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IDENTIFICATION IN AUTOMATIC CONTROL SYSTEMS

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1. INTRODUCTION

System identification is a very wide notion and different authors use it in slightly different ways. We shall not try to find a general definition and, for the purpose of this paper, we shall simply treat the identification as the determination of a mathematical model of the process which is to be controlled. This paper cannot be used as an instruction manual for the practical application of different methods – not even as a reference paper. Its aim rather is to give a reasonably comprehensive picture of the recent development of this special field, to point-out and to compare the principal ideas of different approaches to the solution of the given problem. It should be stressed that such a survey cannot be complete and the expressed opinions are influenced by our knowledge and experience which are limited in many respects.

The literature on system identification is now quite extensive. At the end of the paper, we list only the recent papers which were available for us and which appeared after the third IFAC Congress in 1966. The anterior literature is cited only where it is necessary for our exposition. For a more complete list of references, the reader may consult the survey paper by Eykhoff, Van der Grintern, Kwakernaak and Veltman (1966).

Generally speaking, there are two possible ways to determine the mathematical model of a given physical process:

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1) Mathematico-physical analysis based on general physical laws (process dynamics) and

2) Experimental identification where the main information about the process is obtained by measurements. Only the experimental identification will be considered in this paper. However, when the experimental identification is applied, the black box approach should be considered only as a last possibility. In practical situations, even a rough analysis together with engineering experience usually can provide very valuable a priori information about the process studied, about the most suitable form of the mathematical model and perhaps even approximate values of some parameters.

Actually, the process identification is always only the first step in the solution of a more complex control problem. Both identification and synthesis of the control system should be considered together. This is very easy to say but much more difficult to realize. The principal difficulty is that the mathematical description of the process must be adequate for the conditions under which the system will be operated but, these conditions can be known only after the synthesis for which the identification is required. Perhaps this is the main reason why the identification is so often handled as a separate problem. Nevertheless, the final goal must be always considered.

From this point of view, the following situations should be distinguished:

a) Open-Loop Optimal Control

The final goal in this case is the determination of the input signal so that the resulting process is optimal with respect to some criterion and, may be, to some restrictions. The task of system identification is the determination of a mathematical model, the output of which would be as close as possible to the output of the real system for any inputs and initial conditions which come into the consideration. The system can be usually considered as a

deterministic system, the influence of noise and random disturbances is of minor importance and often only measurement errors can be considered. On the other hand, also knowledge of the initial conditions is often required and their determination can be considered as a part of system identification.

b) Closed-Loop Control of Deterministic Systems

A typical representative of this case is a servomechanism (Figure 1). The problem which is usually solved – and for which the system identification is required – is the determination of the correcting filter F so that the closed-loop is stable and the output signal x follows the input signal w as well as possible. The input signal of the closed-loop w can be deterministic or stochastic one. The mathematical model of the system S must be determined so that its behavior in closed loop is the same as the behavior of the original system. The initial conditions are usually out of our interest in this case.

c) Closed-Loop Control of Noisy Systems

In both foregoing cases, the system could be considered as deterministic and the only stochastic errors, which are to be eliminated in the process of identification are the measurement errors. But many industrial plants cannot be simply considered as deterministic systems. They are very often influenced by stochastic disturbances which cannot be measured and sometimes even the source of disturbances cannot be exactly specified. As an extreme case of this kind, the regulation of a noisy system (in Figure 2) can be given. The purpose of the regulator R is to suppress the influence of the noise and to reduce the deviation of the output to minimum. This situation is often met in mass production (chemical plants, paper making machines, power plants, etc.). Such systems can be treated

as stochastic transformation of the input to the output and the task of system identification is to determine some statistical characteristics of this transformation which are necessary for the design of optimal controller.

We have mentioned only three typical examples to point out the wide variety of real systems and different possible interpretations of system identification.

For the simplicity, we shall consider only single-variable systems (systems with single input and single output). Most methods, which we are going to discuss can be – at least theoretically – extended also for multi-variable systems.

2. CLASSIFICATION OF METHODS FOR SYSTEM IDENTIFICATION

From the mathematical point of view, the experimental system identification can almost always be considered as a problem of finding extrema of functionals. The form of the functional, the extremum of which is to be found, is given by the criterion accepted for the system identification and by the mathematical model of the system. Therefore, the accepted criterion of identification and the accepted form of the mathematical model of the system are apparently the most significant features of every method. Moreover, the criterion of optimality also determines in what situations the given method can be used with some assurance of success.

To achieve the desired extreme different computing technique can be applied. From this point of view, two large groups of methods can be distinguished.

The method of the first group can be denoted as direct methods. They do not use the physical realization of the mathematical model of the system and handle the identification simply as mathematical problem of finding an extreme. According to the form of the functional they produce either the

explicit mathematical relation for the unknown parameters or they apply different hill-climbing and variational numerical procedures.

The model-adjusting methods form the second group. They use the physical realization of the model often in connection with analog computing technique.

The experimental identification of a physical system is to a great extent dependent on the kind of the input signal which excites the unknown system. Where it is possible to choose the input signal, the choice can markedly influence the accuracy of the identification or simplify the whole procedure.

Summarizing what was said above, we can formulate the following criteria for classification of different identification methods:

a) What criterion of optimality for system identification is accepted;
b) What form of the mathematical model of the system is considered;
c) What computing technique is used;
d) What kind or form of the input signal is applied.

By combination of these possibilities, we can get an immense number of different methods for system identification. Some of them, which seem to us as to be the most important in present development of control theory, will be discussed in Section 4. For the reasons mentioned above, we shall use the criterion (a) as the most important. The most frequent forms of mathematical models used for description of controlled systems will be discussed separately in the next section.

3. MATHEMATICAL MODELS OF SYSTEMS

The choice of the mathematical model, or more precisely, the form of the input-output relation adopted, obviously plays a major role in the identification procedure. The most suitable form of the model depends on many factors among which the most important are:

a) The purpose for which the identification is undertaken

b) The physical nature of the process.

c) The prior knowledge about the system studied.

In this section we shall describe the mathematical models which are most commonly used for the description of systems in the time domain.

3.1 Deterministic Systems.

Systems with Finite Memory: Linear Systems. For a linear time-invariant controllable system, the output $v(t)$ is given in terms of the input $u(t)$ by the convolution integral:

$$v(t) = \int_{-\infty}^{t} h(t-\sigma)\, u(\sigma)\, d\sigma = \int_{0}^{\infty} h(\sigma)\, u(t-\sigma)\, d\sigma \qquad (3.1)$$

where $h(t)$ is the impulse response (weighting function) of the system. It is assumed that the system is stable:

$$\int_{0}^{\infty} |h(t)|\, dt < \infty \qquad (3.2)$$

In the time-discrete case (systems with discrete input and output sampled with constant sampling period) similarily:

$$v(k) = \sum_{0}^{\infty} h(i)\, u(k-i) \qquad (3.3)$$

and in (3.2) sum instead of integral appears.

The impulse response $h(\cdot)$ fully determines the dynamical properties of a controllable linear system and when this function is determined, the system is clearly identified.

For inputs with a given bond M, given ϵ arbitrarily small it is clearly possible to find T_M such that

$$\left| \int_{T_M}^{\infty} h(\sigma)\, u(t-\sigma)\, d\sigma \right| < \epsilon, \quad t > T_M$$

This fact enables us to approximate a linear system (with infinite memory) by a system with 'finite' memory and to assume that $h(t) = 0$ for $t > T_M$. Here T_M is also referred to as the 'settling time'. In discrete case we get:

$$v(k) = \sum_{0}^{M} h(i)\, u(k-i) \tag{3.5}$$

where M is the "practical length" of the impulse response (memory of the system). For computational reasons, the formula (3.5) is often used also in the continuous case taking $h(i)$ as $h(i\Delta T)\Delta T$ where ΔT is the step of approximate integration which must be sufficiently small. By this approximation, the problem is reduced to the estimation of $(M + 1)$ unknown parameters $h(i)$, $i = 0, 1, \ldots, M$.

From the computational point of view the very important and favorable feature of the model (3.5) is its linearity in the unknown parameters $h(i)$, $i = 0, \ldots M$. This fact significantly simplifies the procedure of identification and is undoubtedly the main reason ascribable for the wide popularity of this model.

The "settling time" of the impulse response has to be a priori known. It does not need to be very critical in many practical situations where this value can be well estimated from the physical nature of the system. [Taylor and Balakrishnan 1967]

As a generalization of the representation of the impulse response by samples, we may use the Rayleigh-Ritz forms:

$$h(t) = \sum_{i=1}^{M} \beta_i\, \phi_i(t) \tag{3.6}$$

Where the functions $\phi_i(t)$ are predetermined. Usually a set of orthonormal exponentials is used for this purpose, β_i being the parameters which have to be estimated. Apparently, the class of linear systems which can be described by the impulse response of the type (3.6) is restricted by the choice of the base $\phi_i(t)$, $i = 1, \ldots, N$. The disadvantage of the model (3.6) (which is often criticized) is the fact that even a very simple impulse response has to be approximated by relatively large number of members in (3.6) if the base does not suit the given case. This disadvantage can be removed by introducing further adjustable parameters into the base (Izawa and Furuta (1967)). However, it considerably complicates the process of identification.

Nonlinear Systems: Volterra Expansion

Any nonlinear system that is controllable can be described by the Volterra expansion:

$$v(t) = \sum_{n=1}^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} w_n(t_1, \ldots, t_n)\, u(t - t_1) \cdots u(t - t_n)\, dt_1, \ldots, dt_n \tag{3.7}$$

When the system is approximated by a system with finite memory (m) and the functional power series is truncated to finite number (N) of terms and the integration is replaced by summation, we obtain:

$$V(k) = \sum_{n=1}^{N} \left(\sum_{m_1=1}^{M} \cdots \sum_{m_n=1}^{M} h_n(m_1, \ldots, m_n)\, u(k - m_1) \cdots u(k - m_n) \right) \tag{3.8}$$

The Kernels $h_j(\ldots)$ can obviously be taken to be symmetric functions. The Equation (3.8) can be used as a mathematical model for a fairly wide class of nonlinear systems and may be regarded as generalization of (3.5) for the nonlinear-case (Balakrishnan 1963). From the viewpoint of identification, the main advantages of this type of description of nonlinear systems are its generality (the a priori knowledge of the structure is not required) and its lincarity in the parameters $h_j(\ldots)$ which have to be estimated. However, the dimensionality increases rapidly with N.

If the 'odd' symmetry of the system can be assumed, $F\left(-u(t)\right) = -F\left(u(t)\right)$, only the odd terms of expansion (3.6) may be considered (Taylor and Balakrishnan (1967)). Another way to reduce the dimensionality of the problem is to introduce the adjoint space (Balakrishnan (1963)), so that the Kernels may be assumed to have to form:

$$h_j\left(\tau_1, \ldots \tau_j\right) = \int_M^L f\left(t-\tau_1\right) f\left(t-\tau_j\right) dt$$

and only the single function $f(.)$ needs to be determined.

Dynamic Systems

When a dynamic system model is available from energy/force balance equations, the identification problem becomes a parametric one and has the advantage in permitting the use of non-steady-state data. The basic model is the relation:

$$f\left(v^{(n)}(t), \ldots\ v(t);\ u^{(m)}(t), \ldots u(t);\ \beta_1, \ldots \beta_n\right) = 0 \qquad (3.10)$$

where $v^{(i)}(t)$ and $u^{(i)}(t)$ are the derivatives of the output and input signals and $\beta_1 .. \beta_M$ are the parameters to be determined. Often the more general state space representation

$$\begin{aligned} \dot{\underline{Z}} &= F(\underline{Z}, \underline{u}, \underline{\beta}, t) \\ \underline{V} &= H(\underline{Z}, \underline{u}, t) \end{aligned} \qquad (3.11)$$

can be used where $\underline{Z}$ is the vector of suitable chosen state variables and $\underline{u}$, $\underline{v}$ are generally multidimensional input/output signals.

In (3.10) or (3.11) the form of the functions f or F and H as well as the order n (dimension of state space) are supposed to be a priori known. In such a case, the problem can be reduced to that of finding the initial state of a more complicated but known system. In the linear case (3.10) takes the form (for single-input, single-output case)

$$\sum_{o}^{n} a_i \, v^{(i)}(t) = \sum_{o}^{m} b_i \, u^{(i)}(t) \qquad (3.12)$$

and we assume $m \leqq n$.

In state space form; we have:

$$\begin{aligned} \dot{\underline{z}} &= A\underline{Z} + Bu \\ u &= CZ + Du \end{aligned} \qquad (3.13)$$

and a_i, b_i and the elements of the matrices A, B are now the parameters formerly denoted as β. It must be noted that in the form (3.13) the system need not be observable or controllable.

In the discrete case, the past values of particular signals appears in (3.1) instead of the derivatives:

$$\sum_{o}^{n} a_i \, v(k-i) = \sum_{o}^{m} b_i \, u(k-i) \qquad (3.14)$$

or more generally,

$$\begin{aligned} Z(k+1) &= A\,Z(k) + B\,u(k) \\ V(k) &= C\,z(k) + D\,u(k) \end{aligned}$$

The differential/difference models seems to be most convenient from the view point of the number of the parameters to be estimated as well as the relation to the control problem. Of course the order n of the system has to be known; experience shows that most pracitcal plants can be described with good approximation by the differential or difference model of third or less order.* Such models have also been used for distributed parameter systems (Peterka and Vidincev - 1967).

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*If the order of the system is not a priori known, it is usually determined in practice by making several different estimates of different orders and choosing the minimum order which gives a suitable fit to the data. Of course, this procedure can be misleading.

3.2 Noisy Systems

By a noisy system we shall mean one in which the system is blended with an internal noise source quite apart from (and to be distinguished from) errors in the input-output measurements. The noisy system may often be decomposed into a deterministic system and an 'internal' additive noise (such as 'load disturbance') corrupting the ideal output v, as shown in Figure 3. Such a structure is common in 'regulator' problems where the undesirable influence of the noise has to be compensated by feedback control. In the simple linear case, the deterministic system is linear, and a discrete version is illustrated in Figure 4. With reference to Figure 4, the spectral density of the noise is:

$$\phi(z) = \sigma_\epsilon^2 \, F(z) \, F(z^{-1}) \tag{3.16}$$

where σ_ϵ^2 is the variance of the discrete white noise ϵ. If both the spectral density (3.16) and the pulse-transfer function G(z) can be expressed as ratios of polynomials:

$$G(z) = \frac{b_n(z^{-1})}{a_n(z^{-1})} \quad ; \quad F(z) = \frac{\beta_m(z^{-1})}{\alpha_m(z^{-1})} \tag{3.17}$$

where we have used the notation:

$$b_n(z^{-1}) = \sum_0^n b_k \, z^{-k}$$

and similarly for the other polynomials. The noisy system as a whole can be described by the difference equation:

$$\sum_0^N A_i \, x(k-1) = \sum_0^N B_i \, u(k-1) + \sum_0^n C_i \, e(k-1) \tag{3.18}$$

where $N = n + m$ and the coefficients A_i, B_i, and C_i are the coefficients of polynomials

$$A_n(z^{-1}) = a_n(z^{-1})\, d_m(z^{-1}) \; ; \; B_n(z^{-1}) = d_m(z^{-1})\, b_n(z^{-1}) \; ; \; C_n(z^{-1}) = a_n(z^{-1})\, \beta_m(z^{-1})$$

It is useful to choose $A_o = 1$ and the standard deviation σ_e so that also $C_o = 1$. The remaining coefficients A_i, C_i $(i = 1, \ldots N)$ and B_i $(i=0, \ldots N)$ are enough to determine the minimum variance (steady state) control strategy (Astrom 1967a).

4. COMPARISON OF SOME APPROACHES TO THE IDENTIFICATION PROBLEM

According to the kind of criterion which is used as a measure of fit between the mathematical model and the real process, three large groups of identification methods can be distinguished:

a) Minimization of output error (or 'fitting' error)

b) Minimization of equation error

c) Statistical error criterion

4.1 Minimization of Output Error

The principal idea of this approach is to determine the mathematical model so that when excited by the same input signal (Figure 6) its output reproduces the output of the real system as well as possible. Let $u(t)$ denote the observed input and $z(t)$ the observed output. In the simplest of situations, we may assume that these are stationary stochastic processes, and that the system is stable, and that steady state has been attained. If a linear model of the system is assumed, for example, then the fit error is:

$$e(t) = z(t) - \int_0^\infty h(s)\, u(t-s)\, ds \qquad (4.11)$$

and we may seek to minimise: the mean square error:

$$q = \lim_{T \to \infty} \frac{1}{T} \int_0^T e(t)^2\, dt \qquad (4.12)$$

In this case the optimal $h(.)$ satisfies the well-known Wiener-Hopf equation:

$$\phi_{uz}(t) = \int_0^\infty h(s)\, \phi_{uu}(t-s)\, ds \qquad t \geq 0 \qquad (4.13)$$

It is, of course, essential in this that the observed input is also the actual input to the system, and thus this is basically an 'open loop' method.

In practice, even apart from the problem of numerical solution of the Wiener-Hopf equation (Fortova, Kryze, Peterka, 1966), the results using this method are not satisfactory when applied to real systems under normal operating conditions (using operating data, as opposed special test inputs). See also Ehrenburg and Wagner (1966). This is not too surprising considering the many assumptions involved. In fact among the main reasons for the lack of success we may mention: a) the solution of (4.3) is very sensitive to errors in the estimated correlation functions, especially when the autocorrelation function of the input is flat; (b) to obtain the correlation functions with any degree of accuracy, a long interval of observation is required which usually violates the assumed stationarity; and c) short-time correlation functions lead to error even when no noise or measurement errors are present.

As is to be expected, the method provides good results when special test signals are employed, such as pseudo-random input signals to excite the system. (Briggs, Godfrey and Hammond 1967; Krotolica 1967; Davies and Bruce 1967.)

When a finite settling time or finite memory can be assumed, and the input is noise free, it is not necessary to adopt the stochastic point of view or to employ special input signals. Instead, one can directly find the best $h(.)$ (in the linear case) that minimizes

$$\int_M^L \left(z(t) - \int_0^M h(s)\, u(t-s)\, ds \right)^2 dt \qquad (4.14)$$

where L is the total data length. The nonlinear model (cf., 3.7, 3.8) can then also be used. See Taylor 1968 for a concrete application.

The criterion (4.14) is often referred to as 'Least Squares for Fit Error', and in the discrete case can be formulated as that of solving the 'ill-posed' equation

$$Z = F\beta \tag{4.15}$$

leading to the well-known 'pseudo-inverse' solution:

$$\beta = (F^* F)^{-1} F^* Z \tag{4.16}$$

where Z is the observation vector, and β is the vector parameter to be estimated, the (rectangular) matrix F being composed of known elements depending on the input data. It is essential in this method that the input be noise free, otherwise the estimate will be biassed. In the discrete version of the linear case (4.14), we have for example:

$$F = \{f_{ij}\}$$

where

$$f_{ij} = u(i-j) \quad , \quad i = M, \ldots L; \; j = 0, \ldots M.$$

and F^*F has the elements:

$$\sum_{k=m}^{L} u(k-i)\, u(k-j) \qquad 0 \leqq i, j \leqq M \tag{4.17}$$

A numerical problem is the inversion of the matrix F^*F. Obviously any method of factorization is of advantage and in particular when updating of the estimate is required as L increases (as more data is available). It should be noted that if a pseudo random input signal can be used, the inverse matrix can be explicitly calculated, leading to tremendous simplification of the identification algorithm. (Briggs, Godfrey and Hammond 1967.) Other special signals may also be used where permitted, to provide the necessary diagonalization. (Roberts 1967; Mintz and Thorp 1967;

Egorov 1966.) Because of its importance we shall now briefly describe an example of a pseudorandom signal.

Pseudorandom binary input signal

An example of pseudorandom binary signal is given in Figure 8. It is a deterministic periodical signal with the following properties:

$$u(t) = u_k = \pm\kappa \quad \text{for} \quad kT \leqq t \leqq (k+1)T\,, \quad u_k = u_{k+\nu p}$$

$$\frac{1}{\rho p}\sum_{k=1}^{\rho p} u_{k-i}\,u_{k-j} = \begin{cases} \kappa^2 & \text{for } i = j + \nu p \\ -\dfrac{\kappa^2}{p} & \text{for } i \neq j + \nu p \end{cases}$$

$$\frac{1}{\rho p}\sum_{k=1}^{\rho p} u_k = \frac{\kappa}{p}$$

where ν, ρ and p are integers and κ is either positive or negative constant.

When periods of this input signal are applied to the unknown system and the discrete impulse response is considered the $\left((M+1)\times(M+1)\right)$ matrix $\underline{F}^T\underline{F}$ takes the form

$$\underline{F}^T\underline{F} = \begin{bmatrix} p & -1 & -1 & \dots & -1 \\ -1 & p & -1 & \dots & -1 \\ -1 & -1 & p & \dots & -1 \\ -1 & 1- & -1 & \dots & p \end{bmatrix}$$

and

$$\left(\underline{F}^T\underline{F}\right)^{-1} = \frac{1}{\kappa^2(p+1)(p-M)} \begin{bmatrix} p+1-M & 1 & \dots & 1 \\ 1 & p+1-M & \dots & 1 \\ \vdots & \vdots & & \\ 1 & 1 & & p+1-M \end{bmatrix}$$

The pseudorandom binary signal can be very easy generated and simplifies the whole procedure of identification. Because of its spectral characteristics, the pseudorandom binary input signal can be recommended also for other types of models, when its other properties cannot be fully utilized.

Dynamic System (Differential Equation) Models

The minimization of fit error when the system is dynamic (including distributed parameter systems) leads to a nonlinear problem, even when the unknown system is linear. For a general formulation see Balakrishnan 1968a. It is convenient to use the state space formulation because of its generality. We shall first consider the case of an unknown linear system. Thus let the system be represented by:

$$\dot{x}(t) = A\,x(t) + B\,u(t) \tag{4.18}$$

where $u(.)$ is the known noise-free input, and the observation $z(t)$, because of measurement errors, is expressed:

$$z(t) = C\,x(t) + n(t)\,, \quad 0 < t \tag{4.19}$$

where $n(t)$ is the output observation error or noise. The least squares fit error method is to minimize:

$$\int_a^b \| C\,x(t) - z(t) \|^2\, dt \tag{4.20}$$

where $[a, b]$ is the interval over which the fit error is considered and the minimization is over some or all of the parameters in the matrices A, B, and C. In (4.20), $x(t)$ is determined as soon as A, B, C are specified, along with the initial state $x(o)$; indeed:

$$x(t) = T(t)\,x(o) + \int_o^t T(t-s)\,Bu(s)\,ds \tag{4.21}$$

with $T(t) = \exp At$. Often it can be assumed that the system is stable, and 'a' is large enough so that the first term in (4.21) is negligible; otherwise the initial state has to be another unknown also to be determined. In terms of the structure (4.19), the minimization of (4.20) can be interpreted as a maximum likelihood estimate for $n(t)$ assumed to be white Gaussian. (In the continuous case, strictly speaking, the integrals involving the observed data must then be interpreted as Ito integrals).

Unlike the case of a linear system with finite memory, the functional in (4.20) is no longer linear in the unknowns. We have thus all the computational complexity associated with finding the minimum of a nonlinear functional. Gradient methods appear to be slow, and a version of the Newton-Raphson recommended in Balakrishnan 1968a has been employed successfully for the determination of aircraft stability derivatives in Taylor 1963. For white Gaussian noise in (4.19), the maximum likelihood estimate is asymptotically unbiassed.

Of interest in this problem is the on-line or updating method as more data is taken, that is, the estimate as a function of the upper limit b in (4.20). This is already treated in the survey paper of Cuenod and Sage (1967) and reference may be made to that work. No actual industrial problem of any magnitude has been reported using this method, however, as yet.

It is clear that (4.20) can be extended to systems which are not linear. Thus we could easily (in theory) replace (4.18) by the more general form:

$$\dot{x}(t) = F\left(x(t);\, u(t);\, \beta\right) \qquad (4.18a)$$

where β is an unknown vector parameter to be determined. The problem is more complicated in that now a nonlinear equation has to be solved to determine $x(t)$ for given β. Recently a method whereby one may avoid having to solve the dynamic equations has been reported in Balakrishnan 1968b.

Parallel adaptive models

Another method for minimizing the output error is the application of model-adjusting technique (Figure 9). The main feature of this technique is the physical realization of the model and the application of the principles of extremum control for adjusting of unknown parameters. Many procedures for adjusting of models have been proposed (Kokotovic et al. (1966), Reinisch and Wernstedt (1967), Rake (1966), Norkin (1967), Dymock et al.,

(1967), Doganovsky (1968), Plander (1967), Chadeev (1967)). The most popular method for automatic adjustment of parameters is the "Sensitivity method."

Because of theoretical difficulties and lack of more practical experience it is very difficult to compare and rate all the different existing adjusting procedures. An attempt in this direction has been made by Parks (1967) for a simple example and three chosen methods.

The following trends in model adjusting methods seem to be most significant: a) stability analysis of existing schemes, b) simplification of instrumentation, c) noise influence analysis, d) improvement of speed of adaptation, e) increase of the sensitivity according to some parameters by adaptation of the model to the unknown system in closed loop with known feedback.

4.2 Minimization of Equation Error

Let us suppose that the input-output relation of a dynamic system can be described by the differential Equation (3.10) and let us replace the output $v(t)$ and the input $u(t)$ by their observations $\tilde{v}(t)$ and $\tilde{u}(t)$ and the parameters β by their estimates $\hat{\beta}$. The resulting unbalance in the equation

$$f\left(\tilde{v}^{(n)}(t), \ldots \tilde{v}(t), \tilde{u}^{(m)}(t), \ldots \tilde{u}(t), \hat{\underline{\beta}}\right) = g(t)$$

is called the "equation error" and can be used as a measure of the deviation of the behavior of the model from the behavior of the real system. To form the equation error $q(t)$ requires n derivatives of the system output and m derivatives of the input whereas usually only the input and output are directly available. This difficulty can be overcome in the linear case through the use of "drivative filters" operating on the output and the input. However, these filters must have two properties: commutativity with the operators in the differential equation model and sufficiently wide bandwith. An example how this idea can be used in the case of linear lumped parameter system is given in Figure 10 where A(s), B(s), A(s), B(s) and Q(s) are polynomials

in s (differential operator). The polynomial Q(s) ensures the physical realizability of derivations and its degree is equal to or greater than the order of the system. The model operating on both the system input and the system output is sometimes called the "generalized model." The minimization of equation error has also been recently used for identification of nonlinear systems (Sprague and Kohr (1968), Hoberock and Kohr (1967), Lion (1966), Butler and Bohn (1966)).

The scope and limitations of this approach can be illustrated by considering a linear discrete process describable by the difference equation model (3.14). Let us assume that the observed output $x(.)$† is similar to (4.19) with additive noise $\epsilon(.)$:

$$x(k) = v(k) + \epsilon(k)$$

We can then rewrite (3.14) in the form:

$$x(k) + \sum_{1}^{n} a_i x(k-i) - \sum_{o}^{n} b_i u(k-i) = \epsilon(k) + \sum_{1}^{n} a_i \epsilon(k-i)$$

$$= q(k) \qquad (4.22)$$

Here $q(k)$ is the equation error, in our terminology, and we have assumed the coefficient a_o in (3.14) to be nonzero, and taken it to be unity without loss of generality. Suppose now that from the observations we can write N such equations, $k = 0, 1, \ldots N-1$, and N is such that N is much larger than 2n. Let x_o denote the column vector with components $x(i)$, $i=0, \ldots N-1$, let 'a' denote the column vector with components a_i, and similarly 'b' the column vector with components b_i. Then we may represent the N equations in matrix form as

$$x_o - Xa - ub = q = \epsilon a$$

where ϵ, X, U are rectangular matrices with components read off from (4.22) and q is the column vector with components $q(i)$. Let us denote the N-by-(2n+1) matrix L

† we use $x(.)$ here to avoid confusion with the z's in z-transform!

$$L = [-X , U]$$

Then denoting by β the (2n+1) column vector of unknowns:

$$\begin{matrix} a \\ b \end{matrix}$$

we have that

$$x_0 - L\beta = q = \epsilon a \tag{4.23}$$

To minimize the equation error we may use the ' weighted least squares' criterion, namely, we determine β so as to minimize:

$$\left(X_0 - L\beta\right)^* R \left(x_0 - L\beta\right) \tag{4.24}$$

where R is a suitably chosen positive definite NxN matrix. The optimal solution $\hat{q}$ is then given by:

$$\hat{q} = (L^* R L)^{-1} L^* R x_o \tag{4.25}$$

If we substitute into this from (4.23) we have that:

$$q = \hat{q} + (L^* R L)^{-1} L^* \epsilon a \tag{4.26}$$

so that the error in q determined by the second term, whose expected value is not zero, so that the estimate is biassed in general. However the identification problem is reduced to a linear one, unlike the case where we minimize the fit error. Many methods of choosing R have been proposed in an effort to remove the bias and also to aid in the updating or on-line feature. The so-called 'instrumentation variables' method (Wong and Polak (1967)) may be classified as one such. More recently Peterka (1968) has introduced an on-line technique in which the R matrix is determined by the input sequence. It would appear safe to say that one of the current needs is find a method which is bias free and simple to instrument on line.

Stochastic Approximation

When the input can be assumed to be a stationary stochastic process, stochastic approximation is an attractive on-line method for minimizing

the equation error, for example q in (4.23). It has also the advantage that no knowledge of the input or output process statistics is required. However, the results obtained using the method, which is essentially a gradient method with predetermined relaxation factors, have not lived up to earlier claims. Among the major difficulties would appear to be the slow convergence. The general form of the algorithm is:

$$\hat{\beta}_{n+1} = \hat{\beta}_n + \gamma_n \nabla_\beta Q(\hat{\beta}_n) \tag{4.27}$$

where β is the unknown parameter, and $\hat{\beta}_n$ the estimate at the n^{th} stage, $Q(.)$ is the equation error (assuming β to be equal to $\hat{\beta}_n$) at the n^{th} sample. The relaxation factor γ_n is a predetermined sequence, usually

$$n\,\gamma_n = \text{constant} \tag{4.28}$$

Tsypkin (1966), Holmes (1967), Zhivagladov and Kaipov (1967), Chadeev (1967) Imezadze and Lelashvili (1967), Panuska (1968). The method can also be applied to systems with finite memory using a Volterra type expansion. Holmes (1967), Roy and Sherman (1967), Nagumo and Noda (1967). The socalled 'learning technique' is very similar to the stochastic approximation scheme. The theory of convergence is rather involved, and the many assumptions to ensure it difficult to verify in practice. Sakrison (1967), Valis (1968).

Concluding Remarks

It is apparent that the 'Least Squares-Equation Error' techniques have many advantages in practice such as high parameter sensitivity, case of on-line use, and above all simplicity. Simplicity is particularly significant in the case of linear systems, where the equation error is actually linear in the unknown coefficients. However there are many drawbacks. Parameter fit does not necessarily ensure the output fit even using the same input as that in the identification mode, especially when the model assumed is not accurate. This is particularly serious

in open loop control. But the situation may be different in closed loop control. For instance a discrete controller operates only on a finite set of input-output data of the system; and the relation between these neighbouring data can be the most important information about the system which is required. From this point of view the minimization of equation error may well be reasonable.

4.3 STATISTICAL ERROR CRITERIA

As we have seen, the identification problem is essentially an estimation problem. Thus when the necessary statistical description is available, optimal estimates can be based on statistical criteria. Such a procedure not only frees us from ad hoc methods. It also provides us with a quantitative numerical evaluation of the error.

Mean Square Error Criteria:

Let the vector X denote the total observation (including input and output), and let β, as before, denote the system parameter vector to be determined. If the situation is such that the joint distribution of X and β can be specified explicitly or implicityly, then a statistical criterion that has been much studied is the mean square error criterion

$$E\left(\| \hat{\beta} - \beta \|^2 \right)$$

where E denotes expected value, and $\hat{\beta}$ is the estimate, a function of X. The associated theory is well-known when the estimates are constrained to be linear, but considerably more complex when no such restriction is placed Stratanovich (1967); Kushner (1967). Unfortunately, in the identification problem, the estimates are nonlinear even when the system is linear. In fact the identification problem for dynamic systems is no less difficult than the general nonlinear filtering problem for Markov processes. This becomes clear when we note that every such identification problem can be formulated as the problem of estimating the initial state of a known, but albeit more complex, system. (For example see Cuenod and Sage (1967); Balakrishnan 1968). Thus suppose as in (4.18a) we have:

$$x(t) = F\left(x(t);\, u(t);\, \beta\right) \qquad (4.31)$$

$$v(t) = Cx(t) + n(t)$$

where $u(.)$ is the input and $v(.)$ the observed output. If we form a new vector

$$Y = [x, \beta]$$

and add the equation

$$\dot{\beta} = 0$$

we can rewrite (4. 3. 1) in the form:

$$\dot{Y} = \tilde{F}\left(Y\,;\, u(.)\right) \qquad (4.32)$$

$$v(t) = H(Y) + n(t)$$

and the problem of determining β is the same as that of determining $Y(O)$ from $v(s)$, $0 < s < t$. Of course, it is also well known that the optimal mean square estimate is given by; the conditional expectation:

$$E\left[Y(0) \,\middle|\, v(s)\,,\ 0 < s < t\right]$$

But the problem of evaluating the right side is a complex one and at present no effective constructive method has yet been found. Many approximations have been suggested but without any evaluation of the associated error. Kushner (1967).

Method of Maximum Likelihood

When (in the terminology above) the conditional probability density $p(x \mid \beta)$ of X given β is known, (or calculable), we can base the optimization on the method of maximum likelihood. Thus we define the optimal β as one that maximizes for given x

$$p(x|\beta) \quad \text{or} \quad \log p(x|\beta)$$

Unlike the mean square error method, this criterion leads in many cases to a variational problem (of the type met with in optimal control). Although not necessarily confined to this case, it is most useful when the input is noise free, and the output is contaminated by additive Gaussian noise, because in this case it reduces to a least squares criterion (4.20), (4.14). Thus the minimum of the functional has a direct interpretation in terms of the error in the response to the given input, when it can be assumed that the input is noise free. Taylor (1968). Astrom (1967) has also employed it for the linear noisy system described by (3.18), assuming steadystate conditions.

The maximum likelihood estimate has also the desirable property that it is asymptotically unbiassed. Moreover when the conditional distribution of X given β is known, one can use the well-known Cramer-Rao inequality for the estimate variance:

$$E\left[\left(\hat{\beta}-\beta\right)\left(\hat{\beta}-\beta\right)^{*}\right] \geq J^{-1}$$

where J^{-1} is calculable from the conditional distribution, and the right-side can be used as a measure of the achievable accuracy. It is known that the maximum likelihood estimate achieves this lower bound asymptotically under the conditions usually satisfied by identification problems. Such calculations (in the steady state case) have been presented by Astrom (1967) for (3.18).

5. ACCURACY OF IDENTIFICATION

The problem of providing same estimate of the accuracy of determination is obviously an important one, and continues to attract attention. The use of Carmer-Rao inequality has already been noted. Other approaches have also been suggested. The relation between the errors in the time domain and frequency domain for a linear system has been investigated by Unbehauen and Schlegel (1967) and also by Strobel (1967). It was shown that even small errors in step response can cause serious errors in the frequency response. The accuracy of identification has been studied in

detail by Stepan (1967) from the viewpoint of the behavior of the model and the system in closed loop with a proportional controller.

6. CONCLUDING REMARKS

Many remarkable results have been achieved in the field of systems identification but still much remains to be done.

The best results of experimental identification can be expected when the unknown system is investigated in conditions under which it will be controlled. From this point of view the elaboration of methods suitable for application in closed loop seems to be very important.

The significance of accuracy analysis of different methods for systems identifications is indisputable. On the other hand the question what accuracy is actually required remains still unanswered.

The stability of many schemes for model adjusting has not yet been cleared. The same holds for some iterative procedures and hill climbing methods.

The identification of nonlinear systems and also the suitable description of nonlinear noisy system is also a question which is still open.

In foregoing sections we have tried to outline the ideas and trends in the present development of system identification. The limited space did not permit inclusion of much recent work in the field. Some further references are listed below. We cannot hope that all important papers on this subject are known to us and that some of them undoubtedly have not been omitted, albeit inadvertently.

To facilitate the reader's orientation we have keyed the references in the following way:

(a) criterion using output error

(b) criterion using equation error

(c) maximum likelihood

(d) frequency methods

(e) impulse response model

(f) Volterra expansion model

(g) application of orthogonal functions

(h) differential equation model

(i) difference equation model

(j) noisy system described by difference equation model (3, 18)

(k) correlation technique

(l) stochastic approximations

(m) learning technique

(n) model adjusting technique

(o) pseudorandom binary input

(p) analysis of identification accuracy

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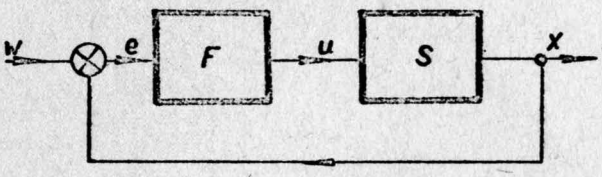

FIG.1

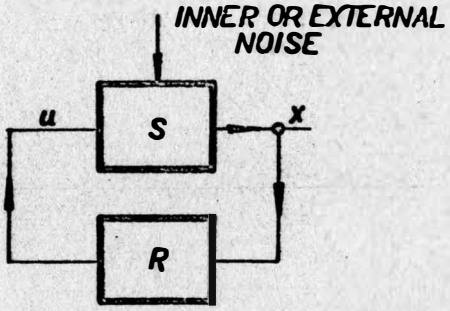

FIG.2

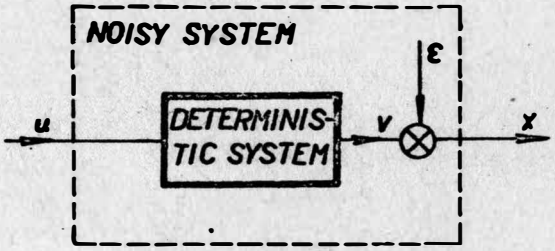

FIG.3

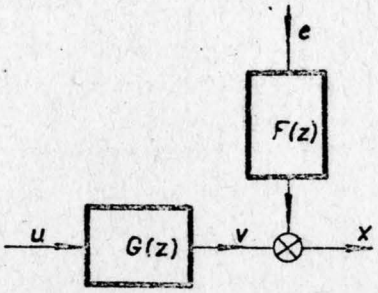

FIG. 4

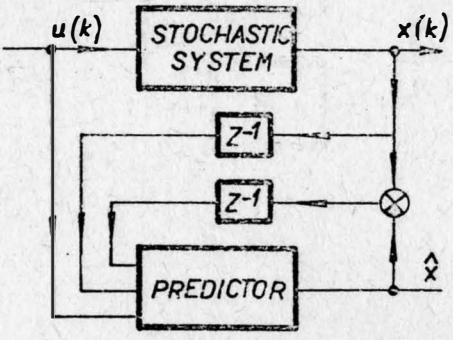

FIG. 5

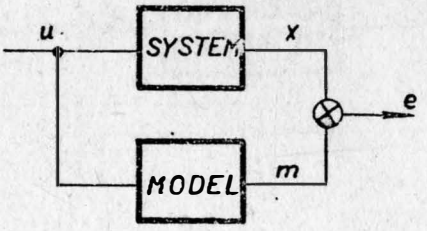

FIG. 6

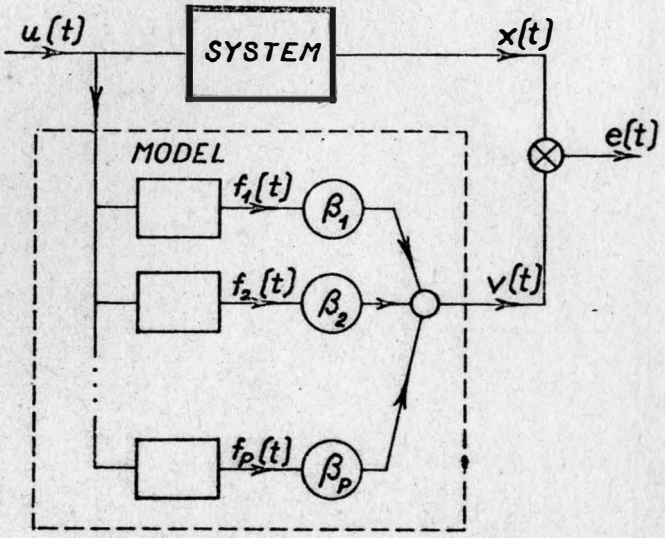

FIG. 7

FIG. 8

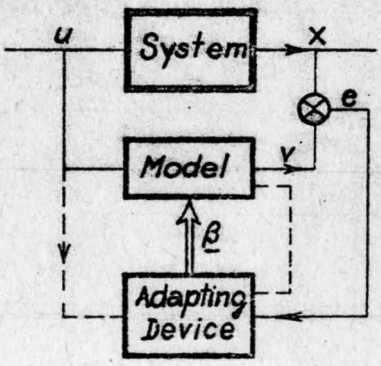

Fig. 9

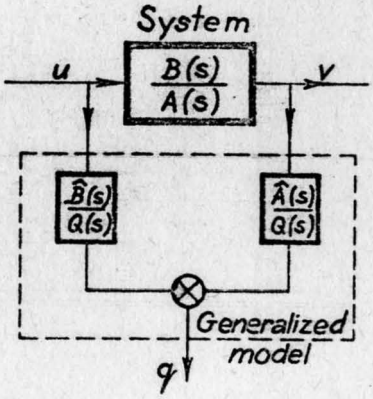

Fig. 10