.84	0.64	0.54
	u_3	1.00	0.99	0.99	0.98	0.98	0.13	0.16	0.28	0.45	0.73	1.00	0.98

TABLE 3

Minimum terminal error for the complete composition profile using Fletcher and Powell's method on the control switching times

I \ t_1	67	70	72	74	75	76
0	5.03, -4	8.76, -5	1.01, -4	2.70, -4	7.29, -4	1.51, -3
1	3.66, -4	3.17, -5	4.03, -5	6.69, -5	3.77, -5	1.41, -4
5	9.91, -5	2.44, -5	8.20, -6	2.30, -6	8.90, -7	3.30, -6
10	7.45, -5	1.79, -5	8.10, -6	1.30, -6	8.40, -7	1.90, -6
15	7.39, -5	1.65, -5		1.10, -6	8.10, -7	1.20, -6
20				1.10, -6	8.10, -7	8.60, -7

Resulting final states:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
Specified	0.0636	0.140	0.258	0.398	0.518	0.641	0.769	0.878	0.9534
$t_1 = 60$	0.0687	0.148	0.255	0.389	0.510	0.641	0.779	0.882	0.9495
$t_1 = 70$	0.0653	0.141	0.259	0.397	0.511	0.639	0.776	0.881	0.9516
$t_1 = 75$	0.0637	0.138	0.259	0.398	0.518	0.640	0.769	0.879	0.9533

Optimum control switching times:

$t_1 = 67$	S	0.00	4.20	37.72	60.12	65.91	66.42			
	u_1	-				+				
	u_2	+					-			
	u_3	-	+	-	+					
$t_1 = 70$	S	0.00	1.97	38.56	61.92	67.12	68.07	69.01	69.41	69.99
	u_1	-				+		-		
	u_2	+					-		+	
	u_3	-	+	-	+					-
$t_1 = 75$	S	0.00	2.01	37.78	62.81	68.23	70.64	71.62	74.04	74.82
	u_1	-					+		-	
	u_2	+				-		+		
	u_3	-	+	-	+					-

TABLE 4

Minimum start-up cost for top product optimization using steepest descent on the control values

Penalty function $t_1 = 135$	N \ λ		10^3	10^4	10^5	10^6
	0	-960.7	-1097.5	-1071.6	-1084.2	-1094.1
	1	-1038.1	-1097.9	-1089.5	-1098.0	-1103.4
	3	-1084.2	-1098.7	-1091.5	-1101.8	-1104.2
	5	-1091.9	-1100.4	-1093.2	-1103.1	-1105.1
	10	-1095.3	-1105.1	-1103.6	-1104.2	-1105.4
	15	-1109.5	-1106.3	-1107.1	-1106.2	-1106.4
Minimum start up cost		-1100.5	-1110.2	-1109.6	-1107.5	-1106.4
Final state error			3.9, -3	2.5, -4	1.3, -5	1.1, -8

Resulting final states:

$t_1 = 135$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
SPECIFIED OPTIMUM STEADY STATE	0.0636	0.140	0.258	0.398	0.518	0.641	0.769	0.878	0.9534
$\lambda = 0$	0.0692	0.136	0.244	0.377	0.497	0.612	0.741	0.860	0.9510
$\lambda = 10^6$	0.0709	0.148	0.267	0.406	0.522	0.644	0.770	0.877	0.9534

Optimum control trajectories:

$t_1=135$, $\lambda=0$	T	0	20	30	43	45	50	70	90	110	120	130	134
	u_1	1.0	1.0	1.0	1.0	0.35	0.35	0.36	0.37	0.38	0.38	0.35	0.33
	u_2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	u_3	0.0	0.002	0.003	0.003	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$t_1=135$, $\lambda=10^6$	T	0	10	20	43	45	50	70	80	110	120	130	134
	u_1	1.0	1.0	1.0	1.0	0.24	0.28	0.31	0.34	0.43	0.44	0.43	0.42
	u_2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	u_3	0.0	0.0	0.02	0.03	1.0	1.0	1.0	1.0	1.0	1.0	0.99	0.97

TABLE 5

Minimum start-up cost for top product optimization using Fletcher and Powell's method on the control switching times

Penalty function t_1 = 135	I \ λ	0	2×10^3	10^4	3×10^4	10^5	10^6
	0	-1155.5	-1126.5	-1124.9	-1128.5	-1130.8	-1126.1
	1	-1156.0	-1130.6	-1129.8	-1131.9	-1133.3	-1129.5
	2	-1156.3	-1135.4	-1131.1	-1132.9	-1133.9	-1129.6
	3	-1157.1	-1135.7	-1131.8	-1134.4	-1134.2	-1129.7
	5	-1159.4	-1137.0	-1132.5	-1134.8	-1134.4	-1130.8
	10	-1161.9	-1140.7	-1134.5	-1135.4	-1134.7	-1130.8
Min.start up cost		-1161.9	-1144.7	-1137.4	-1137.4	-1135.7	-1130.8
Final state error			2.0, -3	2.9, -4	6.6, -5	1.0, -5	0.0

Resulting final states:

t_1 = 135	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
Specified optimum stdy. st.	0.0636	0.140	0.258	0.398	0.518	0.641	0.769	0.878	0.9534
λ = 0	0.0766	0.128	0.219	0.344	0.468	0.574	0.704	0.833	0.9500
$\lambda = 10^6$	0.0886	0.154	0.263	0.399	0.528	0.653	0.775	0.880	0.9536

Optimum control switching times:

t_1 = 135, λ = 0	S	0.00	24.34	38.91	46.65	68.00	74.26	
	u_1	-	+	-		+	-	
	u_2	+						
	u_3	-	+	-	+			
t_1 = 135, λ = 0	S	85.20	91.88	99.86	107.14	119.14	125.94	
	u_1	+	-	+	-	+	-	
	u_2							
	u_3							
t_1 = 135, $\lambda = 10^6$	S	0.00	19.65	43.19	48.01	68.91	75.38	86.86
	u_1	-	+	-		+	-	+
	u_2	+						
	u_3	-	+	-	+			
t_1 = 135, $\lambda = 10^6$	S	94.13	105.50	112.85	123.52	129.27	134.10	
	u_1	-	+	-	+	-		
	u_2							
	u_3						-	

TABLE 6

Minimum start-up cost for bottom product optimization using steepest descent on the control values

Penalty function t_1 = 135	N \ λ	0	10^3	10^4	10^5	10^6
	0	-566.6	-638.1	-625.2	-623.8	-633.0
	1	-568.3	-638.2	-633.0	-633.1	-637.2
	3	-633.5	-638.3	-633.6	-636.1	-637.9
	5	-637.8	-639.2	-634.3	-637.9	-638.9
	10	-638.9	-639.8	-639.1	-638.9	-639.5
	15	-639.7	-641.7	-640.9	-640.1	-639.6
Min.start up cost		-639.7	-643.5	-642.8	-640.9	-639.9
Final state error			1.8, -3	1.9, -4	0.8, -5	0.2, -6

Resulting final states:

t_1 = 135	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
Specified optimum stdy. st.	0.0465	0.105	0.206	0.341	0.473	0.588	0.722	0.846	0.9402
λ = 0	0.0492	0.166	0.225	0.363	0.490	0.604	0.731	0.846	0.9314
λ = 10^6	0.0469	0.106	0.205	0.340	0.471	0.584	0.714	0.838	0.9326

Optimum control trajectories:

t_1=135, λ=0	T	0	20	40	60	67	68	80	90	100	110	120	130	134
	u_1	0.0	0.0	0.0	0.0	0.06	0.50	0.50	0.50	0.50	0.49	0.50	0.50	0.51
	u_2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	u_3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
t_1=135, λ= 10^6	T	0	20	40	60	66	68	70	80	100	120	130	134	
	u_1	0.0	0.0	0.0	0.0	0.0	0.59	0.58	0.55	0.49	0.45	0.44	0.44	
	u_2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
	u_3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.99	

TABLE 7

Minimum start-up cost for bottom product optimization using Fletcher and Powell's method on the control switching times

Penalty function t_1 = 135	I \ λ	0	2×10^3	10^4	3×10^4	10^5	10^6
	0	-597.3	-615.0	-626.8	-636.3	-638.6	-637.7
	1	-626.2	-623.2	-633.3	-637.8	-639.1	-638.5
	2	-650.0	-627.3	-635.1	-638.4	-639.1	-638.5
	3	-650.4	-634.7	-635.7	-639.1	-639.2	-638.5
	5	-650.9	-637.5	-639.2	-639.7	-639.3	
	10	-655.5	-641.2	-639.8	-639.8	-639.4	
Min.start up cost		-655.5	-644.8	-641.6	-640.3	-639.6	-638.8
Final state error			1.8, -3	1.8, -4	1.7, -5	2.0, -6	3.0, -7

Resulting final states:

t_1 = 135	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
Specified optimum stdy. st.	0.0465	0.105	0.206	0.341	0.473	0.588	0.722	0.846	0.9402
λ = 0	0.0501	0.147	0.278	0.416	0.529	0.642	0.759	0.856	0.9182
$\lambda = 10^6$	0.0471	0.100	0.201	0.340	0.472	0.584	0.711	0.830	0.9274

Optimum control switching times:

t_1 = 135, λ = 0	S	0.00	67.36	71.88	79.14	87.73	91.47	93.26	101.42
	u_1	-	+	-	+	-	+	-	+
	u_2	+							
	u_3	+							
t_1 = 135, λ = 0	S	108.07	116.47	121.58	127.43				
	u_1	-	+	-	+				
	u_2								
	u_3								
t_1 = 135, $\lambda = 10^6$	S	0.00	67.50	73.14	78.13	83.91	90.12	95.69	102.73
	u_1	-	+	-	+	-	+	-	+
	u_2	+							
	u_3	+							
t_1 = 135, $\lambda = 10^6$	S	108.41	115.42	121.00	127.54	132.72			
	u_1	-	+	-	+	-			
	u_2								
	u_3								

LEARNING CONTROL SYSTEM FOR A PILOT DISTILLATION COLUMN

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I. Introduction

a/ Definition of Learning Processes

Learning has been defined by G.A. Kimble [1], the psychologist, as "continuous relative changes of possible behaviour".

It is felt that this definition can also be adapted to technological processes in accordance with J. Peschon [2]:

"In general, a learning system is the system which improves its performance in time owing to a better - than in conventional systems-utilization of available information about its past performance".

Definition of learning applied to process control in a distillation column or, more generally to any physical processes, should imply such a structure of the control system which allows to improve gradually the control algorithm. Thus, learning control systems belong to the class of adaptive systems.

The above definition implies at least two phases of system operation:

- teaching phase which makes learning possible. This may be an external activity, in such a case we say about "trainer"

or internal activity which is based on gradual use of formerly obtained results.

- identification and classification phase, which permits to identify the state of the controlled system and to classify inputs and outputs of a learning system in such a way that determining controls for new inputs should be possible.

b/ Application of Learning Processes

Notion of Learning has been employed in studies of many different problems and especially in the following ones:

- Stochastic Optimal Control

The problem is to optimize control of a system, state variables and parameters of which are measured with an error due to noise. In general, in such cases it is assumed that both, the form of equations describing the system and the probability distributions of all random variables are known. One optimizes then the expected value of the performance index which is a quadratic form.

These problems were investigated by Kalman [3] who determined the estimator minimizing the expected value of the square of the difference between the present values of the system state coordinates and their predicted future values. The measurement of the state coordinates bears an error rezulting from noises.

The same problem was also studied by other authors as for example Ho [4], Lee [5], Gunckel and Franklin [6], Joseph and Tou [7] [8], Kwakernaak [9], Feldbaum [10] [11], Aoki [12].

The algorithm for determining the estimator allows to estimate, in a general way, the mean values of the measured quantities after taking into account initial information about the system and the information about former experiments. After that a deterministic control system can be applied.

Although cited authors not always say about learning, above definition is applicable here because in the operation of the system the past experience is utilized to make better use of information about the process as well as for future control.

- Pattern Recognition by Learning

The problem of pattern classification /letters, pictures, sounds etc./ is meant here. On the base of measurements of some physical features of the object to be classified, the codding unit determines such a set of internal variables that patterns belonging to the same class are clustered in certain subregions in the hyperspace of these variables. The subregions are separated from each other by separating surfaces, their coefficients being determined during the learning phase which is carried out by a trainer.
As the trainer a human being is usually employed.

Thus the problem of pattern recognition by learning is now reduced to the classification problem. All this is feasible if the number of classes is known, if measurements of the chosen plant parameters can be carried out, if a separating transformation for different classes can be realized and first of all, if during the learning phase the trainer is able to judge correctness or incorrectness

of the classification done by the learning system.
A number of works deals with the problem of finding out separating surfaces and in spite of saying about learning systems theory the problem reduces to the theory of "classifiers"

Basic studies concerning pattern recognition by learning were carried out by: Rosenblatt [13], Mattson [14], Widrow [15], [16], Selfridge [17], Aizerman [18], [19], Nilsson [20], Highleyman [21], Costan and Perennou [22], [23].

- Intelligent Machines

These are devices the activity of which resemble the human activity as a result of advance teaching which consists in applying a series of reinforcing stimuli. As an example the A.L. Samuel's 24 chess playing machine can be given. The internal local criteria of the machine make possible to choose a strategy which leads to the final victory.

- Learning Control Systems for Physical Processes

Among the authors working on this line are: M.D. Waltz [25], R.H. Raible [26], W.K. Taylor [27], M. Odo and K. Nakmura [28]. The last four were using an extremum-seeking controller as a trainer.

The results of trainer's activity were on different ways stored in memory. If an input situation was recognized as already known from the past, the stored data were utilized.
The authors were only interested in the static optimization of merely static processes. They were trying to reduce searching time and consequently to widen the spectrum of noises which can be compensated by the system.

Contrary to the researches mentioned above, M.D. Waltz

was interested in dynamic optimal control. He studied the problem in a deterministic way assuming that the mathematical model of the system /2nd or 3rd order/ is known and the unknown are the values of parameters. Waltz applies a criterion which results in the $u = \pm 1$ control and searches for optimal switching surfaces in the state space by learning. /The algorithm makes use of local criteria which are proper to a given problem/. After finishing the learning process, the state of the system can be determined by measuring the state coordinates. A suitable optimal control resulting from the learning is provided to the system. The author admits that the problem becomes difficult to solve and learning time lengthens significantly if the control can take on a greater number of values and especially in the case of existing many acceptable controls.

c/ Limitations of Learning Approach to Process Control

A more detailed analysis of the above examples shows that algorithms and structures of the proposed solutions are directly dependent on the assumptions concerning both, the measurable signals and the desired goal.

Thus, there do not exist entirely general solutions.

There exist, however, still much important limitations which are: necessity for trainer's presence and for facilities in appying a series of intensive trials. For this purpose local criteria should be known which assure convergence to a desired goal.

Moreover, if we are interested in optimal control in the presence of disturbances there should exist a possibility

to measure them /directly or indirectly/ and they are desired to reappear in order that similar situations be repeated many times.

A digital comuter with a great storage capacity, operating in real time should also be accessible.

A period of time for learning is also required. The learning time depends first of all on the preliminary knowledge about the system. In case of statistical optimal control it is assumed that the form of a mathematical model is known and that one should simply determine the mean values of some variables or parameters. M.D. Waltz assumes that state variables are known and they can be measured. It is clear that if they are unknown the learning time lengthens.

Learning time also depends on the complexity of the process and on the type of controls. If control can assume just two values, learning time is much shorter than in case of continnous controls. It should not be hidden that the time may be indispensibly long but in general it can be cut down by gaining in the preliminary knowledge about the system.

II. Starting Hypothesis and Final Objective of the Suggested Method

The Control Laboratory of the Faculty of Science in Grenoble have at their disposal a pilot /semitechnical/ distillation column in which separation of tetrachloromethane and toluene is carried out. The column serves the purpose of different experiments. In particular, investigations has been done on static optimization according to a profit criterion [29]. Investigations are continued on dynamic optimal control in

the presence of disturbances caused by changes of the efficieny and concentration of the input product.

The problem was studied in a deterministic manner.

An assumption was made that state variables for the system are unknown and can not be measured whereas disturbances ar measurable and changing in a discontinuous way /or under more general assumption - they increase linearly/, particular jumps of the disturbances level being so apart in time from each other that an extremum-seeking controler is able to reach the statically optimal point during the time interval between two consecutive jumps.

The controls for the column are: heating power Q_B and amount of reflux L_R. Other measured quantities are the disturbances /in our experiments - amount of the input product L_F/. These quantities are related with the concentration of the input product X_d and with profit per unit time I, by formulas [29]:

$$I = 1000\, Q_B - 125\, L_R - 400 \qquad \text{if } X_d - X_o \geqslant -0{,}04\%$$

$$I = -140\, Q_B \qquad \text{if } X_d - X_o < -0{,}04\%$$

where X_o - the assumed concentration of the output product.

Our purpose is to determine the dynamic quasioptimal control solely on the base of the hypotheses presented here and assuming limited learning time. We wish to maximize the performance index

$$J = \int_{t_o}^{t_f} I \, dt$$

The hypoteses assumed here seem to be costraining, however

it should be poited out that they would be necessary /and supplemented by others/ if we wanted to find optimal control applying the Pontryagin's Maximum Principle or the dynamic programming approach. Indeed, in that method all the state coordinates should be measured and disturbances should be known at time t_o for the interval $t_o - t_f$. Moreover, we would have to overcome significant mathematical difficulties since there exist some pure time delays in the system.

III. Proposed Solution

Since our hypotheses differ from ones assumed by the cited authors, the structures and algorithms obtained also differ in both cases. In particular, impossibility of making use of a trainer which would /in the learning time/ determine the searched out final optimal solution, excludes application of pattern recognition methods. These methods would not be applicable anyway because they require measurements of the state coordinates. Moreover, the number of classes /the values of the control vector/ is not known a priori and is very large.

The suggested structure for the optimal learning control system is shown in Fig.1, where: "process" is the distillation column, $\underline{P}$ - disturbance vector /L_F in the experiments/, variations of which are $\Delta\underline{P}$, $\underline{X}$ - vector of controls /Q_B and L_R/, C - auxiliary criterion replacing J, $\underline{X}_s^*$ and I_s^* - optimal values of the control vector and the profit per unit time in static state, $\underline{T}$ - vector of parameters of the adaptive dynamic algorithm the optimal values of which are $\underline{T}^*$.

The learning proceeds in two steps [30]:

- at the begining "learning in static state"takes place [31] on the base of which an optimal control for the static conditions of the process is determined. An extremum-seeking controller is the trainer. A certain number of values of controls and profits which are functions of disturbances and have been determined by the controller i.e., relations for $\underline{X}_s^*(\underline{P})$ and $I_s^*(\underline{P})$ are stored in the memory. Interpolation /or approximation/ allows to determine such controls for arbitrary values of disturbances.

A heuristic procedure allowing to determine proper number of points, which are to be stored in the memory, and to determine the degree of the interpolation polynomial has been worked out.

It should be stressed that in practice capacity of the memory need not be very large because the relations for $\underline{X}_s^*(\underline{P})$ are very simple ones. This fact has been proved for three investigated examples. In the case of the distilation column of the Control Laboratory in Grenoble about one hundred words was sufficient to store the formulas for $L_{RS}(L_F, X_F)$, $Q_{BS}(L_F, X_F)$ and $I_S(L_F, X_F)$ in the memory and to achieve the interpolation accuracy of 1 per cent.

Next, "learning in dynamic state" is carried out, which after a series of trials results in obtaining the optimal /or close to optimal/ dynamic controls for transition paths from one static state to another /after a jump of $\underline{P}$/. Final values of optimal controls are known as a result of learning in static states. Therefore, they can be fed to the process as a jump obtained from an adaptive "function

generator" /dynamical unit in Fig.1/ for which the optimal value $\underline{T}^*$ of the parameter vector $\underline{T}$ is searched out.

Any optimal control $X(t)$ can be defined as belonging to a metric space of continnous functions $C\,[t_o, t_f]$ such that:

$$X(t_o) = X_o \quad \text{(initial value)}$$

$$X(t_f) = X_f \quad \text{(final value - known at time } t_o \text{ from learning in static states)}$$

$$|X(t_2) - X(t_1)| \leq L|t_2 - t_1| \quad \text{- for arbitrary } t_1 \text{ and } t_2 \text{ /Lipschitz conditions/}$$

This is a compact space [32]. Hence, arbitrary function $X(t)$ can be approximated by a finite number of exponential curves or it can be obtained as a step response from a unit of a certain transmittance for the step $(X_f - X_o)$.

The optimal value $\underline{T}$ of the transmittance parameters should maximize the performance index

$$J = \int_{t_o}^{t_f} I \, dt$$

To adjust these parameters an auxiliary criterion C, independent on $\Delta\underline{P}$, which will be defined below, can be utilized.

Let $\underline{Y}$ be the state vector of the distilation column. The profir per unit time I is a function of $\underline{Y}$, $\underline{X}$, $\underline{P}$.

Therefore

$$J = \int_{t_o}^{t_f} I\,(\underline{Y}, \underline{X}, \underline{P})\, dt$$

It is assumed that at the time t_o^- the column is in its statically optimal state, the variables having values: $\underline{Y}$, $\underline{P}_o$, $\underline{X}_o$, $\underline{I}_o$. At the time t_o^- appears the disturbing jump:

$$\underline{P} = \underline{P}_o + \Delta\underline{P} = \underline{P}_f$$

Hence, at the time t_o^+ , $\underline{P}_f$ is known and owing to the learning in steady states the values of $\underline{X}_f$ and $\underline{I}_f$ are also known.

Expansion of J gives:

$$J = \int_{t_o}^{t_f} I_o(\underline{Y}_o, \underline{X}_o, \underline{P}_o)\, dt + \int_{t_o}^{t_f} \left[\left(\frac{\partial I}{\partial \underline{Y}}\right)^T \Delta\underline{Y} + \left(\frac{\partial I}{\partial \underline{X}}\right)^T \Delta\underline{X} + \right.$$

$$\left. + \left(\frac{\partial I}{\partial \underline{P}}\right)^T \Delta\underline{P} \right] dt + \int_{t_o}^{t_f} \Delta^T Q\, \Delta\, dt + \varepsilon \qquad /1/$$

Where

$$\varepsilon \simeq 0$$

$$\Delta = \begin{bmatrix} \Delta\underline{Y} \\ \Delta\underline{X} \\ \Delta\underline{P} \end{bmatrix} \qquad Q = \begin{bmatrix} \frac{\partial^2 I}{\partial \underline{Y}^2} & \frac{\partial^2 I}{\partial \underline{Y}\,\partial \underline{X}} & \frac{\partial^2 I}{\partial \underline{Y}\,\partial \underline{P}} \\ \frac{\partial^2 I}{\partial \underline{X}\,\partial \underline{Y}} & \frac{\partial^2 I}{\partial \underline{X}^2} & \frac{\partial^2 I}{\partial \underline{X}\,\partial \underline{P}} \\ \frac{\partial^2 I}{\partial \underline{P}\,\partial \underline{Y}} & \frac{\partial^2 I}{\partial \underline{P}\,\underline{X}} & \frac{\partial^2 I}{\partial \underline{P}^2} \end{bmatrix}$$

$\underline{A}^T$ in these formulas is the transpose of the vector $\underline{A}$. The vector $\underline{\Delta Y}$ is a function of time. It is assumed that $\underline{Y}(t) = \underline{Y}_o + \underline{\Delta Y}(t)$, and similarly for $\underline{X}$ and $\underline{P}$.

If, for example, the components of the vectors $\underline{Y}$ and $\underline{X}$ are y_1, y_2 and x_1, x_2 - respectively, the expressions $\frac{\partial I}{\partial \underline{Y}}$ and $\frac{\partial^2 I}{\partial \underline{Y} \partial \underline{X}}$ have the form of the following vector and matrix

$$\frac{\partial I}{\partial \underline{Y}} = \begin{bmatrix} \frac{\partial I}{\partial y_1} \\ \\ \frac{\partial I}{\partial y_2} \end{bmatrix} ; \quad \frac{\partial^2 I}{\partial \underline{Y} \partial \underline{X}} = \begin{bmatrix} \frac{\partial^2 I}{\partial y_1 \partial x_1} ; & \frac{\partial^2 I}{\partial y_1 \partial x_2} \\ \\ \frac{\partial^2 I}{\partial y_2 \partial x_1} ; & \frac{\partial^2 I}{\partial y_2 \partial x_2} \end{bmatrix}$$

Note that all the partial derivatives of the Expression /1/ are calculated at points $\underline{Y}_o$, $\underline{X}_o$, $\underline{P}_o$, hence, they are constant in time interval from t_o to t_f.

If we assume that the variation $\underline{\Delta P}$ is small, the column is considered to be a linear unit as far as the dynamics is concerned, hence

$$\underline{\Delta Y} = L_1 \underline{\Delta X} + L_2 \underline{\Delta P}$$

Denote

$$\underline{\Delta Y}_1 = L_1 \underline{\Delta X} \qquad \text{and} \qquad \Delta Y_2 = L_2 \underline{\Delta P}$$

In these expressions $\underline{\Delta Y}$, $\underline{\Delta Y}_1$, $\underline{\Delta Y}_2$, $\underline{\Delta X}$ and $\underline{\Delta P}$ are vector functions of time, L_1, L_2 - lineare operators.

It can be proved that

$$\int_{t_o}^{t_f} \left[\left(\frac{\partial I}{\partial \underline{Y}}\right)^T \Delta \underline{Y}_1 + \left(\frac{\partial I}{\partial \underline{X}}\right)^T \Delta \underline{X} \right] dt = 0$$

As a matter of fact, $\frac{\partial I}{\partial \underline{Y}}$ and $\frac{\partial I}{\partial \underline{X}}$ are independent on $\Delta \underline{P}$ and calculated for the points $\underline{Y}_o$, $\underline{X}_o$, $\underline{P}_o$. Thus, it can be assumed: $\Delta \underline{P} = 0$ and then

$$J = \int_{t_o}^{t_f} \left\{ I_o \left(\underline{Y}_o, \underline{X}_o, \underline{P}_o \right) + \left[\left(\frac{\partial I}{\partial \underline{Y}}\right)^T L_1 + \left(\frac{\partial I}{\partial \underline{X}}\right)^T \right] \Delta \underline{X} \right\} dt$$

Since $\Delta \underline{P} = 0$, the maximal value of the performance index J is equal to J_o and it corresponds to the optimal operation of the system:

$$J_o = \int_{t_o}^{t_f} I_o \left(\underline{Y}_o, \underline{X}_o, \underline{P}_o \right) dt$$

Hence

$$\int_{t_o}^{t_f} \left[\left(\frac{\partial I}{\partial \underline{Y}}\right)^T L_1 + \left(\frac{\partial I}{\partial \underline{X}}\right)^T \right] \Delta \underline{X} \; dt \leqslant 0 .$$

This condition should be satisfied for arbitrary $\Delta \underline{X}$.

Hence

$$\int_{t_o}^{t_f} \left[\left(\frac{\partial I}{\partial \underline{Y}} \right)^T L_1 + \left(\frac{\partial I}{\partial \underline{X}} \right)^T \right] d\underline{X}\, dt = 0$$

In such a case, for the neighbourhood of the optimal operating point $\underline{P}_o$ /corresponding to $\underline{Y}_o$ and $\underline{X}_o$/ we can write

$$J = J_o + \underline{J}_1^T \Delta\underline{P} + \Delta\underline{P}^T [J_2] \Delta\underline{P}$$

where $\underline{J}_1 = \frac{\partial I}{\partial \underline{Y}} L_2 + \frac{\partial I}{\partial \underline{P}}$ - is independent on $\Delta\underline{X}(t)$.

$[J_2]$ is a matrix, some coefficients of which are functions of $\Delta\underline{X}(t)$ or they depend on the adjustable parameters $\underline{T}$. Thus, in any region for which the control does not depend on the direction of the vector $\Delta\underline{P}$, as an auxiliary criterion the following expression can be accepted

$$C = \frac{J - J_o - \underline{J}_1^T \Delta\underline{P}}{\| \Delta\underline{P} \|^2} = \frac{\underline{P}^T J_2 \underline{P}}{\| \Delta\underline{P} \|^2}$$

Such regions can be found and $\underline{J}_1$ calculated by comparing the values of J for $\underline{T}$ = const, for different $\Delta\underline{P}$. As a next step $\underline{T}$ can be adjusted so as to maximize C. It is evident, that examples can be found for which determination of such regions is practically impossible. However, it is not so in the case of the distillation column and the considered criterion. In any case, the criterion J can always be employed, as a function of $\Delta\underline{P}$. However,

the criterion C may not depend on $\underline{\Delta P}$ in the region for which our hypotheses are valid and this fact makes possible comparatively fast learning in dynamic states.

IV. Results of Experiments

The control of the pilot distilation column at the Control Laboratory in Grenoble has not been realized so far because for the last couple of months this column is not accessible. We are presenting here merely results obtained with a mathematical model /ALGOL language, IBM 7044 computer/. The results obtained from the column will be delivered at the Congress.

We will not dwell upon learning performed on the model in static states. Such learning does not create any difficulties and, as we have already mentioned, it requires comparatively small storage capacity. Therefore, we only present the results refering to the learning in dynamic states assuming that the first step has been done. In order to reduce the computing time as compared to the time required for a model of the complete column a simplified dynamical model is employed, Fig.2, which is valid for the neighbourhood of an operating point. The time constants are expressed in minutes and they represent one thenth part of their real values.

In this model X_d follows X_o owing to the changes of the reflux flow, supplied by a predictive control L_R t determined by learning which commences with L^*_{RS}. The controls arrive at the system through a "transmittance generator" which is a unit having the transfer function /chosen for its simplicity/ of the form

$$\frac{1}{1 + T_5 p}$$

In this function the constraints for $L_R(t)$ are included. $Q_B(t)$ is also obtained by learning starting from Q^*_{BS} and through a "transfer function generator" the transmittance of which is

$$\frac{1}{1 + T_6 p}$$

In this term the constraints for $Q_B(t)$ are included.

The flow of the input product L_F is the measurable disturbance acting on the system. We simply mention here the final results which are effected by disturbances

$$L_F = 20 + 2\,u(t) \qquad \text{where} \quad u(t) = 0 \quad \text{if } t < t_o$$
$$u(t) = 1 \quad \text{if } t > t_o$$

The curves of equal value of the index C /equicritere/ in the plane /T_5, T_6/ are shown in Fig.3. It can be seen that in the neighbourhood of the peak, the slope is very large for decreasing values of T_6. This follows from the form of the expression for the profit which gives a large loss if $X_d - X_o < -0{,}004$. In Figs.1 through 7 the quasi optimal relations for $Q_B(t)$, $L_{RT}(t)$, $(X_d - X_o)(t)$ and $I(t)$ are shown.

Analogous results have been obtained for other operating points and for other amplitudes of disturbances L_F.

V. Conclusion

The experimental results obtained with the model of the pilot distillation column and with other models show that the propcsed method gives satisfactory results if the criterion and the process are compatible. Since this method requires little knowledge about the process /it is sufficient to know and be able to measure controls, disturbances and the goal of the control/, learning time is relatively long especially for slow processes. For a complete control of the investigated pilot rectifying column it should be assumed that the learning time is 1000 times greater than the time of response to a jump. Note, however, that this is not necesserily an important limitation because the process goes on all the time. None the less there exists a possibility of reducing the learning time by an increase of the initial information. It can be assumed, for example, that the optimal static states are known /to determine them a static model is sufficient/. This method can also be applied if we know a dynamical model of the column. Learning /or at least its initial phase/ can be carried out rapidly with such a model. This way a dynamic quasi-optimal control can be found out without necessity to measure the state coordinates of the column or to solve Pontriagin's Equations or the dynamic programming equations.

I would like to express here my thanks to Mr A.W. Naylor, Professor of the Michigan University, for numerous and fruitfull discussions and for clarifying hints which I have received from him.

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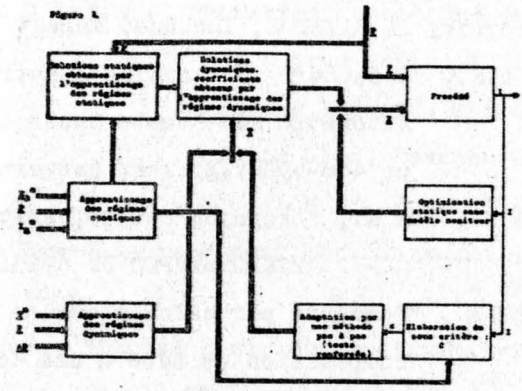

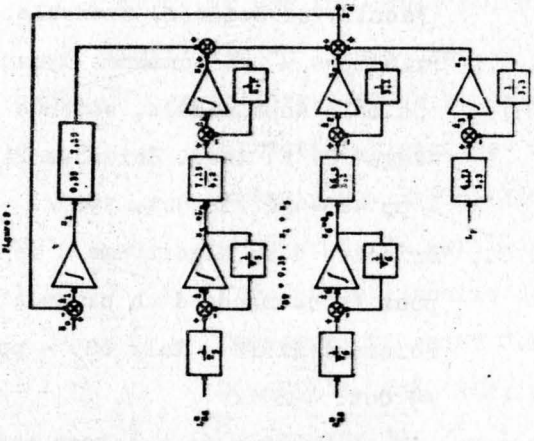

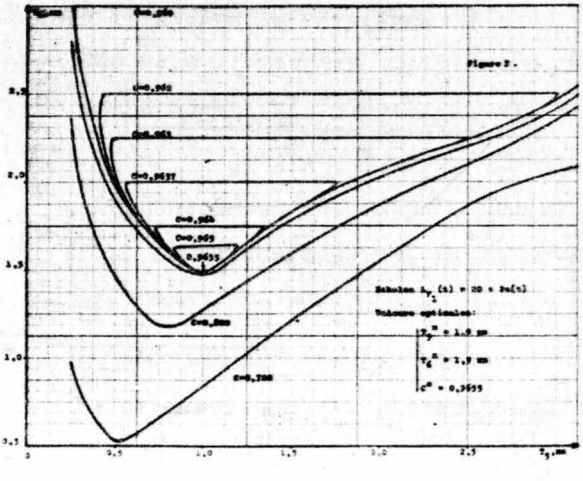

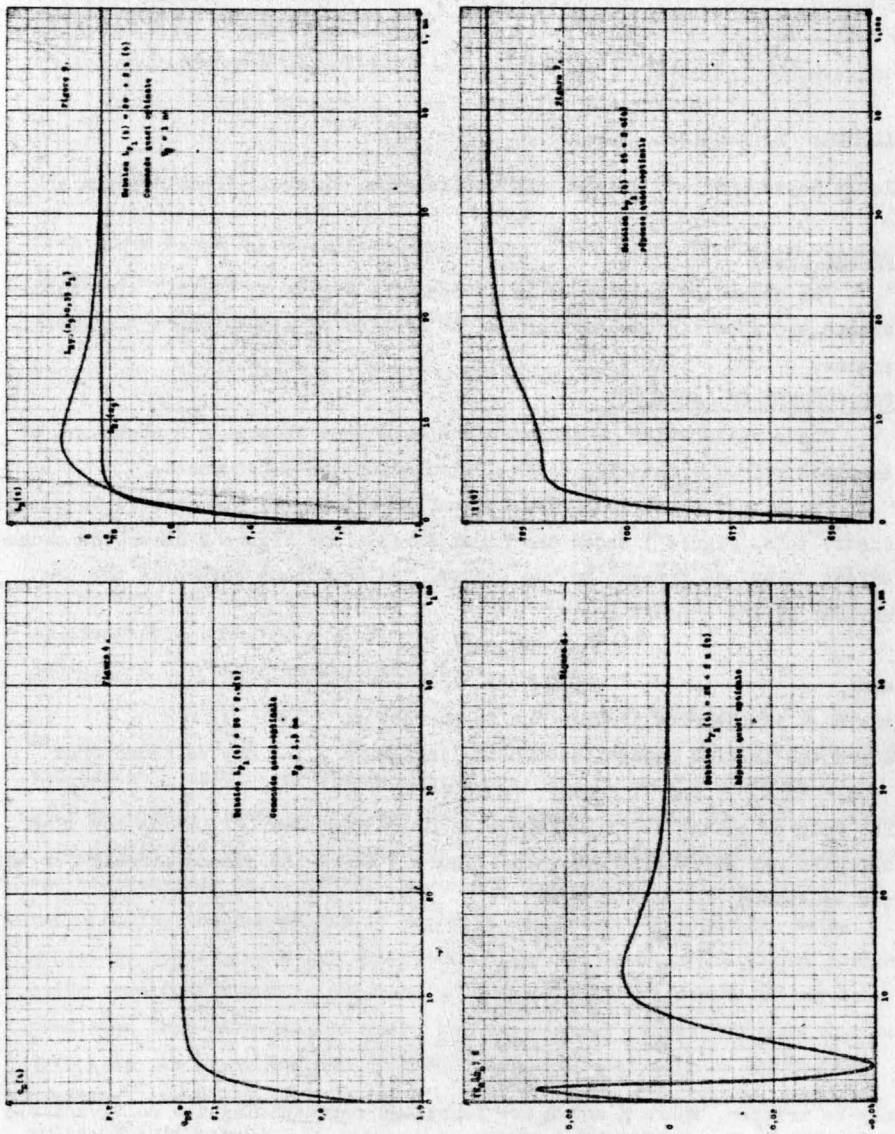

A NONLINEAR DIGITAL SIMULATION METHOD APPLIED TO GAS TURBINE DYNAMICS

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1. INTRODUCTION

The object of this paper is to describe a general digital simulation method and show its application on the problem of simulating a gas turbine engine.

2. THE PROBLEM OF CAUSALITY

The consideration of cause/effect relations through a system must be examined before a detailed digital simulation can be produced.

Two simple electrical circuit configurations are illustrated to demonstrate this. Figure 1 shows the first circuit and Figure 2 shows the cause/effect relations present in the system. The equations governing the behaviour of the circuit are:

$$V_L + i_R = 0 \qquad 1.$$

and

$$d\lambda/dt = V_L \qquad 2.$$

where λ is the flux linkage due to current i.

Given the initial condition of flux linkage, λ, i_L may be found from $i_L = f(\lambda)$ and V_L from $V_L = -R_L$

The rate of change of flux linkage is found from equ.(2), hence the time solution may be readily computed. Figure 3 shows the second circuit for which the equations

$$V_1 = -i_1 R_1 \qquad 3$$

$$V_2 = -i_2 R_2 \qquad 4$$

$$V_1 = d\lambda_1/dt \qquad 5$$

$$V_2 = d\lambda_2/dt \qquad 6$$

$$\lambda_1 = f_1(i_1, i_2) \qquad 7$$

$$\lambda_2 = f_2(i_1, i_2) \qquad 8$$

may be written, where f_1 and f_2 are functions representing the multivariable components. Figure 5 shows a possible representation of equations (7 & 8). From equation (3) - (8), the cause/effect diagram may be drawn, Figure 4. Again given the initial condition of flux linkage, the problem may be solved graphically, as shown below.

The initial conditions of λ_1 and λ_2 are known.

Draw a graph of i_1 against i_2 at $\lambda_1 = \lambda_2$ and superimpose a second plot of i_1 against i_2 with $\lambda = \lambda_2$ as in Figure 6. The point at which the two curves intersect gives the values of i_1 and i_2 which satisfy equations (7) and (8).

The time solution is then found as for Figure 1. If the problem is tackled analytically, an iterative technique, such as the Newton-Raphson method of successive approximation must be employed. This involves the computation of the Jacobean matrix of partial derivation for the system, ie.

$$\underset{\sim}{A} = \begin{bmatrix} \frac{\partial \lambda_1}{\partial i_1} & \frac{\partial \lambda_2}{\partial i_2} \\ \frac{\partial \lambda_1}{\partial i_2} & \frac{\partial \lambda_2}{\partial i_2} \end{bmatrix} \qquad (9)$$

Then given λ_1 and λ_2 as initial conditions i_1^* and i_2^* as first approximations to the solution of equations (7) and (8), $\Delta\lambda_1$ and $\Delta\lambda_2$ may be calculated by evaluating equations (7) and (8) for i_1^* and i_2^*, where $\Delta\lambda_1 = (\lambda_{1_{GIVEN}} - \lambda_{2_{CALC}})$ and $\Delta\lambda_2 = (\lambda_{1_{GIVEN}} - \lambda_{2_{CALC}})$

Second approximations for i_1 and i_2 are found from

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_1^* \\ i_2^* \end{bmatrix} - K \begin{bmatrix} \underset{\sim}{A}^{-1} \end{bmatrix} \begin{bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \end{bmatrix} \qquad (10)$$

The procedure is then repeated until a specified accuracy is reached.

At this point it becomes clear where a digital system simulation will differ from the more conventional analogue simulation for systems having multivariable non-linear elements.

3. NON-LINEAR COMPONENT REPRESENTATION

The representation of multivariable non-linear components for an analogue simulation involves the development and construction of complicated, and probably expensive, items of hardware; it is also the most difficult part of a simulation to produce. When a digital computer is used to effect the simulation of such components, full use of the machine's flexibility and accuracy may be employed. The method employed for this work was that of representing the components by analytic equations, the equation being evaluated to determine the behaviour of the component. This is, of course, quite straightforward in the case of components where behaviour may be predicted from mathematical laws, but immediately raises problems when no such laws are known and the component behaviour may only be known from practical observations. The problem was overcome by the development of a multivariable curvilinear regression analysis programme. Use of this programme enables the user to enter practical observations of a non-linear multivariable component and receive as output a regression equation which may be used to represent the component in question. The output also includes statistical information which may be used to assess the accuracy and validity of the resulting equation. The form of the equation will be as follows:

$$y = A_0 + \sum_{i=1}^{n} A_i Z_i$$

where the Z_i are each unique functions of the m input independent variables. Each Z will consist of the product of p factors ($1 \leq p \leq m$), where each factor is a simple function of one of the input independent variables, viz. x^n, x^{-n}, $x^{1/n}$, $x^{-1/n}$ where n is a positive integer.

Reduction of the practical observations to an equation of the above form fulfills two objectives, the first that evaluation of the equation will predict its behaviour for a given set of input variables, and secondly because of the mathematical nature of the equation it may readily be differentiated to produce all or part of the Jacobean matrix described in Section 1, enabling a technique such as the Newton-Raphson method to be applied.

4. DERIVATION OF A GENERAL SIMULATION METHOD

The method takes the form of a set of rules which, when applied to a system satisfying the following conditions, will result in a simulation of that system.

3.1. System Specification

(a) A system showing no direct solution for the instantaneous values of the state vector of the system, (discussed in Section 1).

(b) A system having non-linear multivariable components which behave in a quasi static manner, i.e. all such components are of zero memory type.

(c) The dynamical behaviour of the system may be predicted from a set of first order differential equations, as distinct from differential-difference equations.

(d) For efficient simulation, all the system time constants should be of the same order of magnitude.

3.2. System Simulation

To construct a model of the system, it is first necessary to choose the state variables of the system. These will be the minimum number variables which, when observed, will enable the future system behaviour to be accurately predicted.

Given the values of the state variables, or vector, it follows that the complete state of the system may be calculated. An equal number of points in the system are then chosen, at which tests may be made to determine the compatibility of the values of the state vector.

This technique may best be shown as an example using the electrical circuit of Figure 3.

The equations of the system may be written down by inspection, equations (3) to (8).

If the united conditions of λ_1 and λ_2 are to be satisfied, the

the two functions f_1 and f_2 may be used to test the compatibility of i_1 and i_2.

The degree of compatibility may be expressed as

$$F[1] = \lambda_{1c} - f_1(i_1, i_2) \tag{11}$$

and

$$F[2] = \lambda_{2c} - f_2(i_1, i_2) \tag{12}$$

To solve the equations and hence produce a model of the system, it is possible to use the Newton-Raphson method as shown below. The partial derivatives of each of the error functions with respect to each of the state variables must first be calculated.

i.e. $\dfrac{\partial F[i]}{\partial X[j]}$ j FROM 1 TO n, i FROM 1 TO n

where n is the number of state variables chosen.

The partial derivatives may then be used to calculate the required change in the state vector to reduce the values of the error functions.

F(1) - F(n)

$$\begin{bmatrix} \Delta X[1] \\ \vdots \\ \Delta X[n] \end{bmatrix} = \begin{bmatrix} \underset{\sim}{A}^{-1} \end{bmatrix} \begin{bmatrix} F[1] \\ \vdots \\ F[n] \end{bmatrix} \tag{13}$$

where A consists of elements

and A^{-1} IS THE INVERSE OF A $\quad a_{ij} = \dfrac{\partial F[i]}{\partial X[j]}$ j FROM 1 TO n, i FROM 1 TO n

The calculation of the partial derivatives a(i,j) is facilitated if the expressions for F(i) may be directly differentiated. (If it is not possible to obtain analytic expressions from mathematical considerations, then if data is obtainable, it is possible to obtain analytic expressions using a programme such as a multivariable curvilinear stepwise regression programme). Successive corrections are made to the values of the state variables until the permitted tolerance is reached. When this has been achieved, the dynamical behaviour may be predicted from the system equations directly, and the procedure repeated. The example of the electrical network is shown completed to illustrate the above

λ_1 and λ_2 are given as initial conditions

i_1 and i_2 are found to satisfy equations (7) and (8)

Then $V_1 = -i_1 R_1$ from equation (3)

and $V_2 = -i_2 R_2$ from equation (4)

$V_1 = d\lambda_1/dt$ from equation (5)

or $V_1 = -i_1 R_1 = \dfrac{\Delta \lambda_1}{\Delta t}$ approximately,

provided that Δt is chosen small enough so that $\dfrac{\Delta \lambda_1}{\Delta t} = \dfrac{d\lambda_1}{dt}$ to a sufficient degree of approximation determined by the problem.

Then $\Delta \lambda_1 = -\Delta t \cdot i_1 \cdot R_1$

and $\Delta \lambda_2 = -\Delta t \cdot i_2 \cdot R_2$

Having calculated $\Delta\lambda_1$ and $\Delta\lambda_2$ the problem becomes the original problem of knowing the system equations and requiring the values of i_1 and i_2 to satisfy the initial conditions of ($\lambda_1 + \Delta\lambda_1$) and ($\lambda_2 + \Delta\lambda_2$).

5. APPLICATION OF THE GENERAL METHOD TO THE SIMULATION OF A SINGLE SHAFT GAS TURBINE ENGINE

Consider a single shaft engine, as shown in block diagram from Figure 7. For a given set of ambient conditions, variables, P_1, T_1, and P_{ac} and a given value of Fe, the forcing variable, a unique steady running point for the engin is defined. If an initial value of N is given, not the final steady value, the the engine will assume the final value of N in a uniquely defined manner. Furthermore, the knowledge of the intake mass flow, pressure ratios accross th compressor and turbine completely specifies the state of engine.

Since it is not possible to calculate directly the values of the three variables, Qo, Prc and Prt, which will be termed the state variables of the engine, it is necessary to derive a method of employing iteration.

To solve for the three unknowns, three independent equations are needed. These take the form of three relationships.

1. Compressor Characteristic
2. Turbine Characteristic
3. Nozzle Calculation

The compressor and turbine characteristics were obtained from test data, and the Multivariable Curvilinear Regression Analysis Programme was used to derive the following expressions $N/\sqrt{T_1} = f_1(Q\sqrt{T_1}/P_1 , P_{RC})$ (14.)

and $Q\sqrt{T_3}/P_3 = f_2(N/\sqrt{T_3} , P_{RT})$ (15)

The nozzle calculation is effectively $Q_5 = f_3(P_4, T_4, P_e , A_T C_D)$ (16)

Each of the above three expressions may be written in a form to express an error, namely:

(predicted value) - (actual value) = error

Therefore:

Equation (14) becomes
$$f_1(Q\sqrt{T_1}/P_1 , P_{RC}) - (N/\sqrt{T_1}) = E[1] \quad (17)$$

Equation (15) becomes
$$f_2(N/\sqrt{T_3} , P_{RT}) - (Q_3\sqrt{T_3}/P_3) = E[2] \quad (18)$$

Equation (16) becomes
$$f_3(P_4, T_4, P_e , A_T C_D) - Q_5 = E[3] \quad (19)$$

Each of equations (17) - (19) are affected by variations in one or more of the three state variables in the following manner.

Then
$$\begin{bmatrix} \Delta E(1) \\ \Delta E(2) \\ \Delta E(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} \Delta x(1) \\ \Delta x(2) \\ \Delta x(3) \end{bmatrix}$$

where $\Delta E(1)$ denotes the variable in E(1) etc., and each 1 of the above matrix infers dependence.

It is, therefore, possible to solve the three equations employing a technique such as the Newton-Raphson method, provided it is possible to calculate the appropriate partial derivatives.

4.1. Engine Calculation and Partial Derivative Evaluation

Consider the engine in component form.

(a) The air intake. The function of this component is to change the kinetic energy of the oncoming air into internal energy prior to the entry into the engine.

Given the aircraft speed and the ambient conditions Va, P_{∞} and T_{∞} one can calculate the compressor intake pressure and temperature P_1 and T_1 for a given intake efficiency.

(b) The engine compressor. The performance of an axial flow compressor may be expressed by means of two characteristics, the first being a plot of non-dimensional mass flow, compressor pressure ratio and non-dimensional rotational speed; the second characteristic is a plot of adiabatic efficiency, pressure ratio and non-dimensional rotational speed. Examples of this form of characteristic are shown in Figures (8) and (9).

(c) The combustion chamber. Representation of this component is usually effected by a simple combustion calculation, equating the total heat before combustion to the total heat after combustion.

(d) The turbine. The high pressure, high temperature gas flow from the combustion chamber is fed through the turbine and partially expanded. The energy obtained from the expansion is converted into mechanical energy by the turbine and is used to power the compressor.

(e) The final nozzle. The gasses exhausting from the turbine are used to produce the engine thrust by expanding the gas through a nozzle, converting the internal energy of the gas to kinetic energy. The change of momentum during this process gives rise to the thrust.

4.2. Computer Implementation

Subroutines are written which facilitate the engine thermodynamic calculation. The subroutines are all based on polynomials for the specific heat at constant volume Kva and for stoichometric products of standard fuel Kvs. The polynomials were obtained by fitting basic data using the chebyshev method, and may also be used to obtain the specific heats at constant pressure, Kpa and Kps.

Subroutine 1.

Calculates the entholpy change ΔG Hp. sec/lb. given the fuel/air ratio and the initial and final temperatures.

Subroutine 5.

Calculates for an insentropic process the final temperature t_2° k given fuel/air ratio, the initial temperature t_1° k and the pressure ratio P_1/P_2.

Iteration is required.

Further procedures employing the above subroutines are used to perform basic engine calculations.

1. The combustion process

Given the entry conditions to the combustion chambers, the fuel flow Fe, the combustion efficiency η_{coms}, the procedure calculates the state of the gas at exit from the combustion process. The principle of total heat is used, ie. total heat at entry + heat addition = total heat at exit.

2. Mixing Calculation

A small amount of air is bled at entry to the combustion chamber for the purpose of cooling the turbine disc. Although this air actually enters the gas stream prior to the turbine expansion, for the purpose of turbine calculation it may be assumed that the air is not truly mixed until after the turbine expansion. The procedure calculates the effect of this mixing on the state of the air at the turbine exit plane.

3. Final Nozzle Calculation

The procedure will calculate the mass flow through a convergent nozzle, given the nozzle area, the entry conditions of the gas stream and the ambient pressure P . A check is made to test if the nozzle is operating in a choked or unchoked mode, and the exhaust pressure and velocity is calculated. The ideal gross thrust is calculated from

$$Q_2 = A_J \cdot P_2 \cdot V_2 / 550 \cdot R \cdot t_2$$

and $$X_9 = Q_2 V_2 / g + (P_2 - P_\infty) \cdot A_J$$

The computer algorithm to calculate the engine state may now be described.

Given -

The engine intake conditions, P_1 and T_1

The exhaust ambient pressure condition P_∞

The fuel flow Fe

The final nozzle area A_j

The air mass flow ratio of turbine cooling bleed to intake mass flow

The combustion efficiency η_{COMB}

The compressor and turbine characteristics in equation form.

Initial estimates of the values of the state variables must be made.

The procedure is as follows:

1. Evaluate the compressor and turbine efficiencies.
2. $P_2 = P_1 \times X(2)$
3. $T_2 = T_1 + (SR5\ (T_1, X\ (2)) - T_1)$ / eff. compressor.
4. Deduct the bleed mass flow.
5. Procedure call to evaluate the combustion process.
6. $P_3 = P_2 \times 0.931$ (combustion pressure loss factor)
7. $P_4 = P_3 / X(3)$

8. $T_4 = T_3 - (T_3 - SR6\ (q3, T_3, X(3))$ x eff. turbine.
9. Calculate turbine work.

   $WT = SR1\ (q3, T_4, T_3)$ x mass flow.
10. Procedure call to evaluate the effect of mixing the turbine cooling bleed.
11. Procedure call to evaluate the mass flow through the nozzle and thrust.
12. Set up the error functions E(1) - E(3) and test if they lie within tolerance. If they do, jump to step 17.
13. Evaluate system partial derivations.
14. Solve for $\Delta X[1] - \Delta X[3]$.
15. Correct X(1) - X(3).
16. Return to step 1.
17. Calculate difference between compressor and turbine powers and given the time increment, compute the new engine speed.
18. Return to step 1.

6. EXPERIMENTAL RESULTS

The engine was represented by regression equations giving the pressure ratio, mass flow, engine speed characteristics of the compressor, the adiabatic efficiency of the compressor, and the turbine mass flow characteristics. Constant turbine efficiency was assumed.

Figure 8 shows how the original data was represented by the regression equation, and Figure 9 shows the associated curves of adiabatic efficiency.

Representation of the turbine was obtained in the same way, the source of the data being a turbine test report by Bristol Siddeley Engines Limited.

Compressor 'Blow Off', a device whereby the first stages of the compressor are allowed to pass more air than the engine requires, the excess being bled off to atmosphere midway through the compressor, was not represented. The object of this device is to shift the steady running line away from the surge line at lower engine speeds, thus allowing more rapid acceleration from the idling condition. The nominal engine speed above which the 'Blow Off' valve is closed is 75% N_D, which coincides with the lowest engine speed considered.

Two steady running lines corresponding to nozzle areas of 141 square inches and 137 square inches, are also shown in Figure 8. Figures (10) and (11) give a further breakdown of steady state performance and comparison is made between the simulation results and a standard production engine.

Figure 13 shows examples of simulated transients drawn as trajectories on the compressor characteristics. Figure 13 shows logarithmic plots of engine speed for large steps in fuel flow, and Figure 14 shows the variation in local time constant for various engine speeds, summarised in Table One.

7. CONCLUSIONS

The main result of the investigation is to show that a reasonably

accurate simulation of a gas turbine engine can be produced from a knowledge of the component characteristic, employing a general simulation method. Further accuracy may be obtained by a more detailed engine representation and calculation, and the production of a two speed engine simulation may be carried out using the same method.

The method is particularly suited to systems having multivariable non-linearities and operating over a wide range. Use may be made of the method to predict the final behaviour of a system from a knowledge of the behaviour of its component parts in the design and production of the system, or in the design of its controlling system.

The simulation accuracy may be described as follows:
The accuracy is firstly determined by the accuracy with which the separate components are represented, and secondly an assessment must be made of the change in component characteristics due to the linking of the components together.

Simulation of a complete powerplant is also possible. Regression analysis may be used to include the effect of variable geometry in the engine compressor, or in the intake, the dynamical behaviour of fuel system may be included, or even the entire control system, each part of the simulation being produced by following the basic simulation method.

8. REFERENCE

" A General Nonlimear Digital Simulation Method Applied To Gas Turbine Dynamics",
C. Dennison, Ph.D.Thesis, London University - 1968.

FIGURE 1

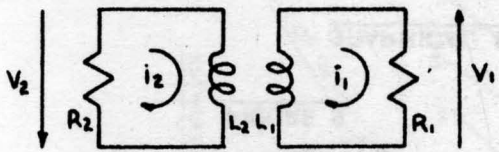


FIGURE3

ELECTRICAL CIRCUIT

EXAMPLES

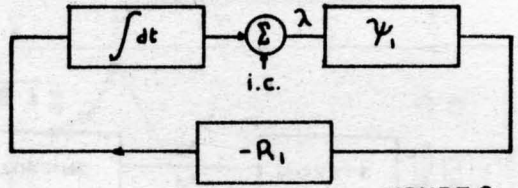


FIGURE 2

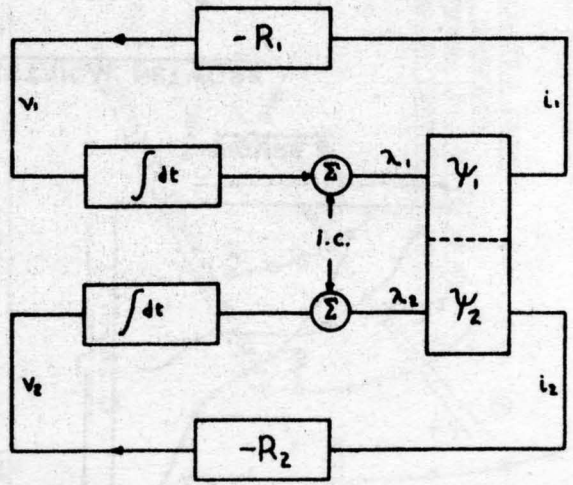


FIGURE 4

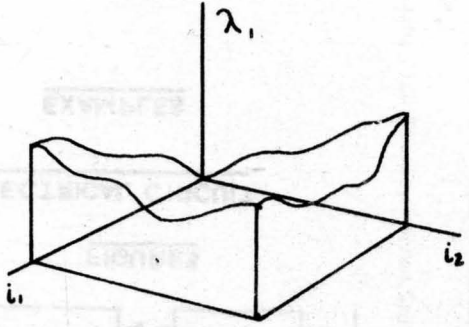


FIGURE 5

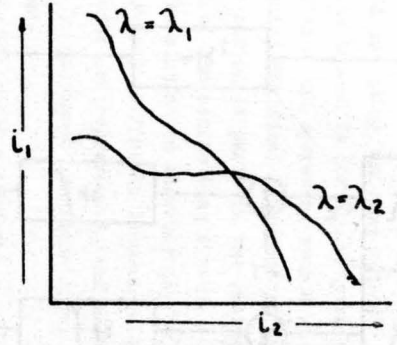


FIGURE 6

GRAPHICAL SOLUTION TO ELECTRICAL NETWORK

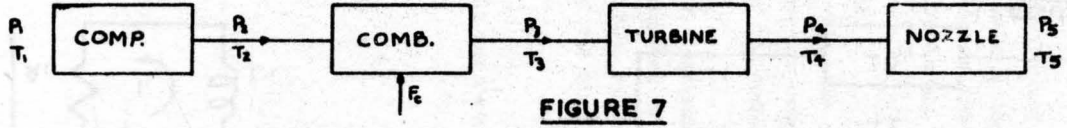


FIGURE 7

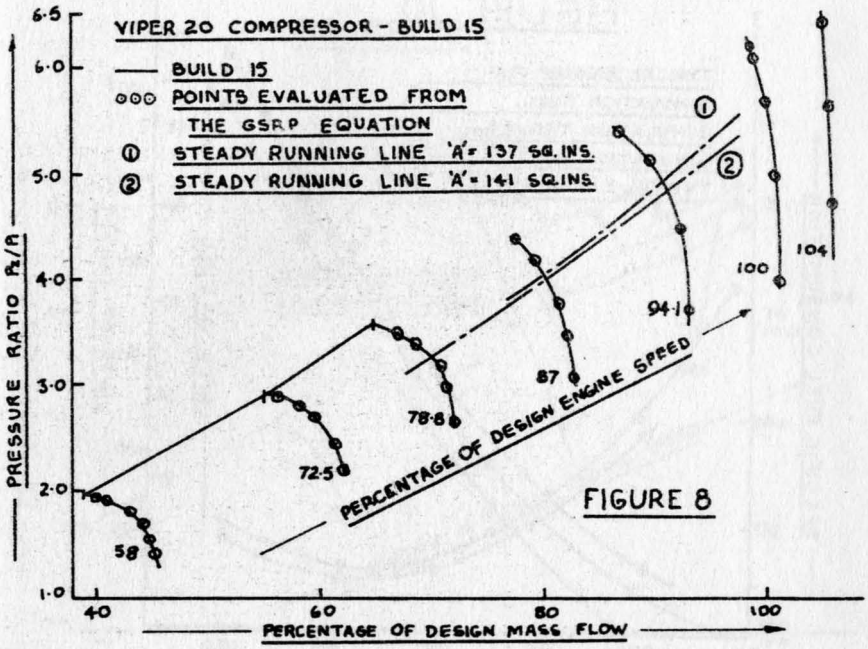


FIGURE 8

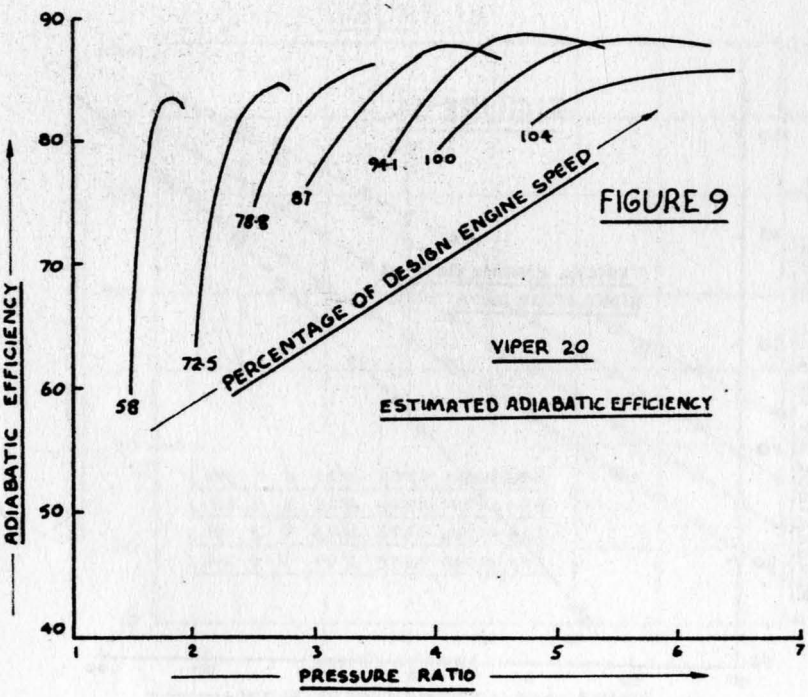


FIGURE 9

FIGURE 10

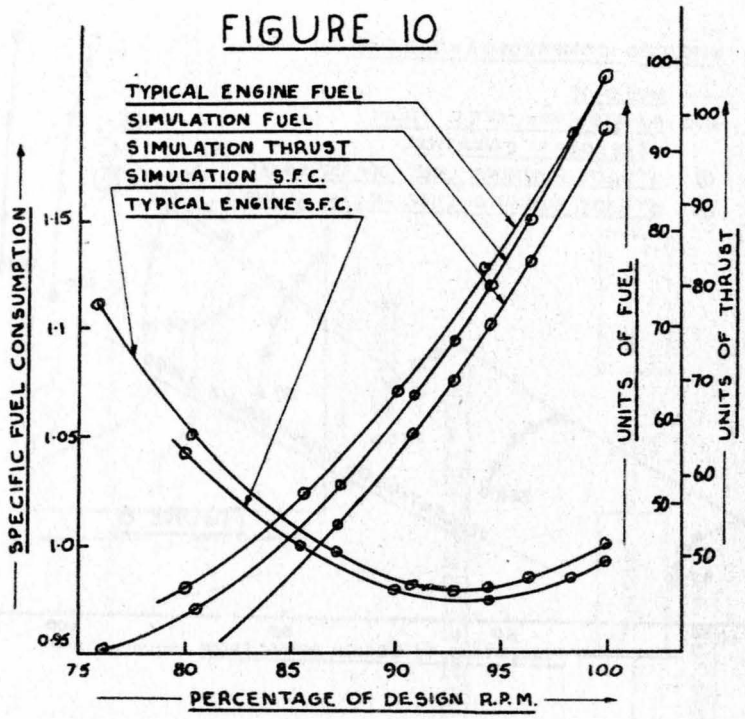


FIGURE 11

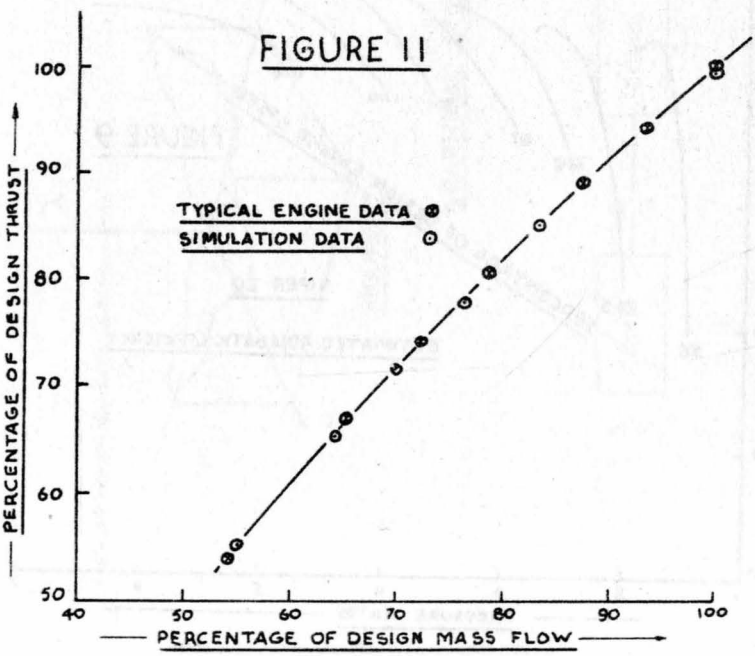


FIGURE 12

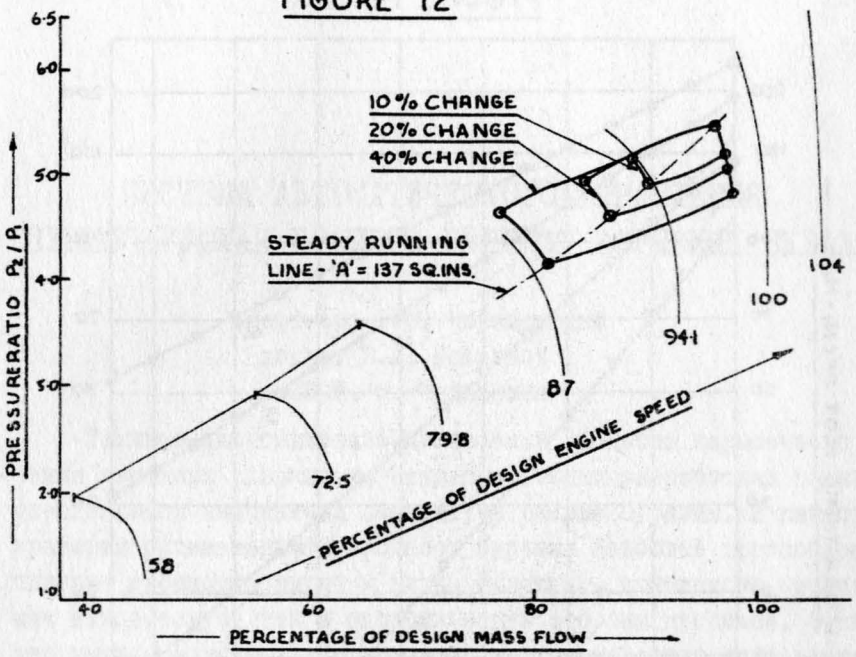
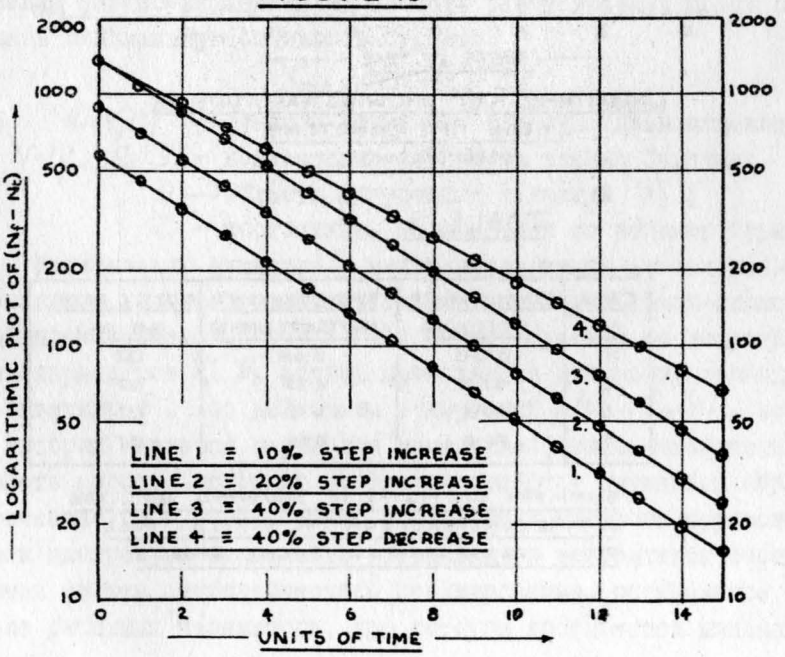
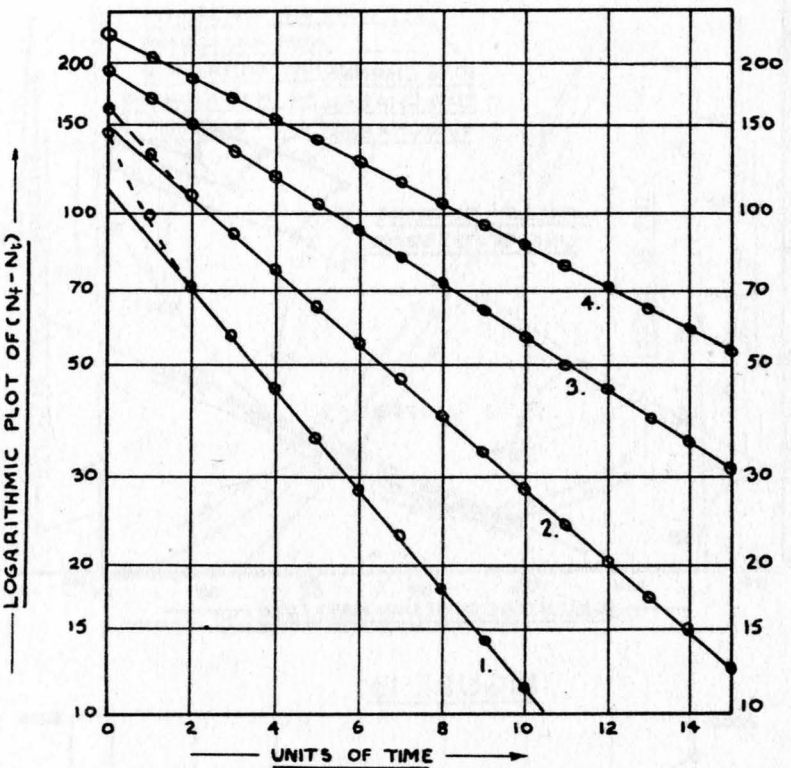


FIGURE 13



PLOT OF LOG (N_f - N_t) V. TIME

FIGURE 14



LOGARITHMIC PLOT SHOWING VARIATION IN LOCAL TIME CONSTANT (SEE BELOW)

TABLE I.

CASE Nº	FINAL ENGINE SPEED(%)	TIME CONSTANT (ARBITRARY UNITS)	NOZZLE AREA (SQ. INS.)
1	100·0	3·68	137
2	87·2	4·32	137
3	80·4	6·92	137
4	76·0	8·83	137

SUMMARY OF FIGURE 14 SHOWING VARIATION IN TIME CONSTANT FOR 5% STEPS IN FUEL FLOW FOR VARIOUS FINAL ENGINE SPEEDS.

СИСТЕМА АВТОМАТИЧЕСКОГО УПРАВЛЕНИЯ, ОПТИМИЗИРУЮЩАЯ ПРОЦЕСС БУРЕНИЯ ВЗРЫВНЫХ СКВАЖИН

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Технико-экономические показатели процесса шарошечного бурения взрывных скважин на открытых горных разработках полностью определяются значениями параметров режима бурения. В качестве критерия оптимальности процесса бурения наиболее целесообразно принять стоимость единицы длины скважины, комплексно отражающую как техническую, так и экономическую стороны процесса. Очевидно, что в этом случае задача системы автоматического управления (САУ) процессом состоит в том, чтобы в условиях случайного изменения физико-механических свойств разбуриваемых пород обеспечивать минимальную стоимость бурения

$$S_{min} = \min \Phi(Y, C),$$

где $\Phi(Y,C)$ - функция стоимости единицы длины скважины;
$Y=(y_1,\dots,y_n); y_i$ - независимые параметры режима бурения;
D - область допустимых значений Y ;
C - постоянные, независящие от режимов бурения.

Минимизацию функции S может осуществить многоканальная экстремальная система автоматического управления, блок-схема которой представлена на рис.I. В процессе бурения по текущим значениям параметров $y_1 \dots y_n$ режима вычисляется стоимость единицы длины скважины S . Под действием возмущений $f_1(t) \dots f_m(t)$, основным из которых является случайное изменение физико-механических свойств пород, экстремум стоимости дрейфует случайным образом. Логический блок формирования управляющих сигналов осуществляет поиск экстремума стоимости, устанавливая посредством соответствующих систем автоматического регулирования, оптимальное соотношение режимных параметров, при котором достигается минимальная

стоимость бурения во встретившейся породе. При этом должны учитываться ограничения H_i , накладываемые на область допустимых изменений параметров Y , определяемые конструктивными особенностями бурового оборудования или технологическими требованиями.

Из практических соображений целесообразно регулировать лишь основные параметры $y_1, \ldots, y_k$, влияние которых на стоимость наиболее существенно.

Трудность технической реализации критерия минимума стоимости единицы скважины обуславливается тем, что стоимость бурения может быть определена лишь по результатам законченного бурения, в то время как режимные параметры должны определяться и устанавливаться непосредственно в ходе процесса и причем так, чтобы по окончании бурения получить минимальную его стоимость.

Указанное обстоятельство вызывается тем, что непосредственная реализация критерия минимальной стоимости бурения предполагает использование оперативной информации о текущем износе бурового инструмента, что не представляется возможным, поскольку устройства, способные выдать эту информацию, в настоящее время отсутствуют.

Учитывая вышесказанное, целесообразно заранее определить оптимальные значения режимных параметров (по критерию минимальной стоимости) для различных физико-механических свойств пород. Для достижения минимальной стоимости бурения основные параметры режима: осевое усилие на забое F и скорость вращения бурового инструмента n должны изменяться [I] в зависимости от крепости разбуриваемой породы f , как показано на рис.2. При использовании этих зависимостей достигается наиболее простая структура САУ процессом бурения (рис.3). Указанная САУ является разомкнутой самонастраивающейся системой с нелинейными компенсирующими связями по основному возмущению [2,3,4]. Точность достижения минимального значения стоимости, очевидно, будет определяться достоверностью априорной информации, используемой для настройки функциональных преобразователей, и точностью оценки физико-механических свойств разбуриваемых пород.

Существующие способы определения физико-механических свойств горных пород основаны на разовых лабораторных испытаниях образцов и, очевидно, для целей автоматического управления процессом не пригодны, поскольку оценка должна производить-

ся непрерывно и непосредственно на рабочей машине, осуществляющей бурение. Оценка физико-механических свойств горных пород, удовлетворяющая указанным выше требованиям, может быть осуществлена следующим образом.

На рис.4 представлены обобщенные зависимости механической скорости бурения v от величины осевого усилия на забое F при механическом бурении пород различной крепости $f(f_1<f_2<f_3)$. На этот же рисунок нанесем внешнюю характеристику системы подачи инструмента на забой. Пусть эта характеристика такова, что имеет место кривая I. Точки пересечения внешней характеристики с зависимостями $v=f(F)$ характеризует режим бурения в породе с определенными физико-механическими свойствами. Очевидно, что в породах различной крепости будет устанавливаться различное осевое усилие на забое и различная механическая скорость бурения. При крепости f_1 - усилие F_{y_1} и скорость v_{y_1}, при f_2 - F_{y_2} и v_{y_2} и т.д.

Таким образом, можно сделать вывод, что величины устанавливающегося осевого усилия F_y или устанавливающейся механической скорости бурения v_y определяют сопротивляемость горной породы разрушению при бурении и могут быть использованы в качестве оценок физико-механических свойств буримых пород. Как устанавливающееся осевое усилие на забое, так и устанавливающаяся скорость бурения являются интегральными оценками физико-механических свойств пород и комплексно отражают такие свойства пород как твердость, хрупкопластические свойства, трещиноватость и др. Выбор конкретного показателя (v_y или F_y) при реализации САУ процессом бурения будет определяться дополнительными соображениями, такими как простота и надежность измерения показателя, вид конкретной внешней характеристики системы подачи станка и т.д. Если внешнюю характеристику системы настроить соответствующим образом (например, подбором сопротивления в гидросистеме станка и настройкой редукционного клапана), то устанавливающиеся усилия на забое будут оптимальными, соответствующими рис.2.

Для случая, когда оценка пород производится по устанавливающемуся усилию на забое, блок-схема САУ представлена на рис.5. При изменении физико-механических свойств породы $f(t)$ устанавливается новое осевое усилие на забое F_y, измеряемое и преобразуемое в электрический сигнал U_F датчиком осевых усилий. Напряжение U_F, являющееся оценкой физико-механических свойств

породы, подается далее на функциональный преобразователь, формирующий на основании заложенной программы, напряжение задания $U_з$ системы автоматического регулирования скорости вращения инструмента.

Таким образом, каждой новой горной породе будет соответствовать оптимальное соотношение режимных параметров, поскольку устанавливающиеся осевые усилия в различных породах являются оптимальными благодаря соответствующей настройке внешней характеристики системы подачи инструмента на забой, а оптимальная скорость вращения устанавливается системой регулирования в функции физико-механических свойств разбуриваемых пород. На рис.6 приведены осциллограммы, иллюстрирующие работу САУ процессом бурения в промышленных условиях. При забуривании скважины, по мере прохождения поверхностного разрушенного слоя, сопротивляемость породы возрастает, увеличивается величина осевого усилия с 3–4 тонн постепенно до 13–14 тонн. Скорость вращения бурового инструмента при этом в соответствии с функциональной зависимостью уменьшается с 140–145 об/мин до 65–70 об/мин (рис.6а). В процессе углубления скважины физико-механические свойства породы вновь меняются, так на рис. 6б зарегистрирован процесс бурения при переходе из породы средней крепости в породу меньшей крепости. При этом осевое усилие на забое уменьшается с 13–14 тонн до 7–8 тонн, а скорость вращения инструмента увеличивается с 60–70 об/мин до 120 об/мин. Процесс бурения поррд со случайно изменяющимися свойствами показан на рис. 6 в.

Как видно, всякий раз при изменении свойств пород происходит перестройка режимных параметров так, что бурение ведется в оптимальном режиме. Как отмечалось выше, соответствие режима онтимальному определяется достоверностью функциональных зависимостей $n=\varphi_1(f)$, $F_y=\varphi_2(f)$ (рис.2) и точностью их реализации в системе. Внедрение САУ обеспечивает ежегодный экономический эффект в сумме 5–8 тыс.рублей на 1 станок. Динамическая модель системы управления процессом бурения [5], построенная для малых приращений величин и соответствующая блок-схеме рис.5 представлена на рис.7, где:

ΔX – приращение положения плунжера дросселя гидросистемы подачи инструмента на забой;

$\Delta P_н$ – приращение давления, развиваемого маслонасосом системы подачи;

$\Delta Q_{гр}$ – приращение выходного расхода гидродросселя системы подачи;

ΔF – приращение движущей силы гидроцилиндра;

$\Delta F_c(\Delta v)$ – приращение сопротивляемости породы за счет изменения механической скорости бурения на величину Δv ;

$\Delta F_c(\Delta n)$ – приращение сопротивляемости породы за счет изменения скорости вращения инструмента на величину Δn ;

ΔF_c – общее приращение сопротивляемости породы;

A – площадь поршня;

V – общий объем сжимаемой жидкости;

β – модуль объемной упругости жидкости;

m – масса подвижных частей;

$K_1, K_2, K_3, K_{ут}, K_{тр}$ – коэффициенты, характеризующие гидросистему подачи инструмента;

$\hat{K}_v = \frac{\partial F_c}{\partial v}, \hat{K}_n = \frac{\partial F_c}{\partial n}$ – случайные коэффициенты, характеризующие изменение сопротивляемости породы при изменении соответственно скорости бурения v и скорости вращения инструмента n ;

K_F – передаточный коэффициент датчика осевых усилий на забое;

$\frac{K_ф}{1+\rho T_ф}$ – передаточная функция фильтра;

$K_{фу}$ – передаточный коэффициент функционального преобразователя;

$W_{сс}(\rho)$ – передаточная функция САР скорости вращения инструмента;

Положим $\Delta x = 0$, $\Delta P_н = 0$, что эквивалентно рассмотрению свободного состояния системы. Анализ динамических свойств САУ проведем с использованием методики[6]. Передаточная функция разомкнутой САУ (рис.8) будет:

$$W(\rho) = \frac{1}{\hat{K}_v}\left\{K_{тр} + \rho m + \frac{A^2}{K_3 + K_{ут} + \rho\frac{V}{\beta}}\left[1 + \frac{\hat{K}_n K_ф K_F K_{фу} \Phi_{сс}(\rho)}{(1+\rho T_c)(1+\rho T_ф)}\right]\right\},$$

где $\Phi_{сс}(\rho) = \frac{1}{K_{ск} + \frac{1}{W_{сс}(\rho)}} = \frac{1}{K_{ск} + Y_{сс}(\rho)}$ – передаточная функция замкнутой следящей системы;

$\hat{K}_v, \hat{K}_n$ – случайные параметры с весьма большой дисперсией численных значений (40 ÷ 160 раз[5]).

Характеристическое уравнение замкнутой САУ

$$F(\gamma)=(K_{тр}+m\gamma)(K_3+K_{ут}+\frac{V}{\beta}\gamma)(1+T_c\gamma)(1+T_\phi\gamma)[K_{ск}+Y_{сс}(\gamma)]+$$
$$+A^2(1+T_c\gamma)(1+T_\phi\gamma)[K_{ск}+Y_{сс}(\gamma)]+A^2\hat{K}_n K_F K_{фу} K_\phi + \qquad (I)$$
$$+\hat{K}_r(K_3+K_{ут}+\frac{V}{\beta}\gamma)(1+T_c\gamma)(1+T_\phi\gamma)[K_{ск}+Y_{сс}(\gamma)]=0,$$

где γ – корни уравнения.

Построим Д-разбиение в координатах параметров $\hat{K}_n$ и $\hat{K}_r$. Условие прохождение кривой Михайлова через нуль выражается следующим образом:

$$F(j\omega)=a(\omega)+jb(\omega)+\hat{K}_n c+[d(\omega)+je(\omega)]\hat{K}_r=0, \qquad (2)$$

где

$$a(\omega)+jb(\omega)=[A^2+(K_{тр}+j\omega m)(K_3+K_{ут}+j\omega\frac{V}{\beta})\times$$
$$\times(1+j\omega T_c)(1+j\omega T_\phi)[K_{ск}+Y_{сс}(j\omega)];$$
$$d(\omega)+je(\omega)=(K_3+K_{ут}+j\omega\frac{V}{\beta})(1+j\omega T_c)[K_{ск}+Y_{сс}(j\omega)](1+j\omega T_\phi);$$
$$c=A^2K_\phi K_F K_{фу}.$$

Уравнение (2) разбивается на два

$$a(\omega)+\hat{K}_n c+d(\omega)\hat{K}_r=0, \quad b(\omega)+\hat{K}_r e(\omega)=0.$$

Полагая, что система этих уравнений совместна, решим ее относительно $\hat{K}_r$ и $\hat{K}_n$, получим:

$$\hat{K}_r=-\frac{b(\omega)}{e(\omega)}, \quad \hat{K}_n=\frac{1}{c}\left[\frac{d(\omega)b(\omega)}{e(\omega)}-a(\omega)\right].$$

Построим для значений коэффициентов кривые Д-разбиения (рис.9). После построения в координатах $\hat{K}_r$ и $\hat{K}_n$ плотности распределения $f(\hat{K}_n, \hat{K}_r)$ этих параметров[5], определим вероятность устойчивости САУ

$$P[\hat{K}_n, \hat{K}_r \subset (R)]=\iint_{(R)} f(\hat{K}_n, \hat{K}_r)d\hat{K}_n d\hat{K}_r < 1, \qquad (3)$$

где (R) – область устойчивости САУ. Полная вероятность устойчивости САУ, при данной плотности вероятности параметров бурения, может быть достигнута соответствующей коррекцией системы.

Покажем, что качество САУ бурением может быть определено только в вероятностном смысле.

Представим уравнение (I) в таком виде:

$$[1+\gamma^2 T_m T_\beta \alpha+\gamma(T_m+T_\beta)\alpha](1+\gamma T_c)(1+\gamma T_\phi)+L\hat{K}_n+ \\ +N\hat{K}_v(1+\gamma T_\beta)(1+\gamma T_c)(1+\gamma T_\phi)=0, \tag{4}$$

где $T_m=\frac{m}{K_{тр}}$; $T_\beta=\frac{V}{\beta K_з}$; $K_з'=K_з+K_{уТ}$; $\alpha=\frac{K_{тр}K_з'}{A^2+K_{тр}K_з'}$;

$$K_F'=K_F K_\phi K_{фу}; \quad L=\frac{A^2 K_F'}{(A^2+K_{тр}K_з')K_{ск}}; \quad N=\frac{K_з'}{A^2+K_{тр}K_з'}. \tag{5}$$

Рассмотрим случай, когда $T_m, T_\beta \ll T_c, T_\phi$. Вместо (4) можно приближенно принять

$$\gamma^2 T_c T_\phi+\gamma(T_c+T_\phi)+\hat{B}+1=0, \tag{6}$$

где

$$\hat{B}=\frac{L\hat{K}_n}{1+N\hat{K}_v}=\frac{L\sigma_{кп}\hat{K}_{n*}}{1+N\sigma_{кv}\hat{K}_{v*}}; \quad \hat{K}_{n*}=\frac{\hat{K}_n}{\sigma_{кп}}; \quad \hat{K}_{v*}=\frac{\hat{K}_v}{\sigma_{кv}}. \tag{7}$$

Уравнение (6) дает приближенные значения главных корней системы:

$$\gamma_{1,2}\cong-\frac{1}{2}\left(\frac{1}{T_\phi}+\frac{1}{T_c}\right)\pm\sqrt{\frac{1}{4}\left(\frac{1}{T_\phi}+\frac{1}{T_c}\right)^2-\frac{(1+\hat{B})}{T_c T_\phi}}. \tag{8}$$

Колебательный процесс будет при:

$$\hat{B}>\varphi\left(\frac{T_c}{T_\phi}\right)=\frac{1}{4}\left(\frac{T_c}{T_\phi}+\frac{T_\phi}{T_c}-2\right). \tag{9}$$

Уравнение (8) после подстановки значений (7) и (5) примет вид:

$$\hat{K}_{n*}>K_{n_0}=\varphi\left(\frac{T_c}{T_\phi}\right)(q+h\hat{K}_{v*}), \tag{10}$$

где безразмерные коэффициенты:

$$q=\left(1+\frac{K_{тр}K_з'}{A^2}\right)\frac{K_{ск}}{K_F'\sigma_{кп}}=\frac{1}{L\sigma_{кп}}; \quad h=\frac{\sigma_{кv}K_з'K_{ск}}{\sigma_{кп}A^2K_F'}=\frac{N\sigma_{кv}}{L\sigma_{кп}}.$$

Нанесем граничную зависимость (10) на плоскость распределения случайных параметров $\hat{K}_{v*}$ и $\hat{K}_{n*}$ (рис.10). Вероятность колебательного процесса, очевидно, определяется следующим образом:

$$P(\Omega>0)=P[\hat{K}_{n*}>K_{n_0}]=\int_{-\infty}^{\infty}\int_{K_{n_0}}^{\infty}f(\hat{K}_{v*},\hat{K}_{n*})\,d\hat{K}_{v*}\,d\hat{K}_{n*} \tag{11}$$

Для определения вероятности колебательного процесса с частотой больше Ω_A получим из (8):

$$\sqrt{\frac{1+\hat{B}}{T_c T_\phi} - \frac{1}{4}\left(\frac{1}{T_\phi}+\frac{1}{T_c}\right)^2} > \Omega_A ,$$

которое после преобразований приобретает вид:

$$\hat{B} > T_c T_\phi \Omega_A^2 + \varphi\left(\frac{T_c}{T_\phi}\right).$$

Данное уравнение, учитывая (5) и (7) запишется так:

$$\hat{K}_{n*} > K_{n1} = \left[T_c T_\phi \Omega_A^2 + \varphi\left(\frac{T_c}{T_\phi}\right)\right](q + h K_{v*}) .$$

Вероятность колебательного процесса с частотой больше Ω_A, аналогично (II) может быть определена (рис.I0)

$$P(\Omega > \Omega_A) = P(\hat{K}_{n*} > K_{n1}) = \int_{-\infty}^{\infty}\int_{K_{n1}}^{\infty} f(\hat{K}_{v*}, \hat{K}_{n*})\, d\hat{K}_{v*}\, d\hat{K}_{n*} . \qquad (I2)$$

Коэффициент затухания при колебательном процессе зависит от постоянных времени T_c и T_ϕ и не зависит от случайных параметров $\hat{K}_v$ и $\hat{K}_n$:

$$\alpha_o = \frac{1}{2T_\phi} + \frac{1}{2T_c} .$$

При апериодическом процессе главное абсолютное значение коэффициента затухания равно:

$$\alpha = \frac{1}{2}\left(\frac{1}{T_\phi}+\frac{1}{T_c}\right) - \sqrt{\frac{1}{4}\left(\frac{1}{T_\phi}+\frac{1}{T_c}\right)^2 - \frac{(1+\hat{B})}{T_c T_\phi}} .$$

Ограничивая это значение предельным $\alpha > \alpha_A$ получим:

$$\hat{B} > \Psi(T_c, T_\phi, \alpha_A) = (T_c + T_\phi)\alpha_A - T_o T_\phi \alpha_A^2 - 1 . \qquad (I3)$$

После подстановки в (I3) значения $\hat{B}$ из уравнения (7) будем иметь:

$$\hat{K}_{n*} > K_{n2} = \Psi(T_c, T_\phi, \alpha_A)(q + h\hat{K}_{v*}) .$$

Вероятность апериодического процесса с главным коэффициентом затухания меньше заданной величины α_A, аналогично выражению (II), может быть определена (рис.I0):

$$P(\alpha < \alpha_A) = P(\hat{K}_{n*} > K_{n2}) \int_{-\infty}^{\infty}\int_{-\infty}^{K_{n2}} f(\hat{K}_{v*}, \hat{K}_{n*})\, d\hat{K}_{v*}\, d\hat{K}_{n*} .$$

Если характеристическое уравнение не приводится ко второму порядку, вероятность заданного качества определяется согласно (3), где (R) рассматривается как область заданного качества.

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к докладу Г.С. Черноруцкого и В.А. Цыганкова

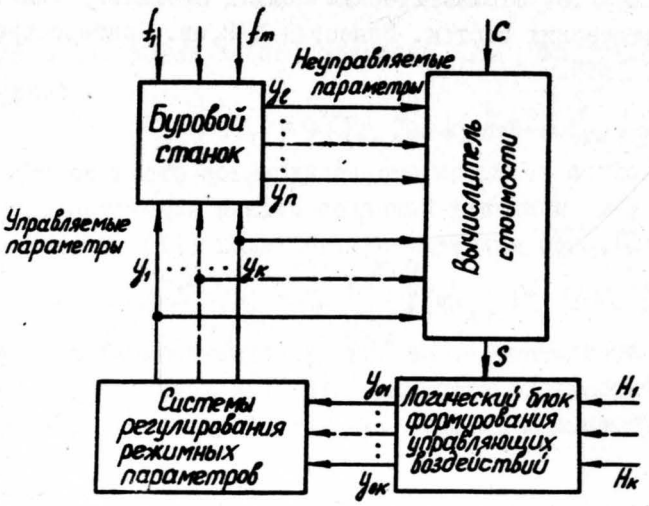
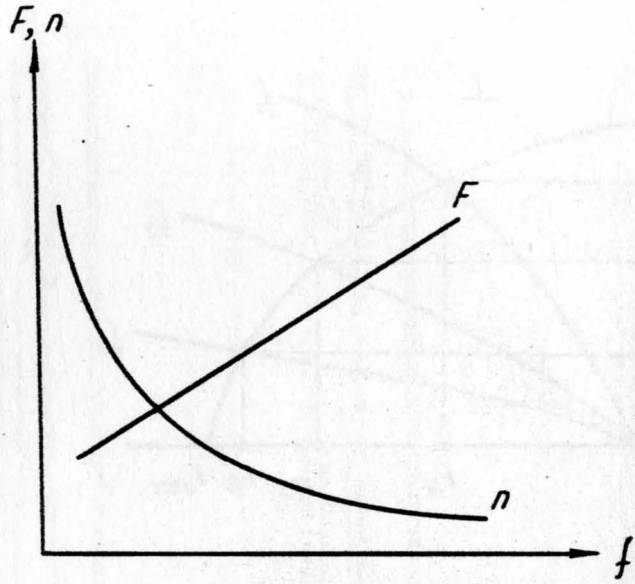


Рис.1


F, n
F
n
f

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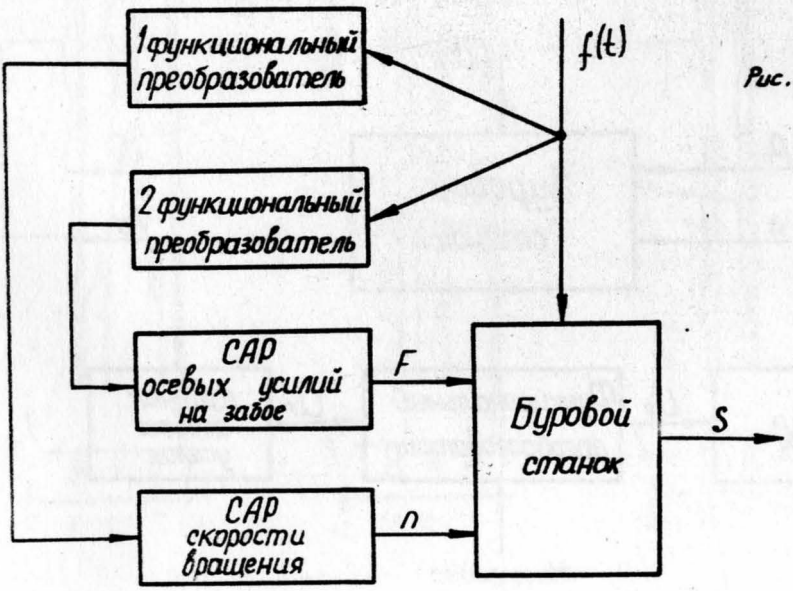
1 функциональный преобразователь
2 функциональный преобразователь
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САР скорости вращения
f(t)
F
n
Буровой станок
S

Рис. 3

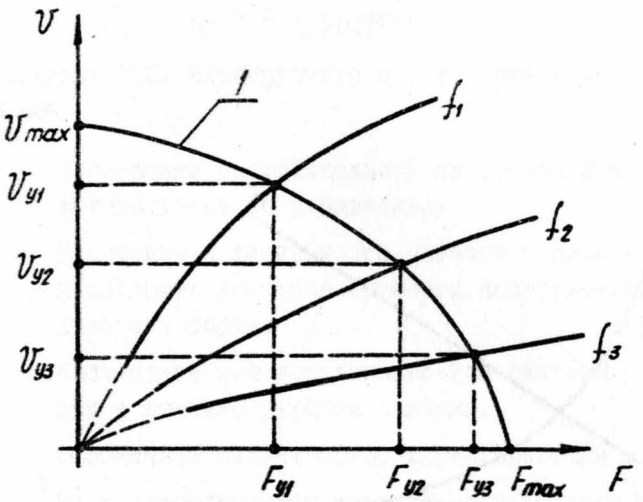
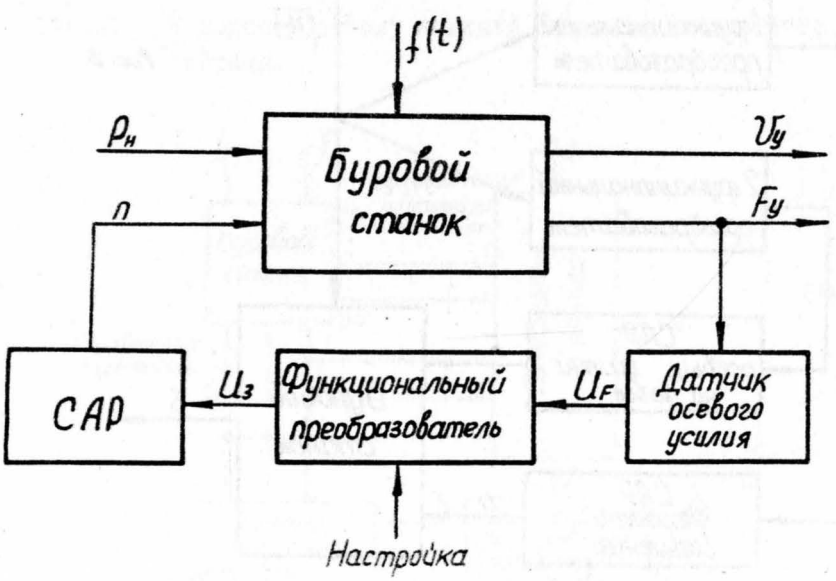
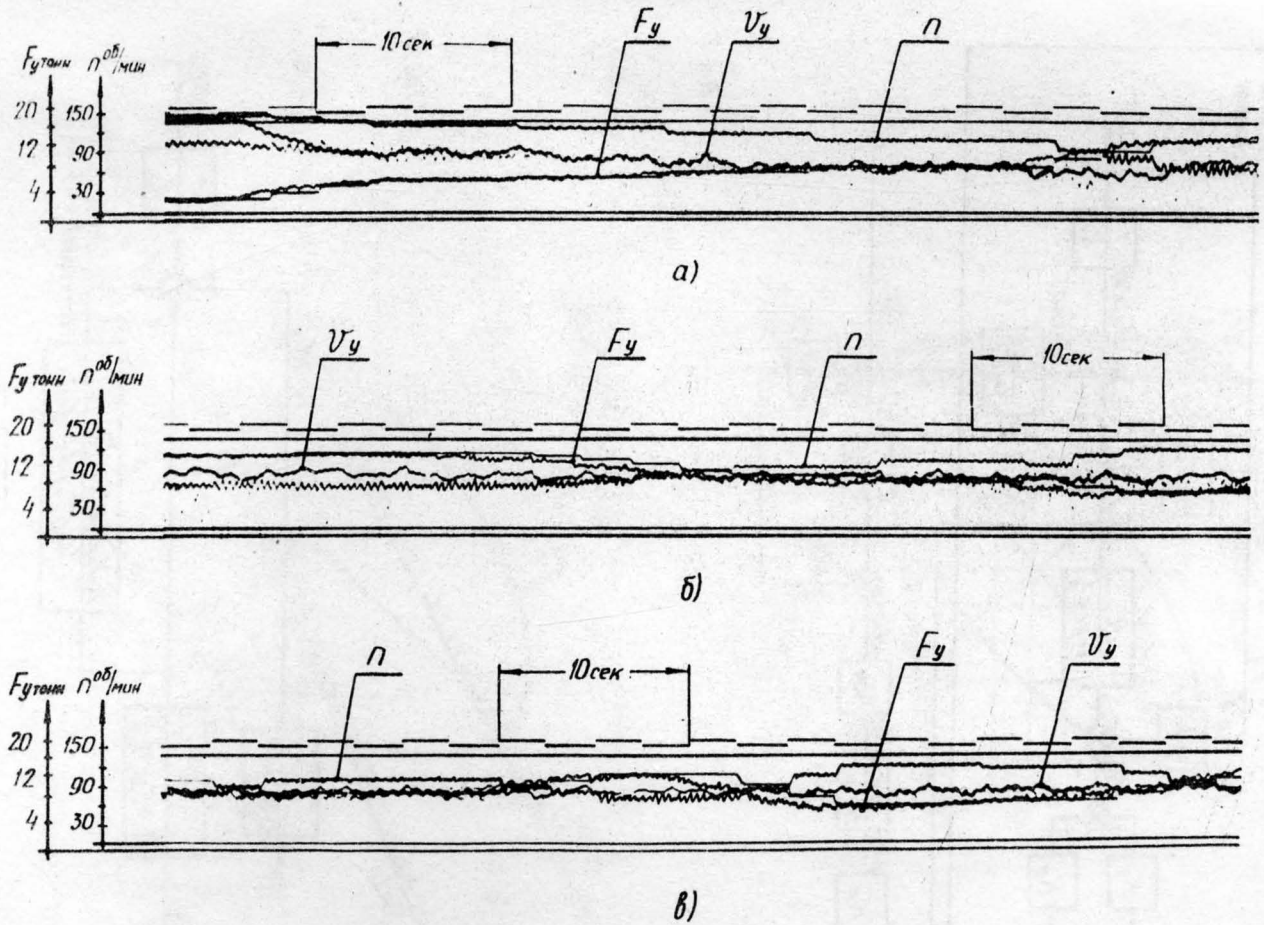

V
1
V_{max}
V_{y1}
V_{y2}
V_{y3}
f_1
f_2
f_3
F_{y1}
F_{y2}
F_{y3}
F_{max}
F

Рис. 4

Рис. 5


f(t)
$P_н$
n
Буровой станок
V_y
F_y
САР
$U_з$
Функциональный преобразователь
U_F
Датчик осевого усилия
Настройка


Fу тонн
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12
4
150
90
30
10 сек
Fу
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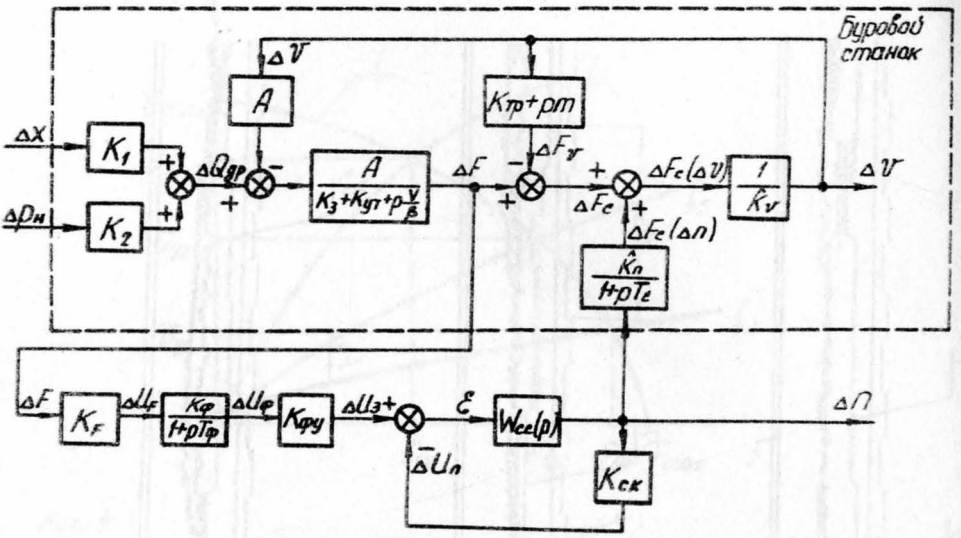
Буровой станок
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ΔX
K1
ΔQqp
A
K3+Kуп+p V/β
ΔF
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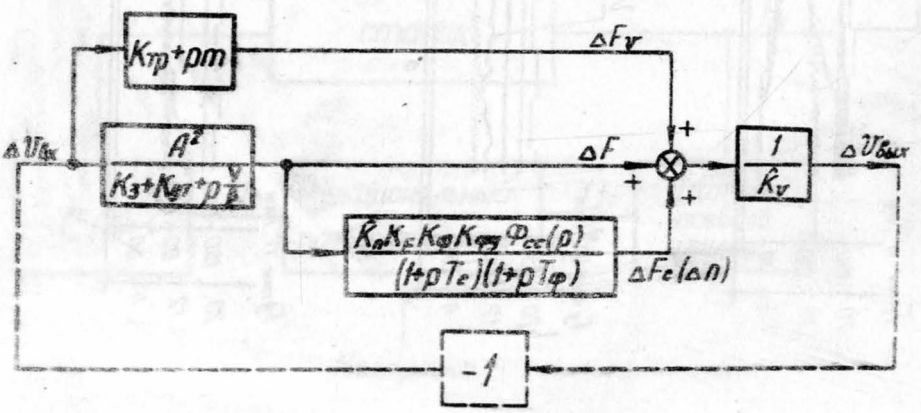
ΔFc(ΔV)
1/Kv
ΔV
Δpн
K2
ΔFc
ΔFc(Δn)
Kn/(1+pTc)
ΔF
KF
ΔUF
Kф/(1+pTф)
ΔUф
Kру
ΔUз
ε
Wcc(p)
Δn
ΔUn
Kск

Рис. 7

Рис. 8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ΔFv
ΔVвх
A²/(K3+Kуп+p V/β)
ΔF
1/Kv
ΔVвых
Kn·Kc·Kф·Kру·Φcc(p)/((1+pTc)(1+pTф))
ΔFc(Δn)
-1

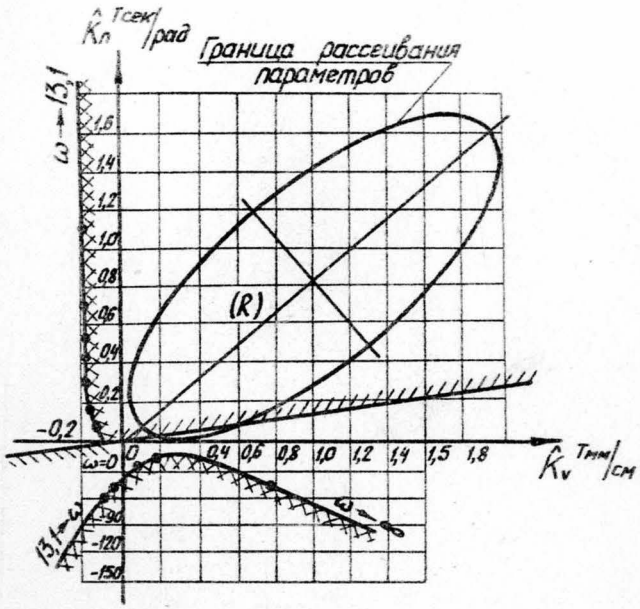

Рис. 9

Рис. 10